Assignment 4

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1 Question 1

The system is an M/M/1/K queue with limited capacity $K = 3$.

- Arrival Rate: $\lambda = 4$ pkt/s
- Service Rate: $\mu = 12$ pkt/s
- System Capacity (Queue + Server): $K = 3$

$$\rho = \frac{\lambda}{\mu} = \frac{4}{12} = \frac{1}{3}$$

1.1 1a

For an M/M/1/K queue where $\rho \neq 1$, the probability of zero packets is:

$$P_0 = \frac{1-\rho}{1-\rho^{K+1}}$$

Substituting $\rho = 1/3$ and $K = 3$:

$$P_0 = \frac{1-\frac{1}{3}}{1-\left(\frac{1}{3}\right)^{3+1}} = \frac{2/3}{1-1/81} = \frac{2/3}{80/81} = \frac{2}{3} \cdot \frac{81}{80} = \frac{27}{40}$$

The probability of N packets is given by $P_N = P_0\rho^N$. The system has 1 packet being served and 1 packet waiting, which means the total number of packets in the system is $N = 2$. We compute P_2:

$$P_2 = P_0\rho^2 = \left(\frac{27}{40}\right)\left(\frac{1}{3}\right)^2$$

$$P_2 = \frac{27}{40} \cdot \frac{1}{9} = \frac{3}{40}$$

So, the probability that the system has 2 packets is $\mathbf{3/40} \approx 0.075$.

1.2 1b

We find out the expected time a packet stays in the system W (queueing + service). We proceed via Little's Law.
Mean number in system L : Using $p_n = P\{N = n\} = P_0 \rho^n$ with $\rho = \frac{1}{3}$ and $K = 3$,

$$p_1 = \frac{27}{40} \cdot \frac{1}{3} = \frac{9}{40}, \qquad p_2 = \frac{27}{40} \cdot \frac{1}{9} = \frac{3}{40}, \qquad p_3 = \frac{27}{40} \cdot \frac{1}{27} = \frac{1}{40}.$$

Hence

$$L = \sum_{n=0}^{3} n\, p_n = 0 \cdot p_0 + 1 \cdot p_1 + 2 \cdot p_2 + 3 \cdot p_3 = \frac{9 + 6 + 3}{40} = \frac{18}{40}.$$

Effective arrival rate λ_{eff}: Arrivals are blocked when the system is full, with blocking probability

$$P_{\text{block}} = p_3 = \frac{1}{40}.$$

Thus the admitted (throughput) rate is

$$\lambda_{\text{eff}} = \lambda\,(1 - P_{\text{block}}) = 4\left(1 - \frac{1}{40}\right) = \frac{156}{40} = 3.9 \text{ pkt/s}.$$

Applying $L = \lambda_{\text{eff}}\, W$,

$$W = \frac{L}{\lambda_{\text{eff}}} = \frac{\frac{18}{40}}{\frac{156}{40}} = \frac{18}{156} = \boxed{\frac{3}{26} \text{ s } \approx 0.11538 \text{ s}}.$$

2 Question 2

The system is a standard M/M/1 queue with infinite capacity.

- Arrival Rate: $\lambda = 6$ pkt/s
- Service Rate: $\mu = 12$ pkt/s
- Traffic Intensity: $\rho = \frac{\lambda}{\mu} = \frac{6}{12} = \frac{1}{2}$

2.1 2a

The expected time a packet spends in the system (**W**) includes both queueing time and service time. The formula for the average time in the system for an M/M/1 queue is:

$$W = \frac{1}{\mu - \lambda}$$

Substituting the rates:

$$W = \frac{1}{12 - 6} = \frac{1}{6} \text{ seconds}$$

The expected time a packet spends in the system is **1/6** seconds.

2.2 2b

The expected time a packet waits in the buffer before getting served is the average queueing time ($\mathbf{W_q}$). The formula for the average queueing time for an M/M/1 queue is:

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

Substituting the rates:

$$W_q = \frac{6}{12(12-6)} = \frac{6}{12(6)} = \frac{6}{72} = \frac{1}{12} \text{ seconds}$$

The expected waiting time in the buffer is $\mathbf{1/12}$ seconds.

3 Question 3

Given Parameters

- Number of applications/streams: $c = 5$
- Individual Arrival Rate: $\lambda_i = 3$ pkt/s
- Individual Service Rate: $\mu_i = 6$ pkt/s

Packet delay is defined as the average time a packet spends in the system (**W**).

3.1 3a

For a single dedicated server:

- $\lambda = 3$ pkt/s
- $\mu = 6$ pkt/s

The average time in the system (W_{ded}) is:

$$W_{\text{ded}} = \frac{1}{\mu - \lambda} = \frac{1}{6-3} = \frac{1}{3} \text{ seconds}$$

For a single Powerful Server (1 Combined M/M/1 Queue), the total rates for the single combined server are:

- Total Arrival Rate: $\lambda_{\text{total}} = 5 \cdot 3 = 15$ pkt/s. (The reason the total arrival rate must be 5λ for the single powerful server is due to the principle of Superposition of Poisson Processes)
- Total Service Rate: $\mu_{\text{total}} = 5 \cdot 6 = 30$ pkt/s

The average time in the system (W_{single}) is:

$$W_{\text{single}} = \frac{1}{\mu_{\text{total}} - \lambda_{\text{total}}} = \frac{1}{30-15} = \frac{1}{15} \text{ seconds}$$

Comparing the delays:

$$W_{\text{single}} = \frac{1}{15} \quad \text{and} \quad W_{\text{ded}} = \frac{1}{3} = \frac{5}{15}$$

Since $W_{\text{single}} < W_{\text{ded}}$, the single powerful server provides a lower packet delay. The single powerful server is better in terms of packet delay.

3.2 3b

This is based on the single powerful server from Case 2, which is an M/M/1 queue with $\rho = 15/30 = 1/2$. We want to compute P_N for $N = 6$. The steady-state probability of having N packets in an M/M/1 system is $P_N = (1-\rho)\rho^N$.

$$P_6 = (1-\rho)\rho^6$$

Substituting $\rho = 1/2$:

$$P_6 = \left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)^6 = \frac{1}{2} \cdot \frac{1}{64} = \frac{1}{128}$$

The probability that there are 6 packets in the system is $\mathbf{1/128}$.

4 Question 4

Given Parameters

- System Type: M/M/1/K
- Fixed Capacity: $K = 2$

The average number of customers in the system (L) is $L = \sum_{N=1}^{K} N \cdot P_N$, where $P_N = P_0 \rho^N$ and $P_0 = \frac{1-\rho}{1-\rho^{K+1}}$.

4.1 4a

In the startup Phase, the parameters are:

- $\lambda = 2$ pkt/s
- $\mu = 10$ pkt/s
- $\rho = 2/10 = 0.2$

Let's find the probabilities of having 0,1,2 customers in the system.

1. $P_0 = \frac{1-0.2}{1-(0.2)^3} = \frac{0.8}{0.992} \approx 0.80645$
2. $P_1 = P_0 \cdot (0.2) \approx 0.16129$
3. $P_2 = P_0 \cdot (0.2)^2 \approx 0.03226$

So the average number of customers is

$$L = 1 \cdot P_1 + 2 \cdot P_2 \approx 0.16129 + 0.06452 \approx 0.2258$$

4.2 4b

In the Adolescent Phase, the parameters are:

- $\lambda = 8$ pkt/s
- $\mu = 10$ pkt/s
- $\rho = 8/10 = 0.8$

Let's find the probabilities of having 0, 1, 2 customers in the system.

1. $P_0 = \frac{1-0.8}{1-(0.8)^3} = \frac{0.2}{0.488} \approx 0.40984$
2. $P_1 = P_0 \cdot (0.8) \approx 0.32787$
3. $P_2 = P_0 \cdot (0.8)^2 \approx 0.26229$

So the average number of customers is

$$L = 1 \cdot P_1 + 2 \cdot P_2 \approx 0.32787 + 0.52458 \approx 0.8525$$

4.3 4c

In the Viral Spike Phase, the parameters are:

- $\lambda = 15$ pkt/s
- $\mu = 10$ pkt/s
- $\rho = 15/10 = 1.5$

Let's find the probabilities of having 0, 1, 2 customers in the system.

1. $P_0 = \frac{1-1.5}{1-(1.5)^3} = \frac{-0.5}{-2.375} \approx 0.21053$
2. $P_1 = P_0 \cdot (1.5) \approx 0.31579$
3. $P_2 = P_0 \cdot (1.5)^2 \approx 0.47368$

So the average number of customers is

$$L = 1 \cdot P_1 + 2 \cdot P_2 \approx 0.31579 + 0.94736 \approx 1.2632$$

Note that since $\rho > 1$, the server is a major bottleneck, and the probability of the system being full (P_2) is the highest.

4.4 4d

We consider the viral-spike parameters $\lambda = 15$ pkt/s, $\mu = 10$ pkt/s so that $\rho = \lambda/\mu = 1.5 > 1$. If we double the buffer from $K = 2$ to $K = 4$ (M/M/1/K), the steady–state empty probability is

$$p_0 = \frac{1-\rho}{1-\rho^{K+1}} = \frac{1-1.5}{1-1.5^5} = \frac{-0.5}{-6.59375} \approx 0.07586.$$

Hence the blocking probability is

$$p_K = p_4 = p_0\rho^4 \;=\; 0.07586 \times 1.5^4 \;=\; 0.07586 \times 5.0625 \;\approx\; 0.384.$$

The admitted (throughput) rate is therefore

$$\lambda_{\text{eff}} = \lambda(1-p_K) = 15(1-0.384) = 15 \times 0.616 = 9.24 \text{ pkt/s}.$$

(For comparison, with the original $K = 2$, $p_2 = 0.4737$ and $\lambda_{\text{eff}} = 15(1-0.4737) \approx 7.89$ pkt/s.)

Increasing the buffer reduces blocking and raises throughput from ≈ 7.89 to 9.24 pkt/s, but it does not fix the overload: the offered load $\lambda = 15$ still exceeds the service capacity $\mu = 10$, so the system remains congested and spends much of the time near full. To handle the spike one must increase service capacity (larger μ or more servers).

General Rule of Networking: The response implies the following general rule:

Buffer Space is Not a Substitute for Service Capacity (Bandwidth).

A system that is unstable ($\rho > 1$) must address the service bottleneck (μ), not the buffer size (K). The fundamental condition for stability is $\lambda < \mu$.

5 Question 5

Given Parameters (M/M/c/c System)

- Number of Servers: $c = 3$
- Arrival Rate: $\lambda = 2$ pkt/s
- Service Rate (per server): $\mu = 4$ pkt/s

The traffic intensity (ρ) is $\rho = \frac{\lambda}{c\mu} = \frac{2}{3 \cdot 4} = \frac{2}{12} = \frac{1}{6}$. Since $\rho < 1$, the system is stable. The term λ/μ is $\alpha = \frac{2}{4} = \frac{1}{2}$.

5.1 5a

We assume the system has reached steady-state. We first find P_0. The steady-state probability of zero packets is:

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!} \frac{1}{1-\rho} \right]^{-1}$$

Substituting $\alpha = 1/2$, $c = 3$, and $\rho = 1/6$:

$$P_0 = \left[\frac{(1/2)^0}{0!} + \frac{(1/2)^1}{1!} + \frac{(1/2)^2}{2!} + \frac{(1/2)^3}{3!} \frac{1}{1-1/6} \right]^{-1}$$

$$P_0 = \left[1 + \frac{1}{2} + \frac{1}{8} + \frac{1/8}{6} \cdot \frac{6}{5} \right]^{-1}$$

$$P_0 = \left[1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{40} \right]^{-1} = \left[\frac{40 + 20 + 5 + 1}{40} \right]^{-1} = \left[\frac{66}{40} \right]^{-1} = \frac{40}{66} = \frac{20}{33}$$

Since $N = 3$ is less than or equal to $c = 3$, we use the formula:

$$P_3 = \frac{(\lambda/\mu)^3}{3!} P_0$$

$$P_3 = \frac{(1/2)^3}{6} \cdot \frac{20}{33} = \frac{1/8}{6} \cdot \frac{20}{33} = \frac{1}{48} \cdot \frac{20}{33} = \frac{5}{12 \cdot 33} = \frac{5}{396}$$

So the probability is $\mathbf{5/396} \approx 0.0126$.

5.2 5b

The probability P_3 represents the steady-state probability that there are exactly 3 customers in the system. Since the number of customers ($N = 3$) equals the number of servers ($c = 3$), this probability specifically represents the state where All 3 Servers are Busy, and the Queue is Empty. The system is operating at maximum utilization without any customer needing to wait in the buffer.