KODUTÖÖ 3

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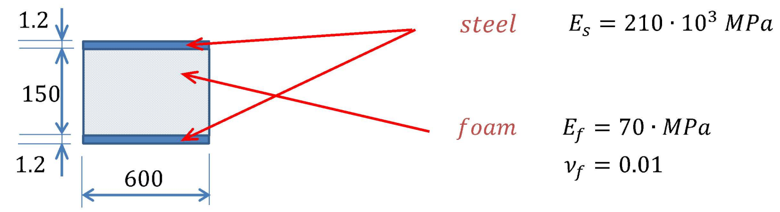
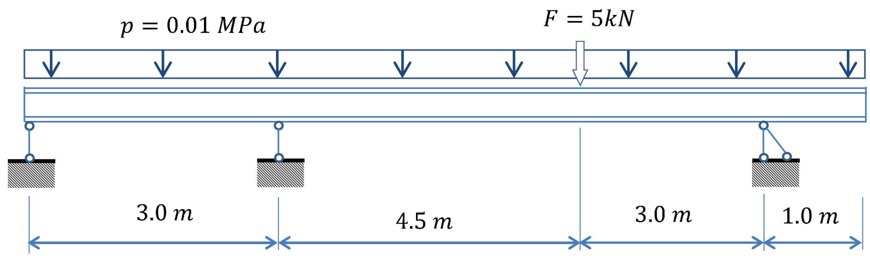
Euleri tala*





Maatriks EL sisaldab endas süsteemi kohta käivat infot.

$EL := READPRN("kohutav\_idee.txt")$

$dim := \max\left(EL^{\langle 2 \rangle}, EL^{\langle 3 \rangle}\right) = 24$

$L := 11500$ $\quad E := 210 \cdot 10^3$ $\quad E_f := 70$ $\quad h_c := 150$

$\nu_f := 0.01$ $\quad b := 600$

$h_e := \dfrac{L}{23}$ $\quad p := 0.01 \cdot 10^3$ $\quad t_p := 1.2$ $\quad A_{shear} := h_c \cdot b$

$q := p \cdot b$

$F := 5$ $\quad q = 6 \times 10^3$ $\quad G_f := \dfrac{E_f}{2 \cdot (1 + \nu_f)}$

Impulsimomendi ja staatilise momendi väärtused

$$I := 2 \cdot \left[\frac{t_p^{\,3} \cdot b}{12} + 2 \cdot t_p \cdot b \cdot \left(\frac{h_c}{2} + \frac{t_p}{2}\right)^2 + \frac{E_f}{E} \cdot \frac{h_c^{\,3} \cdot b}{12}\right] = 1.657 \times 10^7$$

$b_t := 600$ $\quad S_t := 1.2 \cdot 600 \cdot (75 + 0.6) + 75 \cdot 600 \cdot 37.5 = 1.742 \times 10^6$

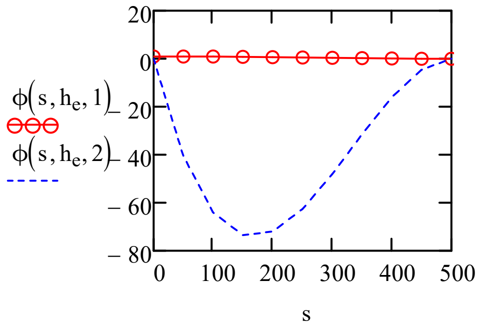
Järgnevaks on ära defineeritud kujufunktsioonid: ϕ , selle esimene tuletis dϕ ja teine tuletis ddϕ.
Lisaks on ka ette antud kaalufunktsioon W.

$$\phi(x,h_e,i) = \left| \begin{array}{l} R \leftarrow 2\cdot\dfrac{x^3}{h_e^{\,3}} - 3\cdot\dfrac{x^2}{h_e^{\,2}} + 1 \quad \text{if } i = 1 \\ R \leftarrow 2\cdot\dfrac{x^2}{h_e} - \dfrac{x^3}{h_e^{\,2}} - x \quad \text{if } i = 2 \\ R \leftarrow 3\cdot\dfrac{x^2}{h_e^{\,2}} - 2\cdot\dfrac{x^3}{h_e^{\,3}} \quad \text{if } i = 3 \\ R \leftarrow \dfrac{x^2}{h_e} - \dfrac{x^3}{h_e^{\,2}} \quad \text{if } i = 4 \\ \text{return } R \end{array} \right.$$

$$dd\phi(x,h_e,i) = \left| \begin{array}{l} R \leftarrow \dfrac{12\cdot x}{h_e^{\,3}} - \dfrac{6}{h_e^{\,2}} \quad \text{if } i = 1 \\ R \leftarrow \dfrac{4}{h_e} - \dfrac{6\cdot x}{h_e^{\,2}} \quad \text{if } i = 2 \\ R \leftarrow \dfrac{6}{h_e^{\,2}} - \dfrac{12\cdot x}{h_e^{\,3}} \quad \text{if } i = 3 \\ R \leftarrow \dfrac{2}{h_e} - \dfrac{6\cdot x}{h_e^{\,2}} \quad \text{if } i = 4 \\ \text{return } R \end{array} \right.$$

$$ddd\phi(x,h_e,i) = \left| \begin{array}{l} R \leftarrow \dfrac{12}{h_e^{\,3}} \quad \text{if } i = 1 \\ R \leftarrow -\dfrac{6}{h_e^{\,2}} \quad \text{if } i = 2 \\ R \leftarrow -\dfrac{12}{h_e^{\,3}} \quad \text{if } i = 3 \\ R \leftarrow -\dfrac{6}{h_e^{\,2}} \quad \text{if } i = 4 \\ \text{return } R \end{array} \right.$$

$$s := 0, 50 .. h_e$$



Kaalufunktsioon W on teise astme polünoom.

$$W := \begin{pmatrix} \dfrac{-1}{\sqrt{3}} & 1 \\ \dfrac{1}{\sqrt{3}} & 1 \end{pmatrix}$$

Kood, mis arvutab igast elemendist sõltuvalt välja jäikusmaatriksi. Selle väärtuste kuvamiseks on vajalikud eelnevalt defineeritud kujufunktsioon ddϕ, kaalufunktsioon W, inertsimoment I ja elastusmoodul E.

```
K_el(E, I, h_e) := | for i ∈ 1..4
                   |   for j ∈ 1..4
                   |     R_{i,j} ← 0
                   | for inte ∈ 1..rows(W)
                   |   | x_a ← 0
                   |   | x_b ← h_e
                   |   | x ← 1/2 · [x_a + x_b + W_{inte,1} · (x_b − x_a)]
                   |   | for i ∈ 1..4
                   |   |   for j ∈ 1..4
                   |   |     R_{i,j} ← R_{i,j} + E · I · W_{inte,2} · ddϕ(x, h_e, i) · ddϕ(x, h_e, j) · (x_b − x_a)/2
                   | return R
```

Elemendi jäikusmaatriksi väärtused:

$$K_{el}(E,I,h_e) = \begin{pmatrix} 3.341\times10^{5} & -8.353\times10^{7} & -3.341\times10^{5} & -8.353\times10^{7} \\ -8.353\times10^{7} & 2.784\times10^{10} & 8.353\times10^{7} & 1.392\times10^{10} \\ -3.341\times10^{5} & 8.353\times10^{7} & 3.341\times10^{5} & 8.353\times10^{7} \\ -8.353\times10^{7} & 1.392\times10^{10} & 8.353\times10^{7} & 2.784\times10^{10} \end{pmatrix}$$

Kood, mis arvutab igast elemendist sõltuvalt välja jõuvektori. Deformatsioonist tuleneva kuju kirjeldab peamiselt kaalufunktsioon W ja kujufunktsioon ϕ.

```
F_el(h_e) := | for i ∈ 1..4
             |   R_i ← 0
             | for inte ∈ 1..rows(W)
             |   | x_a ← 0
             |   | x_b ← h_e
             |   | x ← 1/2 · [x_a + x_b + W_{inte,1} · (x_b − x_a)]
             |   | for i ∈ 1..4
             |   |   R_i ← R_i + W_{inte,2} · ϕ(x, h_e, i) · q · (x_b − x_a)/2
             | return R
```

Jõuvektori väärtused:

$$F_{el}(h_e) = \begin{pmatrix} 1.5\times10^{6} \\ -1.25\times10^{8} \\ 1.5\times10^{6} \\ 1.25\times10^{8} \end{pmatrix}$$

Osa globaalse jäikusmaatriksi K_{gl} (48 x 48) väärtustest.

$K_{gl} =$

	1	2	3	4	5
1	$3.341 \cdot 10^{5}$	$-8.353 \cdot 10^{7}$	$-3.341 \cdot 10^{5}$	$-8.353 \cdot 10^{7}$	0
2	$-8.353 \cdot 10^{7}$	$2.784 \cdot 10^{10}$	$8.353 \cdot 10^{7}$	$1.392 \cdot 10^{10}$	0
3	$-3.341 \cdot 10^{5}$	$8.353 \cdot 10^{7}$	$6.682 \cdot 10^{5}$	0	$-3.341 \cdot 10^{5}$
4	$-8.353 \cdot 10^{7}$	$1.392 \cdot 10^{10}$	0	$5.568 \cdot 10^{10}$	$8.353 \cdot 10^{7}$
5	0	0	$-3.341 \cdot 10^{5}$	$8.353 \cdot 10^{7}$	$6.682 \cdot 10^{5}$
6	0	0	$-8.353 \cdot 10^{7}$	$1.392 \cdot 10^{10}$	0
7	0	0	0	0	$-3.341 \cdot 10^{5}$
8	0	0	0	0	$-8.353 \cdot 10^{7}$
9	0	0	0	0	...

$$F_{gl} = F1_{gl} + F2_{gl}$$

Globaalse jõuvektori (48 x 1) väärtused eraldi välja toodud.

$F1_{gl} =$

	1
1	$1.5 \cdot 10^{6}$
2	$-1.25 \cdot 10^{8}$
3	$3 \cdot 10^{6}$
4	$7.451 \cdot 10^{-8}$
5	$3 \cdot 10^{6}$
6	$7.451 \cdot 10^{-8}$
7	$3 \cdot 10^{6}$
8	$7.451 \cdot 10^{-8}$
9	$3 \cdot 10^{6}$
10	$7.451 \cdot 10^{-8}$
11	$3 \cdot 10^{6}$
12	$7.451 \cdot 10^{-8}$
13	$3 \cdot 10^{6}$
14	$7.451 \cdot 10^{-8}$
15	$3 \cdot 10^{6}$
16	$7.451 \cdot 10^{-8}$
17	$3 \cdot 10^{6}$
18	...

$F2_{gl} =$

	1
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	$-5 \cdot 10^{3}$
17	0
18	...

$F_{gl} =$

	1
1	$1.5 \cdot 10^{6}$
2	$-1.25 \cdot 10^{8}$
3	$3 \cdot 10^{6}$
4	$7.451 \cdot 10^{-8}$
5	$3 \cdot 10^{6}$
6	$7.451 \cdot 10^{-8}$
7	$3 \cdot 10^{6}$
8	$7.451 \cdot 10^{-8}$
9	$3 \cdot 10^{6}$
10	$7.451 \cdot 10^{-8}$
11	$3 \cdot 10^{6}$
12	$7.451 \cdot 10^{-8}$
13	$3 \cdot 10^{6}$
14	$7.451 \cdot 10^{-8}$
15	$3 \cdot 10^{6}$
16	$-5 \cdot 10^{3}$
17	$3 \cdot 10^{6}$
18	...

Kood siirete vektori koostamise jaoks.

$$u = U^T \cdot \left(U \cdot K_{gl} \cdot U^T\right)^{-1} \cdot U \cdot F_{gl}$$

Tala läbipaine w tuleneb järgnevast analüütilisest lahendist, mis võtab arvesse koormuse q, tala pikkuse L, ristlõike A_{sheer}, nihkeelastusmooduli G_f ning sõlmede kaugused x.

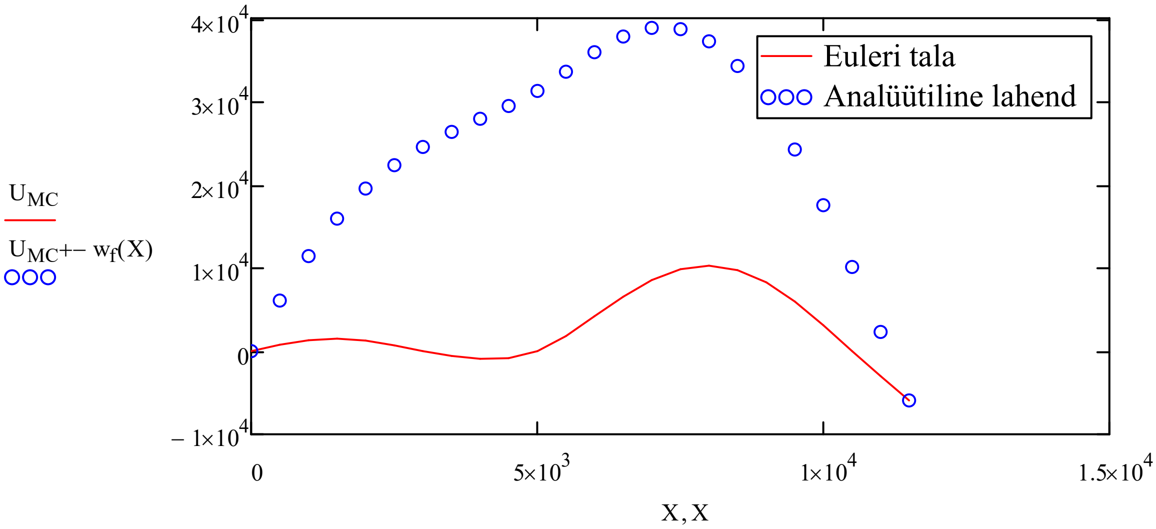
$$w_f(x) = \frac{q \cdot L^2}{G_f \cdot A_{shear} \cdot 2} \cdot \left(\frac{x^2}{L^2} - \frac{x}{L}\right)$$

Graafiku koostamiseks kantakse y-teljele talas toimuvad siirded ning x-teljele tala sõlmede kaugused, kus siirded täpselt toimuvad.

Kood U_{MC} võtab siirete maatriksist u välja paaritud komponendid, ehk need väärtused mis kirjeldavad igas sõlmes toimuvad siiret.

Kood X kirjutab välja iga sõlme kauguse, kui nullpunkt asub tala vasakus otsas.

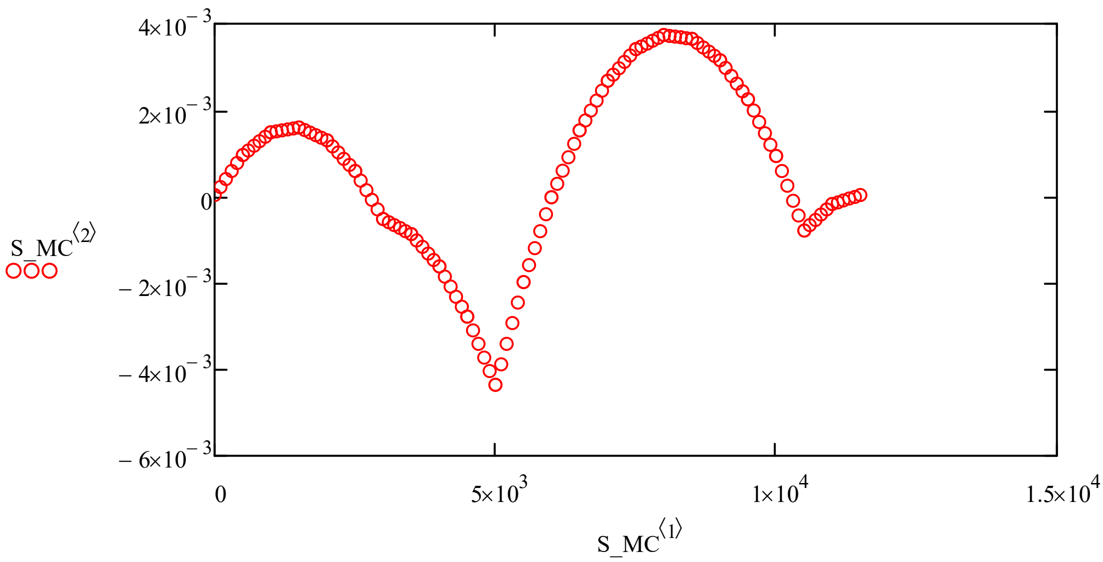
```
U_MC = | N_solm ← max(EL⟨2⟩, EL⟨3⟩)
       | for i ∈ 1 .. N_solm
       |   R_i ← u_(2·i−1)
       | return R

X = | N_solm ← max(EL⟨2⟩, EL⟨3⟩)
    | for i ∈ 1 .. N_solm
    |   R_i ← h_e · (i − 1)
    | return R
```



Kood, mis liigub mööda tala ning arvutab välja selles esinevad sisepinged.

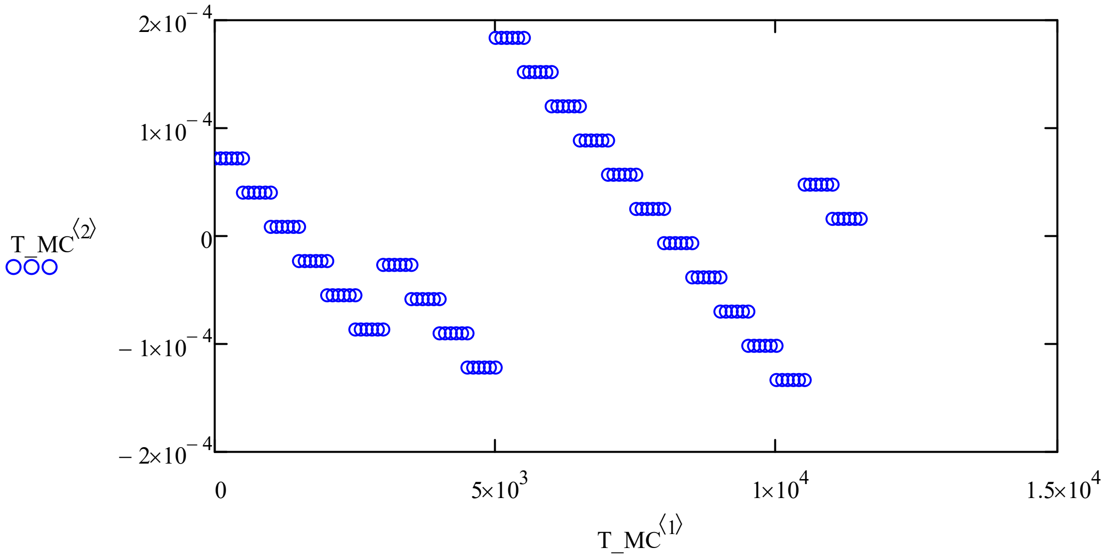
```
S_MC = | h ← 150
       | N_el ← max(EL⟨1⟩)
       | for i_el ∈ 1 .. N_el
       |   | for i ∈ 1 .. 4
       |   |   ue_i ← u_((i_el−1)·2+i)
       |   | for i ∈ 1 .. 6
       |   |   | R_((i_el−1)·6+i, 1) ← (i_el − 1) · h_e + ((i − 1)/5) · h_e
       |   |   | SUM ← 0
       |   |   | for j ∈ 1 .. 4
       |   |   |   | x ← ((i − 1)/5) · h_e
       |   |   |   | SUM ← SUM + ue_j · ddϕ(x, h_e, j)
       |   |   | R_((i_el−1)·6+i, 2) ← −E · (SUM/I) · (h/2)
       | return R
```

PINGEID POLE VAJA



Kood, mis liigub mööda tala ning arvutab välja selles esinevad väändepinged.

$$
T\_MC := \left|
\begin{array}{l}
N\_el \leftarrow \max\left(EL^{\langle 1 \rangle}\right) \\
\text{for } i\_el \in 1..N\_el \\
\quad \left|
\begin{array}{l}
\text{for } i \in 1..4 \\
\quad ue_i \leftarrow u_{(i\_el-1)\cdot 2+i} \\
\text{for } i \in 1..6 \\
\quad \left|
\begin{array}{l}
R_{(i\_el-1)\cdot 6+i,\,1} \leftarrow (i\_el - 1)\cdot h_e + \dfrac{(i-1)}{5}\cdot h_e \\
SUM \leftarrow 0 \\
\text{for } j \in 1..4 \\
\quad \left|
\begin{array}{l}
x \leftarrow \dfrac{(i-1)}{5}\cdot h_e \\
SUM \leftarrow SUM + ue_j \cdot ddd\phi\left(x, h_e, j\right)
\end{array}
\right. \\
R_{(i\_el-1)\cdot 6+i,\,2} \leftarrow -E\cdot \dfrac{SUM}{I}\cdot \dfrac{S_t}{b_t}
\end{array}
\right.
\end{array}
\right. \\
\text{return } R
\end{array}
\right.
$$



Timoshenko tala

Andmed:

$E = 2.1 \times 10^5$

$I = 1.657 \times 10^7$

$L = 1.15 \times 10^4$

$q = 6 \times 10^3$

$h_e = 500$

$A_{red} = h_c \cdot b = 9 \times 10^4$

$E_v = 70$

$\nu_v = 0.01$

$G_v = \dfrac{E_v}{2 \cdot (1 + \nu_v)}$

Mis muudab Timošenko tala erinevaks Euleri omast on see, et Timošenko võtab arvesse lisaks eelnevatele tasakaaluvõrranditele veel kinemaatilised seosed, mis seovad sisejõud deformatsiooni kirjeldatavate suurustega. Selleks on oluline tuua sisse funktsioon ψ, mis kirjeldab tala ristlõike pinna pöördumist y-telje suhtes ning on tingitud ainult paindedeformatsioonist. Samas pole aga mainitud deformatsioon endam võrdeline läbipainde teise tuletisega.

$$\psi(i, h_e, x) = \left| \begin{array}{l} R \leftarrow \left(\dfrac{2 \cdot x^2}{h_e^2} - \dfrac{3 \cdot x}{h_e} + 1 \right) \text{ if } i = 1 \\ R \leftarrow \left(\dfrac{4 \cdot x}{h_e} - \dfrac{4 \cdot x^2}{h_e^2} \right) \text{ if } i = 2 \\ R \leftarrow -\left(\dfrac{x}{h_e} - \dfrac{2 \cdot x^2}{h_e^2} \right) \text{ if } i = 3 \\ \text{return } R \end{array} \right.$$

$$d\psi(i, h_e, x) = \left| \begin{array}{l} R \leftarrow \left(\dfrac{4 \cdot x}{h_e^2} - \dfrac{3}{h_e} \right) \text{ if } i = 1 \\ R \leftarrow \left(\dfrac{4}{h_e} - \dfrac{8 \cdot x}{h_e^2} \right) \text{ if } i = 2 \\ R \leftarrow -\left(\dfrac{1}{h_e} - \dfrac{4 \cdot x}{h_e^2} \right) \text{ if } i = 3 \\ \text{return } R \end{array} \right.$$

$$dd\psi(i, h_e, x) = \left| \begin{array}{l} R \leftarrow \dfrac{4}{h_e^2} \text{ if } i = 1 \\ R \leftarrow \dfrac{-8}{h_e^2} \text{ if } i = 2 \\ R \leftarrow \dfrac{4}{h_e^2} \text{ if } i = 3 \\ \text{return } R \end{array} \right.$$

Elementide jäiksumaatriksi väärtused:

$$K\_tim_{el}\left(E, G_v, I, A_{red}, h_e\right) = \begin{pmatrix} 6.238\times10^{3} & -1.559\times10^{6} & -6.238\times10^{3} & -1.559\times10^{6} \\ -1.559\times10^{6} & 7.35\times10^{9} & 1.559\times10^{6} & -6.571\times10^{9} \\ -6.238\times10^{3} & 1.559\times10^{6} & 6.238\times10^{3} & 1.559\times10^{6} \\ -1.559\times10^{6} & -6.571\times10^{9} & 1.559\times10^{6} & 7.35\times10^{9} \end{pmatrix}$$

Tala globaalne jäikusmaatriks ja jõuvektor kuvatakse praktiliselt samamoodi nagu Euleri tala puhulgi. Ainuksed erinevused seisnevad selles, et elementide jäikusmaatriks ja jõuvektor on arvutatud Timošenko tala valemitega.

```
K_tim_gl :=  | dim ← 2 · max(EL⟨2⟩, EL⟨3⟩)
             | for  i ∈ 1 .. dim
             |   for  j ∈ 1 .. dim
             |     R_{i,j} ← 0
             | for  el ∈ 1 .. rows(EL)
             |   | Kel ← K_tim_el(E, G_v, I, A_red, h_e)
             |   | for  i ∈ 1 .. 2
             |   |   for  j ∈ 1 .. 2
             |   |     | 1
             |   |     | for  i_v ∈ 1 .. 2
             |   |     |   for  j_v ∈ 1 .. 2
             |   |     |     | Ξ ← (EL_{el,i+1} − 1)
             |   |     |     | ϒ ← (EL_{el,j+1} − 1)
             |   |     |     | R_{Ξ·2+i_v, ϒ·2+j_v} ← R_{Ξ·2+i_v, ϒ·2+j_v} + Kel_{(i−1)·2+i_v, (j−1)·2+j_v}
             | return  R
```

Elementide kood on enamjaolt sarnane Euleri omaga, kuid nüüd on sisse toodud funktsioon ψ ning väärtused kuvatakse R1, R2, R3 ja R4 väärtuste omapäraste kombinatsioonide kujul.

$$
F\_tim_{el}(h_e) := \left|\begin{array}{l}
\text{for } i \in 1..3 \\
\quad R_i \leftarrow 0 \\
\text{for } inte \in 1..\text{rows}(W) \\
\quad \left|\begin{array}{l}
x_a \leftarrow 0 \\
x_b \leftarrow h_e \\
x \leftarrow \frac{1}{2}\cdot\left[x_a + x_b + W_{inte,1}\cdot(x_b - x_a)\right] \\
\text{for } i \in 1..3 \\
\quad R_i \leftarrow R_i + W_{inte,2}\cdot\psi(i,h_e,x)\cdot q\cdot\frac{(x_b - x_a)}{2}
\end{array}\right. \\
R1 \leftarrow \begin{pmatrix} R_1 + \frac{1}{2}\cdot R_2 \\ -\frac{1}{8}\cdot R_2\cdot h_e \\ R_3 + \frac{1}{2}\cdot R_2 \\ \frac{1}{8}\cdot R_2\cdot h_e \end{pmatrix} \\
\text{return } R1
\end{array}\right.
$$

Elementide jõuvektori väärtused:

$$
F\_tim_{el}(h_e) = \begin{pmatrix} 1.5\times 10^{6} \\ -1.25\times 10^{8} \\ 1.5\times 10^{6} \\ 1.25\times 10^{8} \end{pmatrix}
$$

Globaalne jõuvektor samamoodi nagu Euleri talaga:

$$
F1\_tim_{gl} := \left|\begin{array}{l}
dim \leftarrow 2\cdot\max\left(EL^{\langle 2\rangle}, EL^{\langle 3\rangle}\right) \\
\text{for } i \in 1..dim \\
\quad R_i \leftarrow 0 \\
\text{for } el \in 1..\text{rows}(EL) \\
\quad \left|\begin{array}{l}
Fel \leftarrow F\_tim_{el}(h_e) \\
\text{for } i \in 1..2 \\
\quad \text{for } i\_v \in 1..2 \\
\quad\quad \left|\begin{array}{l}
\Xi \leftarrow EL_{el,i+1} - 1 \\
R_{\Xi\cdot 2+i\_v} \leftarrow R_{\Xi\cdot 2+i\_v} + Fel_{(i-1)\cdot 2+i\_v}
\end{array}\right.
\end{array}\right. \\
\text{return } R
\end{array}\right.
$$

$$
F2\_tim_{gl} := \left|\begin{array}{l}
dim \leftarrow 2\cdot\max\left(EL^{\langle 2\rangle}, EL^{\langle 3\rangle}\right) \\
\text{for } i \in 1..dim \\
\quad R_i \leftarrow 0 \\
R_{16} \leftarrow -5\cdot 10^{3} \\
\text{return } R
\end{array}\right.
$$

Ka Timošenko tala puhul arvutatakse siire ja läbipaine samade valemitega.

$$u\_tim = U^T \cdot \left(U \cdot K\_tim_{gl} \cdot U^T\right)^{-1} \cdot U \cdot F\_tim_{gl}$$

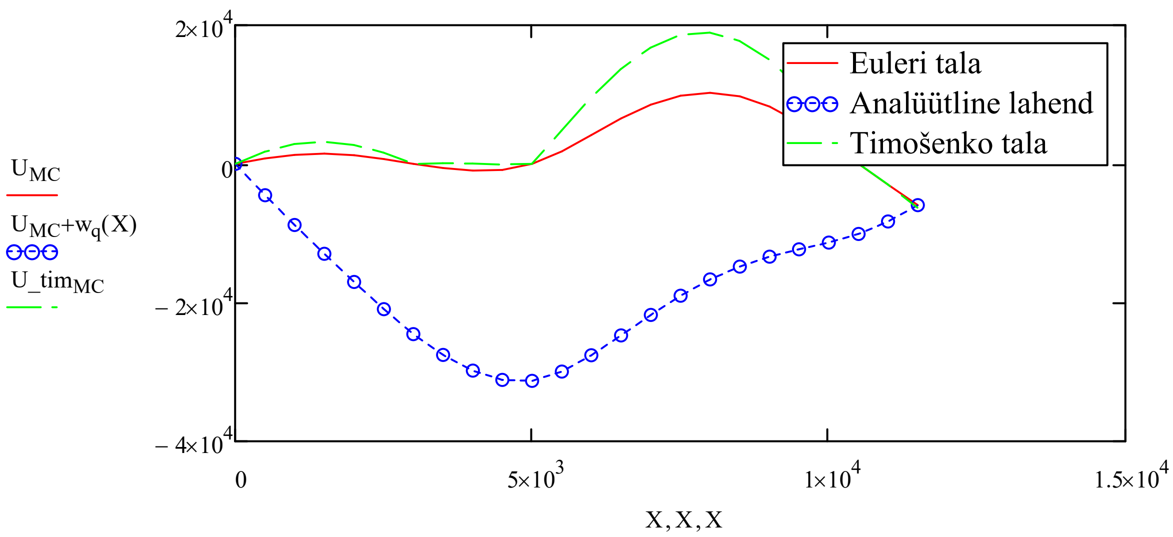
$$U\_tim_{MC} = \begin{array}{|l} N\_solm \leftarrow \max\left(EL^{\langle 2\rangle}, EL^{\langle 3\rangle}\right) \\ \text{for } i \in 1..N\_solm \\ \quad R_i \leftarrow u\_tim_{2\cdot i-1} \\ \text{return } R \end{array}$$

$$X = \begin{array}{|l} N\_solm \leftarrow \max\left(EL^{\langle 2\rangle}, EL^{\langle 3\rangle}\right) \\ \text{for } i \in 1..N\_solm \\ \quad R_i \leftarrow h_e \cdot (i-1) \\ \text{return } R \end{array}$$

$$w_q(x) = \frac{q \cdot L^2}{G_v \cdot A_{red} \cdot 2} \cdot \left(\frac{x^2}{L^2} - \frac{x}{L}\right)$$

u_tim =

	1
1	0
2	-1.766
3	$1.788 \cdot 10^3$
4	-1.532
5	$2.896 \cdot 10^3$
6	-0.975
7	$3.219 \cdot 10^3$
8	-0.309
9	$2.754 \cdot 10^3$
10	0.25
11	$1.609 \cdot 10^3$
12	0.486
13	0
14	0.184
15	96.144
16	-0.434
17	...



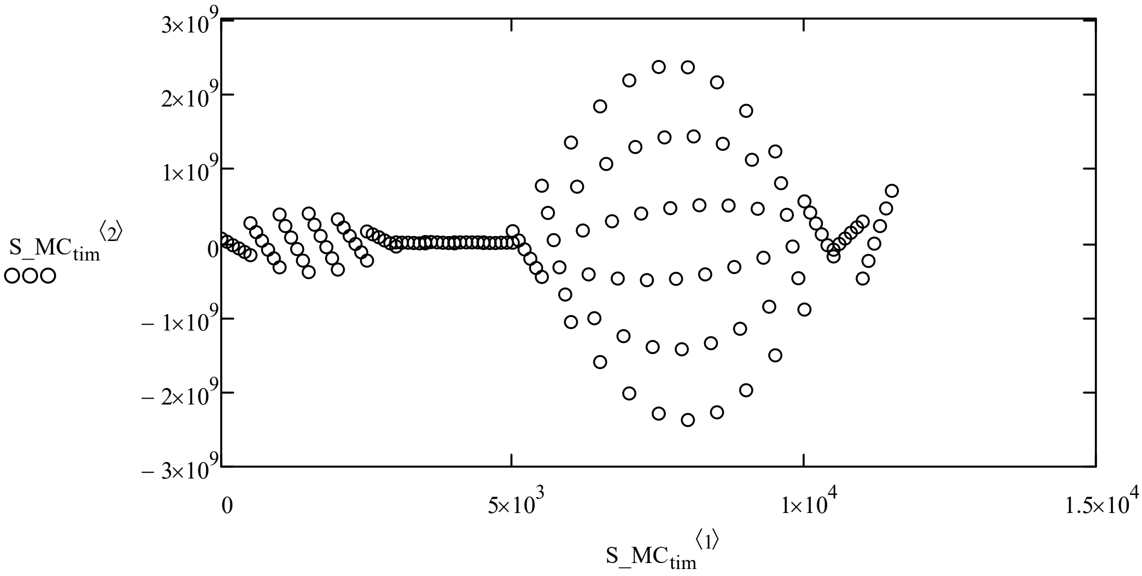
Timošenko tala jaoks arvutatud sisepinged. Nagu väärtustest ja graafikust näha on siis mingil põhjusel tulevad Timošenko tala väärtused kõvasti suuremad Euleri omadest.

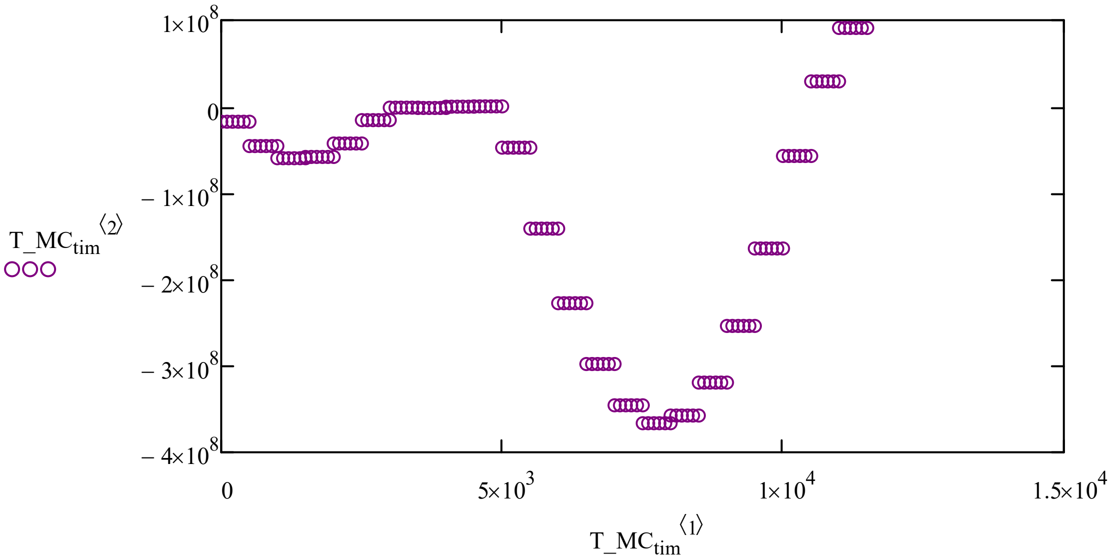
```
S_MC_tim := | h ← 150
            | N_el ← max(EL^⟨1⟩)
            | for i_el ∈ 1 .. N_el
            |   | for i ∈ 1 .. 4
            |   |   ue_i ← u_tim_((i_el−1)·2+i)
            |   | for i ∈ 1 .. 6
            |   |   | R_((i_el−1)·6+i, 1) ← (i_el − 1) · h_e + ((i − 1)/5) · h_e
            |   |   | SUM ← 0
            |   |   | for j ∈ 1 .. 4
            |   |   |   | x ← ((i − 1)/5) · h_e
            |   |   |   | SUM ← SUM + ue_j · dψ(j , h_e, x)
            |   |   | R_((i_el−1)·6+i, 2) ← −E · SUM · (h/2)
            | return R
```

$S\_MC_{tim} =$

	1	2
1	0	$5.653\cdot10^{7}$
2	100	$1.14\cdot10^{7}$
3	200	$-3.374\cdot10^{7}$
4	300	$-7.888\cdot10^{7}$
5	400	$-1.24\cdot10^{8}$
6	500	$-1.692\cdot10^{8}$
7	500	$2.604\cdot10^{8}$
8	600	$1.422\cdot10^{8}$
9	700	$2.413\cdot10^{7}$
10	800	$-9.399\cdot10^{7}$
11	900	$-2.121\cdot10^{8}$
12	$1\cdot10^{3}$	$-3.302\cdot10^{8}$
13	$1\cdot10^{3}$	$3.752\cdot10^{8}$
14	$1.1\cdot10^{3}$	$2.211\cdot10^{8}$
15	$1.2\cdot10^{3}$	$6.693\cdot10^{7}$
16	$1.3\cdot10^{3}$	...

Timošenko tala sise- ja väändepingete graafikud:





Jagasin tala 23-ks osaks

$$3 + 4.5 + 3 + 1 = 11.5$$

$$\frac{11.5}{23} = 0.5$$

$$0.5 \cdot 23 = 11.5$$

Tala elementide maatriksi EL väärtused. Esimene tulp näitab tala numbrit, teine ja kolmas sõlmede numbreid ning viimane tulp iga elemendi pikkust.

EL =

	1	2	3	4
1	1	1	2	0.5
2	2	2	3	0.5
3	3	3	4	0.5
4	4	4	5	0.5
5	5	5	6	0.5
6	6	6	7	0.5
7	7	7	8	0.5
8	8	8	9	0.5
9	9	9	10	0.5
10	10	10	11	0.5
11	11	11	12	0.5
12	12	12	13	0.5
13	13	13	14	0.5
14	14	14	15	0.5
15	15	15	16	0.5
16	16	16	17	0.5
17	17	17	18	0.5
18	18	18	19	0.5
19	19	19	20	0.5
20	20	20	21	0.5
21	21	21	22	0.5
22	22	22	23	0.5
23	23	23	24	0.5

BOUND(n) on kood, mis arvutab ääretingimuste maatriksi.

```
BOUND(n) := | dim ← 2 · max(EL⟨2⟩, EL⟨3⟩)
            | dim_red ← rows(n)
            | for j ∈ 1 .. dim
            |   for i ∈ 1 .. dim − dim_red
            |     R_{i,j} ← 0
            | ind ← 1
            | s ← 0
            | for k ∈ 1 .. dim
            |   | if k ≠ n_ind
            |   |   | 1
            |   |   | R_{k−s,k} ← 1
            |   | if k = n_ind
            |   |   | ind ← ind + 1  if ind ≠ rows(n)
            |   |   | s ← s + 1
            |   | S_k ← s
            | return R
```

Vaadades joonist ja võttes arvesse asjaolu, et siiret kirjeldavad paaritud komponendid, moodustub meil järgmine n maatriks:

$$n = \begin{pmatrix} 1 \\ 13 \\ 21 \\ 43 \end{pmatrix}$$

$U = \text{BOUND}(n) =$

	1	2	3	4	5	6	7	8	9	10
1	0	1	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	...

Kood, mis arvutab globaalse jäikusmaatriksi. Antud globaalse maatriksiga luuakse süsteem, kust lõppude lõpuks saab deformatsioone lugeda välja sellisel kujul, kus paaritud komponendid kirjeldavad siiret ja paariskomponendid pööret.

```
K_gl = | dim ← 2 · max(EL⟨2⟩, EL⟨3⟩)
       | for i ∈ 1 .. dim
       |   for j ∈ 1 .. dim
       |     R_{i,j} ← 0
       | for el ∈ 1 .. rows(EL)
       |   | Kel ← K_el(E, I, h_e)
       |   | for i ∈ 1 .. 2
       |   |   for j ∈ 1 .. 2
       |   |     | 1
       |   |     | for i_v ∈ 1 .. 2
       |   |     |   for j_v ∈ 1 .. 2
       |   |     |     | Ξ ← (EL_{el,i+1} − 1)
       |   |     |     | ϒ ← (EL_{el,j+1} − 1)
       |   |     |     | R_{Ξ·2+i_v, ϒ·2+j_v} ← R_{Ξ·2+i_v, ϒ·2+j_v} + Kel_{(i−1)·2+i_v, (j−1)·2+j_v}
       | return R
```

Kood, mis arvutab globaalse jõuvektori esimese osa, ehk see kood arvutab välja lauskoormuse p. Lauskoormus definitsiooni poolest koosneb kahest osast: äärtesse mõjuvatest lõikejõududest ja paindemomentidest.

```
F1_gl = | dim ← 2 · max(EL⟨2⟩, EL⟨3⟩)
        | for i ∈ 1 .. dim
        |   R_i ← 0
        | for el ∈ 1 .. rows(EL)
        |   | Fel ← F_el(h_e)
        |   | for i ∈ 1 .. 2
        |   |   | 1
        |   |   | for i_v ∈ 1 .. 2
        |   |   |   | Ξ ← EL_{el,i+1} − 1
        |   |   |   | R_{Ξ·2+i_v} ← R_{Ξ·2+i_v} + Fel_{(i−1)·2+i_v}
        | return R
```

Kood, mis arvutab globaalse jõuvektori teise osa ehk lisab sõlme rakendatud jõu.

```
F2_gl = | dim ← 2 · max(EL⟨2⟩, EL⟨3⟩)
        | for i ∈ 1 .. dim
        |   R_i ← 0
        | R_16 ← −5 · 10^3
        | return R
```

Siirete vektori väärtused:

$u =$

	1
1	0
2	-1.647
3	778.11
4	-1.384
5	$1.32\cdot10^{3}$
6	-0.739
7	$1.489\cdot10^{3}$
8	0.073
9	$1.255\cdot10^{3}$
10	0.836
11	696.827
12	1.335
13	0
14	1.354
15	-593.403
16	0.989
17	-941.204
18	0.336
19	-845.341
20	-0.821
21	0
22	-2.698
23	$1.817\cdot10^{3}$
24	...

Tala läbipaine w väärtused:

$w_f(X) =$

	1
1	0
2	$-5.29\cdot10^{3}$
3	$-1.01\cdot10^{4}$
4	$-1.443\cdot10^{4}$
5	$-1.828\cdot10^{4}$
6	$-2.164\cdot10^{4}$
7	$-2.453\cdot10^{4}$
8	$-2.693\cdot10^{4}$
9	$-2.886\cdot10^{4}$
10	$-3.03\cdot10^{4}$
11	$-3.126\cdot10^{4}$
12	$-3.174\cdot10^{4}$
13	$-3.174\cdot10^{4}$
14	$-3.126\cdot10^{4}$
15	$-3.03\cdot10^{4}$
16	$-2.886\cdot10^{4}$
17	$-2.693\cdot10^{4}$
18	$-2.453\cdot10^{4}$
19	$-2.164\cdot10^{4}$
20	$-1.828\cdot10^{4}$
21	$-1.443\cdot10^{4}$
22	$-1.01\cdot10^{4}$
23	$-5.29\cdot10^{3}$
24	0

S_MC väärtused eraldi välja toodud. Esimene tulp näitab sõlme kaugust, teine aga sisepinge.

S_MC =

	1	2
1	0	$3.413 \cdot 10^{-5}$
2	100	$2.204 \cdot 10^{-4}$
3	200	$4.066 \cdot 10^{-4}$
4	300	$5.929 \cdot 10^{-4}$
5	400	$7.792 \cdot 10^{-4}$
6	500	$9.654 \cdot 10^{-4}$
7	500	$9.654 \cdot 10^{-4}$
8	600	$1.07 \cdot 10^{-3}$
9	700	$1.174 \cdot 10^{-3}$
10	800	$1.278 \cdot 10^{-3}$
11	900	$1.383 \cdot 10^{-3}$
12	$1 \cdot 10^{3}$	$1.487 \cdot 10^{-3}$
13	$1 \cdot 10^{3}$	$1.487 \cdot 10^{-3}$
14	$1.1 \cdot 10^{3}$	$1.51 \cdot 10^{-3}$
15	$1.2 \cdot 10^{3}$	$1.532 \cdot 10^{-3}$
16	$1.3 \cdot 10^{3}$	$1.554 \cdot 10^{-3}$
17	$1.4 \cdot 10^{3}$	$1.577 \cdot 10^{-3}$
18	$1.5 \cdot 10^{3}$	$1.599 \cdot 10^{-3}$
19	$1.5 \cdot 10^{3}$	$1.599 \cdot 10^{-3}$
20	$1.6 \cdot 10^{3}$	$1.54 \cdot 10^{-3}$
21	$1.7 \cdot 10^{3}$	$1.48 \cdot 10^{-3}$
22	$1.8 \cdot 10^{3}$	$1.421 \cdot 10^{-3}$
23	$1.9 \cdot 10^{3}$	$1.361 \cdot 10^{-3}$
24	$2 \cdot 10^{3}$	$1.302 \cdot 10^{-3}$
25	$2 \cdot 10^{3}$	$1.302 \cdot 10^{-3}$
26	$2.1 \cdot 10^{3}$	$1.16 \cdot 10^{-3}$
27	$2.2 \cdot 10^{3}$	$1.019 \cdot 10^{-3}$
28	$2.3 \cdot 10^{3}$	$8.775 \cdot 10^{-4}$
29	$2.4 \cdot 10^{3}$	$7.361 \cdot 10^{-4}$
30	$2.5 \cdot 10^{3}$	$5.946 \cdot 10^{-4}$
31	$2.5 \cdot 10^{3}$	$5.946 \cdot 10^{-4}$
32	$2.6 \cdot 10^{3}$	$3.713 \cdot 10^{-4}$
33	$2.7 \cdot 10^{3}$	$1.48 \cdot 10^{-4}$
34	$2.8 \cdot 10^{3}$	$-7.537 \cdot 10^{-5}$
35	$2.9 \cdot 10^{3}$	...

T_MC väärtused eraldi välja toodud. Esimene tulp näitab sõlme kaugust, teine aga väändepinget.

T_MC =

	1	2
1	0	$7.21 \cdot 10^{-5}$
2	100	$7.21 \cdot 10^{-5}$
3	200	$7.21 \cdot 10^{-5}$
4	300	$7.21 \cdot 10^{-5}$
5	400	$7.21 \cdot 10^{-5}$
6	500	$7.21 \cdot 10^{-5}$
7	500	$4.039 \cdot 10^{-5}$
8	600	$4.039 \cdot 10^{-5}$
9	700	$4.039 \cdot 10^{-5}$
10	800	$4.039 \cdot 10^{-5}$
11	900	$4.039 \cdot 10^{-5}$
12	$1 \cdot 10^{3}$	$4.039 \cdot 10^{-5}$
13	$1 \cdot 10^{3}$	$8.679 \cdot 10^{-6}$
14	$1.1 \cdot 10^{3}$	$8.679 \cdot 10^{-6}$
15	$1.2 \cdot 10^{3}$	$8.679 \cdot 10^{-6}$
16	$1.3 \cdot 10^{3}$	$8.679 \cdot 10^{-6}$
17	$1.4 \cdot 10^{3}$	$8.679 \cdot 10^{-6}$
18	$1.5 \cdot 10^{3}$	$8.679 \cdot 10^{-6}$
19	$1.5 \cdot 10^{3}$	$-2.303 \cdot 10^{-5}$
20	$1.6 \cdot 10^{3}$	$-2.303 \cdot 10^{-5}$
21	$1.7 \cdot 10^{3}$	$-2.303 \cdot 10^{-5}$
22	$1.8 \cdot 10^{3}$	$-2.303 \cdot 10^{-5}$
23	$1.9 \cdot 10^{3}$	$-2.303 \cdot 10^{-5}$
24	$2 \cdot 10^{3}$	$-2.303 \cdot 10^{-5}$
25	$2 \cdot 10^{3}$	$-5.474 \cdot 10^{-5}$
26	$2.1 \cdot 10^{3}$	$-5.474 \cdot 10^{-5}$
27	$2.2 \cdot 10^{3}$	$-5.474 \cdot 10^{-5}$
28	$2.3 \cdot 10^{3}$	$-5.474 \cdot 10^{-5}$
29	$2.4 \cdot 10^{3}$	$-5.474 \cdot 10^{-5}$
30	$2.5 \cdot 10^{3}$	$-5.474 \cdot 10^{-5}$
31	$2.5 \cdot 10^{3}$	$-8.645 \cdot 10^{-5}$
32	$2.6 \cdot 10^{3}$	$-8.645 \cdot 10^{-5}$
33	$2.7 \cdot 10^{3}$	$-8.645 \cdot 10^{-5}$
34	$2.8 \cdot 10^{3}$	$-8.645 \cdot 10^{-5}$
35	$2.9 \cdot 10^{3}$	$-8.645 \cdot 10^{-5}$
36	$3 \cdot 10^{3}$	...

EL maatriks jääb seliseks nagu ennegi.

	1	2	3	4
1	1	1	2	0.5
2	2	2	3	0.5
3	3	3	4	0.5
4	4	4	5	0.5
5	5	5	6	0.5
6	6	6	7	0.5
7	7	7	8	0.5
8	8	8	9	0.5
9	9	9	10	0.5
10	10	10	11	0.5
11	11	11	12	0.5
12	12	12	13	0.5
13	13	13	14	0.5
14	14	14	15	0.5
15	15	15	16	0.5
16	16	16	17	0.5
17	17	17	18	0.5
18	18	18	19	0.5
19	19	19	20	0.5
20	20	20	21	0.5
21	21	21	22	0.5
22	22	22	23	0.5
23	23	23	24	0.5

Timošenko tala elementide jäikusmaatriksis on lisaks omapärastele valemitele veel juurde lisatud nihkeelastsusmaatriks G_v ja ristlõike pindala A_{red}.

$$K\_tim_{el}(E, G_v, I, A_{red}, h_e) = \left| \begin{array}{l} \lambda_e \leftarrow \dfrac{E \cdot I}{G_v \cdot A_{red} \cdot h_e^2} \\ \mu_0 \leftarrow 12 \cdot \lambda_e \\ R \leftarrow \dfrac{2 \cdot E \cdot I}{\mu_0 \cdot h_e^3} \cdot \begin{bmatrix} 6 & -3 \cdot h_e & -6 & -3 \cdot h_e \\ -3 \cdot h_e & h_e^2 \cdot (1.5 + 6 \cdot \lambda_e) & 3 \cdot h_e & h_e^2 \cdot (1.5 - 6 \cdot \lambda_e) \\ -6 & 3 \cdot h_e & 6 & 3 \cdot h_e \\ -3 \cdot h_e & h_e^2 \cdot (1.5 - 6 \cdot \lambda_e) & 3 \cdot h_e & h_e^2 \cdot (1.5 + 6 \cdot \lambda_e) \end{bmatrix} \\ \text{return } R \end{array} \right.$$

$F\_tim_{gl} = F1\_tim_{gl} + F2\_tim_{gl}$

Globaalse jäikusmaatriksi (48 x 48) ja jõuvektori (48 x 1) arvulised väärtused:

$K\_tim_{gl} =$

	1	2	3	4	5
1	$6.238\cdot10^{3}$	$-1.559\cdot10^{6}$	$-6.238\cdot10^{3}$	$-1.559\cdot10^{6}$	0
2	$-1.559\cdot10^{6}$	$7.35\cdot10^{9}$	$1.559\cdot10^{6}$	$-6.571\cdot10^{9}$	0
3	$-6.238\cdot10^{3}$	$1.559\cdot10^{6}$	$1.248\cdot10^{4}$	0	$-6.238\cdot10^{3}$
4	$-1.559\cdot10^{6}$	$-6.571\cdot10^{9}$	0	$1.47\cdot10^{10}$	$1.559\cdot10^{6}$
5	0	0	$-6.238\cdot10^{3}$	$1.559\cdot10^{6}$	$1.248\cdot10^{4}$
6	0	0	$-1.559\cdot10^{6}$	$-6.571\cdot10^{9}$	0
7	0	0	0	0	$-6.238\cdot10^{3}$
8	0	0	0	0	$-1.559\cdot10^{6}$
9	0	0	0	0	0
10	0	0	0	0	0
11	0	0	0	0	0
12	0	0	0	0	0
13	0	0	0	0	0
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	0	0	...

$F\_tim_{gl} =$

	1
1	$1.5\cdot10^{6}$
2	$-1.25\cdot10^{8}$
3	$3\cdot10^{6}$
4	0
5	$3\cdot10^{6}$
6	0
7	$3\cdot10^{6}$
8	0
9	$3\cdot10^{6}$
10	0
11	$3\cdot10^{6}$
12	0
13	$3\cdot10^{6}$
14	0
15	$3\cdot10^{6}$
16	...

Sarnane probleem tekib ka väändepingete arvutamisel.

$$
T\_MC_{tim} = \left|
\begin{array}{l}
N\_el \leftarrow \max\left(EL^{\langle 1 \rangle}\right) \\
\text{for } i\_el \in 1..N\_el \\
\quad \left|
\begin{array}{l}
\text{for } i \in 1..4 \\
\quad ue_i \leftarrow u\_tim_{(i\_el-1)\cdot 2+i} \\
\text{for } i \in 1..6 \\
\quad \left|
\begin{array}{l}
R_{(i\_el-1)\cdot 6+i,\,1} \leftarrow (i\_el - 1)\cdot h_e + \dfrac{(i-1)}{5}\cdot h_e \\
SUM \leftarrow 0 \\
\text{for } j \in 1..4 \\
\quad \left|
\begin{array}{l}
x \leftarrow \dfrac{(i-1)}{5}\cdot h_e \\
SUM \leftarrow SUM + ue_j \cdot dd\psi\left(j, h_e, x\right)
\end{array}
\right. \\
R_{(i\_el-1)\cdot 6+i,\,2} \leftarrow -E\cdot SUM \cdot \dfrac{S_t}{b_t}
\end{array}
\right.
\end{array}
\right. \\
\text{return } R
\end{array}
\right.
$$

$T\_MC_{tim} =$

	1	2
1	0	$-1.747\cdot10^7$
2	100	$-1.747\cdot10^7$
3	200	$-1.747\cdot10^7$
4	300	$-1.747\cdot10^7$
5	400	$-1.747\cdot10^7$
6	500	$-1.747\cdot10^7$
7	500	$-4.572\cdot10^7$
8	600	$-4.572\cdot10^7$
9	700	$-4.572\cdot10^7$
10	800	$-4.572\cdot10^7$
11	900	$-4.572\cdot10^7$
12	$1\cdot10^3$	$-4.572\cdot10^7$
13	$1\cdot10^3$	$-5.967\cdot10^7$
14	$1.1\cdot10^3$	$-5.967\cdot10^7$
15	$1.2\cdot10^3$	$-5.967\cdot10^7$
16	$1.3\cdot10^3$	...