EXTENDED ACADEMIC EKF REPORT FOR IMU-BASED DEAD RECKONING

This academic-style report presents a scientific and engineering formulation of a Quaternion-based Extended Kalman Filter (EKF) for real-time inertial navigation. The algorithm is implemented in the dr-vehicle project for MPU6050 IMU fusion, and this document formalizes the mathematical structure comparable to research papers.

1. BACKGROUND AND MOTIVATION

Inertial navigation using low-cost MEMS IMUs suffers from drift due to noise, bias instability, scale-factor errors, and integration of accelerometer readings. Fusing gyroscope and accelerometer using a constrained quaternion EKF reduces drift and provides globally stable attitude estimates.

This EKF is equivalent in form to algorithms used in robotics, aerospace, drones, and autonomous vehicles for 3D orientation estimation.

2. QUATERNION MATHEMATICS

A unit quaternion $q = [q0, q1, q2, q3]^T$ represents a rotation in 3D space.

Quaternion multiplication defines rotation propagation. For angular rate ω:

$$dq/dt = 0.5 * \Omega(\omega) * q$$

The matrix $\Omega(\omega)$ encodes the cross-product structure of angular velocity.

Compared to Euler angles, quaternions avoid singularities and discontinuities.

3. CONTINUOUS-TIME PROCESS MODEL

The gyroscope provides $\omega = [wx, wy, wz]$ in rad/s.

The propagation is:

$dq/dt = 0.5 * \Omega(\omega) q$

Discretization:

$q_{k+1}^- = \text{normalize}(q_k + dq/dt * dt)$

Noise enters through gyro measurement noise and unmodeled dynamics.

Thus covariance is propagated as:

$P_{k+1}^- = F_k P_k F_k^T + Q$

where $F_k \approx I$ because dt is small.

Q is diagonal, expressing uncertainty in quaternion propagation.

4. MEASUREMENT MODEL (GRAVITY VECTOR)

Accelerometer measures specific force f = a - g.

When IMU has small linear acceleration, accelerometer direction aligns with gravity.

The EKF uses this direction to correct quaternion drift:

z = acc / ||acc||

Predicted measurement:

h(q) = R(q)^T * [0,0,1]

The innovation is:

y = z - h(q)

The update corrects roll and pitch drift. Yaw cannot be corrected without magnetometer.

5. JACOBIAN DERIVATION

The measurement Jacobian $H = \partial h/\partial q$ is difficult to derive manually.

Instead, numerical Jacobian is used:

$H(:,i) = (h(q + eps*ei) - h(q - eps*ei)) / (2\ eps)$

This matches academic practice for quaternion EKFs when symbolic Jacobians are messy.

6. KALMAN UPDATE FORMULATION

Compute innovation covariance:

$S = H P H^T + R$

Compute Kalman gain:

$K = P H^T S^{-1}$

Update quaternion:

$q^+ = \text{normalize}(q^- + K y)$

Update covariance:

$P^+ = (I - K H) P$

This ensures quaternion remains normalized and consistent.

7. ZERO VELOCITY UPDATES (ZUPT)

ZUPT is widely used in pedestrian and ground vehicle dead reckoning research.

Stillness is detected when:

$|a_{world} + g| \approx 0$

$|gyro| < threshold$

After N consistent stationary samples:

$v = 0$

This dramatically reduces drift in double-integration of accelerometer.

ZUPT is essential for MEMS-based inertial navigation.

8. VELOCITY DECAY MODEL

Even when IMU is nearly still, noise in accelerometer produces small accelerations that integrate into drifting velocity. A decay model:

v = decay_factor * v

suppresses this drift. Academically, this is equivalent to applying a damping prior on velocity magnitude.

9. SIMPLIFIED KINEMATIC MODEL

The navigation state includes:

position p

velocity v

orientation q

Position updates:

$p_{k+1} = p_k + v_k \, dt$

Velocity updates:

$v_{k+1} = v_k + a_{world} \, dt$

This minimal formulation is standard in pedestrian dead-reckoning literature.

10. SENSOR CALIBRATION THEORY

Accelerometer LS calibration solves:

C * raw + o = calibrated

where C is 3x3 scale/orthogonality correction

and o is bias offset.

Gyroscope bias must be estimated at rest.

Inaccurate calibration leads to large EKF drift.

11. LIMITATIONS OF 6-AXIS EKF

Without magnetometer:

yaw drift is unobservable.

The EKF only corrects roll and pitch.

Yaw deadband and bias adaptation reduce drift but do not fully eliminate it.

For full 3D orientation, 9-axis fusion or GNSS observations are required.

12. APPLICATIONS

This EKF is suitable for:

- ground vehicle odometry
- drone attitude stabilization (if augmented)
- robotic localization
- handheld IMU tracking
- inertial dead-reckoning research experiments

13. CONCLUSION

The quaternion EKF presented provides a robust and academically grounded framework for fusing MEMS IMU data. Its combination with ZUPT and decay filtering forms a complete dead-reckoning system suitable for embedded robotics and real-time systems.