Market Risk

一. Parametric 计算 VaR

1. normal VaR: assumption: $R = \frac{P_t - P_{t-1}}{P_{t-1}} \sim$ normal.

   • formula: $VaR_s = -(\mu - z_\alpha \cdot \sigma) P_{t-1}$ (single & 组合)

   $VaR = z_\alpha \times \sigma \times V \times Delta.$

   $\downarrow$ one-tail

   (portfolio) $VaR_p = \sqrt{VaR_1^2 + VaR_2^2 + 2 \cdot \rho \cdot VaR_1 \cdot VaR_2}$ $(\mu = 0)$

   • VaR表示loss. • dollar VaR vs % VaR.

2. lognormal $VaR = [1 - e^{\mu - z \cdot \sigma}] \times P_{t-1}$

   • assumption: $r = \ln(\frac{P_t}{P_{t-1}}) \sim$ normal.

   • formula: $VaR = 1 - e^{(\mu - z\sigma)}$ (%) $VaR = [1 - e^{\mu - z\sigma}] \cdot P_{t-1}$ (dollar VaR)

3. QQ plot: 判断是否为正态分布 steeper slope ⇒ fat-tail

二. Non-parametric:

① 定义: collect data → 排序 → 查数

② assumption: the near future will be sufficiently like the recent past. 历史重演

③ pros & cons:

优点: can deal with variance-covariance metrics. curse of high dimensionality.

缺点: Basic HS historical simulation → discrete confidence interval

↓

Non-parametric density estimation: 线性插值.

ghost effect, hard to handle shift, extreme value.

④ VaR vs ES: Expected Shortfall = mean (VaR > VaR Benchmark)

5. Coherent risk measures:

(1) monotonicity: $x_1 \leq x_2 \rightarrow \underset{\substack{\downarrow \\ \text{risk}}}{\rho}(x_1) \geq \rho(x_2)$

(2) translation invariance: $\rho(X+k) = \rho(X) - k$ → cash.

(3) homogenerity: $\rho(bX) = b\rho(X)$

(4) Subadditivity: $\underbrace{\rho(X_1+X_2)}_{\text{risk of portfolio}} \leq \rho(X_1) + \rho(X_2)$

三. Semi-parametric approach.

① BRW: age-weighted HS: 调weight, 不变数据.

$\rightarrow w_{(i)} = \lambda^{(i-1)} \cdot \frac{(1-\lambda)}{(1-\lambda^n)}$

→ 优点：no ghost effect, no jumps

② HW：volatility-weighted HS：改变数据，不变权重.

→ $r_t^* = \frac{\sigma_T (今天)}{\sigma_t (t天前)} \cdot r_t$ (t天前 return)

→ 优点：可以超过原来数据最大值. / 考虑风险.

③ CWHS：correlation-weight：基于HW. → 考虑 σ, ρ

④ Filted historical simulation.：考虑 σ, ρ 与不对称性.

→ combine HS with GARCH and AGARCH.

Ⅲ. Extreme value：

① Block maxima：丢失有价值数据

→ n↑ → Generalized extreme-value (GEV)

3 parameter：μ (mean)：location parameter

σ (volatility)：scale / dispersion

ξ (tail index) ⟶ $\begin{cases} >0 \text{ fat tail. Frechet.} \\ =0 \text{ (normal) Gumbel} \\ <0 \text{ (thin tail) Weibull.} \end{cases}$

② Peak-over-threshold (POT) Approach：

→ threshold 难确定

→ Generalized Pareto distribution：$\begin{cases} \text{scale parameter: } \beta \\ \text{tail index: } \xi \end{cases}$

③ POT vs GEV.

④ Using POT (Peaking-over-threshold) to calculate VaR and ES：

$VaR = \mu + \frac{\beta}{\xi}\left(\left[\frac{n}{N_u}(1-\alpha)\right]^{-\xi} - 1\right)$ ξ↑ ⇒ ES, VaR↑

$ES = \frac{VaR}{1-\xi} + \frac{\beta - \xi\mu}{1-\xi}$

五. Backtesting：VaR.

① binomial distribution：$f(x) = C_n^x p^x (1-p)^{n-x}$.

• Model Verification Based on Failure rates：

$z = \frac{x - pT}{\sqrt{p(1-p)T}} \approx N(0,1)$ $\begin{cases} x \sim \text{\# of exceptions.} \\ p \sim \text{significant level} \end{cases}$

② kupiec VaR：对二项修正：fat tail.

→ LRuc > 3.841. → H_0：VaR is true

③ Christofferson：加3 2个 parameter：考虑时间 (数据聚集性)

④ Basel：Capital = VaR (3 + k) k：# exception.

→ market VaR：99% over the past year (250×1%)

六. VaR Mapping:

① Bond: 1> principal: 1年 \$200, 5年 \$300 ⇒ $1\times\frac{200}{500}+5\times\frac{300}{500}=3$(年)

→ 找到到期时间相同的 zero-coupon Bond's VaR

→ $VaR_p = VaR_B = T\cdot VaR(factor)$

2> Duration (考虑 coupon): 找到与 duration 相同的 zero-coupon Bond's VaR

→ 线性查值.

undiversified / diversified → $VaR_p = VaR_B = D\cdot VaR(factor)$

3> cash-flow: $VaR_p = \sum D_i^* \cdot VaR(t_{yr}) \cdot P_t$ → 对应时间点 cash flow 折现

② Derivative: 1> Currency forward contracts: $\uparrow F_t = (\overset{\uparrow}{S_t}\, e^{-\overset{\downarrow}{y}t})\, e^{\overset{\uparrow}{r}t}$

- long foreign currency spot, long foreign currency bill
- short U.S dollar bill.

2> FRA 6×12: long 6-month bill + short 12-month bill.

3> Interest rate swap: = buy float-rate bond + short fixed-rate bond. / a series of FRA

4> Option: BSM model: $call = S\,N(d_1) - k\cdot e^{-rt}N(d_2)$

$put = ke^{-rt}N(-d_2) - SN(-d_1)$

9. type I error: α significant level 拒真, typeII error: 取伪 {type I.II 此消彼长. n↑, type I,II↓

type I + confidence interval = 1   type II error + power of test = 1 $z=\frac{x-PT}{\sqrt{P(1-P)T}}$

- 提高 power of test: 1> confidence interval↓ → α↑ → type I↑ II↓ → power of test↑

2> n↑ → typeII↓

10. Basel: VaR, stressed VaR ⟶ stressed ES measure.

↓ Basel I. Basel II.5

FRTB: stressed ES ⟶ determine market risk capital.

11. General risk factor: in market risk:

① equity risk factor. ② Interest rate risk

③ Exchange rate risk. ④ Commodity price risk

11. Risk management for trading books:

① VaR: disregard 尾部损失, lack of subadditivity.

② Expected loss (ES):

→ account for 尾部损失.

→ subadditive and coherent. (一致性)

→ complex & computationally intensive.

③ Spectral risk measures：给ES配权重.
→ smoothness properties
→ risk aversion of investors.
→ little effort if simulations-based.

• Basel framework: "Building Block": non-integrated approach to risk management.

• FRTB 修正 Basel: → fundamental review of trading Book.
→ 99% VaR + 99% Stress VaR ✗
97.5% ES ✓. (12 month stress period)
→ calculate ES: Internal models-based approach (IMA)
Revised standardized approach.
→ two types of risk: credit spread risk. (10 days 99% VaR)
Jump-to-default risk (1yr. 99.9% VaR)

12. Calculate correlation swap:

$$\text{payoff} = \text{Principal} \times (\underline{\rho_{realized}} - \rho_{fixed})$$

$$\rho_{realized} = \frac{2}{n^2-n} \sum \rho_{ij}$$

• Buy correlation: buy correlation swap. (fixed rate payer is buying correlation)
(gain from $\rho\uparrow$) → buy put option on an index + sell put option on individual stocks.

• Credit Crisis from CDO (ρ between tranches of CDS ↑)
Fail 1: long equity tranches, short mezzaine tranch (get spread)
Fail 2: default on equity tranches
Fail 3: loss from senior tranches
→ 结论: ① $\rho_{行业内} > \rho_{行业间}$
② Time dependency of credit risk.
③ β 与 ρ highly dependent, but not significant.

13. Mean reversion of variables:

$$S_t - S_{t-1} = \underset{\text{mean reversion coefficient}}{\underline{a}} (\mu_s - S_{t-1})$$

$$S_t - S_{t-1} = -a S_{t-1} + a\mu_s$$
$$Y \quad = \beta \; X + \alpha$$

• autocorrelation = $1-a$. (persistence: positive correlation)

13. Copulas: joining of multiple univariate distribution ⟶ single multivariate distribution.
→ $C[G_1(\mu_1), \cdots, G_n(\mu_n)] = F_n[F_1^{-1}(G_1(\mu_1)), \cdots, F_n^{-1}(G_n(\mu_n)); \rho_F]$
F_n: joint cummulative distribution function.

13. Equity correlation behaviour:

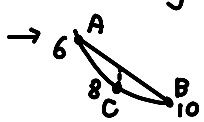
- $\rho\downarrow$ lowest: strong economic growth time.
- correlation volatility lowest: economic expansion.
  highest: normal economic states.
- $\rho(\rho, \sigma) > 0$
- Before every recessions a downturn (衰退) in correlation volatility occurred

14. Risk metrics and Hedge:

- $F^{Real} = -F^{Nominal} \times \frac{DV01^N}{DV01^R} \times \hat{\beta}$ Real ~ tips, Nominal ~ Bond.
  $F^{Real} \times DV01^R = F^{Nominal} \times DV01^N \times \hat{\beta}$
- selling 20-yr swaps → combination of 10- and 30-year swaps.
- PCA: $\sum_1^3 \text{Variance} \rightarrow \sum^{\text{total}} \text{Variance}$

15. Jensen's inequality: $E\left[\frac{1}{(1+r)}\right] > \frac{1}{E(1+r)}$

→ convexity increases with maturity and volatility.

→ 

$E\left(\frac{1}{1+r}\right) = \frac{1}{1+6\%} \times 0.5 + \frac{1}{1+10\%} \times 0.5$
$\vee$
$\frac{1}{E(1+r)} = \frac{1}{1+8\%}$

16. Term Structure model:

(1) model 1: $dr = \sigma dw \quad (dw = \varepsilon\sqrt{dt})$ → assume constant volatility (无 drift)

- 问题: short-term rate can become negative.
  → non-normal distribution.
  → use shadow rate.

(2) model 2: $dr = \lambda dt + \sigma dw$ (均衡模型) (对 drift 项进行调整)

- Ho-lee model: $dr = \lambda(t)dt + \sigma dw$

$$r_0 \begin{cases} r_0 + \lambda_1 dt + \sigma\sqrt{dt} \begin{cases} r_0 + (\lambda_1+\lambda_2)dt + 2\sigma dw \\ r_0 + (\lambda_1+\lambda_2)dt \end{cases} \\ r_0 + \lambda_1 dt - \sigma\sqrt{dt} \begin{cases} r_0 + (\lambda_1+\lambda_2)dt \\ r_0 + (\lambda_1+\lambda_2)dt - 2\sigma dw \end{cases} \end{cases}$$

- Vasicek model: $dr = \underbrace{\kappa(\theta - r)dt}_{\text{mean reversion feature}} + \underline{\sigma dw}$ → drift.

  → decreasing volatility of future short-term rate

- basic point volatility: volatility of dr.

→ $\underbrace{\tau}_{\text{half-life of interest rate}} = \frac{\ln 2}{\kappa}$ → speed of reversion

(3) model 3: $dr = \lambda(t)dt + \sigma(t)dw$ / $dr = \lambda(t)dt + \sigma \cdot e^{-at} dw$

→ 对drift和volatility做出调整.

→ CIR: cox-ingersoll-Ross Model:

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dw$$

- basic point volatility of short-rate will be proportional to the square root of the rate.
- mean reversion
- yield volatility is specified as being constant while basis-point volatility is allowed to vary.

(4) model 4: $\frac{dr}{r} = a dt + \sigma dw$. (lognormal model)

→ with determinstic drift: $d(\ln r) = a(t)dt + \sigma dw$

→ with mean reversion: $d(\ln r) = k(t)[\ln(\theta_t) - \ln(r)]dt + \sigma(t)dw$.

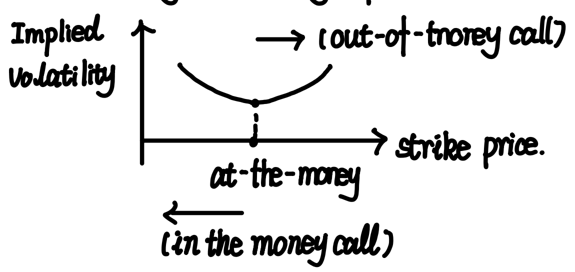
17. Volatility smiles:

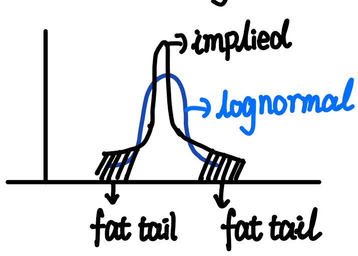
1> put-call parity: Call(market) − Call(BSM) = Put(market) − Put(BSM)

2> volatility frowns result when jumps occur in asset prices.

3> Protective puts will increase the price of puts → increase their volatility.

4> Foreign currency options:





(5) Equity option: