ML
LECTURE-5

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Multiple linear regression

- We assume that there are N independent variables $x_1, x_2, \cdots, x_N$. Let the dependent variable be y.
- Let there also be n observed values of these variables:

Variables (features)	Values (examples)			
	Example 1	Example 2	$\cdots$	Example n
x_1	x_{11}	x_{12}	$\cdots$	x_{1n}
x_2	x_{21}	x_{22}	$\cdots$	x_{2n}
$\cdots$				
x_N	x_{N1}	x_{N2}	$\cdots$	x_{Nn}
y (outcomes)	y_1	y_2	$\cdots$	y_n

Table 7.3: Data for multiple linear regression

- The multiple linear regression model defines the relationship between the N independent variables and the dependent variable by an equation of the following form:

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_N x_N$$

Multiple linear regression

- As in simple linear regression, here also **we use the ordinary least squares (OLS) method to obtain the optimal estimates of β_0, β_1, ⋯ , β_N.** The method yields the following procedure for the computation of these optimal estimates. Let

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & \cdots & x_{N1} \\ 1 & x_{12} & x_{22} & \cdots & x_{N2} \\ \vdots & & & & \\ 1 & x_{1n} & x_{2n} & \cdots & x_{Nn} \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad B = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_N \end{bmatrix}$$

- Then it can be shown that the regression coefficients are given by

$$\mathbf{B = (X^T X)^{-1} X^T Y}$$

Multiple linear regression Example

- **Example:**
- Fit a multiple linear regression model to the following data:

x_1	1	1	2	0
x_2	1	2	2	1
y	3.25	6.5	3.5	5.0

Table 7.4: Example data for multi-linear regression

- **Solution:**
- In this problem, there are two independent variables and four sets of values of the variables. Thus, in the notations used above, we have n = 2 and N = 4. The multiple linear regression model for this problem has the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

- The computations are shown below.

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 0 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 3.25 \\ 6.5 \\ 3.5 \\ 5.0 \end{bmatrix}, \quad B = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

Multiple linear regression Example

$$X^T X = \begin{bmatrix} 4 & 4 & 6 \\ 4 & 6 & 7 \\ 6 & 7 & 10 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} \frac{11}{4} & \frac{1}{2} & -2 \\ \frac{1}{2} & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix}$$

$$B = (X^T X)^{-1} X^T Y$$

$$= \begin{bmatrix} 2.0625 \\ -2.3750 \\ 3.2500 \end{bmatrix}$$

- The required model is

$$y = 2.0625 - 2.3750x_1 + 3.2500x_2$$

Multiple linear regression Example

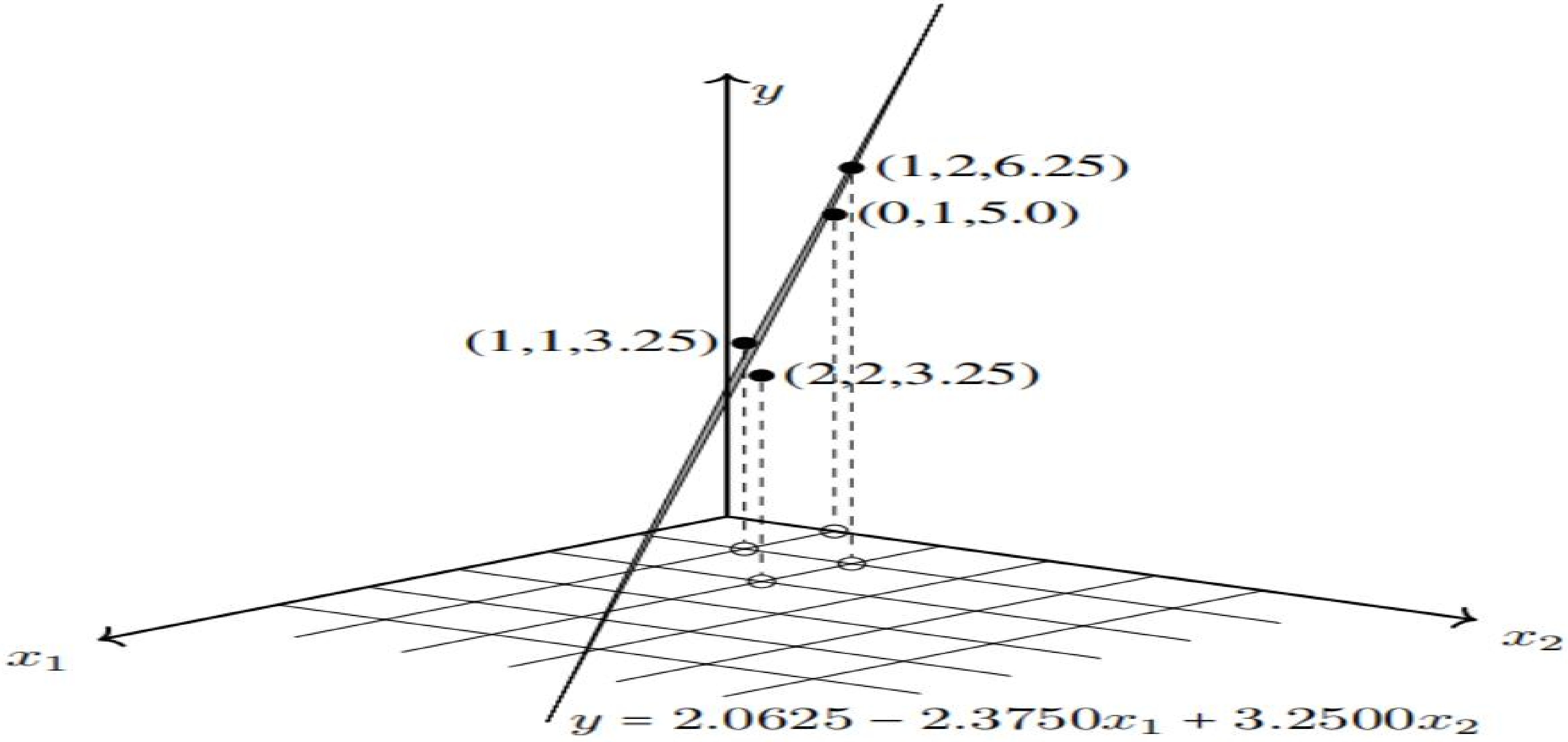
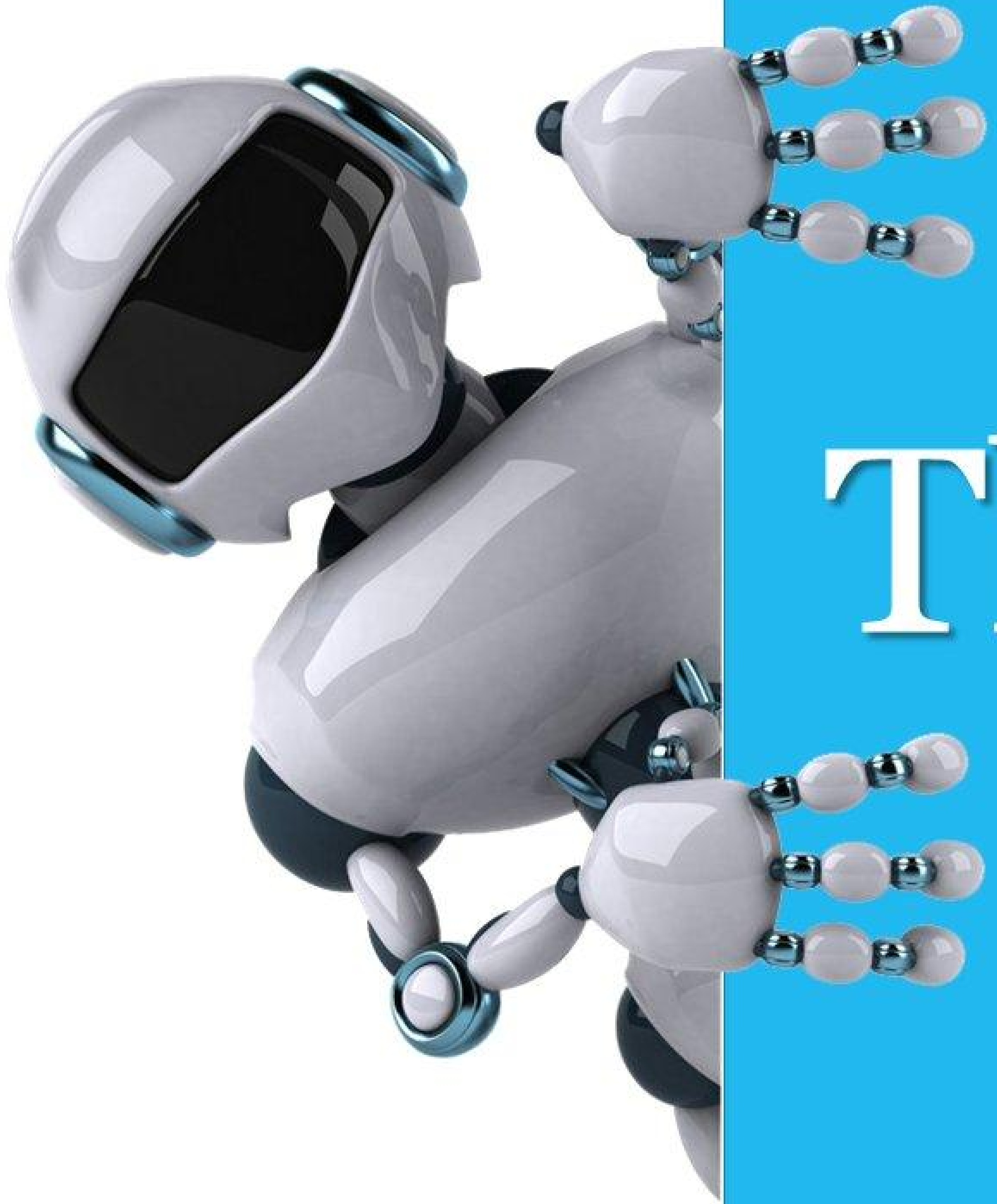


Figure 7.4: The regression plane for the data in Table 7.4



Thank you