Kontrolltöö

1. Allugu aparaadi eluiga T eksponentjaotusele. Olgu keskmine eluiga $E(T) = 8$ aastat. Leidke 6 sellise põhjal seadud tõenäosus, et lühima kvaliteediga neist ulatub/peale vastu üle 4 aasta ja parima kvaliteediga üle 16 aasta

$$T \sim \mathcal{E}(\alpha) \; ; \quad \alpha = 8 \; ; \quad n = 6 \; ; \quad \lambda = \frac{1}{\alpha} = \frac{1}{8}$$

$$f(t) = \begin{cases} 0, & \text{kui } t < 0 \\ \lambda \exp(-\lambda t), & \text{kui } t \geq 0 \end{cases}$$

Suurima ja vähima vahe jaotus

$$P\left(T_{(1)} > x_1, T_{(n)} \leq x_2\right) = \left[F(x_2) - F(x_1)\right]^n$$

$$P\left(T_{(1)} > 4, T_{(6)} \leq 16\right) = \left[F(16) - F(4)\right]^6 = *$$

i) $$F(4) = \int_0^4 \lambda e^{-\lambda t} dt = \int_0^4 \frac{1}{8} \cdot e^{-\frac{1}{8}t} dt =$$

$$= \frac{1}{8} \int_0^4 e^{-\frac{t}{8}} dt = \left[\begin{array}{l|l|l} -\frac{t}{8} = u & t & u \\ & 4 & -\frac{1}{2} \\ -\frac{1}{8} dt = du & 0 & 0 \end{array} \right] =$$

$$= \frac{1}{8} \int_0^{-\frac{1}{2}} e^u \cdot (-8) du = -\int_0^{-\frac{1}{2}} e^u du = \int_{-\frac{1}{2}}^0 e^u du =$$

$$= \left[e^u\right]_{-\frac{1}{2}}^0 = 1 - e^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{e}} \approx 0.39347$$

$$\text{ii)}\ F(16) = \int_0^{16} \frac{1}{8} e^{-\frac{t}{8}}\, dt = \left[\begin{array}{l|l} -\frac{t}{8} = u & \begin{array}{c|c} t & u \\ \hline 16 & -2 \\ 0 & 0 \end{array} \\ -\frac{1}{8}dt = du & \end{array} \right] =$$

$$= -\int_0^{-2} e^u\, du = \int_{-2}^{0} e^u\, du = 1 - e^{-2} = 1 - \frac{1}{e^2} \approx$$

$$\approx 0.86466$$

$$* = \left[\left(1 - \frac{1}{e^2}\right) - \left(1 - \frac{1}{\sqrt{e}}\right) \right]^6 \approx \underline{\underline{0.010945}}$$

(2.) Olgu juhusliku vektori $X = (X, Y)^T$ tihedusfunktsioon

$$f(x,y) = \begin{cases} 6x, & \text{kui } 0 < x < y < 1, \\ 0, & \text{mujal}. \end{cases}$$

Leidke keskmised regressioonisõltuvused $y = g_1(x)$ ja $x = g_2(y)$.

$$f(x,y) = \begin{cases} 6x, & \text{kui } 0 < x < y < 1, \\ 0, & \text{mujal}. \end{cases}$$

$$x = g_2(y) = E(X \mid Y) = \int_{-\infty}^{+\infty} x\, f_1(x \mid y)\, dx$$

$$y = g_1(x) = E(Y \mid x) = \int_{-\infty}^{+\infty} y\, f_2(y \mid x)\, dy$$

$$f_1(x \mid y) = \frac{f(x,y)}{f_2(y)}; \quad f_2(y \mid x) = \frac{f(x,y)}{f_1(x)}$$

$$f_2(y) = \int_{-\infty}^{+\infty} f(x,y)\, dx; \quad f_1(x) = \int_{-\infty}^{+\infty} f(x,y)\, dy$$

$$f_2(y) = \int_0^y 6x\,dx = \left[6 \cdot \frac{x^2}{2}\right]_0^y = 3(y^2 - 0^2) = 3y^2$$

$$f_1(x) = \int_x^1 6x\,dy = 6x\,[y]_x^1 = 6x(1-x)$$

$$f_1(y|x) = \frac{6x}{6x(1-x)} = \frac{1}{1-x}$$

$$f_2(x|y) = \frac{6x}{3y^2} = \frac{2x}{y^2}$$

$$x = g_2(y) = \int_0^y x \cdot \frac{2x}{y^2}\,dx = \frac{2}{y^2}\int_0^y x^2\,dx =$$

$$= \frac{2}{y^2} \cdot \left[\frac{x^3}{3}\right]_0^y = \frac{2}{3} \cdot \frac{1}{y^2}(y^3 - 0) = \underline{\underline{\frac{2}{3}y}}$$

$$y = g_1(x) = \int_x^1 y \cdot \frac{1}{1-x}\,dy = \frac{1}{1-x}\int_x^1 y\,dy =$$

$$= \frac{1}{1-x} \cdot \left[\frac{y^2}{2}\right]_x^1 = \frac{1}{2(1-x)}(1^2 - x^2) =$$

$$= \frac{\cancelto{1}{(1-x)}(1+x)}{2\cancelto{1}{(1-x)}} = \underline{\underline{\frac{1+x}{2}}}$$

3.) Allugu õnnetuste arv kuus Poissoni jaotusele parameetriga λ. Uuriti 5 juhtumit ja saadi järgmised õnnetuste sagedused kuus:

$$x = (0, 2, 2, 1, 3)^T.$$

a) Leidke suurima tõepära (STP) hinnang parameetrile λ.

b) Kui suur on leitud hinnangu ning 5 uuritud juhtumi põhjal tõenäosus, et õnnetuste hulk on kuus alla 2.

a) $X \sim Poi(\lambda)$

$$f(x_i, \lambda) = \frac{\lambda^{x_i}}{x_i!} \cdot e^{-\lambda}$$

STP

$$L(x_i, \lambda) = \prod_{i=1}^{n} f(x_i, \lambda)$$

Logaritmime $L(x_i, \lambda)$:

$$\ln L(x_i, \lambda) = \ln \prod_{i=1}^{n} f(x_i, \lambda) = \ln \prod_{i=1}^{n} \frac{\lambda^{x_i}}{x_i!} \cdot e^{-\lambda} =$$

$$= \sum_{i=1}^{n} \ln\left(\frac{\lambda^{x_i}}{x_i!} \cdot e^{-\lambda}\right) = -n\lambda + \ln\lambda \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \ln(x_i!)$$

Võtame tuletise λ järgi:

$$\frac{d\,L(x_i, \lambda)}{d\lambda} = -n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i + 0 \Rightarrow \lambda = \overbrace{\frac{1}{n}\sum_{i=1}^{n} x_i}^{\text{arit. keskmine}}$$

b) $P(X < 2) = P(X = 0) + P(X = 1)$, $x = \begin{pmatrix} 0 \\ 2 \\ 2 \\ 1 \\ 3 \end{pmatrix}$

$n = 5$

$$\lambda = \frac{1}{n} \sum_{i=1}^{n} x_i = \overline{x},$$

$$\lambda = \overline{x} = \frac{1}{5}(0+2+2+1+3) = \frac{8}{5} = 1.6$$

$$P(X = m) = \frac{\lambda^m}{m!} e^{-\lambda}$$

$$P(X < 2) = \frac{(1.6)^0}{0!} \cdot e^{-1.6} + \frac{1.6^1}{1!} \cdot e^{-1.6} =$$

$$= e^{-1.6}(1 + 1.6) \approx \underline{\underline{0.52493}}$$

4.) Sündmuse A toimumist katsetati 5 korda. Olgu tõenäosus $X = P(A)$ juhuslik suurus tihedusfunktsiooniga

$$f(x,y) = \begin{cases} 2x, & \text{kui } x \in [0;1], \\ 0, & \text{mujal.} \end{cases}$$

Leidke tõenäosus $P(X < 0.5)$, kui on teada, et sündmus A toimus 2 korral.

<u>Bayesi valem</u>: