Assignment 3

(1)

a

Expected time packet needs to go through in a system $= \frac{1}{\mu - \lambda}$

$= \frac{1}{8-4}$

$= \frac{1}{4}$

$= 0.25s$

(2)

a

average inter arrival time

$= \frac{1}{\lambda}$

$= \frac{1}{4}$

$= 0.25 s$

b

P (next inter-arrival time is greater than 2)

$= P(T > 2)$

$= e^{-\lambda t}$

$= e^{-4 \cdot 2}$

$= e^{-8}$

c

$P(A(2) = 4)$

$$= \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

$$= \frac{(4 \cdot 2)^4 \cdot e^{-2 \cdot 4}}{4!}$$

$$= \frac{8^4 e^{-8}}{24}$$

$$= \frac{512}{3} e^{-8}$$

3.

<u>a</u>

avg time packet spends in service

$$= \frac{1}{\mu} = \frac{1}{8} = 0.125 \text{ s}$$

<u>b</u>

Assuming infinite number of servers with each having $\mu_i = 8$, the avg time a packet spends in service is still $\frac{1}{\mu_i}$ because it's a property of packet, not of system.

$$= \frac{1}{\mu_i} = \frac{1}{8} = 0.125 \text{ s}$$

<u>c</u>

With infinite servers, there's no waiting time. So, avg no. of packets

$$= \lambda \cdot \frac{1}{\mu} = \frac{\lambda}{\mu} = 4/8 = 0.5 \text{ s}$$

d

With packet dropping probability $p_d = 0.75$,

$$\lambda_{eff} = \lambda(1 - p_d) = \lambda(1 - 0.75)$$

$$= 4 \cdot (0.25)$$

$$= 1$$

avg number of packets

$$= \frac{\lambda_{eff}}{\mu}$$

$$= \frac{1}{8}$$

$$= 0.125 \text{ s}$$

4.

a

Expected no. of packets that router will receive in 1 min is

$$E[N(60)] = \lambda \cdot 60$$
$$= 10 \cdot 60$$
$$= 600$$

b

packets in next ~~t~~ 15s : $N(15) \sim \text{poisson}(\lambda \cdot 15)$

$$\sim \text{poisson}((10+20+30) \cdot 15)$$
$$\sim \text{poisson}(60 \cdot 15)$$
$$\sim \text{poisson}(900)$$

$$P(N(15) = 1000) = e^{-900} \frac{900^{1000}}{1000!}$$
$$= \frac{e^{-900} \cdot (900)^{1000}}{(1000)!}$$

c

packets in 30s:

$$N(30) \sim \text{poisson}((10+20+30) \cdot 30)$$
$$\sim \text{poisson}(60 \cdot 30)$$
$$\sim \text{poisson}(1800)$$

so, the mean arrival in 30s is 1800 which is 300 greater than the buffer size 1500. So, we can expect to see a buffer overflow.

5.

<u>a</u>

packets at switch at $t = 3$

= arrivals − blocked − departures

= $100 - 8 - 50$

= 42

<u>b</u>

long term arrival rate,

$A(t) = 12t^2 \sin\left(\frac{1}{t}\right)$

avg rate $= \lim_{t \to \infty} \frac{A(t)}{t}$

$= \lim_{t \to \infty} 12t \sin\left(\frac{1}{t}\right)$

let, $x = \frac{1}{t}$

so, $t \to \infty$ means $x \to 0$

so, $\lim_{x \to 0} \frac{12}{x} \sin(x)$

$= 12 \lim_{x \to 0} \frac{\sin(x)}{x}$

We know that $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

so, avg rate $= 12 \cdot 1 = 12$ packets

c

$$D(t) = (13t \cdot e)\left(1 - \left(\frac{1}{t}\right)\right)^t$$

$$\text{Throughput} = \lim_{t \to \infty} \frac{D(t)}{t}$$

$$= 13e\left(\lim_{t \to \infty} \left(1 - \left(\frac{1}{t}\right)\right)^t\right)$$

$$= 13 \cdot e \cdot \frac{1}{e}$$

$= 13$ packets