

CP2410 Pracital 03

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Q1. Describe a recursive algorithm for finding the maximum element in a sequence, S, of n elements. What is your running time and space usage?

```
def find_maximum(arr, n):  
    if n == 1:  
        return arr[0]  
    else:  
        return max(arr[n - 1], find_maximum(arr, n - 1))
```

Answer:

The code above will run recursively to find out maximum value.

Time complexity:

The code will run till n(the length of array) hits 1, in total it will execute n times, which gives it time complexity of $O(n)$.

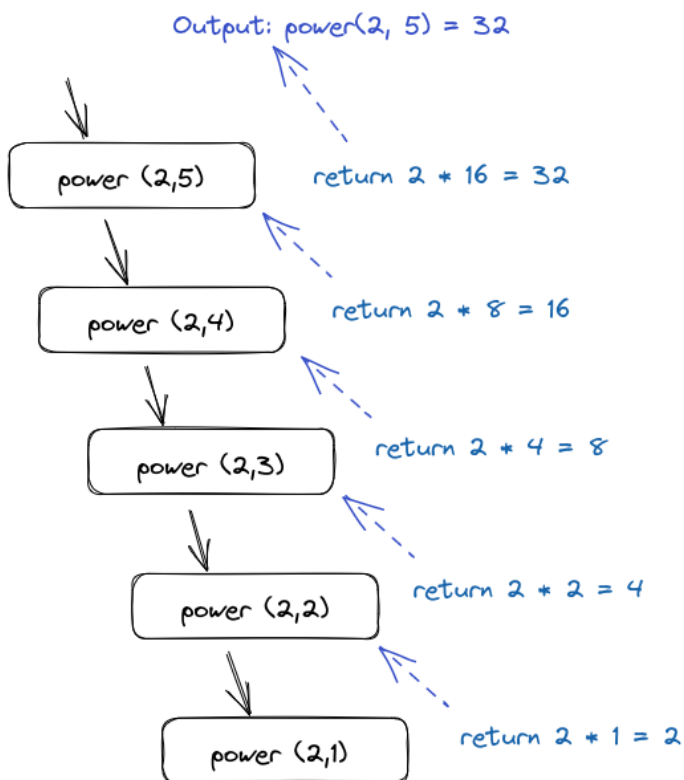
Space complexity:

The code run one time for base case, and n times for recursion, so the total space complexity is $O(n+1)$

Q2. Draw the recursion trace for the computation of $\text{power}(2,5)$, using the traditional function implemented below

```
def power(x, n):  
    """ Compute the value  $x^n$  for integer n."""  
    if n == 0:  
        return 1  
    else:  
        return x * power(x, n - 1)
```

Answer:

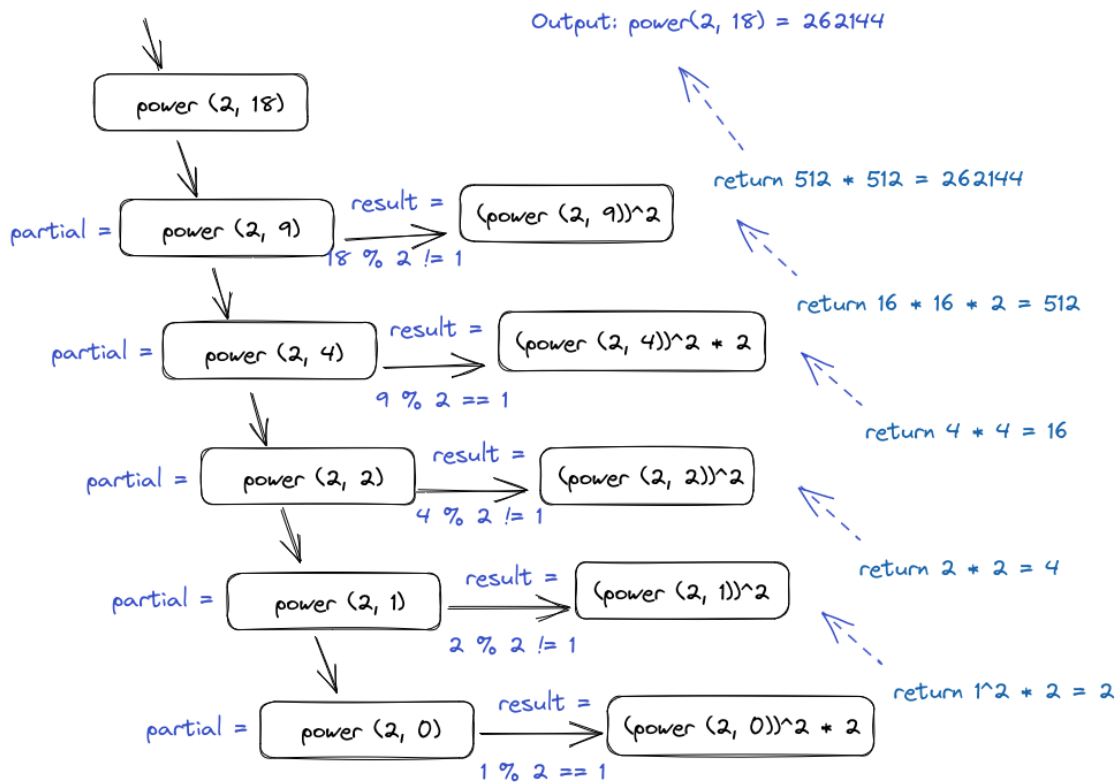


Q3. Draw the recursion trace for the computation of $\text{power}(2,18)$, using the repeated squaring algorithm, as implemented below:

```

def power(x, n):
    """ Compute the value x**n for integer n. """
    if n == 0:
        return 1
    else:
        partial = power(x, n // 2)
        result = partial * partial
        if n % 2 == 1:
            result *= x
        return result
  
```

Answer:



Q4. Give a recursive algorithm to compute the product of two positive integers, m and n, using only addition and subtraction

Answer:

```

def get_product(m, n):
    if n == 1:
        return m
    else:
        return m + get_product(m, n - 1)

```

The above code will calculate products of m and n. The base case is n equal to 1, just return m($m*1$). If n is not 1, code run recursively till n eventually decrease to 1. Each time adding m to result. The code will run $\text{len}(n)$ times, so the final result will be $m * n$.

Q5. Modify ch05/experiment_list_append.py to investigate the time taken by append operations for DynamicArray (ch05/dynamic_array.py).

Answer:

Modification: import DynamicArray module, and for `data` object, initiate dynamic array instead of using list.

```

14 import sys
13 from time import time
12 from dynamic_array import DynamicArray
11
10 try:
9     maxN = int(sys.argv[1])
8 except:
7     maxN = 10000000
6
5 def compute_average(n):
4     """Perform n appends to an empty list and return average time elapsed."""
3     data = DynamicArray()
2     start = time()                # record the start time (in seconds)
1     for k in range(n):
36     data.append(None)
1     end = time()                  # record the end time (in seconds)
2     return (end - start) / n      # compute average per operation
3
4 n = 10
5 while n <= maxN:
6     print('Average of {:.3f} for n {}'.format(compute_average(n)*1000000, n))
7     n *= 10
~
~
~
5 NORMAL experiment_dynamic_array_append.py unix | u

```

Running result comparison:

```

> python experiment_list_append.py
Average of 0.596 for n 10
Average of 0.160 for n 100
Average of 0.059 for n 1000
Average of 0.058 for n 10000
Average of 0.057 for n 100000
Average of 0.046 for n 1000000
Average of 0.040 for n 10000000
> python experiment_dynamic_array_append.py
Average of 8.297 for n 10
Average of 0.770 for n 100
Average of 0.306 for n 1000
Average of 0.335 for n 10000
Average of 0.366 for n 100000
Average of 0.293 for n 1000000
Average of 0.344 for n 10000000

```

Q6. Create a modified version of DynamicArray (ch05/dyanmic_array.py) that takes a parameter, `resize_factor`, which it uses to determine the new size (rather than doubling in the original code - `self._resize(2 * self._capacity)`). Using different values of `resize_factor`, examine if and how the average time to append changes.

Answer:

Modification: create a new class called `DynamicArrayWithResize`, which inherit from `DynamicArray` and takes in a `resize_factor` value, override `append` method to multiply array size by `resize_factor`.

```

31 import sys
30 from time import time
29 from dynamic_array import DynamicArray
28
27 try:
26     maxN = int(sys.argv[1])
25 except:
24     maxN = 10000000
23
22 class DynamicArrayWithResize(DynamicArray):
21     def __init__(self, resize_factor):
20         super().__init__()
19         self._resize_factor = resize_factor
18
17     def append(self, ele):
16         # Not enough space in dynamic array
15         if self._n == self._capacity:
14             # add one in case resize_factor is set to 1
13             self._resize(self._resize_factor * self._capacity + 1)
12             self._A[self._n] = ele
11             self._n += 1
10
9     def compute_average(n, i):
8         """Perform n appends to an empty list and return average time elapsed."""
7         data = DynamicArrayWithResize(i)
6         start = time() # record the start time (in seconds)
5         for k in range(n):
4             data.append(None)
3         end = time() # record the end time (in seconds)
2         return (end - start) / n # compute average per operation
1
53 n = 10
1 while n <= maxN:
2     for i in range(2,5):
3         print(f"Resize factor: {i}")
4         print(f"Average of {0:.3f} for n {1}'.format(compute_average(n, i)*1000000, n))
5         n *= 10
experiment_dynamic_array_append.py

```

Running program

```
python experiment_dynamic_array_append.py >> q6.txt
```

Here is the result:

Resize factor: 2 Average of 7.415 for n 10

Resize factor: 3 Average of 3.910 for n 10

Resize factor: 4 Average of 3.576 for n 10

Resize factor: 2 Average of 0.811 for n 100

Resize factor: 3 Average of 0.601 for n 100

Resize factor: 4 Average of 0.679 for n 100

Resize factor: 2 Average of 0.332 for n 1000

Resize factor: 3 Average of 0.271 for n 1000

Resize factor: 4 Average of 0.255 for n 1000

Resize factor: 2 Average of 0.396 for n 10000

Resize factor: 3 Average of 0.340 for n 10000

Resize factor: 4 Average of 0.296 for n 10000

Resize factor: 2 Average of 0.325 for n 100000

Resize factor: 3 Average of 0.315 for n 100000

Resize factor: 4 Average of 0.304 for n 100000

Resize factor: 2 Average of 0.279 for n 1000000

Resize factor: 3 Average of 0.284 for n 1000000

Resize factor: 4 Average of 0.206 for n 1000000

Resize factor: 2 Average of 0.345 for n 10000000

Resize factor: 3 Average of 0.282 for n 10000000

Resize factor: 4 Average of 0.242 for n 10000000