Assignment 1

Sihat Afnan

Student ID: 35227032

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a) $P(X_2 > 6) = e^{-4 \cdot 6} = e^{-24}$

b) $P(X_1 <= 2) = 1 - e^{-2 \cdot 8} = 1 - e^{-16}$

c) $P(\min(X_1, X_2) <= 10)$

$= 1 - P(\min(X_1, X_2) > 10)$

$= 1 - P(X_1 > 10)\, P(X_2 > 10)$

$= 1 - e^{-8 \cdot 10} \cdot e^{-4 \cdot 10}$

$= 1 - e^{-(8+4)10}$

$= 1 - e^{-120}$

d) $P(X_2 < X_1)$

$= P(X_2 < X_1 \mid X_2 = x) f_2(x)\, dx$

$= \int_0^\infty e^{-\lambda_1 x} \cdot \lambda_2 e^{-\lambda_2 x}\, dx$

$= \lambda_2 \int_0^\infty e^{-(\lambda_1 + \lambda_2)x}\, dx$

$= \dfrac{\lambda_2}{\lambda_1 + \lambda_2}$

(2)

let, $T \sim Exp(\lambda)$

here, $\lambda = 30/5 = 6$

$\therefore P(T \leq t) = 1 - e^{-\lambda t}$

a) $P(\text{Packet in} < 4s) = P(T \leq 4)$

$= 1 - e^{-24}$

b) If one Packet was sent at $t = 0$, then due to memoryless property, the distribution of time to the next packet doesn't depend on the past.

So, P(no packet by 4)

$= P(T > 4)$

$= e^{-\lambda \cdot 4}$

$= e^{-6 \cdot 4}$

$= e^{-24}$

c) Now, the $\lambda = 120/5 = 24$

So, $P(T < 4) = 1 - e^{-24 \cdot 4} = 1 - e^{-96}$

$(1 - e^{-96})$ is larger than $(1 - e^{-24})$.

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a) Next Packet from node 2,

$P(T_2 < T_1) = \frac{\lambda_2}{\lambda_1 + \lambda_4} = \frac{4}{3+4}$

$= 4/7$

b) time to the first of two arrival is

$$T_{min} = \min(T_1, T_2) \sim Exp(\lambda_1 + \lambda_2)$$

$$= Exp(3+4)$$

$$= Exp(7)$$

So, $P(T_{min} > 5) = e^{-7.5} = e^{-35}$

c) After those seconds with no packets, due to memoryless property, the clock will be reset. So, now

$T_1 \sim Exp(3)$

$T_2 \sim Exp(4)$

$T_3 \sim Exp(5)$

So, $P(T_3 < T_1 \text{ and } T_3 < T_2)$

$$= \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$= \frac{5}{3+4+5}$$

$$= 5/12$$

④

Let, service time comes from $S \sim Exp(\mu)$
arrival time comes from $T \sim Exp(\lambda)$

a) $P($B arrives while A still being served$)$

$= P(T < S)$

$= \int_0^\infty P(T < t) f_S(t)\, dt$

$= \int_0^\infty (1 - e^{-\lambda t}) \mu e^{-\mu t} dt$

$= \dfrac{\lambda}{\lambda + \mu}$

b) If $T < S$, packet B must wait for the remaining service time of packet A. Due to memoryless property, this ~~memo~~ remaining service time is also a $Exp(\mu)$.

So, $E[\text{Remaining } S] = \dfrac{1}{\mu}$

and $P(T < S) = \dfrac{\lambda}{\lambda + \mu}$

and if $T > S$, wait time is 0 and the prob. is $P(T>S) = \dfrac{\mu}{\lambda+\mu}$

so, avg time packet B stays in the queue

$$= \left(\frac{\lambda}{\lambda+\mu}\right)\cdot\left(\frac{1}{\mu}\right) + \left(\frac{\mu}{\lambda+\mu}\right)\cdot 0$$

$$= \frac{\lambda}{\mu(\lambda+\mu)}$$

c) system time $= E[\text{wait time}] + E[\text{service time}] + E[\text{arrival time}]$

$$= \frac{\lambda}{\mu(\lambda+\mu)} + \frac{1}{\mu} + \frac{1}{\lambda}$$

$$= \frac{\mu+2\lambda}{\mu(\lambda+\mu)} + \frac{1}{\lambda}$$

$$= \frac{2\lambda^2+2\lambda\mu+\mu^2}{\lambda\mu(\lambda+\mu)}$$