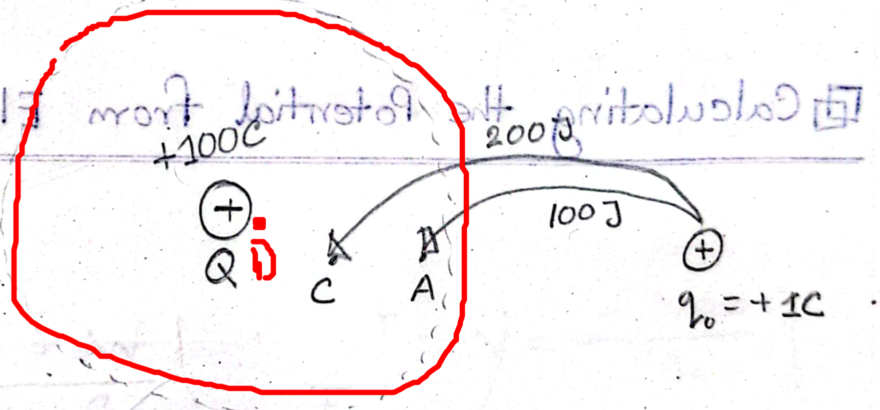
Potential

☐ Electric Potential :

Electric potential is the amount of electric potential energy per unit charge at ~~a~~ a specific point in an electric field.



Potential, $\boxed{V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}}$ $[U = -W]$

if $q_0 = +1C$, $\boxed{V = -W_\infty = U}$

* $\left(\text{electric potential energy}\right) = \left(\text{particle's charge}\right)\left(\dfrac{\text{electric poten. energy}}{\text{unit charge}}\right)$

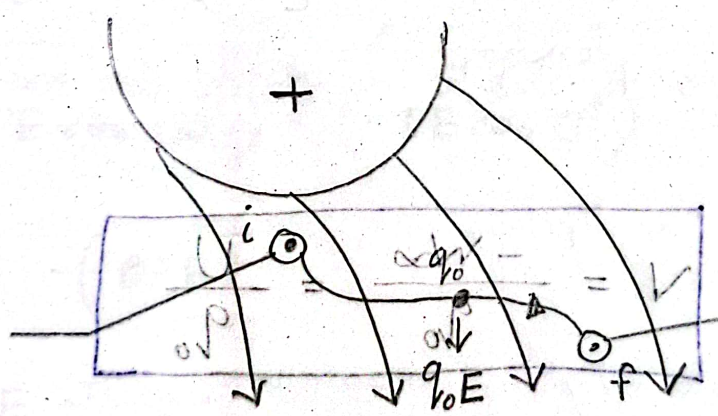
$$\boxed{U = qV}$$

$$\left[V = \frac{U}{q_0}\right]$$

V E

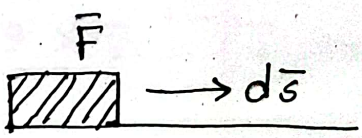
CO2 □ Calculating the Potential from Electric Field:

higher potential. (চার্জের বেশি কাছে। তাই বিকর্ষণ বেশি।)



lower potential

$\boxed{U}$



$\therefore dw = \bar{F} \cdot d\bar{s}$

Now,

$$dW = \vec{F} \cdot d\vec{s}$$

$$= (q_0 \vec{E}) \cdot d\vec{s} \qquad \left[\because \vec{F} = q_0 \vec{E}\right]$$

$$\therefore W = \int dW$$

$$= \int_i^f q_0 \vec{E} \cdot d\vec{s} = \boxed{q_0 \int_i^f \vec{E} \cdot d\vec{s}}$$

Now, $\boxed{\Delta V = \frac{-W}{q_0}}$ $\qquad \left[\because V = \frac{-W_\infty}{q_0}\right]$

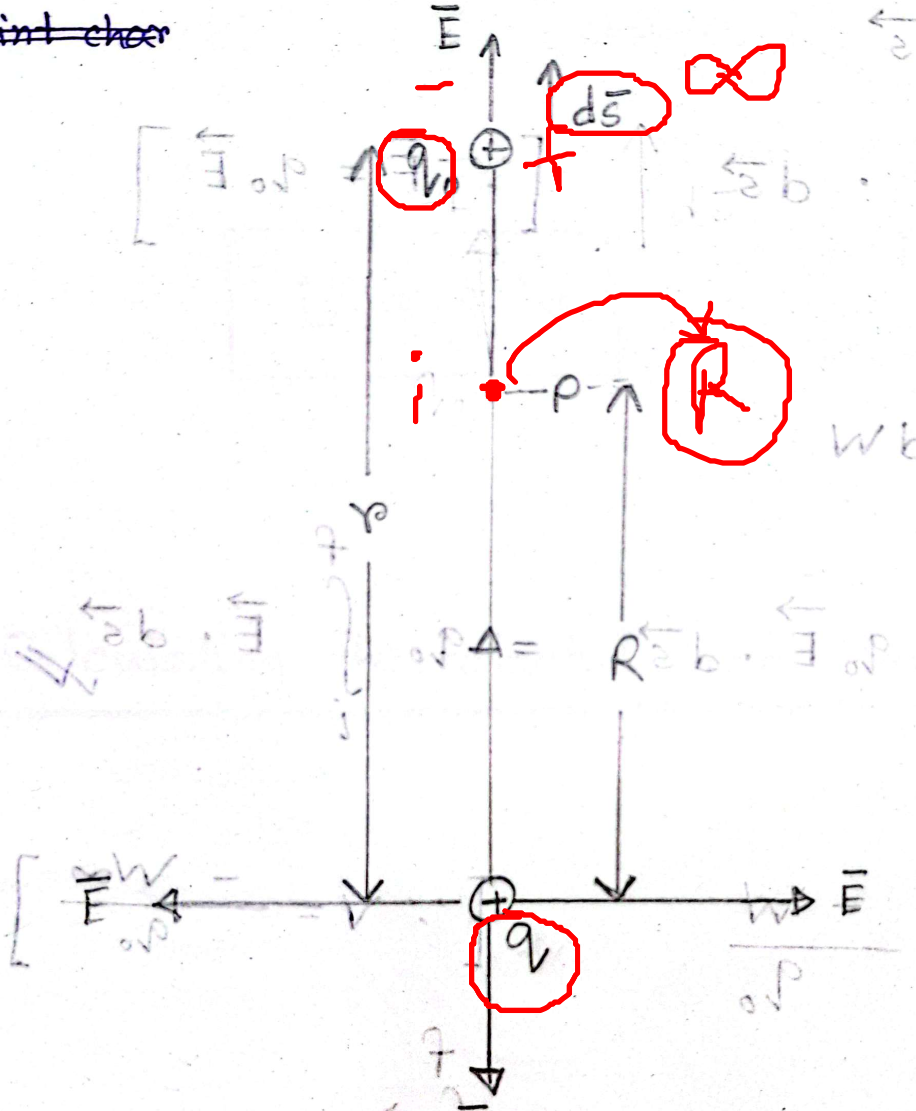
$$\Rightarrow V_f - V_i = \frac{1}{\cancel{q_0}} \cdot -\cancel{q_0} \int_i^f \vec{E} \cdot d\vec{s}$$

$$\Rightarrow \boxed{V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}}$$

spring 23

Potential due to a Charged Particle:

~~(Point char~~



The particle with positive charge q produces an electric field $\bar{E}$ and an Electric potential V at point P (which is at distance R from the particle)

Now, to find the potential, we move a test charge q_0 from P to infinity, with a displacement of $d\vec{s}$.

$\bar{A} \cdot \bar{B}$

$AB \cos\theta$

Now, we know,

$$V_f - V_i = \boxed{-\int_i^f \bar{E} \cdot d\bar{s}}$$

$$= -\int_i^f E\, ds \cos\theta$$

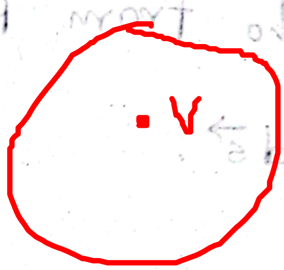
$$= -\int_i^f E\, ds \quad \text{—— (i)}$$

The differential displacement $d\vec{s}$ of the test particle along its path has the same direction as $\vec{E}$.

$\therefore \theta = 0°$

Now, because the path is radial, let us write ds as dr. Then, substituting the limits R and ∞ we rewrite eq^n (i) :

$$V_f - V_i = -\int_R^\infty E\,dr \qquad \text{---} \text{(ii)}$$



Now, potential at infinity $V_f = 0$

potential at distance R, $V_i = V$

$$\therefore \underset{V_F}{0} - \underset{V_i}{V} = -\int_R^\infty E\,dr$$

$$\Rightarrow \cancel{-}V = \cancel{-}\int_R^\infty \underbrace{\left(\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}\right)}\,dr$$

test charge কে ~~বর্তমানে~~ infinity তে নিয়ে যাওয়ার সময় কোনো এক সময়ের Electric field. যেখানে দূরত্ব r.

$$\Rightarrow V_i = \frac{q}{4\pi\epsilon_0}\int_R^\infty \frac{1}{r^2}\,dr$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \int_R^{\infty} r^{-2} \, dr$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{r^{-1}}{-1} \right]_R^{\infty}$$

$$= -\frac{q}{4\pi\epsilon_0} \left[r^{-1} \right]_R^{\infty}$$

$$= -\frac{q}{4\pi\epsilon_0} \left[\frac{1}{\infty} - \frac{1}{R} \right]$$

$$\Rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}}$$

$$F = c \cdot \frac{q_1 q_2}{r^2}$$

$$E = c \cdot \frac{Q}{r^2}$$

$$V = c \cdot \frac{q}{r}$$

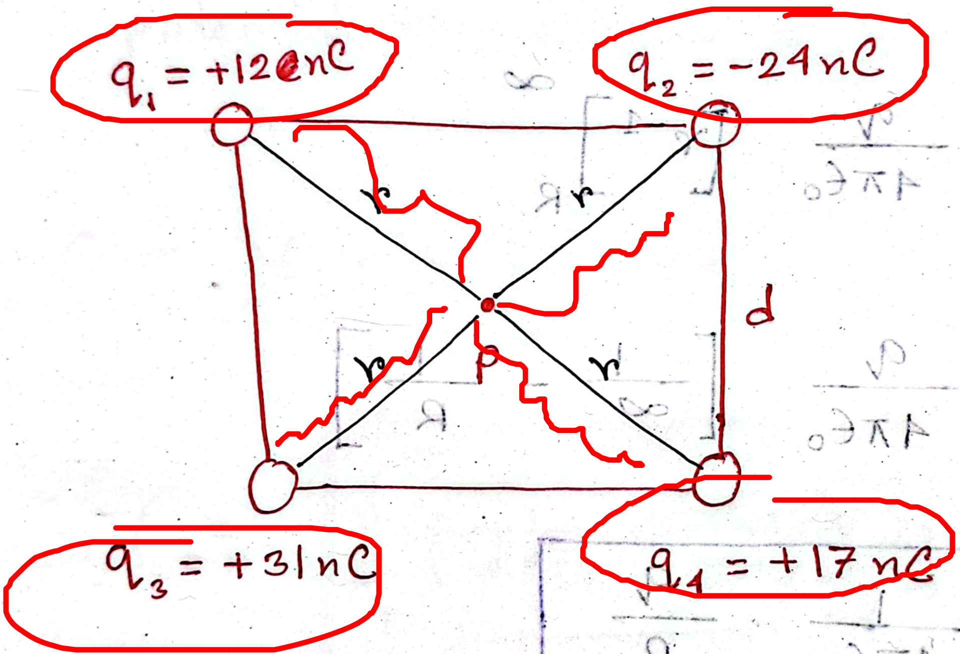
Solving for V and switching R to r,

Potential due to a charged particle q at any radial distance r from the particle :

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}}$$

$$c$$

Q] What is the electric potential at point P, located at the center of the square of charged particles shown below? The distance d is 1.3m. ~~and the~~



Sol:

Distance between P and each charge.

$$r = \frac{d}{\sqrt{2}} = \frac{1.3}{\sqrt{2}} = 0.919 \text{ m}$$

$C \cdot \frac{Q}{V}$ $\qquad$ E / F

Now, net potential,

$$V_{net} = V_1 + V_2 + V_3 + V_4$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r} (q_1 + q_2 + q_3 + q_4)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r} \{ (12 - 24 + 31 + 17) \times 10^{-9} \}$$

$$= 9 \times 10^{9} \cdot \frac{1}{0.919} (36 \times 10^{-9})$$

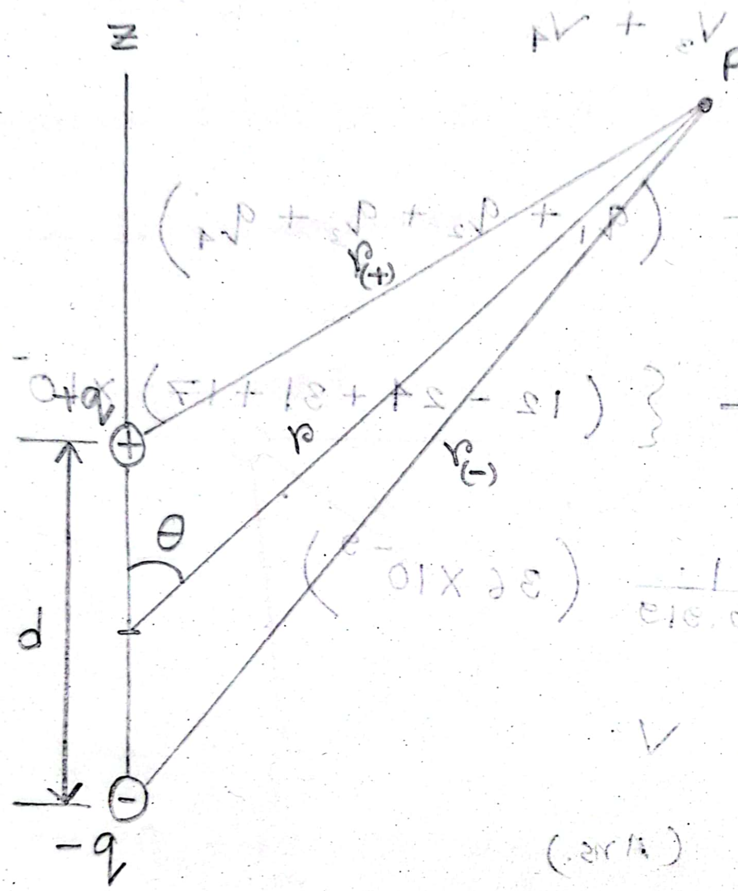
$$= 352.56 \text{ V}$$

(Ans.)

* Check slide and CW for more questions to practice.

Spring 23

☐ Potential due to an Electric Dipole :



We know, potential $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

Using this eq^n to an ~~dip~~ electric dipole to find the potential at an arbitrary point p :

$$V = V_{(+)} + V_{(-)}$$

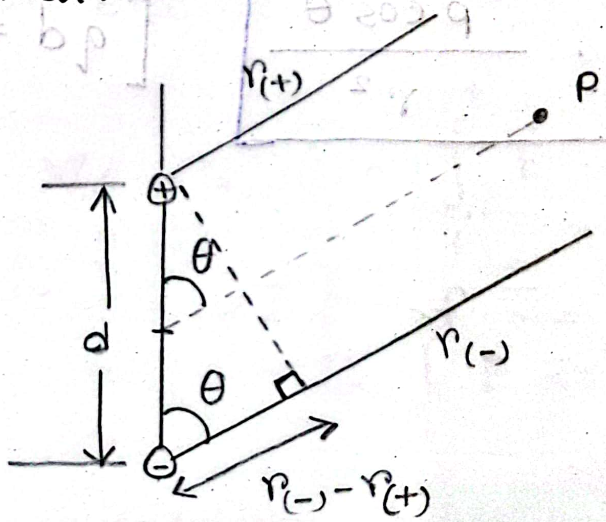
$$= \frac{1}{4\pi\epsilon_0}\left(\frac{+q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_{(+)}} - \frac{1}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left\{\frac{r_{(-)} - r_{(+)}}{r_{(+)}\, r_{(-)}}\right\}$$

Now, the distance between the charges of the dipole are quite small. So, $r \gg d$

So, we can approximate the two lines to P as being parall.



$\therefore$ Now,

$$\cos\theta = \frac{r_{(-)} - r_{(+)}}{d}$$

$$\Rightarrow r_{(-)} - r_{(+)} = d\cos\theta$$

and,

$$r_{(+)} \approx r \qquad \therefore r_{(+)} r_{(-)} = r \cdot r = r^2$$

$$r_{(-)} \approx r$$

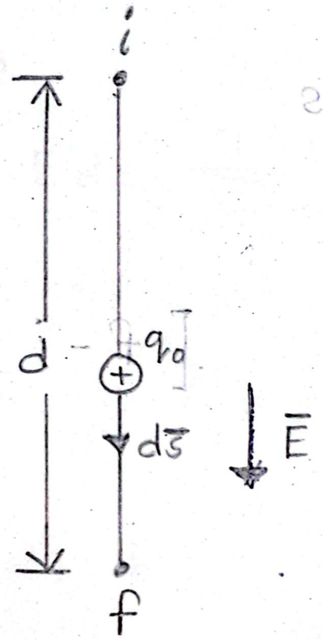
$$\therefore V = \frac{q}{4\pi\epsilon_0} \cdot \frac{d\cos\theta}{r^2}$$

$$\Rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p\cos\theta}{r^2}} \qquad [qd = p]$$

GOnm

Fall 23

Relation between potential difference and path :



Scenario - a

In this scenario, a test charge q_0 moves in a straight line from point i to point f, along the direction of electric field $\bar{E}$.

We know, $V_f - V_i = - \int_i^f \bar{E} \cdot d\bar{s}$

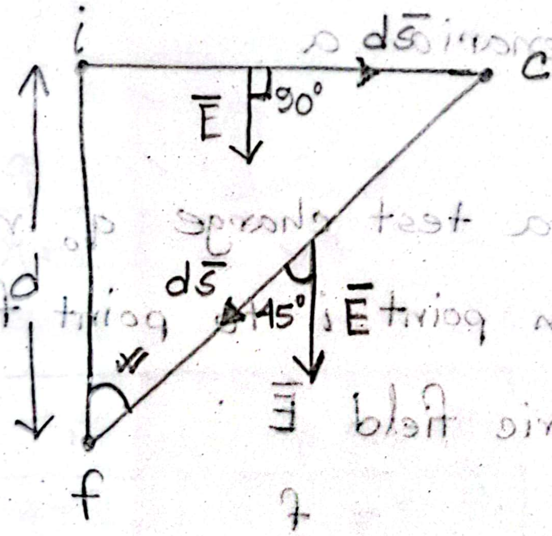
$= - \int_i^f E \, ds \cos 0^\circ$ [both is same dir. $d\bar{s}$ and $\bar{E}$]

$$= -\int_i^f E\,ds$$

$$= -E\int_i^f ds$$

$$\therefore V_f - V_i = -Ed \qquad [f - i = d]$$

<u>Scenario - b :</u>



$$V_c - V_i = -\int_i^c \bar{E}\cdot d\bar{s} = -\int E\,ds\cos 90^\circ = 0$$

Then,

$$V_f - V_c = -\int_c^f \overline{E} \cdot d\overline{s} = -\int_c^f E\, ds \cos 45^\circ$$

$$= -\int_c^f E\, ds \cdot \frac{1}{\sqrt{2}}$$

$$= -\frac{E}{\sqrt{2}} \int_c^f ds$$

$$= -\frac{E}{\sqrt{2}} (cf)$$

$$= -\frac{E}{\sqrt{2}} (d\sqrt{2})$$

$$= -Ed$$

Here,

$$\sin 45^\circ = \frac{d}{cf}$$

$$\therefore cf = \frac{d}{\sin 45^\circ}$$

$$= \frac{d}{1/\sqrt{2}}$$

$$= d\sqrt{2}$$

Finally,

$$V_f - V_i = (V_c - V_i) + (V_f - V_c)$$

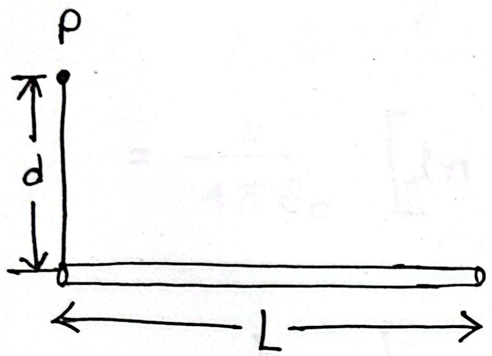
$$= 0 + (-Ed)$$

$$= -Ed$$

∴ Potential difference in ~~seco~~ both scenario is the same.

Moral: When you want to find the potential difference between two points by moving a test charge between them, you can save time and work by choosing a direct path between those points.

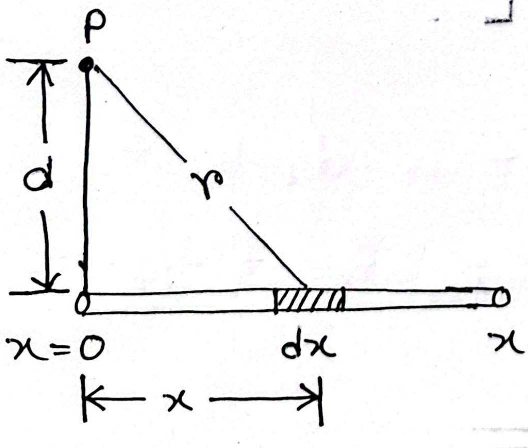
Potential due to a Continuous Charge Distribution:



Let, charge per unit length $= \lambda$

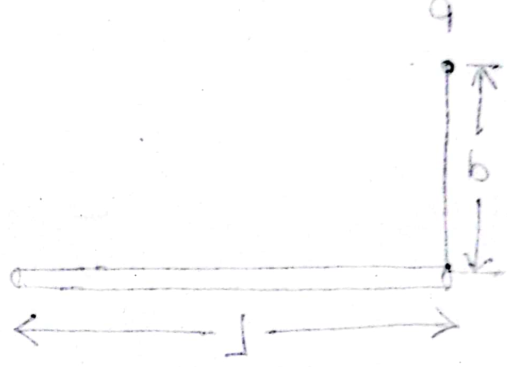
Let, a thin nonconducting rod of length L.

Let us determine the electric potential V due to the rod at point P, a perpendicular distance d from the left end of the rod.



$\therefore$ Charge in differential element dx, $dq = \lambda \cdot dx$

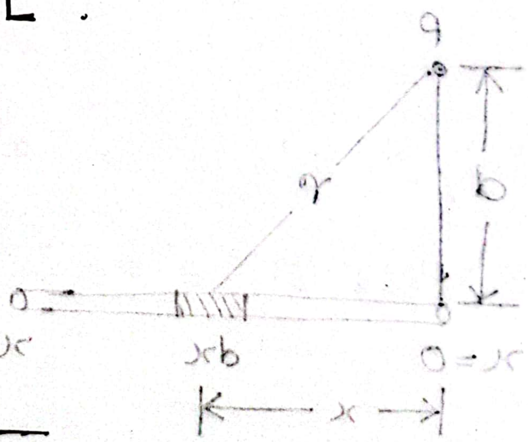
Treating the element as a point charge, we can write the potential dV as



$$dV = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda\, dx}{\sqrt{x^2+d^2}} \quad \text{———(i)}$$

Now, we'll find the total potential V produced by the rod at point P by integrating (i) along the length of the rod, from $x = 0$ to $x = L$.



$$V = \int dV$$

$$= \int_0^L \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda\, dx}{\sqrt{x^2+d^2}}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{1}{\sqrt{x^2+d^2}}\, dx$$

$$= \frac{\lambda}{4\pi\epsilon_0} \left[\ln\left(x + \sqrt{x^2+d^2}\right) \right]_0^L$$

$$= \frac{\lambda}{4\pi\epsilon_0} \left[\ln\left(L + \sqrt{L^2+d^2}\right) - \ln d \right]$$

$$\boxed{V = \frac{\lambda}{4\pi\epsilon_0} \cdot \ln\left(\frac{L+\sqrt{L^2+d^2}}{d}\right)}$$

Equipotential surface:

An equipotential is a surface where every point on it has the same electric potential. This means that, no work is required to move a charge along the surface because the electric potential difference between any two points on the surface is zero.