Assignment 2

Sihat Afnan

Student ID: 35227032

1

a) Let T_1 be the service time of A at server 1 (rate μ_1)

and let T_2 be the service time of B at server 2 (rate μ_2)

So, the Probability that A exits system before B reaches server 3 is the

$$P(T_1 < T_2) = \frac{\mu_1}{\mu_1 + \mu_2}$$

$$= \frac{1}{3}$$

b) Similarly, Probability of B exits system before any other packet moves to server 3 is $P(T_2 < T_1) = \frac{\mu_2}{\mu_1 + \mu_2} = \frac{2}{3}$

c) Expected time T for packet A to exit the system,

T = Service time in server 1

+ Service time in server 3

$= T_{S_1} + T_{S_3}$

$$E[T] = E[T_{S_1}] + E[T_{S_3}]$$

$$= \frac{1}{\mu_1} + \frac{1}{\mu_3}$$

$$= 1/1 + 1/3$$

$$= \frac{4}{3} \text{ second}$$

2

Poisson Process $\{N(t), t \geq 0\}$ with $\lambda = 4$

a) P(2 packets in Next second)

$= P(N(1) = 2)$

$$= \frac{(4 \cdot 1)^2 e^{-4 \cdot 1}}{2!} = \frac{16e^{-4}}{2} = 8e^{-4}$$

b) Expected no of packets received upto time $t = 2s$ is

$$E(N(2)) = \lambda \cdot 2$$

$$= 4 \cdot 2$$

$$= 8 \text{ packets.}$$

c) $P(N(6) - N(4) = 4)$

$$= P(N(2) = 4)$$

$$= \frac{(4 \cdot 2)^4 e^{-4 \cdot 2}}{4!}$$

$$= \frac{8^4 e^{-8}}{24}$$

$$= \frac{512}{3} e^{-8}$$

3

$$\lambda = \frac{\text{Avg. \# of packet}}{\text{time}} = 15/3 = 5 \text{ packet/sec}$$

time bet'n two packets sent

→ exp. rand. variable

# of packets sent out → Poisson Process

a) P(next packet sent after 5 sec)

$= P(T > 5)$

$= e^{-\lambda . 5}$

$= e^{-5.5}$

$= e^{-25}$

b) P(5 packets leave in next sec)

$= P(k=5)$

$$= \frac{(\lambda.1)^5 e^{-\lambda.1}}{5!}$$

$$= \frac{5^5 e^{-5}}{5!} = \frac{625}{24} e^{-5}$$

c) P(5 packets leave in next 2 sec)

$\lambda t = 5 \cdot 2 = 10$

$= P(k=5)$

$= \dfrac{(10)^5 \cdot e^{-10}}{5!}$

$= \dfrac{10^5 \cdot e^{-10}}{120}$

$= \dfrac{2500}{3} e^{-10}$

d) P(more than 4 packets but less than 7 packets in next 2 sec)

$\lambda \cdot t = 5 \cdot 2 = 10$

$= P(4 < k < 7)$

$= P(k=5) + P(k=6)$

We already have $P(k=5) = \dfrac{10^5}{5!} e^{-10}$

and $P(k=6) = \dfrac{10^6 \cdot e^{-10}}{6!}$

$\therefore P(4<k<7) = \left(\dfrac{10^5}{5!} + \dfrac{10^6}{6!}\right) e^{-10}$

(4)

a) client 1 rate $\lambda_1 = 1$ packet/s

client 2 rate $\lambda_2 = 2$ packet/s

P(Next packet from client 1)

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

$$= \frac{1}{1+2}$$

$$= \frac{1}{3}$$

b) P(3 packets in the next 4 sec)

$(\lambda_1 + \lambda_2) \cdot 4$

$= 3 \cdot 4 = 12$

$= P(k = 3)$

$$= \frac{(12)^3 e^{-12}}{3!}$$

$$= \frac{1728}{6} e^{-12}$$

$$= 288 e^{-12}$$

c) $(\lambda_1 + \lambda_2) \cdot t = 3 \cdot 2 = 6$

$$P(k \geq 1) = 1 - P(k = 0)$$

$$= 1 - \frac{(6)^0 e^{-6}}{0!}$$

$$= 1 - e^{-6}$$