Ül. 0.2 Allugu patarei eluiga aastates eksponentjaotusele, mille parameetriks on ν. Uuriti mitu patareid [...] Leidke STP hinnang parameetrile ν. Kui suur on tn (STP põhjal), et patarei elab üle aasta?

STP

$$f(x) = \begin{cases} 0, & x < 0 \\ \nu e^{-\nu x}, & x \geq 0 \end{cases}$$

$$L(\nu, \mathcal{X}) = \prod_{i=1}^{n} \nu e^{-\nu x_i} \to \max \qquad \nu - ?$$

$$\mathcal{X} = \{x_1, x_2, \dots, x_n\}$$

$$L'(\nu, \mathcal{X}) = 0$$

Logaritmime

$$\ln L(\nu, \mathcal{X}) = \ln \prod_{i=1}^{n} f(x_i) = \sum_{i=1}^{n} \ln f(x_i) =$$

$$= \sum_{i=1}^{n} \left[\ln\left(\nu e^{-\nu x}\right) \right] = \sum_{i=1}^{n} \left(\ln \nu + \ln e^{-\nu x_i} \right) =$$

$$= n \cdot \ln \nu + \sum_{i=1}^{n} (-\nu x_i) \cdot \underbrace{\ln e}_{1} = \underline{\underline{n \cdot \ln \nu - \sum_{i=1}^{n} \nu x_i}}$$

Tuletis ν järgi

$$\frac{\partial \left(n \ln \nu - \sum_{i=1}^{n} x_i \nu\right)}{\partial \nu} = n \cdot \frac{1}{\nu} - \sum_{i=1}^{n} (1 \cdot x_i - \nu \cdot 0) =$$

$$= \frac{n}{\nu} - \sum_{i=1}^{n} x_i = 0 \Rightarrow \frac{n}{\nu} = \sum_{i=1}^{n} x_i \Rightarrow \frac{1}{\nu} = \frac{1}{n} \sum_{i=1}^{n} x_i \to ?$$

$$x_i = \frac{1}{n} \sum_{i=1}^{n} x_i \Rightarrow \frac{1}{\nu} = \bar{x} \Rightarrow \boxed{\nu = \frac{1}{\bar{x}}}$$

Ül. 0.2

Lambipirni eluead aastates

$$\{0.9,\ 1.2,\ 1.6,\ 0.8,\ 1.1\}$$

STP

$$f(x) = \begin{cases} 0, & x < 0 \\ \nu e^{-\nu x}, & x \geq 0 \end{cases}$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 1{,}12 \qquad \nu^* = \frac{1}{1{,}12} \approx 0.893$$

Tõenäosus, et patarei kestab kauem kui aasta

$$P(T > 1) = 1 - P(T \leq 1) = 1 - F(1) =$$

$$= 1 - (1 - e^{-\nu^* \cdot 1}) = e^{-0{,}893} \approx \underline{\underline{0{,}41}}$$

Ül. 0.3

Allugu patarei eluiga aastates eksponentjaotusele, mille parameetri m jaotust kirjeldab tihedusfunktsioon

$$f(m) = \begin{cases} \frac{m}{2}, & \text{kui } m \in (0;2) \\ 0, & \text{mujal} \end{cases}$$

5 patarei vastupidavus aastates

$$(0.9,\ 1.2,\ 1.6,\ 0.8,\ 1.1)$$

Täistõenäosuse valem

$$f(t) = \int_0^2 f(t \mid m)\, p(m)\, dm = \int_0^2 m e^{-mt} \cdot \frac{m}{2}\, dm = \int_0^2 \frac{m^2}{2} e^{-mt}\, dm =$$

$$= -e^{-mt} \cdot \frac{1}{2 \cdot t} m^2 + \frac{-mte^{-mt} - e^{-mt}}{t^3} \Bigg|_0^2$$

$$= -\frac{2e^{-2t}}{t} + \frac{-2e^{-2t}t - e^{-2t} + 2t + 1}{t^3} = \beta$$

$$P(X > 1) = 1 - P(X = 0) = 1 - F(0)$$

Ül. 0.4 Küsitlus linnapea populaarsuse kohta. Pooldajate osakaaluks saadi 0.55. Leidke minimaalne küsitletute hulk n, mille puhul pooldajate osakaalu $\alpha = 0{,}95$ alumine piir on üle 0.5.

Tsentraalne piirteoreem

$$\frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)}}\sqrt{n} \sim N(0,1) \qquad , n - ?$$

$\hat{p} = 0{,}55$, pooldajate osakaal

$p_0 = 0{,}5$, $\underbrace{0.95\text{-usaldusintervalli alumine piir}}$

$$\hat{p} - 1{,}96\frac{\sqrt{p_0(1-p_0)}}{\sqrt{n}} \qquad \left[\bar{x} - \Phi^{-1}\left(\frac{\alpha}{2}\right)\frac{\sigma}{\sqrt{n}};\ \bar{x} + \Phi^{-1}\left(\frac{\alpha}{2}\right)\frac{\sigma}{\sqrt{n}}\right]$$

$$0{,}55 - 1{,}96\frac{\sqrt{0{,}5 \cdot 0{,}5}}{\sqrt{n}} = 0{,}55 - \frac{1{,}96 \cdot 0{,}5}{\sqrt{n}} > 0{,}5$$

$$\frac{1{,}96 \cdot 0{,}5}{\sqrt{n}} < 0{,}05$$

$$\sqrt{n} > \frac{1{,}96 \cdot 0{,}5}{0{,}05}$$

$$n > \left(\frac{1{,}96 \cdot 0{,}5}{0{,}05}\right)^2$$

$$n > \underline{\underline{384}}$$

Ül. 0.6 Olgu lambipirni keskmine eluiga $E(T) = 10$ kuud. Leidke 8 sõltumatult põleva lambipirni max ja min elua tihedusfunktsioon.

$X \sim Exp(\lambda)$, $\lambda = \frac{1}{a}$ // keskmine eluiga

$T \sim \mathcal{E}\left(\frac{1}{10}\right)$ Järkstatistikute jaotus

$T_{(1)} < T_{(2)} < \ldots < T_{(8)}$

min max

$$F_{T_{(1)}}(x) = 1 - \left(1 - F(x)\right)^8$$

$\cancel{P(X \leq a)} \cancel{= 1 - P(X \leq a)}$

$$F_{T_{(8)}}(x) = F^8(x)$$

$$f_{T_{(8)}}(x) = \left(F^8(x)\right)' = 8F^7(x)\, f(x)$$

$$f_{T_{(1)}}(x) = 8\left(1 - F(x)\right)^7 f(x)$$

$$f(x) = \lambda e^{-\lambda x} = \frac{1}{10} e^{-\frac{1}{10}x}$$ Tihedusfunktsioon

$$F(x) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{10}x}$$ Jaotusfunktsioon

Ül. 0.11

Olgu meil juhuslik suurus $X \sim B(N, p)$, kus sõltumatute katsete hulk N allub Poissoni jaotusele parameetriga 2. Kui suure tõenäosusega oli sõltumatute katsete hulk 3, kui on teada, et katse õnnestus 1 korral.

$X \sim B(N, p)$

$N \sim Poi(2)$

$p = 0.7$

Bayesi valem

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{P(A)}$$

$$P(A) = \sum_{j=1}^{n} P(B_j)P(A|B_j)$$

$$P(N=3|X=1) = \frac{P(N=3, X=1)}{P(X=1)} =$$

$$= \frac{P(N=3)P(X=1|N=3)}{P(X=1)} =$$

$$= \frac{P(N=3)P(X=1|N=3)}{\sum_{n=1}^{\infty} P(N=n)P(X=1|N=n)} = \frac{0{,}034}{0{,}345} = \underline{0{,}0985}$$

$$P(X=1|N=3)P(N=3) = \underbrace{C_3^1 p^1 (1-p)^2}_{\text{kolmest katsest 1 õnnestub}} \cdot \underbrace{e^{-2} \cdot \frac{2^3}{3!}}_{\text{kui keskmine on 2, siis kui suur on tn kui see juhtub 3 korda}} =$$

$$= C_3^1 \cdot 0{,}7 \cdot 0{,}3^2 \cdot e^{-2} \cdot \frac{2^3}{3!} =$$

$$= 0{,}034$$

$$P(X=1) = \sum_{n=1}^{\infty} C_n^1 p^1 (1-p)^{n-1} \cdot \frac{2^n}{n!} \cdot e^{-2} =$$

$$= 2 \cdot e^{-2} \cdot p \underbrace{\sum_{n=1}^{\infty} \frac{[2(1-p)]^{n-1}}{(n-1)!}}_{\text{Eksponendi astmerida}} = 2e^{-2} p e^{2(1-p)} = 2 \cdot e^{-2} \cdot 0{,}7 e^{2(1-0{,}7)} =$$

$$= 1{,}4 e^{-1{,}4} \approx \underline{0{,}35}$$

Olgu meil juhuslik suurus $X \sim B(N, 0.4)$, kus

Ül. 0.7 $N \sim Po(2)$. Leidke tõenäosus $P(X=1)$.

$X \sim B(N, 0.4)$

$N \sim P_o(2)$

$$P(X=1) = \sum_{n=1}^{\infty} C_n^1 \, p(1-p)^{n-1} \cdot \frac{2^n}{n!} \cdot e^{-2} =$$

$$= \sum_{n=1}^{\infty} \frac{\cancel{n!}}{1!(n-1)!} \cdot 0{,}4 \cdot 0{,}6^{n-1} \cdot \frac{2^n}{\cancel{n!}} \cdot e^{-2} = \qquad \frac{2}{2} = 2^1 \cdot 2^{-1}$$

$$= 0{,}4 \cdot e^{-2} \sum_{n=1}^{\infty} \frac{2^n \cdot 2^1 \cdot 2^{-1} \cdot 0{,}6^{n-1}}{(n-1)!} = \qquad e^z = \sum_{n=1}^{\infty} \frac{z^{n-1}}{(n-1)!}$$

$$= 0{,}4 \cdot 2 \cdot e^{-2} \sum_{n=1}^{\infty} \frac{2^{n-1} \cdot 0{,}6^{n-1}}{(n-1)!} = \left[e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \right] =$$

$$= 0.8 \cdot e^{-2} \sum_{n=1}^{\infty} \frac{(2 \cdot 0{,}6)^{n-1}}{(n-1)!} = 0.8 \cdot e^{-2} \cdot e^{1{,}2} =$$

$$= \underline{0.8 \cdot e^{-0{,}8} \approx 0{,}359}$$

Taylori rida

$$T_n(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \ldots +$$
$$+ \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$e^z = e^0 + \frac{e^0}{1!}(z-0) + \frac{e^0}{2!}(z-0)^2 + \frac{e^0}{3!}(z-0)^3 + \ldots =$$

$$= 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots = \underline{\sum_{n=1}^{\infty} \frac{z^{n-1}}{(n-1)!}} = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

Leidke 8 väljamakse põhjal saadud tn, et min kahjusumma
Ül. 0.8 on üle 1000 euro ja max kahjusumma samal ajal ei ületaks 15 000 €.

$$f(x) = \begin{cases} 0, \text{ kui } x \notin [0;40] \\ \frac{3}{32\,000}(40x - x^2), \text{ kui } x \in [0;40] \end{cases}$$

$$P(X_{(1)} > x_1, X_{(n)} \leq x_2) = [F(x_2) - F(x_1)]^n$$

8 väljamakse põhjal saadud tõenäosus, et minimaalne kahjusumma on üle 1000 ja maksimaalne kahjusumma samal ajal ei ületaks 15 000 eurot.

$$n = 8, \quad \overset{\text{min}}{x_1} = 1000, \quad \overset{\text{max}}{x_2} = 15\,000$$

$$P(X_{(1)} > 1000, X_{(8)} \leq 15\,000) = [F(15) - F(1)]^8 = *$$

// Suurima ja vähima vahe jaotus

$$F(x) = \int_0^x \frac{3}{32000}(40x - x^2)\,dx = \frac{3}{32000}\left(40 \cdot \frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^x =$$

$$= \frac{3}{32000}\left(20 \cdot x^2 - \frac{x^3}{3}\right) = \frac{60x^2}{32\,000} - \frac{x^3 \cdot \cancel{3}^{1}}{32000 \cdot \cancel{3}_{1}} =$$

$$= \frac{60x^2 - x^3}{32\,000}$$

$$* = \left(\frac{10\,066}{32\,000}\right)^8 \approx \underline{\underline{9{,}586 \cdot 10^{-5}}}$$

$$F(15) = \frac{60 \cdot 15^2 - 15^3}{32000} = \frac{10\,125}{32\,000}$$

$$F(1) = \frac{60 \cdot 1^2 - 1^3}{32\,000} = \frac{59}{32000}$$

Ül. 0.9

Leidke konstant c ning keskmised regressiooni-sõltuvused $y = g_1(x)$ ja $x = g_2(y)$.

$$f(x,y) = \begin{cases} cx^2, & \text{kui } 0 < x < y < 1 \\ 0, & \text{mujal} \end{cases}$$

$$c \int_0^y \int_x^1 x^2 \, dx \, dy = 1$$

$$c \int_0^y dx \int_x^1 x^2 \, dy = 1 \Rightarrow c \int_0^y dx \left[x^2 y \right]_x^1 = 1 \Rightarrow$$

$$\Rightarrow c \int_0^y dx \left(x^2 - x^3 \right) = 1 \Rightarrow c \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^y = 1$$

$$\Rightarrow \frac{y^3}{3} - \frac{y^4}{4} = \frac{1}{c} \Rightarrow \frac{4y^3 - 3y^4}{12} = \frac{1}{c} \Rightarrow$$

$$\frac{y^3(4-3y)}{12} = \frac{1}{c}; \quad \begin{matrix} 4 - 3y = 1 \\ y = 1 \end{matrix} \quad \frac{4-3}{12} = \frac{1}{c} \Rightarrow \underline{c = 12}$$

$$y^3 = 1 \Rightarrow$$

Tinglikud tihedused

$$f_2(x \mid y) = \frac{f(x,y)}{f_2(y)} \qquad f_1(y \mid x) = \frac{f(x,y)}{f_1(x)}$$

$$y = g_1(x) = E(Y \mid X) = \int_{-\infty}^{+\infty} y \, f_1(y \mid x) \, dy$$

$$x = g_2(y) = E(X \mid Y) = \int_{-\infty}^{+\infty} x \, f_2(x \mid y) \, dx$$

Marginaalid

$$f_2(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx$$

$$f(x,y) = 12x^2$$

$$f_1(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy$$

$$f_2(y) = \int_0^y 12x^2\,dx = 12 \cdot \frac{x^3}{3}\Big|_0^y = 4(y^3 - 0^3) =$$
$$= 4y^3$$

$$f_1(x) = \int_x^1 12x^2\,dy = 12x^2\,[y]_x^1 =$$
$$= 12x^2(1-x) = 12x^2 - 12x^3$$

$$f_2(x|y) = \frac{f(x,y)}{f_2(y)} = \frac{12x^2}{4y^3} = 3\frac{x^2}{y^2} = 3 \cdot \left(\frac{x}{y}\right)^2$$

$$f_1(y|x) = \frac{f(x,y)}{f_1(x)} = \frac{12x^2}{12x^2(1-x)} = \frac{1}{1-x}$$

$$y = g_1(x) = \int_x^1 y \cdot \frac{1}{1-x}\,dy = \frac{1}{1-x}\int_x^1 y\,dy = \frac{1}{1-x}\left[\frac{y^2}{2}\right]_x^1 =$$

$$= \frac{1}{2(1-x)}(1-x^2) = \frac{(1-x^2)}{(1-x)} = \frac{\cancel{(1-x)}(1+x)}{2\cancel{(1-x)}} = \underline{\underline{\frac{1+x}{2}}}$$

$$x = g_2(y) = \int_0^y x \cdot 3 \cdot \frac{x^2}{y^2}\,dx = \frac{3}{y^2}\int_0^y x^3\,dx = \frac{3}{y^3}\left[\frac{x^4}{4}\right]_0^y =$$

$$= \frac{3}{4y^3}(y^4 - 0^4) = \frac{3}{4} \cdot \frac{y^4}{y^3} = \underline{\frac{3}{4}y}$$