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📁 **Complete Source Code and Model Output — is available on GitHub:**
🔗 View the Code and Results on GitHub

Report: Cointegration-Based Pairs Trading Model Development and Risk Management

In today's financial markets, trading strategies must strike a balance between profitability and risk control. This project presents a cointegration-based pairs trading, which integrates statistical testing (Johansen cointegration test), regression analysis (OLS hedge ratio), technical risk control mechanisms (ATR-based stop-loss), and dynamic leverage adjustment (target volatility scaling). The model also employs multi-stage parameter optimization techniques—Grid Search, Random Search, and Bayesian Optimization—to determine the optimal parameter combination. After six years of backtesting, the strategy demonstrated a stable risk-return trade-off. More importantly, it maintained and even improved its performance during out-of-sample testing.

Model Architecture and Operational Principles

Data Acquisition and Preprocessing

The model begins by downloading historical data for GOOG and MSFT via yfinance. The adjusted closing prices are transformed using the natural logarithm to enhance statistical robustness. This transformation preserves essential information from the price series while laying the necessary groundwork for subsequent cointegration testing.

Cointegration Testing and Hedge Ratio Calculation

In a pairs trading strategy, confirming the existence of a long-term equilibrium relationship between two assets is critical. This implementation employs the Johansen cointegration test on the log-transformed price series of GOOG and MSFT (Figure 1). If the test indicates a cointegrated relationship, the model proceeds to perform an Ordinary Least Squares (OLS) regression to calculate the hedge ratio. This hedge ratio is then used to construct the price spread between the two stocks. The spread serves as the foundation for generating trading signals, ensuring that the strategy does not rely on market direction but rather captures relative price divergence and convergence opportunities.

Figure 1

The Johansen cointegration test

```
✅ Johansen test: GOOG & MSFT appear cointegrated.
✅ Hedge Ratio: 0.7513
```

Risk Management Mechanism and Dynamic Leverage Adjustment

This model incorporates the Average True Range (ATR) to determine stop-loss triggers. ATR dynamically adjusts the stop-loss threshold based on recent asset volatility, effectively limiting losses on individual trades. Furthermore, the model employs a target annualized volatility approach for dynamic leverage adjustment. It calculates a leverage factor based on the rolling volatility of past returns. This allows the model to scale up positions during stable market conditions and automatically reduce leverage in periods of high volatility.

Multi-Stage Parameter Optimization

To identify the most suitable parameter configuration for the model, three optimization techniques are used: Grid Search, Random Search, and Bayesian Optimization (Figure 2). The optimized parameters include the rolling window, quantile window, exit threshold, ATR multiplier, maximum drawdown limit, target annual volatility, and maximum leverage (Figure 3). This multi-stage approach reduce the risk of overfitting due to parameter tuning, while ensuring stable performance during the training phase and providing robust parameter settings for out-of-sample testing.

Figure 2

Three optimization techniques are used

```
--- Optimization Results ---
Grid Search Best Params: (90, 30, 0.2, 0.5, 0.1, 0.08, 1.5), Cost: -0.68
Random Search Best Params: (60, 37, 0.1658693631618945, 0.37220082031107227, 0.0535284373700215, 0.12496934947444209, 1.5397669658124327), Cost: 999.83
Bayesian Optimization Best Params: (61, 39, 0.18656322312112822, 0.5326302867754906, 0.0827880150215764, 0.14401495104184592, 2.323477820045084), Cost: -0.45
```

Figure 3

The optimized parameters

```
--- Selected Parameter Set ---
Rolling Window: 90 days
Quantile Window: 30 days
Exit Threshold: 0.2
ATR Multiplier: 0.5
Max Drawdown Limit: 10.0%
Target Annual Volatility: 0.08
Max Leverage: 1.5
Selected Cost (objective): -0.68
```

It is worth noting that during Grid Search optimization in the out-of-sample test, the best parameter set produced an unusually high-cost value of 999.61(Figure 4). This does not indicate a failure in the model or the code. Rather, it reflects the intentional design of the objective function, which imposed a strict penalty for parameter combinations yielding a total return lower than 20%**.** If the return fell below this threshold, the function added a penalty of +1000 to the negative Sharpe Ratio, filtering out conservative strategies.

Figure 4

An unusually high-cost value of 999.61 in the out-of-sample test

```
--- Training Optimization Results ---
Best Params: (90, 40, 0.1, 0.5, 0.1, 0.08, 1.5), Cost: 999.61
Training set best parameters:
Rolling Window: 90 days
Quantile Window: 40 days
Exit Threshold: 0.1
ATR Multiplier: 0.5
Max Drawdown Limit: 10.0%
Target Annual Volatility: 0.08
Max Leverage: 1.5
```

Backtesting Results and Out-of-Sample Evaluation

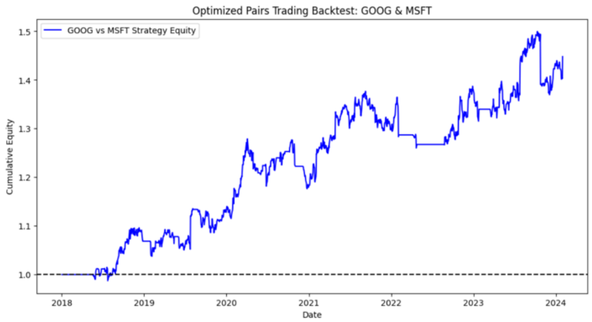
Six-Year Backtest Performance

During the historical backtesting period, the strategy achieved an annualized return of 6.27%, with an annualized volatility of only 9.21%. The Sharpe Ratio stood at 0.68, the Sortino Ratio at 0.78, and the maximum drawdown was limited to -8.67% (Figure 5). These metrics indicate that the model was able to operate with low risk and stable returns, even under uncertain market conditions.

Figure 5

Pairs Trading Strategy Backtest

```
--- Final Performance ---
Rolling Window: 90 days
Quantile Window: 30 days
Exit Threshold: 0.2
ATR Multiplier: 0.5
Max Drawdown Limit: 10.0%
Target Annual Volatility: 0.08
Max Leverage: 1.5
Total Return: 44.80%
Annualized Return: 6.27%
Annualized Volatility: 9.21%
Sharpe Ratio: 0.68
Sortino Ratio: 0.78
Max Drawdown: -8.67%
```



Out-of-Sample Test Results

After entering the out-of-sample testing period from 2023 to 2024, the strategy not only did not deteriorate in performance, but actually achieved better results. The annualized return increased to 8.64%, the Sharpe Ratio rose to 0.80, and the maximum drawdown remained stable (Figure 6). This indicates that the model performs well not only on training data, but also maintains high stability on unseen data—validating the strategy's generalization ability and market-neutral characteristics.

Figure 6

Out-Of-Sample Testing Period Performance

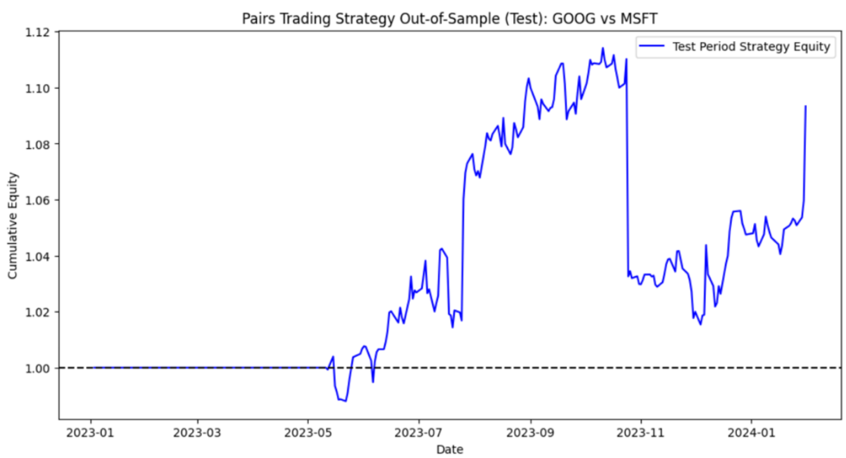
```
--- Test Performance Metrics ---
Total Return: 9.33%
Annualized Return: 8.64%
Annualized Volatility: 10.75%
Sharpe Ratio: 0.80
Sortino Ratio: 0.67
Max Drawdown: -8.87%
```

Although the model performed better during the out-of-sample test period (2023–2024) compared to the training period, there was an unusual period of volatility and sharp decline in Q4 2023 that deviated from the strategy's typically stable style (Figure 7).

This does not indicate a flaw in the strategy's logic, but more likely reflects that unexpected events or structural changes occurred in the market during that time—or that the current risk management mechanisms (such as ATR-based stop loss or leverage controls) still have room for optimization under extreme market conditions. This period of abnormal performance highlights the need to strengthen the model's defensive capabilities during times of high volatility. Enhancements could include event filtering, excluding earnings announcements, or raising thresholds for extreme volatility.

Figure 7

Unusual Q4 2023 Drawdown



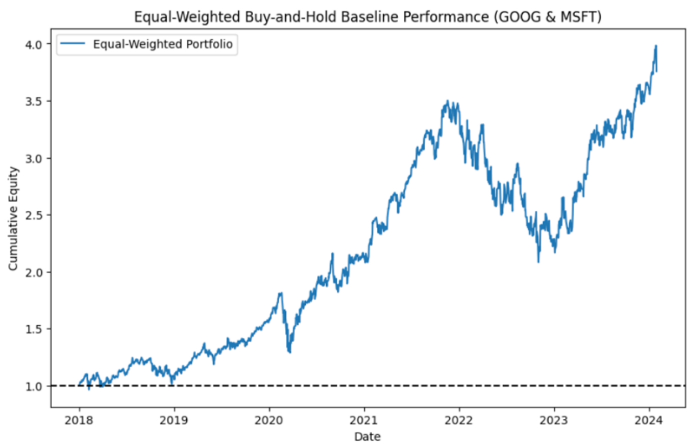
Comparison with Baseline Strategy

Although this pairs trading strategy delivers an annualized return of only 6.27%—significantly lower than the 24.32% achieved by an equal-weighted buy-and-hold strategy of GOOG and MSFT—its true strength lies not in chasing eye-catching absolute returns, but in offering the stability and downside protection that institutional investors prioritize. With an annualized volatility of just 9.21% (versus 28.74% for buy-and-hold) and a maximum drawdown of only -8.67% (compared to -40.54%), the strategy is fully market-neutral, capable of mitigating portfolio risk and reducing overall volatility during periods of systemic market stress (Figure 8).

Figure 8

Equal-Weighted Buy-and-hold Baseline Performance

```
--- Equal-Weighted Buy-and-Hold Baseline Performance ---
Total Return: 275.68%
Annualized Return: 24.32%
Annualized Volatility: 28.74%
Sharpe Ratio: 0.85
Sortino Ratio: 1.15
Max Drawdown: -40.54%
```



This means that even during sharp market downturns, the strategy has the potential to preserve capital and maintain steady net asset value, largely unaffected by broad directional trends.

Conclusion

By integrating statistical testing, backtesting techniques, and multi-stage parameter optimization, this project built a market-neutral and risk-controlled pairs trading model. While its annualized return may not match that of a Buy-and-Hold strategy, its low volatility and drawdown allow it to deliver stable, risk-adjusted returns over the long run—making it a viable strategy for sophisticated portfolio construction.