## § 5.7 Paramagnetic propertioes of free electrons.

Let's start with the classical set of magnetic molecules with a magnetic moment $\mu_0$. The probability that the magnetic moment will be directed at an angles region from θ ,dθ to the magnetic field is (disorder occurs due to thermal motion):

$$dw = A\, e^{\frac{\mu_0 H \cos(\theta)}{(kT)}} \sin(\theta)\, d\theta$$

The constant A can be calculated from the normalization of the probability by one:

$$\int dw = A \int_0^{\pi} A e^{\frac{\mu_0 H \cos(\theta)}{(kT)}} \sin(\theta)\, d\theta = \int_{-1}^{+1} e^{\alpha x}\, dx = 1, here\ \alpha = \frac{\mu H}{kT} \text{ and } x = \cos(\theta)$$

The average value of the projection of the magnetic moment on B is:

$$<\mu> = \int \mu_0 \cos(\theta)\, dw = \mu_0 \frac{\int_{-1}^{+1} x e^{\alpha x} dx}{\int_{-1}^{+1} e^{\alpha x} dx} \ .$$

after integration

$$<\mu> = \mu_0 L\left(\frac{\mu_0 H}{kT}\right) \quad \text{here L is so called Lengaven function}$$

$$L(\alpha) = \coth(\alpha) - \frac{1}{\alpha}$$

For low temperature α<<1 this function can be written as L(α)≈α/3. Then for magnetization vector (can be calculated as multiplication $<\mu>$ and n-concentration of magnetic particles ) we have:

$$M = \frac{\mu_0^2 n}{3kT} H$$

Magnetic susceptibility χ=M/H= $\frac{\mu_0^2 n}{3kT}$ and corresponds to the Curie law for paramagnets.

In framework of quantum mechanics we need to take into account the discreteness of physical quantities. The magnetic moment of an atom in quantum mechanics is related to the orbital and spin degrees of freedom. The magnetic moment of atom then is equal:

$\mu_j = \mu_B j g$ here $\mu_B = \frac{e\hbar}{2mc} - Bohr \quad magneton$ . Physically, the Bohr magneton is equal to the ratio of the magnetic moment to the angular momentum for the orbital motion of an electron:

$$\frac{M_l}{L_z} = \mu_B \hbar$$

for spin motion this ratio is two times larger $\frac{M_s}{s_z} = 2\mu_B \hbar$ . Here **j** is a quantum number for total magnetic moment of electron $\vec{J} = \vec{L} + \vec{S}$ , $g = 1 + \frac{j(j+1) + s(s+1) - l(l-1)}{2j(j+1)}$ is Lande parameter.

The electron energy in  external magnetic field is equal:

$$E_p = -\mu_j H \cos(\widehat{\vec{J}\vec{H}}) = -\mu_B H g m$$

here m-magnetic quantum number for total momentum **J** and range of possible **m** values is [-j,…,0,...+j]. The average projection of magnetic momentum on external magnetic field in this case is:

$$<\mu> = \frac{\sum_{m=-j}^{m=+j} \mu_j \cos(\widehat{\vec{J}\vec{H}}) e^{-\frac{E_p}{kT}}}{\sum_{m=-j}^{m=+j} e^{\frac{-\mu_p}{kT}}} = \mu_B g \frac{\sum_{m=-j}^{m=+j} m e^{\alpha m}}{\sum_{m=-j}^{m=+j} e^{\alpha m}}$$ here $\alpha = \frac{\mu_B g H}{kT}$ . For weak external magnetic α<<1

$$<\mu> = \frac{\mu_B^2 g^2 j(j+1)}{3kT} H \quad .$$

And magnetic susceptibility χ=M/H= $\frac{\mu_0^2 n}{3kT}$ .This result coincided with classical results and Curie law for paramagnetic too.

It seems that this approach can be used for free electrons in metals. But experiments give quite different results. The magnetic susceptibility for metals does not depend on temperature and is about 1000 times less than expected.

To describe the magnetic properties of free electrons, it is necessary to take into account only the electron spin. Free electrons can be divided into two parts. The first part is electrons with spins in the direction of the external magnetic field. This interaction reduces the energy of the electron and the corresponding magnetic moment can be calculated in this way:

$$M_+ = \mu_B \int f_{fd}(E - \mu_b H) G(E) dE$$

analogically for electrons with opposite spins:

$$M_- = \mu_B \int f_{fd}(E + \mu_b H) G(E) dE$$

The total magnetic moment

$$M = M_+ - M_- = \mu_B \int \{f_{fd}(E - \mu_B H) - f_{fd}(E + \mu_B H)\} G(E) dE$$

for weak magnetic field the FD function can be expanded in a series of small H:

$$M = 2 H \mu_B^2 \int (-\frac{d f_{fd}(E)}{dE}) G(E) dE \quad .$$

It is clear that $2\int (-\frac{d f_{fd}(E)}{dE}) G(E) dE = 2\int \frac{d f_{fd}(E)}{d\mu} G(E) dE = 2\frac{d}{d\mu}\int f_{fd}(E) G(E) dE = \frac{dn}{d\mu}$ .

Then $M = \mu_b^2 \frac{dn}{d\mu}$ . Exact calculations give us:

$$M = H \mu_B^2 G(E_F)[1 - \frac{\pi^2}{12}(\frac{T}{T_F})]$$

and magnetic susceptibility

$$\chi = \mu_B^2 G(E_F)[1 - \frac{\pi^2}{12}(\frac{T}{T_F})] \quad .$$