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Homework 1 Summary

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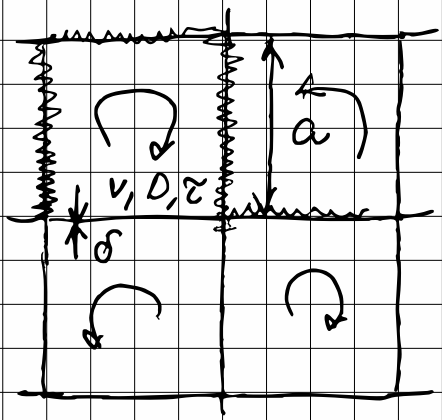
Time of one rotation inside the vortex:

$$\tau \sim \frac{a}{v}$$

a - size of vortex
V - velocity



Sleep and awake times

The movement is in so-called "sleep" mode when a tracer is inside the vortex. Alternatively, it is "awake" if it's inside the near-separatrix region.

t' - awake time

t - total time

Ergodicity: average over time = average over space

$$\frac{t'}{t} = \frac{A_y}{A_t} \; ; \quad A_y \sim a\delta \; ; \quad A_t \sim a^2$$

δ - thickness of near-separatrix region

$$t' = t \cdot \frac{A_y}{A_t} = \frac{\alpha\delta}{a^2} \cdot t = \frac{\delta}{a} \cdot t$$

Distance travelled

$$\langle x^2 \rangle \simeq \frac{a^2}{\tau} \cdot t' \simeq \left[t' = t \cdot \frac{\delta}{a} \right] \simeq \frac{a^2}{\tau} \cdot t \cdot \frac{\delta}{a} \simeq$$

$$= \underbrace{\frac{a\delta}{\tau}}_{D_{eff}} \cdot t$$

Effective Turbulence

$$D_{eff} \simeq \frac{a\delta}{\tau} \simeq \left[\tau \sim \frac{a}{v} \Rightarrow v \sim \frac{a}{\tau} \right] \simeq$$

$$\simeq v\delta \simeq \left[\delta \sim \sqrt{D \cdot \frac{a}{v}} \right] \simeq v \cdot \sqrt{\frac{D \cdot a}{v}} =$$

$$= \sqrt{\frac{v^2 \cdot D \cdot a}{v}} = \sqrt{Dav} = D \sqrt{\frac{a \cdot v}{D}} =$$

$$= D \sqrt{Pe}$$

Pe - Péclet number; dimensionless number that characterizes the ratio of advection to molecular diffusion.