Modelica-based simulation of building and district energy systems

Session 5: Heat transfer in Modelica and object-oriented features

- 1D discretization of heat conduction (solving PDEs)
- Exercise: Constructing a RC lumped model

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SESSION 5 – 1D Discretization of heat conduction

Modelica is a language for representing **lumped** systems. What this means is that the behavior must be expressed in terms of ordinary differential equations – ODEs (or DAEs)

Modelica does not include any means for describing partial differential equations (*i.e.,* equations that involve the gradient of variables in spatial directions).

The unsteady 1D conductive heat transfer equation in x-direction is (PDE):

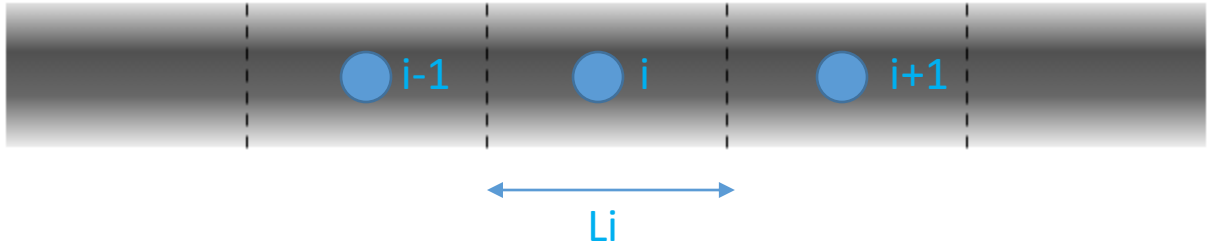
$$\rho c_p \frac{dT}{dt} = k \frac{\partial^2 T}{\partial x^2}$$

ρ: density
c: specific heat capacity
K: thermal conductivity
T: temperature

How can we solve this equation in Modelica?

SESSION 5 – 1D Discretization of heat conduction

Numerical approximation by spatial discretization (control volumes).

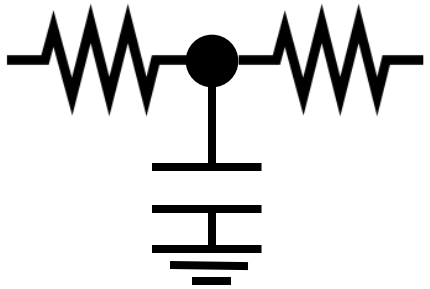


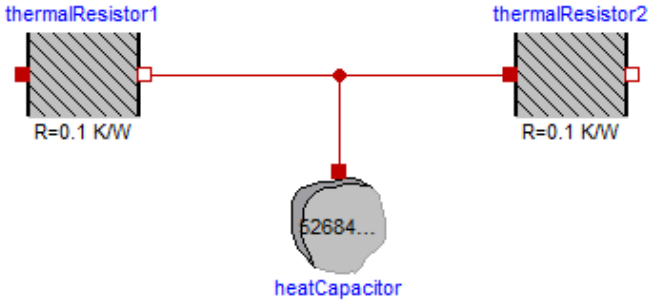
$$\rho A_c L_i C \frac{\mathrm{d}T_i}{\mathrm{d}t} = -kA_c \frac{T_i - T_{i-1}}{L_i} - kA_c \frac{T_i - T_{i+1}}{L_i}$$

Heat stored at node *i*

Heat transfer between node i and i-1

Heat transfer between node i and i+1

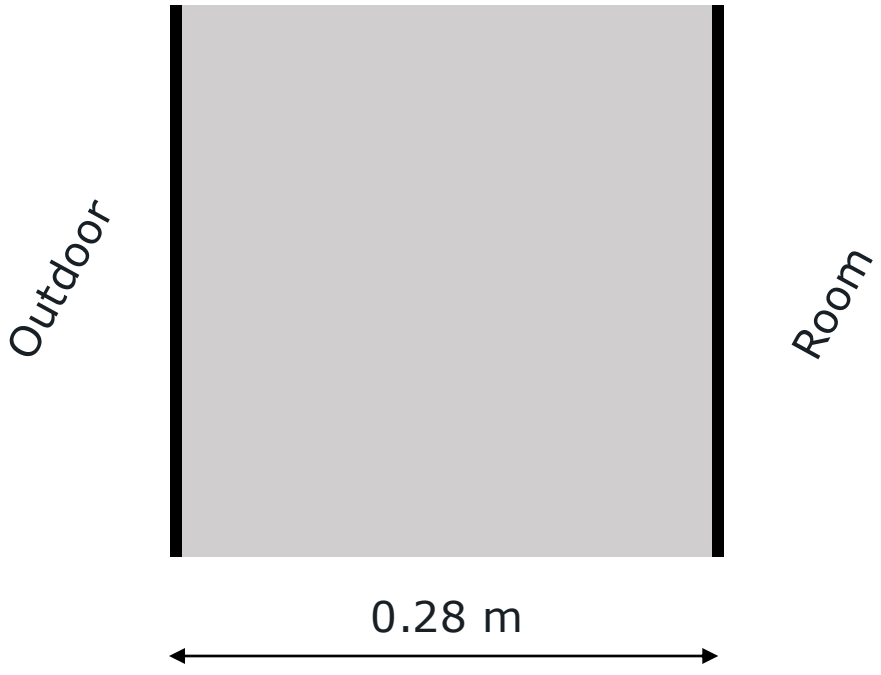




Exercise: – Discretization of heat conduction

Consider a 0.28 m slab of concrete with area of 1 m^2 that acts as a wall for a room with heat capacity of 32000 J/K. The outdoor air temperature is represented by a step block (see parameters below). Calculate the temperature profile of the room for a week period using three approaches:

1. The wall has no control volume (only thermal resistance)
2. The wall is discretized by one control volume
3. The wall is discretized by four control volumes



Step block

Height= -20
Offset = 273.15 + 20
StartTime = 86400 s

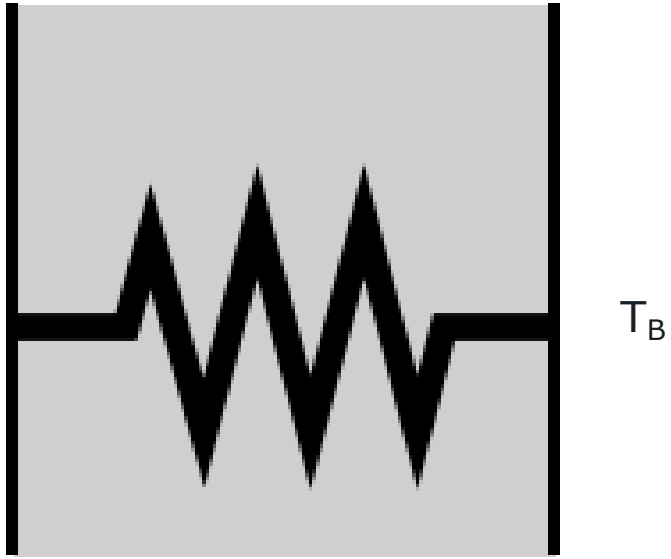
Wall thermal properties

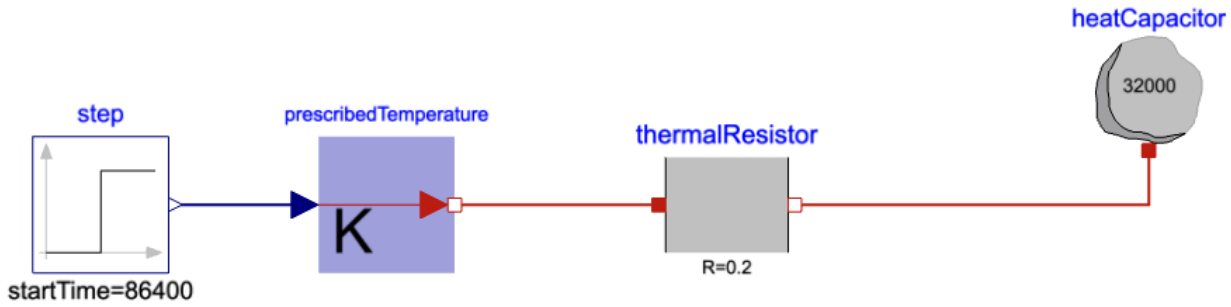
ρ = 2240 kg/m3
c = 840 J/KgK
k = 1.4 W/mK

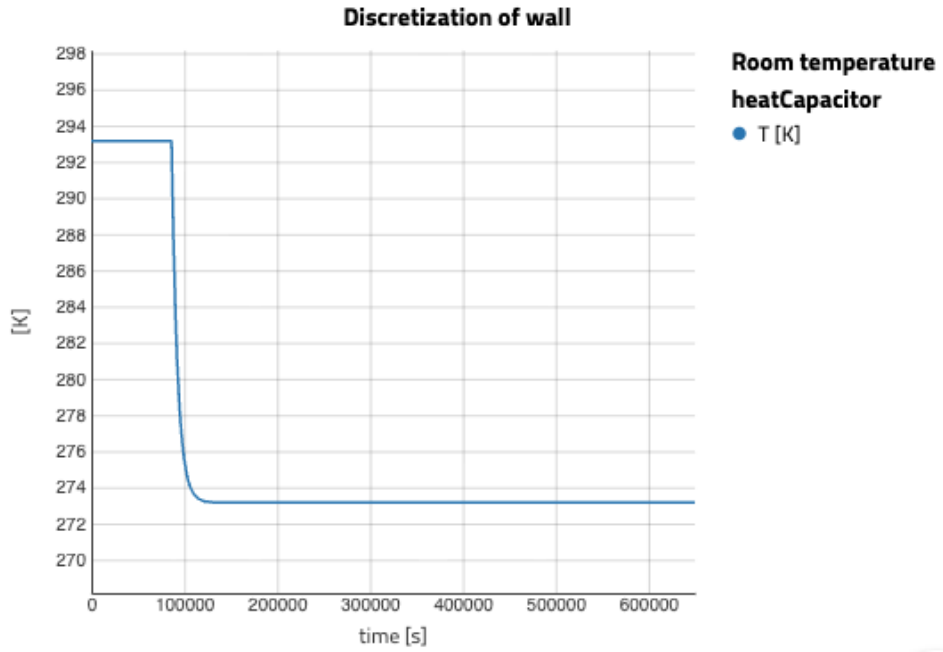
To solve the exercise, you first need to calculate the thermal resistance and the heat capacity of the wall (solutions in the next slides)

SOLUTION 1

No control volume





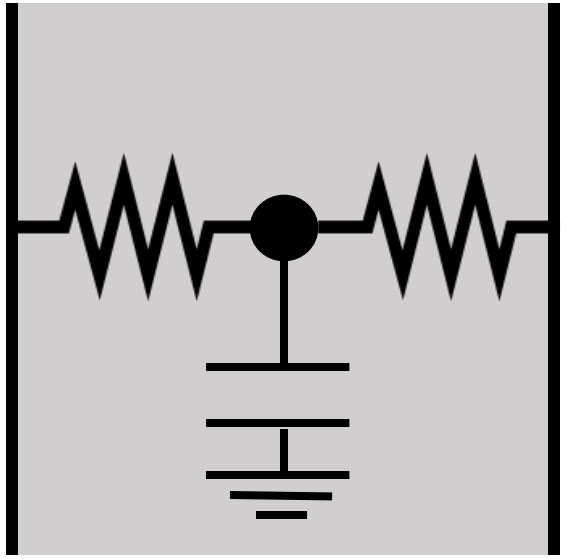


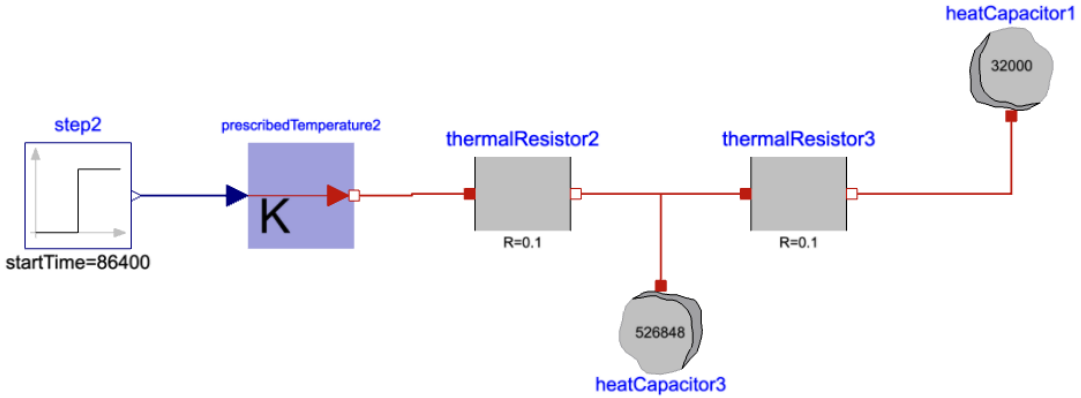
Models required:

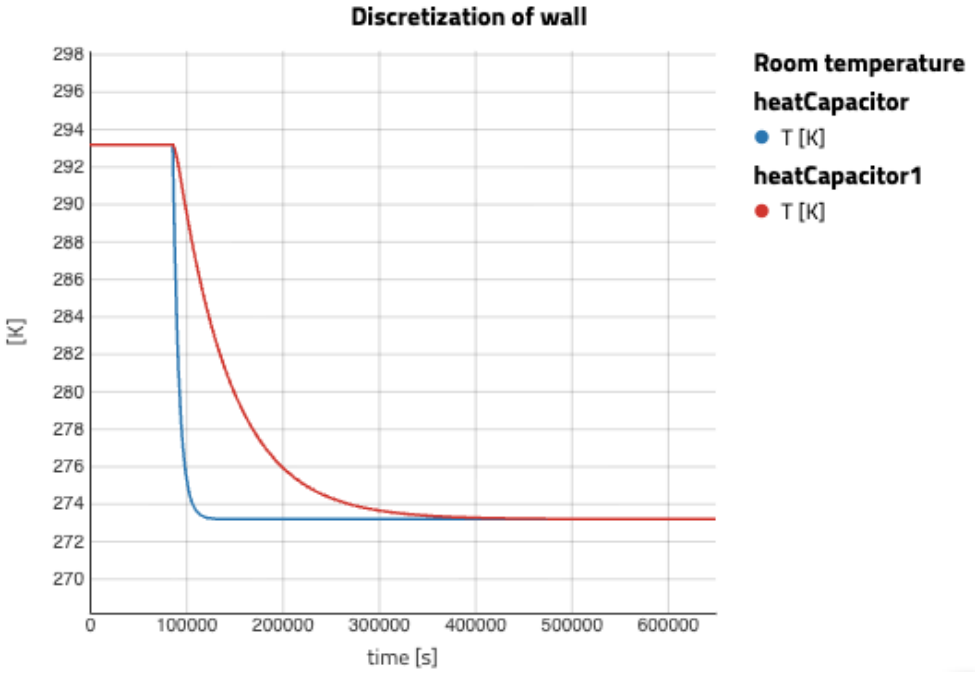
- `Modelica.Thermal.HeatTransfer.Components.ThermalResistor (1)`
- `Modelica.Thermal.HeatTransfer.Components.HeatCapacitor (1)`
- `Modelica.Thermal.HeatTransfer.Sources.PrescribedTemperature (1)`
- `Modelica.Blocks.Sources.Step (1)`

SOLUTION 2

One control volume





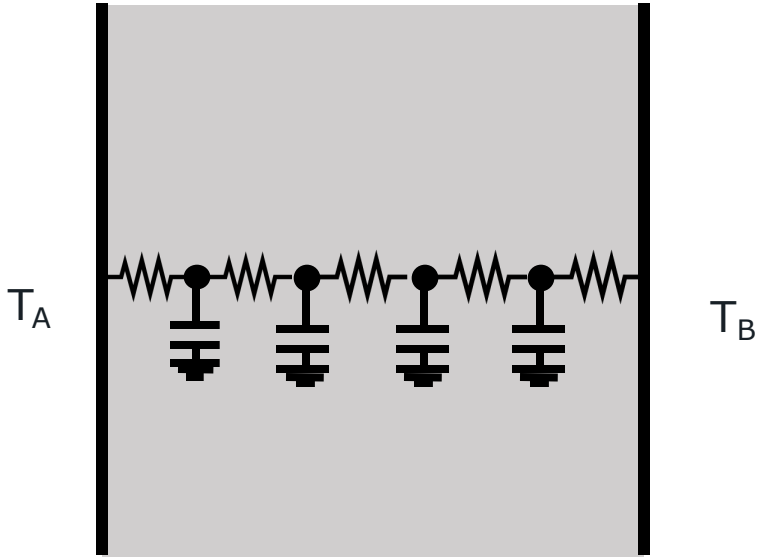


Models required:

- `Modelica.Thermal.HeatTransfer.Components.ThermalResistor (2)`
- `Modelica.Thermal.HeatTransfer.Sources.PrescribedTemperature (1)`
- `Modelica.Blocks.Sources.Step (1)`
- `Modelica.Thermal.HeatTransfer.Components.HeatCapacitor (2)`

SOLUTION 3

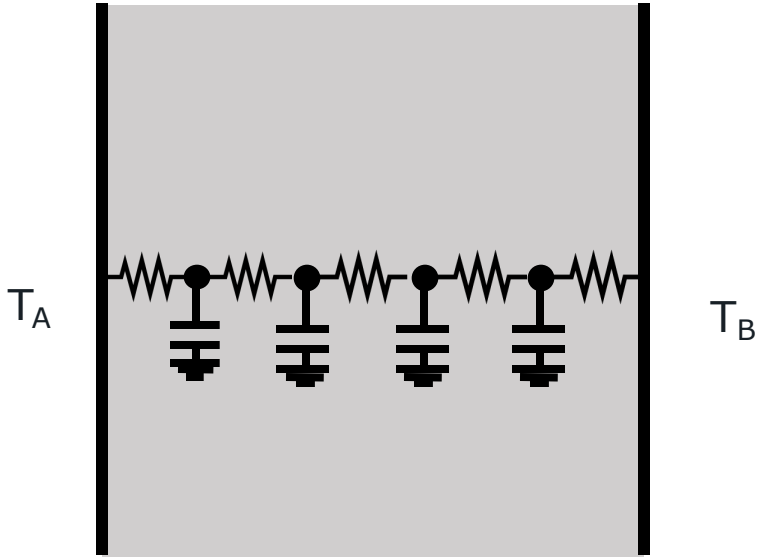
Four control volumes

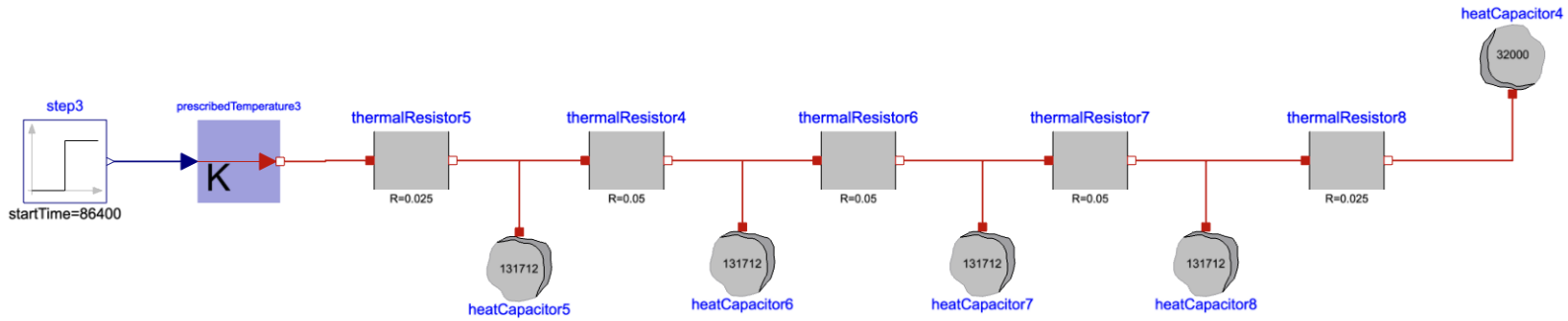


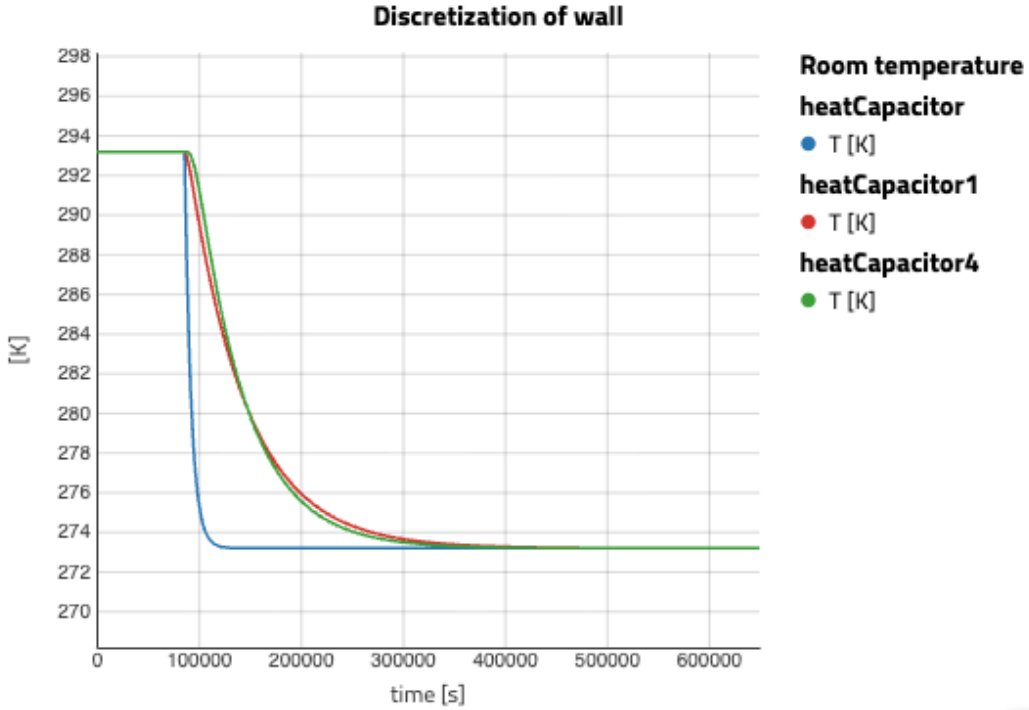
SOLUTION ON THE NEXT PAGE!

SOLUTION 3

Four control volumes







Models required:

- `Modelica.Thermal.HeatTransfer.Components.ThermalResistor (5)`
- `Modelica.Thermal.HeatTransfer.Sources.PrescribedTemperature (1)`
- `Modelica.Blocks.Sources.Step (1)`
- `Modelica.Thermal.HeatTransfer.Components.HeatCapacitor (5)`