control chart problems

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15 sample s

Sample	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5
1	10.68	10.689	10.776	10.798	10.714
2	10.79	10.86	10.601	10.746	10.779
3	10.78	10.667	10.838	10.785	10.723
4	10.59	10.727	10.812	10.775	10.73
5	10.69	10.708	10.79	10.758	10.671
6	10.75	10.714	10.738	10.719	10.606
7	10.79	10.713	10.689	10.877	10.603
8	10.74	10.779	10.11	10.737	10.75
9	10.77	10.773	10.641	10.644	10.725
10	10.72	10.671	10.708	10.85	10.712
11	10.79	10.821	10.764	10.658	10.708
12	10.62	10.802	10.818	10.872	10.727
13	10.66	10.822	10.893	10.544	10.75
14	10.81	10.749	10.859	10.801	10.701
15	10.66	10.681	10.644	10.747	10.728

1. Calculate sample means, sample ranges, mean of means, and mean of ranges.

Sample	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5	Avg	Range
1	10.68	10.689	10.776	10.798	10.714	10.732	0.116
2	10.79	10.86	10.601	10.746	10.779	10.755	0.259
3	10.78	10.667	10.838	10.785	10.723	10.759	0.171
4	10.59	10.727	10.812	10.775	10.73	10.727	0.221
5	10.69	10.708	10.79	10.758	10.671	10.724	0.119
6	10.75	10.714	10.738	10.719	10.606	10.705	0.143
7	10.79	10.713	10.689	10.877	10.603	10.735	0.274
8	10.74	10.779	10.11	10.737	10.75	10.624	0.669
9	10.77	10.773	10.641	10.644	10.725	10.710	0.132
10	10.72	10.671	10.708	10.85	10.712	10.732	0.179
11	10.79	10.821	10.764	10.658	10.708	10.748	0.163
12	10.62	10.802	10.818	10.872	10.727	10.768	0.250
13	10.66	10.822	10.893	10.544	10.75	10.733	0.349
14	10.81	10.749	10.859	10.801	10.701	10.783	0.158
15	10.66	10.681	10.644	10.747	10.728	10.692	0.103
					Averages	10.728	0.220400

Example of x-bar and R charts: Step 2. Determine Control Limit Formulas and Necessary Tabled Values

$\bar{x}$ Chart Control Limits

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

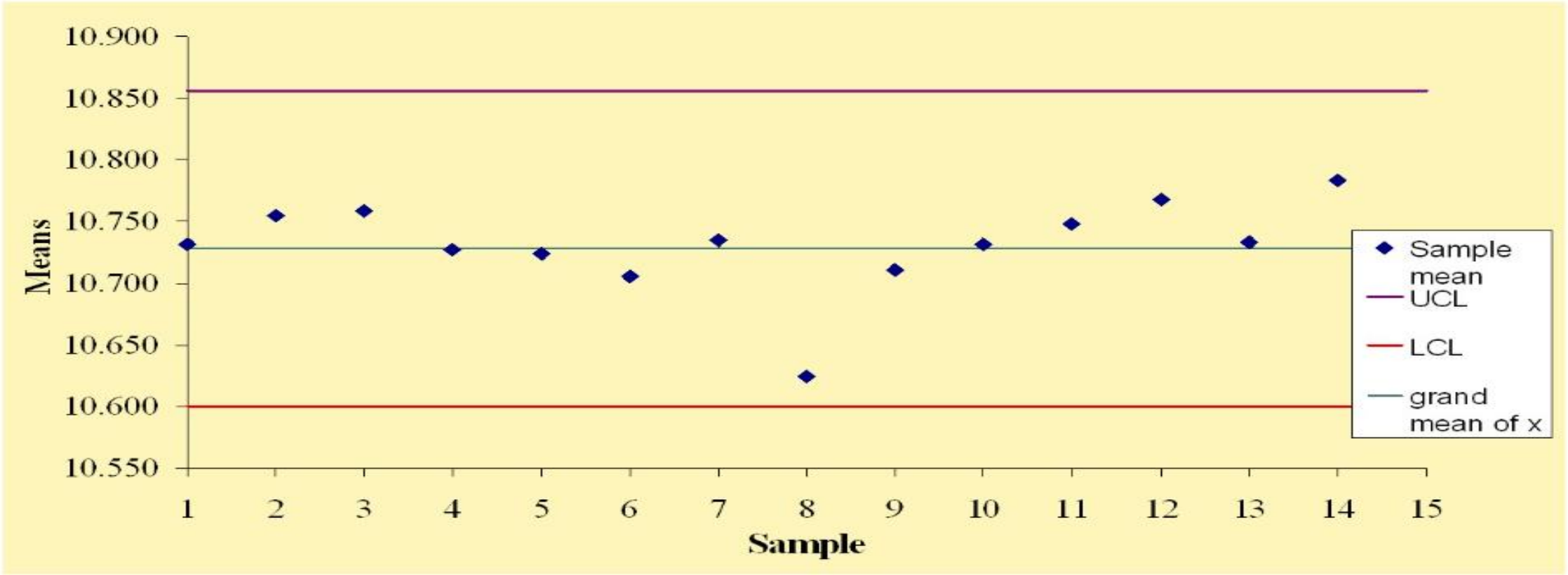
R Chart Control Limits

$$UCL = D_4\bar{R}$$

$$LCL = D_3\bar{R}$$

n	A2	D3	D4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74

Steps 3&4. Calculate x-bar Chart and Plot Values



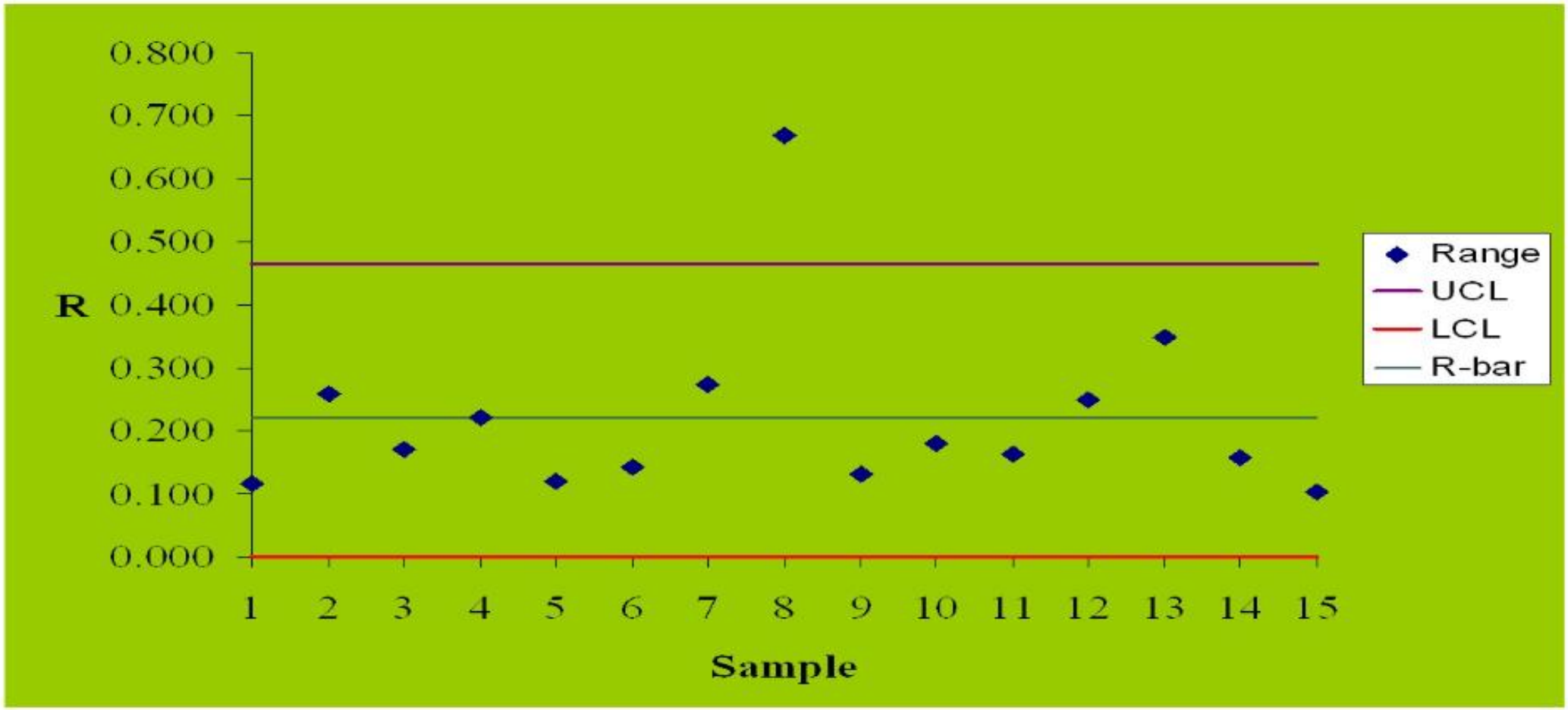
$$UCL = \bar{\bar{x}} + A_2 \bar{R} = 10.728 + .58(0.2204) = 10.856$$

$$LCL = \bar{\bar{x}} - A_2 \bar{R} = 10.728 - .58(0.2204) = 10.601$$

Steps 5&6: Calculate R-chart and Plot Values

$$UCL = D_4\overline{R} = (2.11)(0.2204) = \mathbf{0.46504}$$

$$LCL = D_3\overline{R} = (0)(0.2204) = \mathbf{0}$$



Q-2

TABLE 7-2 Coil Resistance Data

Sample	Observation (Ω)	$\bar{X}$	R	Comments
1	20, 22, 21, 23, 22	21.60	3	
2	19, 18, 22, 20, 20	19.80	4	
3	25, 18, 20, 17, 22	20.40	8	New vendor
4	20, 21, 22, 21, 21	21.00	2	
5	19, 24, 23, 22, 20	21.60	5	
6	22, 20, 18, 18, 19	19.40	4	
7	18, 20, 19, 18, 20	19.00	2	
8	20, 18, 23, 20, 21	20.40	5	
9	21, 20, 24, 23, 22	22.00	4	
10	21, 19, 20, 20, 20	20.00	2	
11	20, 20, 23, 22, 20	21.00	3	
12	22, 21, 20, 22, 23	21.60	3	
13	19, 22, 19, 18, 19	19.40	4	
14	20, 21, 22, 21, 22	21.20	2	
15	20, 24, 24, 23, 23	22.80	4	
16	21, 20, 24, 20, 21	21.20	4	
17	20, 18, 18, 20, 20	19.20	2	
18	20, 24, 22, 23, 23	22.40	4	
19	20, 19, 23, 20,19	20.20	4	
20	22, 21, 21, 24, 22	22.00	3	
21	23, 22, 22, 20, 22	21.80	3	
22	21, 18, 18, 17, 19	18.60	4	High temperature
23	21, 24, 24, 23, 23	23.00	3	Wrong die
24	20, 22, 21, 21, 20	20.80	2	
25	19, 20, 21, 21, 22	20.60	3	

$$\bar{R} = \frac{\sum_{i=1}^{g} R_i}{g} = \frac{87}{25} = 3.48$$

For a sample of size 5, Appendix A-7 gives $D_4 = 2.114$ and $D_3 = 0$. The trial control limits for the R-chart are calculated as follows:

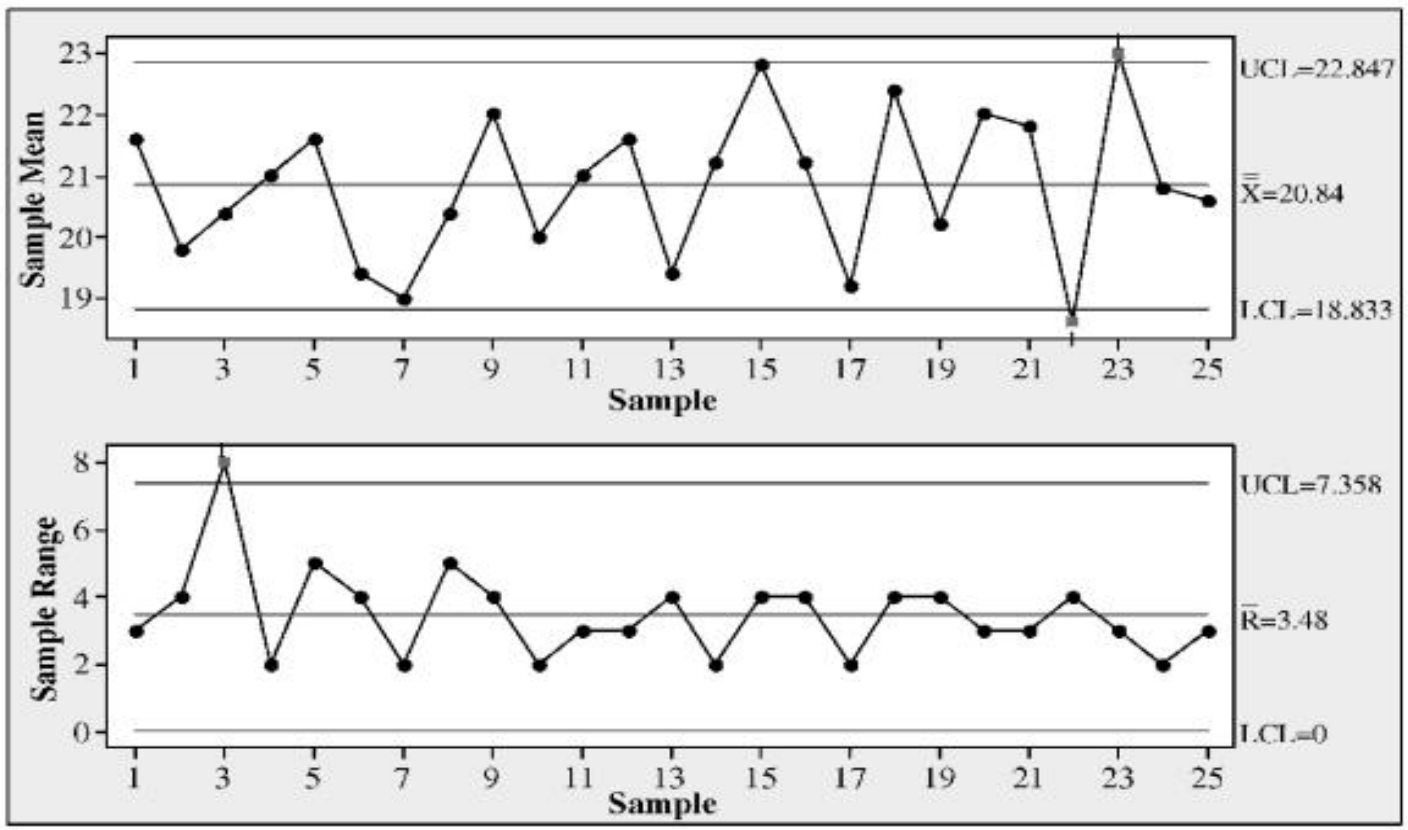
$$\text{UCL}_R = D_4\bar{R} = (2.114)(3.48) = 7.357$$
$$\text{LCL}_R = D_3\bar{R} = (0)(3.48) = 0$$

The centerline on the $\bar{X}$-chart is obtained as follows:

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{g} \bar{X}_i}{g} = \frac{521.00}{25} = 20.840$$

Appendix A-7, for $n = 5$, gives $A_2 = 0.577$. Hence, the trial control limits on the $\bar{X}$-charts are

$$\text{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 20.84 + (0.577)(3.48) = 22.848$$
$$\text{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 20.84 - (0.577)(3.48) = 18.832$$



Revised chart Revised range=87-(8+4+3)=72

Sample=25-3=22.Grand X revised=521-(20.40+18.60+23.00)=521-62=459(do revised chart

With samples 3, 22, and 23 deleted, the revised centerline on the *R*-chart is

$$\bar{R} = \frac{72}{22} = 3.273$$

The revised control limits on the *R*-chart are

$$UCL_R = D_4\bar{R} = (2.114)(3.273) = 6.919$$

$$LCL_R = D_3\bar{R} = (0)(3.273) = 0$$

The revised centerline on the $\bar{X}$-chart is

$$\bar{\bar{X}} = \frac{459}{22} = 20.864$$

The revised control limits on the $\bar{X}$-chart are

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 20.864 + (0.577)(3.273) = 22.753$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 20.864 - (0.577)(3.273) = 18.975$$

7-21 Flight delays are of concern to passengers. An airline obtained observations on the average and range of delay times of flights (in minutes), each chosen from a sample of size 4, as shown in Table 7-20. Construct appropriate control charts and comment on the performance level. What are the chances of meeting a goal of no more than a 10-minute delay?

TABLE 7-20

Observation	Average Delay	Range	Observation	Average Delay	Range
1	6.5	2.1	14	9.2	3.5
2	11.1	3.8	15	7.8	2.2
3	15.8	4.6	16	10.6	4.1
4	10.9	4.2	17	10.7	4.2
5	11.2	4.0	18	8.8	3.8
6	5.6	3.5	19	9.8	3.6
7	10.4	4.1	20	10.2	3.6
8	9.8	2.0	21	9.0	4.2
9	7.7	3.2	22	8.5	3.3
10	8.6	3.8	23	9.8	4.0
11	10.5	4.2	24	7.7	2.8
12	10.2	3.8	25	10.5	3.2
13	10.5	4.0			

7-19 A soft drink bottling company is interested in controlling its filling operation. Random samples of size 4 are selected and the fill weight is recorded. Table 7-19 shows the data for 24 samples. The specifications on fill weight are 350 ± 5 grams (g). Daily production rate is 20,000 bottles.

(a) Find the trial control limits for the $\overline{X}$- and R-charts.

(b) Assuming special causes for out-of-control points, find the revised control limits.

TABLE 7-19

Sample	Observations (g)				Sample	Observations (g)			
1	352	348	350	351	13	352	350	351	348
2	351	352	351	350	14	356	351	349	352
3	351	346	342	350	15	353	348	351	350
4	349	353	352	352	16	353	354	350	352
5	351	350	351	351	17	351	348	347	348
6	353	351	346	346	18	353	352	346	352
7	348	344	350	347	19	346	348	347	349
8	350	349	351	346	20	351	348	347	346
9	344	345	346	349	21	348	352	351	352
10	349	350	352	352	22	356	351	350	350
11	353	352	354	356	23	352	348	347	349
12	348	353	346	351	24	348	353	351	352

7-29 The level of dissolved oxygen in water was measured every 2 hours in a river where industrial plants discharge processed waste. Each observation consists of four samples, from which the sample mean and range of the amount of dissolved oxygen in parts per million are calculated. Table 7-22 shows the results of 25 such observations. Discuss the stability of the amount of dissolved oxygen. Revise the control limits, if necessary, assuming special causes for the out-of-control points. Suppose that environmental standards call for a minimum of 4 ppm of dissolved oxygen. Are these standards being achieved? Discuss.

TABLE 7-22

Observation	Average Level of Dissolved Oxygen	Range	Observation	Average Level of Dissolved Oxygen	Range
1	7.4	2.1	14	4.3	2.0
2	8.2	1.8	15	5.8	1.4
3	5.6	1.4	16	5.4	1.2
4	7.2	1.6	17	8.3	1.9
5	7.8	1.9	18	8.0	2.3
6	6.1	1.5	19	6.7	1.5
7	5.5	1.1	20	8.5	1.3
8	6.0	2.7	21	5.7	2.4
9	7.1	2.2	22	8.3	2.1
10	8.3	1.8	23	5.8	1.6
11	6.4	1.2	24	6.8	1.8
12	7.2	2.1	25	5.9	2.1
13	4.2	2.5			

7-30 In a gasoline-blending plant, the quality of the output as indicated by its octane rating is measured for a sample taken from each batch. The observations from 20 such samples are shown in Table 7-23. Construct a chart for the moving range of two successive observations and a chart for individuals.

TABLE 7-23

Sample	Octane Rating	Sample	Octane Rating	Sample	Octane Rating	Sample	Octane Rating
1	89.2	6	87.5	11	85.4	16	90.3
2	86.5	7	92.6	12	91.6	17	85.6
3	88.4	8	87.0	13	87.7	18	90.9
4	91.8	9	89.8	14	85.0	19	82.1
5	90.3	10	92.2	15	91.5	20	85.8

7-33 The average time (minutes) that a customer has to wait for the arrival of a cab after calling the company has been observed for random samples of size 4. The data for 20 such samples are shown in Table 7-25. Previous analysis gave the upper and lower control limits for an $\overline{X}$-chart when the process was in control as 10.5 and 7.7 minutes, respectively. What is your estimate of the standard deviation of the waiting time for a customer? Construct a moving-average control chart using a span of 3. What conclusions can you draw from the chart?

TABLE 7-25

Sample	Average Waiting Time	Sample	Average Waiting Time	Sample	Average Waiting Time	Sample	Average Waiting Time
1	8.4	6	9.4	11	8.8	16	9.9
2	6.5	7	10.2	12	10.0	17	10.2
3	10.8	8	8.1	13	9.5	18	8.3
4	9.7	9	7.4	14	9.6	19	8.6
5	9.0	10	9.6	15	8.3	20	9.9