

Credit Risk

Credit Risk Measurement and Management

FRM二级培训讲义-强化班

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10% Contribution Breeds Professionalism



Topic Weightings in FRM Part II

Session NO.	Contents	%
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Framework

- A. > Credit Risk Identification
 - Classifications and Analysis
- B. > Credit Risk Measurement
 - Probability of Default
 - Counterparty Exposures
 - Pricing Counterparty Risk
- C. > Credit Risk Management
 - Mitigating Approaches
 - Credit Derivatives
 - Securitization
 - Retail Credit Risk

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Credit Risk Identification

Topic 1: Classifications and Analysis

1. Classifications of Credit Risk
2. Key Indicators
3. Portfolio Credit VaR
4. Capital Structure
5. Credit Decision and Analyst

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◆ Classifications of Credit Risk

➤ Classifications

● Default Risk

- ✓ Related to borrower's default

● Recovery Risk

- ✓ Related to the possibility that, in the event of default, the recovered amount is lower than the full amount due

● Exposure Risk

- ✓ Linked to the possible increase in the exposure at the time of default compared to the current exposure

➤ Three Drivers $RR + LR = 100\%$

- Probability of Default (PD) (recovery rate)
- Exposure at Default (EAD)
- Loss given Default (LGD)

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◆ Key Indicators

➤ Expected Loss (EL) (价格体现)

- The expected value of the credit loss, and represents the portion of loss a creditor should provision for. If the only possible credit event is default, expected loss is equal to:

$$= PD \times LGD \times ED$$

$$EL = PD \times (1 - RR) \times EAD = PD \times LGD \times EAD$$

➤ Unexpected Loss (Credit VaR)

- Is typically defined in terms of unexpected loss (UL) as the worst-case portfolio loss at a given confidence level over a specific holding period, minus the expected loss.

$$WCL(X\%) - EL$$

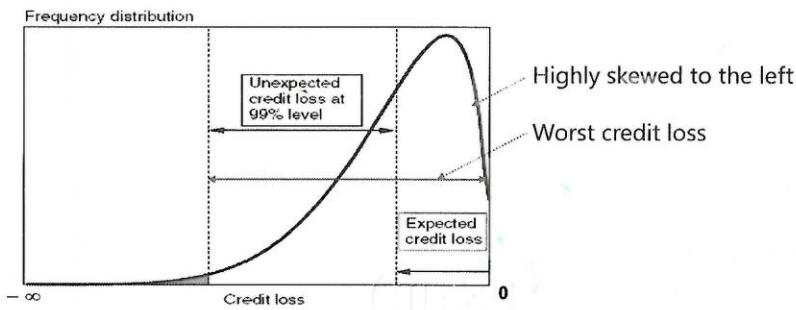
→ Portfolio 的 EL = $\sum EL_i$ (assume PD, LR一样)

$$EL_p$$

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◆ Key Indicators



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$$x = \begin{cases} 1(\pi), \\ 0(1-\pi) \end{cases}$$

◆ Portfolio Credit VaR

➤ Default Correlation 考虑两个资产的违约: $x=0,1 \rightarrow$ 伯努利分布.

● Assumption

- ✓ Two firms (or countries, if we have positions in sovereign debt).
- ✓ With probabilities of default π_1 and π_2 .
- ✓ And a joint default probability – the probability that both default over τ – equal to π_{12} .

● Calculation

$$\text{if } \rho=0 \Rightarrow \pi_{12}=\pi_1 \cdot \pi_2.$$

Outcome	X_1	X_2	$X_1 X_2$	Probability
No default	0	0	0	$1 - \pi_1 - \pi_2 + \pi_{12}$
Firm 1 only defaults	1	0	0	$\pi_1 - \pi_{12}$
Firm 2 only defaults	0	1	0	$\pi_2 - \pi_{12}$
Both firms default	1	1	1	π_{12}

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$$\begin{aligned} E(X_i) &= \pi_i \\ E(X_1 X_2) &= \pi_{12} \\ V(X_i) &= \pi_i(1 - \pi_i) \quad i=1,2 \\ \text{Cov}(X_1, X_2) &= \pi_{12} - \pi_1 \pi_2 \\ p &= \frac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1(1 - \pi_1)} \sqrt{\pi_2(1 - \pi_2)}} \end{aligned}$$

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$$\rho_{X_1, X_2} = \frac{\text{Cov}(X_1, X_2)}{\sigma_{X_1} \sigma_{X_2}}$$

$$\text{cov}(X_1, X_2) = E(X_1 X_2) - E(X_1) E(X_2)$$

$$\sigma^2_{X_i} = E(X_i^2) - E(X_i)^2$$

◆ Portfolio Credit VaR

➤ Default Correlation Impact

- Default correlation affects the volatility and WCL rather than EL.
- If default correlation in a portfolio of credits is equal to 1, then the portfolio behaves as if it consisted of just one credit. No credit diversification is achieved.
- If default correlation is equal to 0, then the number of defaults in the portfolio is a binomially distributed random variable. Significant credit diversification may be achieved.

$\Rightarrow P=1$, 视为 one credit

$\Rightarrow P=0$, 2项分布.

发生概率用 binomial distribution 建立起来. C_{50} 如果一个 default.

找到相累积概率大于 confidence interval 的点

loss 小 \rightarrow confidence interval 95%

loss 大 \rightarrow 1% \rightarrow significant level 5%

方法2: 第2种方法建立非预期损失: σ_U

◆ Capital Structure (第一种方法: $UL_P - EL$)

➤ Steps to Derive Economic Capital for Credit Risk

- Expected Losses (EL)
- Unexpected Losses (UL-Standalone)
- Unexpected Loss Contribution (ULC)
- Economic Capital

① 如果2 assets:

$$UL_P^2 = UL_1^2 + UL_2^2 + 2\rho \cdot UL_1 \cdot UL_2.$$

$$\sigma_U^2 = \sigma_1^2 + \sigma_2^2 + 2\rho \cdot \sigma_1 \cdot \sigma_2.$$

• Where σ_{LR} = standard deviation of the loss rate LR

$$(UL_1)^2 + (UL_2)^2 + 2\rho \cdot \sigma_1 \cdot \sigma_2 \cdot \rho_{PD}.$$

σ_{PD}^2 = standard deviation of the default probability PD

$$\sigma_{PD}^2 = PD \times (1 - PD)$$

⇒ 组合的经济资本 economic capital:

$$EC_P = UL_P \times CM.$$

capital multiplier.

◆ Capital Structure

➤ Unexpected Loss Contribution ULC.

$$(边际贡献). \quad ULMC_i = \frac{\partial UL_P}{\partial UL_i} = \frac{1}{2UL_P} \times \frac{\partial (UL_P^2)}{\partial UL_i}$$

$$= \frac{1}{2UL_P} \times \frac{\partial (\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} UL_i UL_j)}{\partial UL_i} = \frac{\sum_{j=1}^n UL_j \rho_{ij}}{UL_P}$$

$$ULMC_i = \frac{\partial UL_P}{\partial UL_i}$$

$$\Rightarrow ULMC_i = \frac{UL_1 + \rho UL_2}{UL_P}.$$

$$\Rightarrow ULC_i = ULMC_i \times UL_i.$$

$$\Rightarrow ULC_i = \frac{UL_1^2 + \rho \cdot UL_1 \cdot UL_2}{UL_P}.$$

$$\Rightarrow UL_P = UL_1 + UL_2.$$

$$ULC_1 \times CM$$

$$UL_P = \sum_{i=1}^n ULMC_i \times UL_i$$

$$ULC_i = ULMC_i \times UL_i = \frac{\sum_{j=1}^n UL_j \times \rho_{ij} \times UL_i}{UL_P}$$

$$\text{①计算 } ULMC_i = \frac{UL_1 + \rho UL_2}{UL_P}$$

$$ULMC_1 = \frac{UL_1 + \rho UL_2}{UL_P} \quad ULMC_2 = \frac{UL_2 + \rho UL_1}{UL_P}$$

• 3个资产:

$$ULMC_i = \frac{UL_1 + \rho_1 \cdot UL_2 + \rho_2 \cdot UL_3}{UL_P}.$$

$$ULC_i = ULMC_i \times UL_i$$

➤ Economic Capital

- As defined previously, the amount of economic capital needed is the distance between the expected outcome and the unexpected outcome at a certain confidence level.

- Unexpected loss is translated into economic capital for credit risk in three steps:

✓ First, the standalone unexpected loss is calculated.

✓ Then, the contribution of the standalone UL to the UL of the bank portfolio is determined.

✓ Finally, this unexpected loss contribution (ULC) is translated into economic capital.

Credit Decision and Analyst

- Key Differences between the Analysis of the Creditworthiness of consumers, versus that of nonfinancial and financial firms.

- For individuals, income and net worth provide the fundamental criteria and credit score are used to measuring the capacity to meet obligations.
- For nonfinancial companies, liquidity, cash flow together with earnings capacity and profitability, and solvency or capital position are used.
- Similar data is used for financial firms in addition to some bank-specific measures such as capital adequacy and asset quality. Moreover, qualitative analysis is also important for financial firms. (cash flow 不重要) liquidity management.
- Sovereigns: more subjective (主权国家)

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Exercise 1



- Given a portfolio with a notional value of \$1,000,000 with 50 positions, each of which has a default probability of 2% and a recovery rate of 0. Each position is an obligation from the same obligor so that the credit portfolio has a default correlation equal to 1. What is the credit value at risk at the 99% confidence level for this credit portfolio? $\rho = 1$ 看为 single asset

- \$0
- \$1,000
- \$20,000
- \$980,000

Answer: D

$$\begin{array}{l} 1000,000 \\ 2\% \text{ (default)} \rightarrow 0 \\ 98\% \text{ (no default)} \rightarrow 1000,000 \end{array}$$

$$C_{50}^0 (98\%)^{50} = 0.07694 = 7.694\% \text{ of } 100,000$$

$$EL = \frac{PD}{2\%} \times LR \times 1000,000 = \underline{120,000}$$

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Exercise 2



- Given a portfolio with a notional value of \$1,000,000 with 50 credits, each of which has a default probability of 2% and a 0 recovery rate, the default correlation is 0. Suppose each credit is equally weighted. The number of defaults can be treated as binomially distributed with $n = 50$ and $p = 0.02$, and the 95th percentile of the number of defaults based on this distribution is 3. What is the credit VaR at the 95% confidence level?

- \$20,000
- \$40,000
- \$60,000
- \$80,000

Answer: B

$$C_{50}^2 = \frac{50 \times 49}{2} = 1225$$

$$C_{50}^0 \cdot 0\% \cdot (98\%)^49 = 0.3716$$

$$3 \times 1000,000 \times \frac{3}{50} = WCL$$

$$C_{50}^0 \times (98\%)^{50} = 0.3641$$

$$\frac{C_{50}^2 \times (98\%)^{48} (2\%)}{1225}$$

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◆ Exercise 3



- Each of the following is true about key features of credit analysis with respect to borrower type, except which is not true?
 - ✓ A. Individuals: Credit analysis is amenable to automation and the use of scoring models and statistical tools to correlate risk to limited number of variables.
 - ✓ B. Non-financial firms: more detailed and "hands-on"; key variables include liquidity, cash flow, earnings capacity and profitability, solvency or capital position.
 - C. Financial firms: qualitative analysis and asset quality are ~~not~~ important, but cash flow is a highly important (a "key indicator").
 - D. Sovereigns: Includes analysis of country risk, which is primarily political dynamics and state of the economy; and systematic risk, which includes the regulatory regime and the financial system.

➤ Answer: C

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Credit Risk Measurement

Topic 1: Probability of Default

1. Basic Approaches used to Predicting Default
2. Rating System
3. Measurement from Market Prices
4. Exponential Distribution ? *Bond*
5. Single Factor Model *Equity.*

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◆ Basic Approaches used to Predicting Default

➤ Experts-Based, Statistical-based and Numerical Approaches

- Experts-Based *Rating* (启发的)

- Statistical-Based

- Heuristic and Numerical Approach (AI)

➤ Structural Approaches and Reduced-Form Approaches

- Structural Approaches: based on economic and financial theoretical assumptions describing the path to default. Model building is an estimate of the formal relationships that associate the relevant variables of the theoretical model. (e.g., Merton) *Capital*

- Reduced-Form Approaches: the final solution is reaches using the most statistically suitable set of variables and disregarding the theoretical and conceptual causal relations among them.

(*纯统计模型*). *Z-score*

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①直接使用

◆ Rating System ②借鉴计算

➤ Key Features of a Good Rating System (判断是否能用)

- ① Measurability and Verifiability
- ③ Objectivity and Homogeneity ④
- Specificity ⑤ ①评级 \rightarrow PD.
- ② 市场数据验证
- ③ 反应信用风险相关。
- ④ 可比 (同质性).
- ⑤ 独立 (特异性).

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◆ Rating System

➤ Rating Agencies' Assignment Methodologies

- Three competitors have different rating definitions.
- ① Moody's releases mainly issues ratings and far less issuers' rating. (产品的评级)
- ② S&P concentrates on providing a credit quality valuation referred to the issuer, despite the fact that the counterparty could be selectively insolvent on public listed bonds or on private liabilities. (发行主体的评级)
- ③ Fitch adopts an intermediate solution, offering an issuer rating, limited to the potential insolvency on publicly listed bonds, without considering the counterparty's private and commercial bank borrowings.
- Rating released by the three international rating agencies are not directly comparable.

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◆ Rating System ①累积PD(历史数据)

② Transition Matrix

- Rating agencies also assess changes in ratings. Probability estimates are summarized in transition matrices, which show the estimated likelihood of a rating change for a company within a specified time period.

		Final Rating Class (%)									
		Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	Default	WR
Initial Rating Class	Aaa	89.1	7.1	0.6	0.0	0.0	0.0	0.0	0.0	0.0	3.2
	Aa	1.0	87.4	6.8	0.3	0.1	0.0	0.0	0.0	0.0	4.5
	A	0.1	2.7	87.5	4.9	0.5	0.1	0.0	0.0	0.0	4.1
	Baa	0.0	0.2	4.8	84.3	4.3	0.8	0.2	0.0	0.2	5.1
	Ba	0.0	0.1	0.4	5.7	75.7	7.7	0.5	0.0	1.1	8.8
	B	0.0	0.0	0.2	0.4	5.5	73.6	4.9	0.6	4.5	10.4
	Caa	0.0	0.0	0.0	0.2	0.7	9.9	58.1	3.6	14.7	12.8
	Ca_C	0.0	0.0	0.0	0.0	0.4	2.6	8.5	38.7	30.0	19.8
		Source: Moody's (2008).									

Source: Moody's (2008).

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◆ Rating System 评估 rating system.

➤ Measurement of PD in Rating System

① Cumulative Default Probability C. (累积指标)

✓ Probability that a borrower will default over a specified multi-year period.

$$PD_k^{\text{cumulated}} = \frac{\text{Def}_t}{\text{Names}_t}$$

累积违约数
期初存活着数

✓ Names: the number of issuers

Def: the number of names that have defaulted in the time horizon

② Marginal Default Probability MDP_{t,T} (某段时间)

Probability that a borrower will default in any given year.

$$PD_k^{\text{marg}} = PD_{t+k}^{\text{cumulated}} - PD_t^{\text{cumulated}}$$

- $MDP_{0,1} = C_1$
- $MDP_{1,2} = C_2 - C_1 = (1-d_1) d_2$

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联合概率率 < 前期不违约
后期违约

cumulative.

$$C_1 = d_1 = MDP_{0,1}$$

$$C_2 = d_1 + (1-d_1)d_2.$$

$$MDP_{1,2} = C_2 - C_1$$

$$= (1-d_1)d_2. \quad (\text{联合概率率})$$

◆ Rating System

➤ Measurement of PD in Rating System (cont'd)

③ Forward Probability (Contingent to the Survival Rate)

$$PD_{t+k}^{\text{Forw}} = \frac{\text{Def}_{t+k} - \text{Def}_t}{\text{Names survived}_t}$$

当期违约数
当期期初存活数

④ Survival Rate $S_t = (1-d_1)(1-d_2)$

Probability that a borrower will not default over a specified multi-year period.

$$SR_{t+k}^{\text{Forw}} = (1 - PD_{t+k}^{\text{Forw}}) (1 - PD_t^{\text{cumulated}}) = \prod_{i=1}^t SR_i^{\text{Forw}}$$

$S_1 = (1-d_1)$
 $S_2 = (1-d_1)(1-d_2)$

⑤ Annualized Default Rate (ADR) (平均).

- ✓ If it is necessary to price a credit exposed transaction on a five year time horizon, it is useful to reduce the five-year cumulated default rate to an annual basis for the purposes of calculation.

$$(1 - PD_t^{\text{cumulated}}) = \prod_{i=1}^t SR_i^{\text{Forw}} = (1 - ADR_i)^t$$

$$(1 - PD_t^{\text{cumulated}}) = e^{-ADR \times t}$$

按年累积 $S_2 = 1 - C_3 = (1 - ADR)^3$

按每分钟累积: $S_3 = 1 - C_3 = e^{-(ADR)^3}$

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◆ Measurement from Market Prices - Bond

用市场价格来估算 PD.

➤ Risk-Neutral Probability of Default

假设: zero coupon, T=1, face value=1, YTM未知.

$$P(YTM) = \frac{PD \times RR + (1-PD) \times 1}{1+R_f}$$

精确式 ①

$$PD = \frac{1}{1+R_f} \left(1 - \frac{1+R_f}{1+YTM} \right) \quad ②$$

$\Rightarrow PD \times R_f \approx YTM - R_f = \text{credit spread}$ 近似式 ③

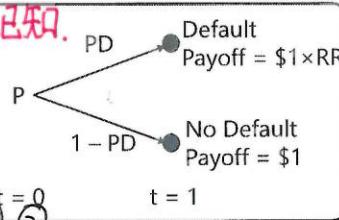
1) 旧时, 用①式

2) face value ≠ 1, 用①②

3) 付息, 用③式

4) YTM未知, market price 已知, 试 ③

5) YTM与CS Price未知, CS已知 ②. 注: 用③式期限调整.



$$\begin{aligned} p &= \frac{\$1}{1+YTM} = \frac{\$1 \times PD \times RR + \$1 \times (1-PD)}{(1+R_f)} \\ \Rightarrow PD &= \frac{1}{LGD} \frac{[YTM - R_f]}{1+YTM} \\ \Rightarrow YTM - R_f &\approx PD \times LGD \end{aligned}$$

近似式

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股票价格法 { Merton
KMV } (补充).

◆ Measurement from Market Prices - Equity

- **Merton Model** Distance to default: ① 股东 E: $V > k$ $V - k \} \max(V - k, 0)$.
Bondholder d_2 . $V < k$ 0.

✓ D = Risk-free debt - put option on firm

$$\text{Debt} = VN(-d_1) + Ke^{-rT}N(d_2)$$

● Shareholder

✓ E = call option on firm

$$\text{Equity} = VN(d_1) - Ke^{-rT}N(d_2)$$

● PD

✓ $N(-d_2)$ is the probability of default

✓ Risk-Neutral vs. Real-World: Risk-Free Rate vs. Asset Drift

✓ Merton-KMV

● Credit Spread

$$\text{Credit Spread} = -\left(\frac{1}{T-t}\right) \ln\left(\frac{D}{F}\right) - r$$

$$d_2 = \frac{\ln\left(\frac{V}{k}\right) + rT - \sigma^2 T}{\sigma\sqrt{T}}$$

$$E = VN(d_1) - Ke^{-rT}N(d_2) \quad (\text{RF})$$

$$d_{1,2} = \frac{\ln\left(\frac{V}{k}\right) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \pm \frac{\sigma\sqrt{T}}{2}$$

- ③ Bondholder D: $V > k$, $k \min(V, k) = k - \max(k - V, 0)$
 $V < k$, V

Put, 标的 V, 执行价 k.

$$D = VN(1-d_1) + ke^{-rT}N(d_2) \quad (\text{RF})$$

$$= V - \text{Equity} \Rightarrow \text{Debt} = ke^{-rT} - \text{Put}$$

- ③ BSM假设: $\Rightarrow V \sim \text{lognormal}$

⇒ 欧式期权, 可以分析提前违约).

⇒ 连续过程 (不可以解释突发的违约).

◆ Measurement from Market Prices - Equity

- **Moody's KMV Model** 假设) 分 LT, ST,

$$DD = \frac{V - K}{\sigma_V}$$

firm's value
↓
value of short-term liabilities
+ some fraction of long-term debt value
volatility of firm's value

● KMV is Merton-based, but abandons the $PD = N(-DD)$ in favor of $PD = \text{historical default rate corresponding to DD}$.

● KMV tweaks the default threshold from total face value of debt to all short-term plus some fraction of long-term debt.

✓ Default = ST + 50% × LT, if $LT/ST < 1.5$.

✓ Default = ST + $(0.7 - 0.3 \times ST/LT)LT$, if otherwise.

计算 DD ⇒ 计算 DD + 历史数据 ⇒ PD.

$$k = ST + 0.5LT \quad \left(\frac{LT}{ST} < \frac{3}{2}\right)$$

$$= 0.75LT + 0.75LT \quad \left(\frac{LT}{ST} > \frac{3}{2}\right)$$

◆ Exponential Distribution

- The exponential distribution is often used to model the time it takes a company to default. The cumulative default time distribution $F(t)$ represents the probability of default over $(0, t)$:

$$P(t^* < t) = F(t) = 1 - e^{-\lambda t}$$

➤ The survival distribution is:

$$P(t^* > t) = 1 - F(t) = e^{-\lambda t}$$

➤ The rate parameter λ measures the rate at which it will take an event to occur. In the context of waiting for a company to default, it is known as the hazard rate and indicates the rate at which default will arrive.

➤ Example: For a company with a five-year spread of 300 bps, with a recovery rate R = 0.40, we have hazard rate of:

$$\lambda \approx \frac{0.03}{1 - 0.4} = 0.05$$

④ Stakeholder: 股东 E $V > k$

call, 标的 V, 执行价 k $P(V > k) = N(d_2)$

$$\Rightarrow P(V > k) = 1 - N(d_2)$$

default Pro

→ k: default point

→ d_2 : distance to default.

→ 用 RF 计算: risk-neutral PD.

用 asset return: physical PD.

$$\Rightarrow d_2 = \frac{\ln\left(\frac{V}{k}\right) - \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$

近似估计: $\text{if } T=1, d_2 \approx \frac{\ln V - \ln k}{\sigma\sqrt{t}}$

$$PD = N(-d_2)$$

$d_2 \uparrow \Rightarrow -d_2 \downarrow \Rightarrow PD \downarrow$.

⑤ credit spread 估计:

Merton: $\rightarrow \text{Debt Value} = D$

face value = F.

$$D = Fe^{-rT}f(t) (T-t)$$

$$CS = -\frac{1}{T-t} \ln\left(\frac{D}{F}\right) - rT$$

$$P(X=i) = \frac{\lambda^i e^{-\lambda t}}{i!}$$

$$P(X=t) = \frac{\lambda^t \cdot e^{-\lambda t}}{t!}$$

◆ Single Factor Model (简化相关性计算)

- The firm's asset return is represented as a function of two random variables: the return on a "market factor" m that captures the correlation between default and the general state of the economy, and a shock ε capturing idiosyncratic risk. Assume that m and ε are standard normal variates, and are not correlated with one another. Under these assumptions, α is a standard normal variate:

(资产收益的计算)

$$\alpha = \beta m + \sqrt{1 - \beta^2} \varepsilon$$

$$E(\alpha) = 0; \text{Var}(\alpha) = \beta^2 + 1 - \beta^2 = 1$$

$$\rho_{1,2} = \frac{\text{Cov}(\alpha_1, \alpha_2)}{\sigma_{\alpha_1} \cdot \sigma_{\alpha_2}} \Rightarrow \alpha_1 = \beta_1 m + \sqrt{1 - \beta_1^2} \varepsilon_1$$

$$\alpha_2 = \beta_2 m + \sqrt{1 - \beta_2^2} \varepsilon_2.$$

$\alpha \sim \text{Normal distribution}$

$$E(\alpha) = \beta E(m) + \sqrt{1 - \beta^2} E(\varepsilon) \quad P_{1,2} = \frac{\text{Cov}(\alpha_1, \alpha_2)}{\sigma_{\alpha_1} \cdot \sigma_{\alpha_2}} = \beta_1 \cdot \beta_2. \quad 28-106$$

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$$\Rightarrow E(d) = 0$$

$$\sigma(\alpha) = 1.$$

◆ Single Factor Model

- Unconditional default distribution is a standard normal distribution.
- Both the market factor and the idiosyncratic shocks are assumed to have unit variance, the beta of each credit i to the market factor is equal to β_i .
- The correlation between the asset returns of any pair of firms i and j is $\beta_i \beta_j = \rho_{ij}$

- Conditional distribution given m is a normal distribution.

- With a mean of $\beta_i \bar{m}$ and a standard deviation of $\sqrt{1 - \beta_i^2}$

$$p = \Phi \left(\frac{K_i - \beta_i \bar{m}}{\sqrt{1 - \beta_i^2}} \right) \quad i = 1, 2, \dots$$

if $\alpha \sim N(\mu, \sigma^2)$.

$$y = \frac{x - \mu}{\sigma} \sim N(0, 1).$$

$$\Rightarrow PD = P(\alpha < K) = P\left(\alpha < \frac{K - \beta \bar{m}}{\sqrt{1 - \beta^2}}\right). \quad 29-106$$

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◆ Exercise 1



- A credit analyst at a bank has been asked to produce an exposure analysis for three of the loans in the bank's portfolio. Loan information assembled by the analyst as well as the bank's internal default

Loan	Tenor (Years)	Notional (USD)	Loss Given Default	S&P Rating	PD
1	2	30,000,000	X 0.75	X BB	0.1
2	3	100,000,000	X 0.90	X A	0.03
3	1	100,000,000	X 0.70	X B+	0.05

Probability of Default (PD)			
	Tenor (Years)		
Loan Quality	1	2	3
Investment Grade	0.01	0.02	0.03
Non-Investment Grade	0.05	0.10	0.20

◆ Exercise 1

- There is no collateral provided by the borrower for these loans, so the analyst uses the notional amount provided above as the Exposure at Default. Which of the following correctly orders the expected loss for each loan from lowest to highest?
- A. Loan 1 < Loan 2 < Loan 3 $E_L = PD \times LR \times EA = \text{本金} \times \text{loss rate}$
- B. Loan 1 < Loan 3 < Loan 2
C. Loan 2 < Loan 3 < Loan 1
D. Loan 2 < Loan 1 < Loan 3
- Answer: A

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金融 · 创新 · 债值

◆ Exercise 2

- The 1-year risk-free rate is 4%, and the yield on a 1-year zero-coupon corporate bond is 7% per year. Assuming a recovery rate of zero, what is the implied probability of default?
- A. 2.80%
B. 3.23%
C. 11.00%
D. 11.28%
- $R_f = 4\%$ $YTM = 7\%$ $RR = 0\%$
- $\text{Bond: } T=1, \text{零息}$
- $$\Rightarrow PD = \frac{1}{1+R_f} \left(1 - \frac{1+R_f}{1+YTM} \right) = 1 \times \left(1 - \frac{1+4\%}{1+7\%} \right) \rightarrow 4\%$$
- 用债券法: $\frac{1}{1+R_f} \left(1 - \frac{1+R_f}{1+YTM} \right)$
- Answer: A

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金融 · 创新 · 债值

◆ Exercise 3

- C. Let firm value equal \$1 billion with face value of debt equal to \$800 million. The debt is zero-coupon and matures in four years ($T = 4$). The riskless rate is 5%. The estimate of the volatility of the firm is 20% per annum. The firm's assets are expected to grow at 10% per annum. What does the Merton model return for the value of the firm's equity?
- $V = \$1,000 \text{ m.}$ $D = \$800 \text{ m.}$
- $(N(1.25786) = 0.8958; N(0.85786) = 0.8045)$ $\text{Equity} = V N(d_1) - e^{-rT} \cdot k N(d_2)$
- A. \$200 million
B. \$330 million
 C. \$369 million
D. \$399 million
- $d_1, d_2 = \frac{\ln(\frac{V}{D}) + e^{-rT}}{6 \times \sqrt{T}} + \frac{20\% \times T}{\sqrt{T}} \Rightarrow d_1 = 1.2579, d_2 = 0.8579$
- $D = V + E \Rightarrow E = D - V$
- $\text{physical PD} \Rightarrow \text{asset return.}$
- $\text{risk neutral PD} = N(-d_2) = 1 - N(d_2)$
- Answer: C

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◆ Exercise 4



- The capital structure of HighGear Corporation consists of two parts: one 5-year zero-coupon bond with a face value of USD 100 million and the rest is equity. The current market value of the firm's asset (MVA) is USD 130 million and the risk-free rate is 5%. The firm's assets have an annual volatility of 30%. Assume that firm value is log-normally distributed with constant volatility. The firm's risk management division estimates the distance to default (in terms of number of standard deviations) using the Merton Model, or

$$\frac{\ln\left(\frac{FV_B}{MVA}\right) - \left(\delta - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A T^{0.5} \sqrt{T}} \quad Rf \geq N(-d_2) = 1 - N(d_2).$$

Given the distance to default, the estimated risk-neutral default probability is: $N(1.9191) = 0.9724 \Rightarrow DD = d_2 \Rightarrow d_2 = 1.9191$

- A. 2.74% B. 12.78% C. 12.79% D. 30.56%

- Answer: A

$$= -\left(\frac{\ln\left(\frac{FV_B}{MVA}\right) - \left(\delta - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A T^{0.5} \sqrt{T}} - \frac{6}{5}\right).$$

敬业·创新·增值

◆ Exercise 5



- An analyst is using Moody's KMV model to estimate the distance to default of a large public firm, Shoos Inc., a firm that designs, manufactures and sells athletic shoes. The firm's capital structure consists of USD 40 million in short-term debt, USD 20 million in long-term debt, and there are one million shares of stock currently trading at USD 10 per share. The asset volatility is 20% per year. What is the normalized distance to default for Shoos Inc.?

$$A. 0.714 \quad ① L/T / S/T < 1.5 \Rightarrow k = S/T + L/T = 50$$

$$\checkmark B. 1.430 \quad ② E = 10m.$$

$$C. 2.240$$

$$D. 5.000 \quad ③ PD = \frac{V-k}{6t\%) \times V} = \frac{(10 - 50)}{20\% \times 70} = 1.4286$$

- Answer: B

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敬业·创新·增值

◆ Exercise 6



- A firm has an value of \$400 million with expected return of 14% per annum and volatility of 36% per annum. The firm's only debt is a short-term zero-coupon bond with face value of \$300 million due in one year. The riskless rate is 4%. Which is nearest to the firm's (normal returns-based) distance to default when derive the physical PD?

$$A. 1.0 \quad DD = d_2 \text{ if Merton.}$$

$$B. 2.7$$

$$C. 3.3 \quad (\text{用 } 14\%)$$

$$D. 8.5 \quad \frac{\ln\left(\frac{V}{F}\right) - \left(r_f - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

- Answer: A
- $$d_2 = \frac{\ln\left(\frac{V}{F}\right) - \left(r_f - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

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V: lognormal

↓
Merton

$\Rightarrow DD = d_2$

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敬业·创新·增值

◆ Exercise 7



- An analyst estimates that the hazard rate for a company is 0.1 per year.

The probability of survival in the first year followed by a default in the second year is closest to:

$$MDP_{1,2} = C_2 - C_1 = (1 - e^{-0.1 \times 2}) - (1 - e^{-0.1 \times 1}).$$

- A. 8.61%
- B. 9.00%
- C. 9.52%
- D. 19.03%

- Answer: A

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商业·金融·增值

◆ Exercise 8



- A single firm has a beta of 0.6 and a $\kappa = -1.645$. The firm's unconditional default probability is therefore 5%. If we enter an economic downturn, such that the market factor (m) shifts to a value of -1.41, what is the economic-downturn conditional default probability?

$$B=0.6 \rightarrow \text{临界值 } P(\alpha < 1.645) = 5\%$$

- A. 7.83%
- B. 10.67%
- C. 15.90%
- D. 22.75%

- Answer: C

$$M\beta + \sqrt{1-\beta^2}\varepsilon$$

$$\alpha' \sim N(\bar{\mu}, \frac{-1.645 - \beta\bar{m}}{\sqrt{1-\beta^2}})$$

$$\alpha \sim N(\beta\bar{m}, \sqrt{1-\beta^2})$$

$$P(\alpha' < -1.645) = P(\alpha' < \frac{-1.645 - \beta\bar{m}}{\sqrt{1-\beta^2}}).$$

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商业·金融·增值

Credit Risk Measurement

Topic 2: Counterparty Exposures

1. Exposure Metrics
2. Exposure Profiles of Different Security Types

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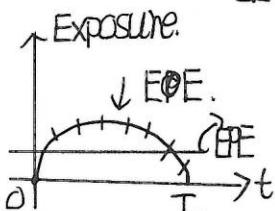
商业·金融·增值

◆ Exposure Metrics

➤ Current Exposure $\max(V, 0)$

- Larger of zero and the market value of a transaction or portfolio of transactions within a netting set, with a counterparty that would be lost upon the default of the counterparty, assuming no recovery on the value of those transactions in bankruptcy.

EE. ➤ Expected Exposure (每个关键时点上, 尚敞口的平均水平)



EE: ① 到期归0.
② Exposure 可何下.

- The mean (average) of the distribution of exposures at any particular future date before the longest-maturity transaction in the netting set matures.

➤ Expected Positive Exposure (EPE)

- The weighted average over time of expected exposures where the weights are the proportion that an individual expected exposure represents of the entire time interval.

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⇒ Effective EE, Effective EPE

◆ Exposure Metrics

➤ Negative Exposure 占在我的角度, 对于所面对的 Exposure.

- The exposure from a counterparty's point of view. Define measures such as negative expected exposure (NEE) and expected negative exposure (ENE), which are the precise opposite of EE and EPE.

↓ 对应EE
↓ 对应EPE.

➤ Peak Exposure/Potential Future Exposure

- A high-percentile (typically 95% or 99%) of the distribution of exposures at any particular future date before the maturity date of the longest transaction in the netting set. (一定置信水平下的平均敞口).

➤ Maximum PFE 整个存续期间最大的 PFE.

- Represents the highest PFE value over a given time interval, thus representing the worst-case exposure over the entire interval.

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◆ Exposure Metrics

➤ Effective Expected Positive Exposure (更接近于短期敞口).

- Measures such as EE and EPE may not capture properly roll-over risk.
- Effective EE is simply a non-decreasing EE.
- Effective EPE is the average of the effective EE.

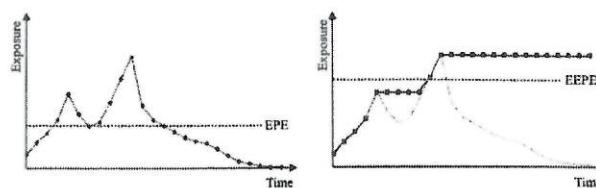


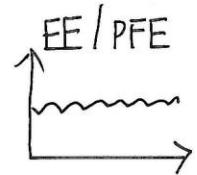
FIGURE 12-7 Illustration of effective EE and effective EPE.

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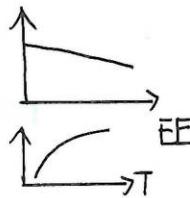
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◆ Exposure Profiles of Different Security Types

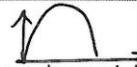
➤ **Bonds:** Typically pay a fixed rate and therefore will have some additional uncertainty since, if interest rates decline, the exposure may increase.



➤ **Loans:** Exposure may decline over time due to the possibility of prepayments. (可以提前偿付)



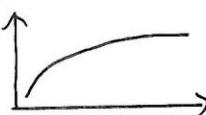
➤ **Forward:** Simple increasing function reflecting the fact that, as time passes, there is increasing uncertainty about the value of the final exchange.



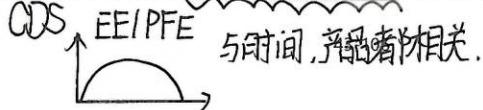
➤ **Interest Rate Swap:** Characterized by a peaked shape arises from the balance between future uncertainties combined with the roll-off of swap payments over time. (尚如与期间现金流交换)



➤ **Cross-Currency Swap:** monotonically increasing exposures results mainly from the uncertainty regarding the final notional payment. (Currency swap 外币互换有本金交换)



➤ **Credit Derivatives:** hard to characterize due to discrete payoffs.



专业·创新·增值

Credit Risk Measurement

交易对手风险

Topic 3: Pricing Counterparty Risk (基于场外衍生品) OTC derivatives.

1. Challenges of Pricing Counterparty Risk CVA, BCVA, Assumption: no wrong way risk.
2. CVA and DVA
3. Wrong-Way Risk and Right-Way Risk

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◆ Challenges of Pricing Counterparty Risk

CVA

BCVA

➤ Challenges of Pricing Counterparty Risk

- More recently, accounting requirements have meant that CVA has become defined via an "exit price" concept and computed with market-implied (risk-neutral) parameters. (脱手价格).
- The use of market-implied default probabilities may be questioned for a number of reasons:

- ✓ Market-implied default probabilities are significantly higher than their real-world equivalents;
- ✓ A default cannot, in general, be hedged since most counterparties do not have liquid single-name credit default swaps referencing them;
- ✓ The business model of banks is generally to 'warehouse' credit risk, and therefore they are only exposed to real-world default risk. (没有脱手).

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◆ CVA and DVA

➤ Credit Value Adjustment

- Credit valuation adjustment refers to the pricing of counterparty risk.
- Standard equation for CVA is:

• collateral \Rightarrow exposure ↓.

$$CVA = LGD \cdot \sum_{i=1}^m EE(t_i) \times PD(t_{i-1}, t_i) \xrightarrow{\text{marginal default}} \text{Probability}$$

- CVA as a Spread:
(近似)

$$CVA = EPE \times \text{Spread}$$

- ✓ The formula assumes that the EE is constant over time and equal to its average value (EPE).

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◆ CVA and DVA

➤ Incremental and Marginal CVA

● Incremental CVA

- ✓ Change (or increment) in CVA that a new trade will create, taking netting into account.

● Marginal CVA

- ✓ By using a marginal CVA measure, it will be possible to break down a CVA for any number of netted trades into trade level contributions that sum to the total CVA.
- ✓ The calculation is identical to that for the standalone CVA, except for the substitution of marginal EE for initial EE.
- ✓ This metric is an appropriate way to calculate the trade-level CVA contributions at a given time.

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◆ CVA and DVA



➤ Definition of DVA

- Key assumption above was that the party making itself could not default.
- Debt Value Adjustment (DVA) represents counterparty risk from the point of view of a party's own default.

➤ Definition of BCVA

- Consideration of a party's own default, together with that of its counterparty, leads to bilateral CVA (BCVA).

$$BCVA = LGD_C \sum_{i=1}^m EE(t_i) \times S_P(t_{i-1}) \times PD_C(t_{i-1}, t_i) \xrightarrow{\text{CVA}}$$

$$- LGD_P \sum_{i=1}^m NEE(t_i) \times S_C(t_{i-1}) \times PD_P(t_{i-1}, t_i) \xrightarrow{\text{DVA}}$$

① 标准

- Estimating BCVA as a Spread: $BCVA = EPE \times \text{Spread}_C - ENE \times \text{Spread}_P$

② 近似

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◆ CVA and DVA

①

➤ Impact of Credit Spread $CS \propto PD \times LR$ (CS 与 PD 之间同向变动) $\Rightarrow CVA \uparrow$

- CVA generally increases with increasing credit spread.
- The upwards-sloping curve gives the largest value, mainly due to having the largest extrapolated credit spread. The inverted curve gives the smallest CVA for the opposite reason.

downward-sloping.

➤ Impact of Recovery Rate

②

- If $LGD_{actual} = LGD_{mkt}$, changing LGD has a reasonably small impact on CVA since there is a cancellation effect: increasing LGD reduces the market-implied default probability but increases the loss in the event of default.

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◆ CVA and DVA

③

➤ Impact of Margin

- The impact of margin on CVA follows directly from the assessment of the impact of margin on exposure. The influence of margin on the standard CVA formula is straightforward: Margin only changes EPE, and hence the same formula may be used with EPE based on assumptions of collateralization.
- A threshold can be seen to be a negative initial margin and vice versa. The initial margin will reduce exposure – and therefore CVA/DVA. For high thresholds, CVA tends to the uncollateralized value, whilst for high initial margin it tends to zero.

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◆ Wrong-Way Risk and Right-Way Risk

➤ Wrong-Way Risk (WWR)

- Wrong-way risk is the phrase generally used to indicate an unfavorable dependence between exposure and counterparty credit quality—i.e., the exposure is high when the counterparty is more likely to default and vice versa. The presence of wrong-way risk will (unsurprisingly) increase CVA.

$EPE \uparrow, PDT \uparrow$

➤ Right-Way Risk (RWR)

- Right-way risk can also exist in cases where the dependence between exposure and credit quality is a favorable one. Right-way situations will reduce counterparty risk and CVA.

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◆ Wrong-Way Risk and Right-Way Risk

1) 对CS模拟 → PD
↓
历史相关性 ↓
RF → Exposure

2) PD分布

Exposure distribution
Copulas

⇒ exposure 分布不用额外建模
simple, functional relationship. ① 历史数据 ② 情景.

- Intensity Approach Simulation. ↗ interest rate.
- ✓ Introduce a stochastic process for the credit spread and correlate this with the other underlying processes required for modelling exposure.
- Structural Approach ↗ Copulas model (历史相关性)

specify a dependence directly between the counterparty default time and the exposure distribution.

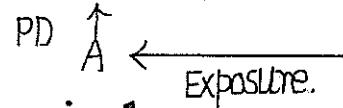
- Parametric Approach ⇒ 函数关系 Exposure = f(PD)

Linking the default probability parametrically to the exposure using a simple, functional relationship.

- Jump Approaches (跳跃).

First proposed to model FX exposures with WWR. This assumes that the relevant FX rate jumps at the counterparty default time.

主权国家 default ⇒ 外汇汇率 jump.



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◆ Exercise 1



- CityBank enters a long position in an over-the-counter (OTC), out-of-the-money (OTM) put option with a five year term. The strike price of the put is \$50 while the current asset price is \$70 with asset volatility of 30%. The risk-free rate is 4% with continuous compounding. $N(d_1) = 0.87$ and $N(d_2) = 0.68$. CityBank assumes the present-valued expected exposure to the counterparty equals the option's present value. The probability of default by the counterparty is 8% with loss given default of 75%. Which is nearest to the credit risk-adjusted value of the long option position, where credit risk-adjusted refers to incorporating an approximate credit valuation adjustment (CVA)?

A. \$3.76

B. \$4.25

C. \$6.99

D. \$8.51

- Answer: A

$$\text{Put} = Ke^{-rT} N(d_2) - SN(-d_1) = 4 \text{ (long put).}$$

$2 \cdot CVA = LR \times PD \times PVEE$ Put.

$$75\% \times 8\% \times 4 = 0.24$$

专业·创新·诚信

$$3. \text{ CVA value } 4 - 0.24 = 3.76.$$

◆ Exercise 2



PD ↑ → Price ↓ , ↓ Exposure

- Which characterizes the exposure, to the option buyer, implied by a company that writes call options on its own stock?

A. Expected exposure

B. Right-way exposure

C. Wrong-way exposure

D. Credit risk mitigants

A long call on B B.

- Answer: B

Credit Risk Management

OTC derivative counterparty risk 简释
Topic 1: Mitigating Approaches

1. Netting → netting 时负为0.
2. Collateralization
3. Bilateral Margin Requirement
4. Termination and Other Terms

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◆ Netting

➤ Types of Netting (日间交易)

- **Payment netting:** Gives a party the ability to net cash flows occurring on the same day. This typically relates to settlement risk.
- **Close-out netting:** Cancellation of all transaction under the master agreement in the event of bankruptcy or other specified default event. The trades are then netted at market value.

➤ Influence Factor (P1, 最差)

Impact	Positive	Negative (P=1 最好)
Correlation	Lower netting benefits	Stronger netting benefits

➤ Netting Factor (两手交易)

$$\begin{aligned} \text{netting factor} &= \frac{\text{EE(netting)}}{\text{EE(no netting)}} \\ &= \frac{\sqrt{n + n(n-1)\bar{p}}}{n} \end{aligned}$$

结论: $\rightarrow 1$ (netting 效果差)
 $\rightarrow 0$ (netting 效果好)

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◆ Collateralization

➤ Provision

- **Haircut:** The amount of the collateral will exceed the funds owed by an amount known as the haircut. (资产的折扣率)
- **Threshold:** The level of exposure below which collateral will not be called and only the incremental amount above the threshold would be collateralized. if $\text{Exposure} > \text{threshold}$, 及 $\text{Exposure} - \text{threshold}$.
- **Minimum Transfer Amount:** The smallest amount of collateral that can be transferred. if $\text{Exposure} > \text{threshold} + \text{MTA}$, 及 $\text{Exposure} - \text{threshold} > \text{MTA}$.
- **Independent Amount:** an additional collateral requirement independent of the exposure. initial margin.
- **Rounding 取整:** collateral.



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Gain Loss 专业·创新·诚信

◆ Collateralization

- Example: Let us consider a collateral calculation assuming a two-way CSA with the threshold, minimum transfer amount and rounding equal to \$1,000,000, \$100,000 and \$25,000, respectively.

	Collateral Calculation		Collateral Calculation
Portfolio Value	\$1,754,858	Portfolio Value	\$1,623,920
Collateral held	-1000,000	Collateral held	\$775,000
Required Collateral	\$754,858	Required Collateral	-\$151,080
Above minimum transfer amount?	Yes	Above minimum transfer amount?	Yes
Rounded amount	\$775,000	Rounded amount	-\$150,000

(向下取整).

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◆ Collateralization

➤ Collateral and Funding

- The traditional role of collateral for bilateral OTC derivatives has been as a counterparty risk mitigants. However, there is another role of collateral, which is as provision of funding. (规定).
- Rehypothecation 用抵押 would seem to be obvious in OTC derivatives markets. From the point of view of funding, rehypothecation is important. However, from the point of view of counterparty risk, rehypothecation is dangerous since it creates the possibility that rehypothecated collateral will not be received in a default scenario.
- Segregation of collateral is designed to reduce counterparty risk and entails collateral posted being legally protected in the event that the receiving counterparty becomes insolvent.

credit exposure = value - margin.
(考虑 default)

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专业·创新·增值

◆ Collateralization

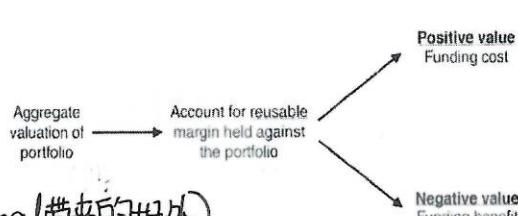
➤ Funding Exposure

(不考虑 default)

$$\text{funding} = \text{value} - \text{margin}$$

funding exposure.

(多收或少收 collateral 带来的好处)



➤ Differences Between Funding and Credit Exposure

- ① Definition of value. (credit : default 的 value) default 清算.
- ② MPoR. margin period of risk. (collateral)
- ③ Aggregation. (组合)
- ④ Wrong-way risk (WWR). default 与敞口 credit exposure 中有.
- ⑤ Segregation. 只与 credit 关系. funding (不利) 与 credit (有利) exposure 有关.

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credit exposure

(考虑 close-out)

netting)

④ Wrong-way risk (WWR). default 与敞口 credit exposure 中有.

⑤ Segregation. 只与 credit 关系. funding (不利) 与 credit (有利) exposure 有关.

⑥ Segregation. 只与 credit 关系. funding (不利) 与 credit (有利) exposure 有关.

◆ Bilateral Margin Requirement

➤ Bilateral Margin Requirements

- Uncleared margin requirements (UMR) (BCBS-IOSCO)
- on most major participants when transacting noncentrally-cleared derivatives with one another.
- Parties are required to meet strict delivery timing requirements for margin, in most cases requiring it to be provided within the same business day as the date of the calculation.
- The UMR apply to all OTC derivatives with the exception of FX swaps and forwards, which are exempt.

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◆ Bilateral Margin Requirement

➤ Bilateral Margin Requirements

- Standards state that covered entities for non-centrally-cleared derivatives must exchange:

① ✓ Variation Margin

- Must be exchanged bilaterally on a regular basis (e.g. daily).
- Full margin must be used (i.e. zero threshold).
- The minimum transfer amount must not exceed €500,000.
- Can be rehypothecated and netted. (可以被再抵押)
- Must be posted in full from the start of the rules.

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◆ Bilateral Margin Requirement

➤ Bilateral Margin Requirements

② Initial Margin

- To be exchanged by both parties with no netting of amounts.
- Should be based on an extreme but plausible move in the underlying portfolio value at a 99% confidence level. (貌似有理的)
- A 10-day time horizon should be assumed on top of the daily variation margin exchanged. ↗用自己的模型. ↗用监管模型
- Can be calculated based on internal (validated) models/regulatory tables.
- Must be exchanged on a gross basis (i.e. amounts posted between two parties cannot cancel), must be segregated and cannot be rehypothecated, repledged, or reused.
- Follows a phased-in implementation. (逐步展开)

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◆ Bilateral Margin Requirement

➤ Eligible Assets for Initial Margin

- Regarding the quality of initial margin, the margin should be:
 - ✓ Highly Liquid.
 - ✓ Hold its value in a stressed market (accounting for the haircut)
 - ✓ Risk-sensitive haircuts should be applied. 稳定
 - ✓ Margin should not be exposed to excessive credit, market, or FX risk.
 - ✓ Margin must not be "wrong-way" meaning correlated to the default of the counterparty. PD↑ → collateral↓ → exposure↑

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专业·创新·增值

◆ Termination and Other Terms

➤ Termination (终止条款). 不直接影响 exposure

● Additional Termination Event (ATE)

- ✓ Allows a party to terminate derivative transactions in certain situations.

● Break Clauses or Mutual Puts

- ✓ May be considered advantageous to attach such clause to a long-dated transaction; May be mandatory, optional or trigger-based, and may apply to one or both parties in a transaction.

➤ Other Terms

● Walkaway Feature

- ✓ Allows an institution to cancel transactions in the event that their counterparty defaults. It is benefit if a party has a negative MtM.
(对手方 default 时, 中止合约)

65-106

专业·创新·增值

◆ Exercise 1



➤ Why do not collateral agreements eliminate counterparty risks?

- A. Exposure may exist below threshold.
- B. Market movement between collateral exchange and default.
- C. Collateral depreciation during close-out period. ✓
- D. All of the above. 时常

➤ Answer: D

66-106

专业·创新·增值

Credit Risk Management

Topic 2: Credit Derivatives

1. Credit Default Swap CDS. \rightarrow default risk.
2. Total Return Swap
3. Credit Linked Note

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专业·创新·增值

Credit Default Swap (CDS)

\hookrightarrow 买方不需要持有 underlying asset.

➤ Characteristic of CDS

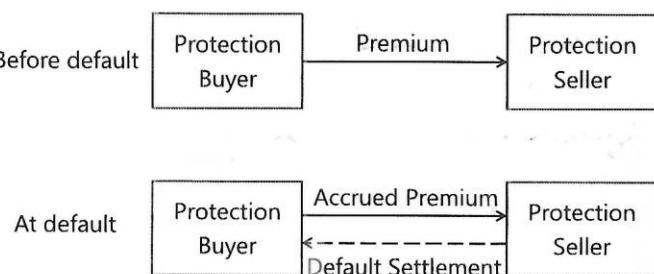


Illustration of a typical CDS contract on a single reference entity.

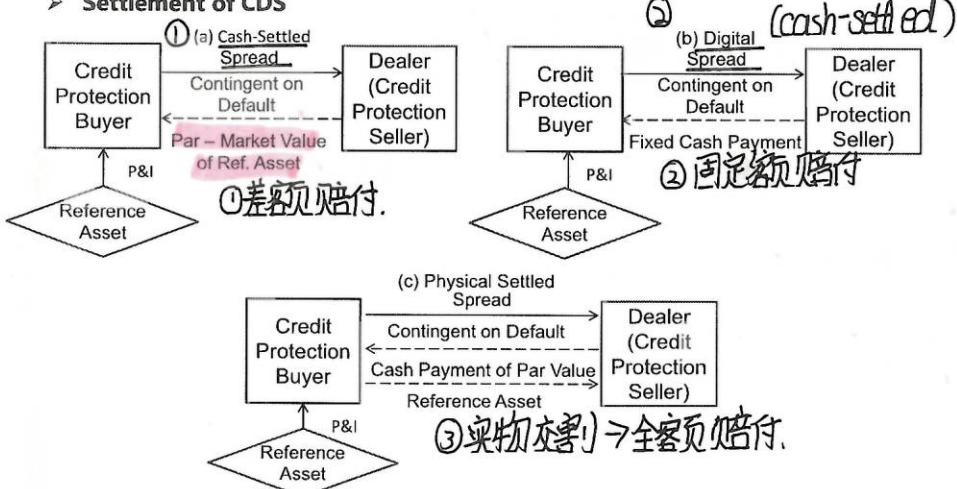
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专业·创新·增值

Credit Default Swap (CDS)

①② 无实物资产的转移.

➤ Settlement of CDS



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专业·创新·增值

{ Single-name CDS
Basket CDS

◆ Credit Default Swap (CDS)

➤ Basket CDS

- **Nth to default CDS:** An nth to default CDS pays off when the nth default occurs in the reference asset portfolio. For this kind of swap, whenever the nth default occurs in the reference basket, the buyer stops paying the premium and receives the difference of the principal amount of the latest (nth) defaulted entity and the recovered value.
- For example, a reference portfolio consists of bonds issued by 100 different companies.
- ✓ A **first to default CDS** pays off if and when the first default occurs. After the payout on the first default, the CDS terminates.
- ✓ A **second to default CDS** will pay off when the second default occurs. This CDS does not pay anything for the first default, and terminates following the payout associated with the second default.

$$P_{t-1} \longrightarrow P_t$$

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专业·创新·增值

1st to default $V_{高}$ (default)

$V_{相同}$ ($V \downarrow$)

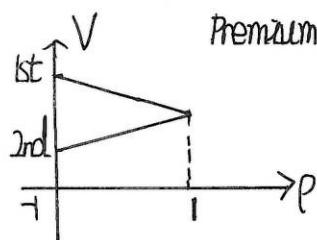
2nd to default. $V_{低}$ (not default). ($V \uparrow$)

◆ Credit Default Swap (CDS)

➤ Basket CDS (cont'd)

(受到 default correlation 影响)

● Correlation Impact in Nth-to-default Swap



- ✓ If the reference assets are perfectly positive correlated, the value of the first-to-default CDS will be the same as the Nth-to-default ($n > 1$) CDS because the number of defaults will likely be either 0 or all assets.
- ✓ If the default correlation is low, small number of defaults is more likely. Therefore, first-to-default is more preferable.
- ✓ When default correlation increase, there is an increased probability of more defaults, and the value of the Nth-to-default ($n > 1$) goes up accordingly.

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专业·创新·增值

◆ Total Return Swap (TRS) (总收益互换)

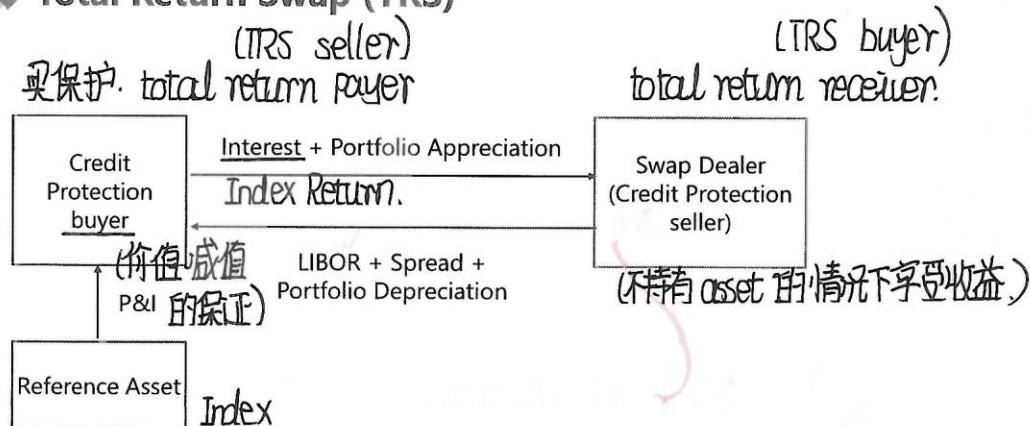
- TRS: Contracts where one party, called the protection buyer, makes a series of payments linked to the total return on a reference asset.
- Whereas a CDS compensates the credit protection buyer for only a loss resulting from an actual default, a TRS protects the buyer from the risk of defaults or declines in value associated with downgrades or other adverse credit events. Among the credit derivatives, the TRS is the more complete hedge, the TRS hedges against:

- Default
- Credit deterioration
- Market risk

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专业·创新·增值

◆ Total Return Swap (TRS)



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专业·创新·增值

the Lender (note holder) receives an enhanced coupon as compensation for bearing the credit risk of issuer.

◆ Credit Linked Note (CLN) : 本质仍是CDS.

➤ CLN: Combines a regular coupon-paying note with some credit risk feature.

A CLN is thus economically equivalent from the issuer's perspective to issuing a normal note plus buying credit protection from the bond investor through a CDS.

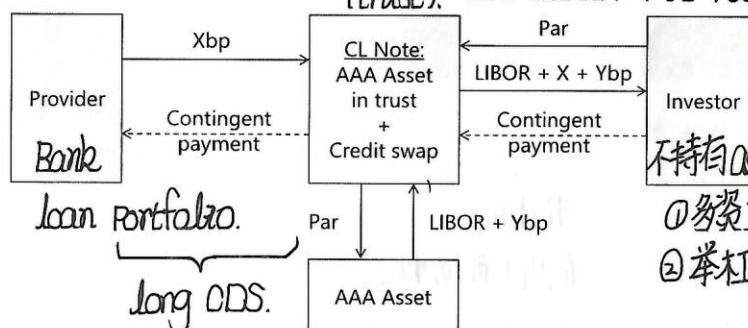
(trust). : 买无风险资产+卖一个CDS = Bond_(risk-free) - CDS.

= Bond risky

不持有 asset 享受收益.

① 多资产的.

② 举杠杆的.

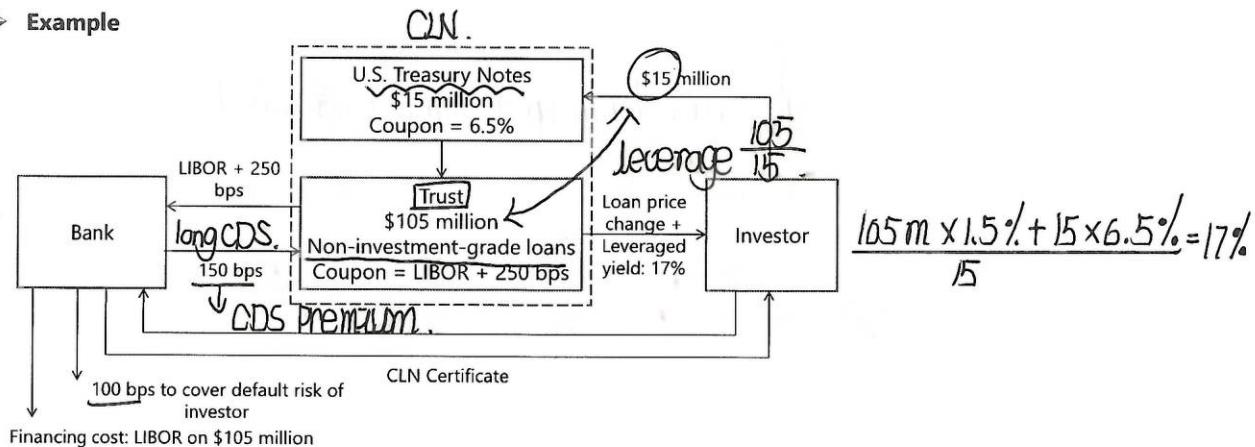


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专业·创新·增值

◆ Credit Linked Note (CLN)

➤ Example



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专业·创新·增值

◆ Exercise 1



- A risk manager is advising the trading desk about entering into a digital credit default swap as a way to obtain credit protection. Which cash flow and delivery requirement will the desk most likely experience in the event of a default of the underlying reference asset?
- A. Receive the pre-agreed cash payment; delivering nothing ✓
B. Receive [(Par Value) – (Market Value of Reference Asset)]; deliver the reference asset.
C. Receive [(Par Value) – (Market Value of Reference Asset)]; deliver nothing ✓ 差额现金结算.
D. Receive the pre-agreed cash payment; deliver the reference asset.

➤ Answer: A

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专业·创新·增值

Credit Risk Management

Topic 3: Securitization

Break clause: "liquidity put"
early termination option (ETO)

1. Securitization and Structured Product
2. Performance Analysis 指标.
3. Credit Scenario Analysis
4. Credit Enhancement 信用+流动性.
5. Subprime Securitization

↓问题+处理方法,

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专业·创新·增值

表外

◆ Securitization and Structured Product

➤ **Structured Finance:** Involves the financial engineering of the firm's liabilities to achieve specific financing and/or risk management objectives.

● **Securitization** True sale, bankruptcy remote (破产隔离)

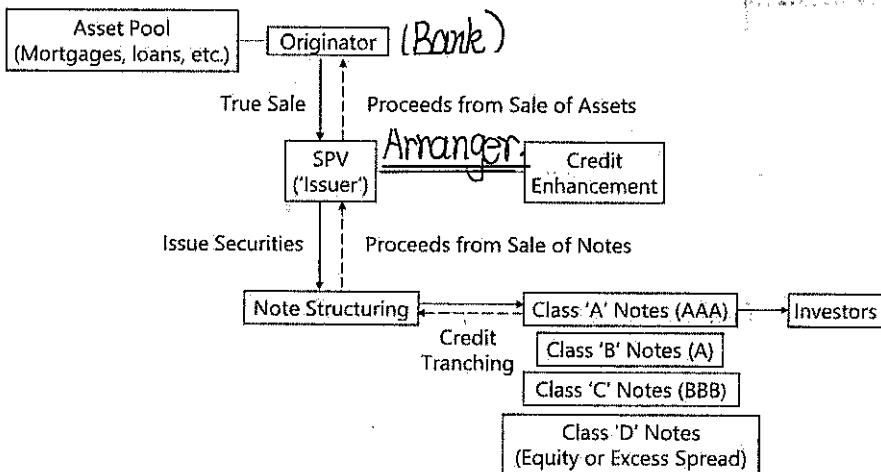
- ✓ Process of selling credit-sensitive assets to a third party that subsequently issues securities backed by the pooled cash flows (principal and interest) of the same underlying assets.
- ✓ Financial institutions benefit from securitization by funding assets, balance sheet management, and risk management. Securitization benefits investors by providing access to liquid assets that were previously not available to them.

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专业·创新·增值

◆ Securitization and Structured Product

> Securitization



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类型·创新·增值

◆ Securitization and Structured Product

> Securitization (cont'd)

- **SPV** (结构化)
Amortizing Structure (本息摊还一期). mortgage. *locked out period.*
- ✓ Revolving Structures credit card receivable < *Revolving period.*
- ✓ Master Trust frequent issuer. *Amortising period.*
- **Tranching** (分层)
 - ✓ Claims with the most seniority have implicit protection from the layer of investors below them. *Senior, junior, equity.* ①
 - **Waterfall** *o/c trigger: 留在 o/c account / or 给到 Equity* ②
 - ✓ The term "waterfall" is used because the capital structure is paid in a "top down" sequence with the senior debt receiving all of its promised payments before any lower tranche receives any monies.

80-106

类型·创新·增值

(归属)

◆ Securitization and Structured Product

信用增级 (次级层对上级的保护)

79.35% is senior (AAA rated)!

Class	Notional	Width	Subordination	S&P
A-1	\$239,618,000	27.18%	72.82%	AAA
A-2A	\$214,090,000	24.29%	48.53%	AAA
A-2B	\$102,864,000	11.67%	36.86%	AAA
A-2C	\$99,900,000	11.33%	25.53%	AAA
A-CD	\$42,998,000	4.88%	20.65%	AAA
M-1	\$35,700,000	4.05%	16.60%	AA+
M-2	\$28,649,000	3.25%	13.35%	AA
M-3	\$16,748,000	1.90%	11.45%	AA-
M-4	\$14,986,000	1.70%	9.75%	A+
M-5	\$14,545,000	1.65%	8.10%	A
M-6	\$13,663,000	1.55%	6.55%	A-
M-7	\$12,341,000	1.40%	5.15%	BBB+
M-8	\$11,019,000	1.25%	3.90%	BBB
M-9	\$7,052,000	0.80%	3.10%	BBB-
B-1	\$6,170,000	0.70%	2.40%	BB+
B-2	\$8,815,000	1.00%	1.40%	BB
X	\$12,340,995	1.40%	0.00%	NR

Equity Tranche (Class X) of 1.4% is the overcollateralization (O/C).

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类型·创新·增值

◆ Securitization and Structured Product

➤ Structured Product

● Covered Bond (不是证券化产品)

没有剥离出资产负债表) 提 securitization.

✓ In a covered bond structure, [mortgage loans] are aggregated into a [cover pool], by which a bond issue is secured. The cover pool remains on balance sheet but is segregated from other assets in the event the bank defaults.

● CDO (Collateralized Debt Obligations) 证券化产品

CLOs and CBOs are simply securities that are collateralized by means of high-yield bank loans and corporate bonds (CLOs and CBOs are also sometimes referred to generically as collateralized debt obligations, or CDOs.)

It allows the cash flows from a pool of loans (or bonds) rated at below investment grade to be pooled together and prioritized, so that some of the resulting securities can achieve an investment-grade rating.

✓ The main differences between CLOs and CBOs are the assumed recovery values for, and the average life of, the underlying assets.

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专业·创新·增值

CMO → 分层结构.

◆ Performance Analysis (资产池特征的核心指标)

Prepayment

提前偿付.

Performance Measure	Calculation	Typical Asset Class
Public Securities Association (PSA)	$PSA = [CPR/(0.2)(months)] * 100$	mortgages, home-equity, student loans
Constant prepayment rate (CPR)	$1 - (1 - SMM)^{12}$	mortgages, home-equity, student loans
Single monthly mortality (SMM)	Prepayment / Outstanding pool balance	mortgages, home-equity, student loans
Weighted average life (WAL)	$\Sigma(a/365) \cdot PF(s)$ Where PF(s)	mortgages
Weighted average maturity (WAM)	Weighted maturity of the pool	mortgages
Weighted average coupon (WAC)	Weighted coupon of the pool	mortgages
Debt service coverage ratio (DSCR)	Net operating income / Debt payments	commercial Mortgages
Monthly payment rate (MPR)	Collections / Outstanding pool balance	all non-amortising asset classes
Default ratio (written-off)	Defaults / Outstanding pool balance	credit cards
Delinquency ratio (90天拖欠)	Delinquents / Outstanding pool balance	credit cards
Absolute prepayment speed (ABS)	Prepayments / Outstanding pool balance	auto loans, truck loans
Loss curves	Show expected cumulative loss	auto loans, truck loans

790天拖欠.

Written-off 贷款坏账部分.

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专业·创新·增值

◆ Performance Analysis prepayment - 本金.

➤ CPR and PSA

- There are a number of methods used to estimate prepayments, two commonly used ones are the **constant prepayment rate (CPR)** and the **Public Securities Association (PSA)** method.

- The CPR approach is: \rightarrow (转化为年化).

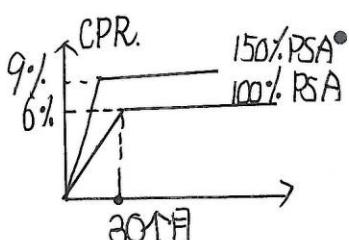
(年化)

$$SMM = \frac{\text{Prepayment}}{\text{Balance-schedule principal repayment}} \quad \text{CPR} = 1 - (1 - SMM)^{12} \quad 1 - CPR = (1 - SMM)^{12}$$

where **single monthly mortality (SMM)** is the single-month proportional prepayment.

反向CPR 情景假设.

- A pool of mortgages is said to have **100% - PSA** if its CPR starts at 0 and increases by 0.2% each month until it reaches 6% in month 30. It is constant 6% after that. Other prepayment scenarios can be specified as multiples of 100% PSA.



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专业·创新·增值

Credit Scenario Analysis

> The mezzanine effect is mixed

$\begin{cases} \text{PD高} & M \rightarrow \text{Equity} \\ \text{PD低} & M \rightarrow \text{Senior} \end{cases}$

> Constant Correlation

不变, PD↑

- Increasing the probability of default will negatively impact the cash flows and, thus, the values of all tranches. $VE \downarrow, VS \downarrow$ (损失可能性)

- Increasing default probability generally decreases the VaR for the equity tranches (less variation in returns) and increases the VaR for the senior tranches (more variation in returns).

↓
损失不确定性:

$VaRE \downarrow, VaRs \uparrow$

senior

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简化信用风险分析: 所有asset PD一样.

1. $0 \rightarrow T$, PD不变, 忽略P.

2. interim期间现金流分析

PD → 不违约 → interest flow

→ 违约 → 回收部分 senior/junior

利差利息,

/

Equity

O/c account

① excess spread

② 回收

③ 前期票息

专业·创新·增值

3. terminal 到期现金流

PD → 不违约 → 本息流入 → senior 本息 / junior

→ 违约 → 回收 O/c 风险

↓
Equity

Credit Scenario Analysis

> Constant Probability of Default PD不变, P↑ \hookrightarrow PD高, VS↓, VE↑, VaRs↑, VaRE↑.

- At low default rates, the impact of an increase in correlation is relatively

low. But when default rates are relatively high, the equity benefits from

high correlation, while the senior bond is hurt by it.

- Senior VaR increases with correlation.

- When default correlation approaches 1, the equity VaR increases steadily

since the returns are more variable.

资产: Senior(50%) Equity(50%)
 $P = 1$ 无损 全损
 $P =$ (无损 or 全损)

86-106

Senior (85%) Equity (15%)

100 asset.	PD=10%	无损	有损
	PD=15%	无损	全损
	PD=20%	有损	全损

① 不变, PD↑ → Equity / Senior 损失可能性↑.

↓ VE↓, Vsenior↓

2) VaR 损失的不确定性:

VaRs↑, VaREquity↓

② PD不变, P↑: Vsenior↓ Vequity↑.
 VaRs↑, VaREquity↓.

Credit Enhancement 信用增级+流动性增级 (结构)

> Internal Credit Enhancement

① Subordination

Creating tranches of differing priority levels.

② Overcollateralization

The pool offers claims for less than the amount of the collateral. (超额抵补).

③ Excess Spread

Gross: difference between interest earned on the collateral assets and interest paid on the debt liabilities of the SPE.

Net: gross excess spread minus fees and expenses.

It is held in a reserve account (also called Cash Collateral Account) to cover against future losses. O/c account.

The step-up feature was introduced as an added incentive for investors, to convince them from the outset that the economic cost of paying a higher coupon is unacceptable and that the issuer would seek to refinance by exercising its call option. 购回条款.

Margin Step-Up

浮息给investor

利息逐步上升.

Credit Enhancement

主要保护 senior 投资人。

Internal Credit Enhancement (cont'd)

Shifting Interest

前3年只给 senior branch

Requires that all principal payments to be applied to senior notes over a specified period of time before being paid to mezzanine bondholders.

Performance Triggers

After the lockout period, subject to passing performance tests, the O/C is released and principal is applied to mezzanine notes from the bottom of the capital structure up until target levels of subordination are reached.

External Credit Enhancement

买保险, 买 CLS.

Pool Insurance

Provided by a composite insurance company to cover the risk of principal loss in the collateral pool.

External Liquidity Enhancement

Interest Rate Swap

Agreed to accept a sequence of fixed payments in return for promising to send a sequence of adjustable-rate payments.

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专业·创新·增值

Subprime Securitization

(发生问题与解决方案)

(7个问题, 5个是主要的)

Key Frictions in Subprime Mortgage Securitization

● Friction 1: Between Borrower (mortgagor) & Originator Bank

✓ Predatory lending / borrowing.

● Friction 2: Between Originator and Arranger

✓ Mortgage Fraud (originator 信息优势) → originator 自身持有部分

✓ Originator has an information advantage over the arranger in regard to the quality of the borrower. Without safeguards, the originator might be incented to collaborate with a borrower in order to falsify the loan application.

✓ Due diligence of the arranger.

{①不按还款能力, 只看 asset

②诱导再融资

③欺诈.

↓
留存充足资本

Bank (originator).

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专业·创新·增值

Subprime Securitization

持有 equity ①

Key Frictions in Subprime Mortgage Securitization (cont'd)

● Friction 3: Between Arranger and Third-Parties (i.e., asset manager, warehouse lender, credit rating agencies)

✓ Arranger has more information than the third parties, which creates an adverse selection problem. (因为信息不对称带来的逆向选择问题)

✓ Due diligence conducted by the portfolio manager on the arranger and originator. 正证券化现金流管理 ②

Friction 4: Between Servicer and Borrower (Mortgagor)

For property near foreclosure, the borrower has less incentive to: (1) pay property taxes; (2) pay insurance; (3) Maintain the property.

✓ Moral hazard: refers to changes in behavior in response to redistribution of risk.

取消抵押品回权

escrow account

(托管账户)

不是主要

原因

of risk.

SN

专业·创新·增值

◆ Subprime Securitization

➤ Key Frictions in Subprime Mortgage Securitization (cont'd)

- Friction 5: Between Servicer & Third-Parties (Asset manager, credit rating agencies)
- ✓ Moral hazard. (servicer).
- ✓ Between servicer & asset manager: two key points of tension between investors and the servicer: (1) reasonable reimbursable expenses, and (2) the decision to modify and foreclose. (收回的)
- ✓ Between servicer & credit agencies: Given the impact of servicer quality on losses, the accuracy of the credit rating placed on securities issued by the trust is vulnerable to the use of a low quality servicer.

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专业·创新·增值

Summary:

1. Securitization 特点.

- financing.
- unlevered. (top-down, o/c trigger).
-

2. Structured product:

(covered bond) 与 securitization 区别.

3. Asset pool performance 分析

▷ Prepayment: SMM, GPR.

4. Credit scenario ① 简化分析.



interim, terminal).

专业·创新·增值

▷ Simulation: PD↑, P↑ → Value Var.

5. Credit enhancement

▷ subordination,
overcollateralization,
excess spread,

6. Subprime: 5 frictions 及应对策略

◆ Subprime Securitization

➤ Five Frictions most Responsible for the Subprime Breakdown

- Predatory borrowing and predatory lending – Friction 1.
- Other frictions worsened the friction between originator and arranger:
Mortgage Fraud – Friction 2.
- If the asset manager does not conduct due diligence, the arranger's incentives to conduct due diligence are reduced: adverse selection – Friction 3.
- Principal-agent problem – Friction 6.
- Credit ratings were assigned to subprime MBS with significant error.
Although the agencies publicly disclosed their rating criteria for subprime, investors lacked the ability to evaluate the efficacy of these models – Friction 7.

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专业·创新·增值

◆ Exercise 1



- Which of the following structures is most likely to be used by an infrequent issuer of securities whose asset pool consists of credit card debt; i.e., short-dated assets with a relatively high pre-payment speed?
- A. Master trust
 - B. Amortizing structure
 - C. Revolving structure
 - D. None of the above

➤ Answer: C

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专业·创新·增值

◆ Exercise 2



- Consider the following three pairs of performance indicators:
- I. Delinquency ratio and default ratio (非摊销结构)
 - II. Loss curves and absolute prepayment speed
 - III. Debt service coverage ratio and weighted average maturity (WAM)
- Which sequence below correctly associates the collateral type with its primary performance indicators?
- A. I. Auto loans, II. Commercial mortgages, III. Credit cards
 - B. I. Auto loans, II. Residential mortgages, III. Credit cards
 - C. Credit cards, II. Auto loans, III. Commercial Mortgages
 - D. I. Commercial mortgages, II. Credit cards, III. Auto loans

➤ Answer: C

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专业·创新·增值

◆ Exercise 3



- A hedge fund is considering taking position in various tranches of a collateralized debt obligation (CDO). The fund's chief economist predicts that the default probability will decrease significantly and that the default correlation will increase. Based on this prediction, which of the following is a good strategy to pursue? PD↓ P↓ set value.
- A. Buy the senior tranche and buy the equity tranche.
 - B. Buy the senior tranche and sell the equity tranche. PD↓ P↓ set value
 - C. Sell the senior tranche and sell the equity tranche.
 - D. Sell the senior tranche and buy the equity tranche. 波动不大

➤ Answer: D

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专业·创新·增值

◆ Exercise 4



- An investor has sold default protection on the most senior tranche of a CDO. If the default correlation between assets held in the CDO P.V. decreases sharply, assuming everything else is unchanged, the investor's position:
- ✓ A. Will gain significant value, since the probability of exercising the protection falls.
 - B. Will lose significant value, since his protection will gain value.
 - C. Will neither gain nor lose value, since only expected default losses matter and correlation does not affect expected default losses.
 - D. Can either increase or decrease, depending on the pricing model used and the market conditions.
- Answer: A

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专业·创新·增值

◆ Exercise 5



- Which of the following statements was not one of the frictions occurred during credit crisis in 2007?
- ✓ A. By using originate-to-distribute model, a strong profit motive took precedence over ethical lending.
 - ✓ B. Arranger has more information than the third parties, which creates an adverse selection problem.
 - C. Structured investment vehicles were used to enhance the risk discovery process for regulators SIV(机构)
 - ✓ D. Given the impact of servicer quality, the accuracy of the credit rating is vulnerable to the use of a low quality servicer.

- Answer: C

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专业·创新·增值

Credit Risk Management

Topic 4: Retail Credit Risk

1. Mortgage Credit Assessment
2. Credit Scoring Model
3. Cutoff Scores

用 score model.

① Z-score > 2.675.

Score → (度量)

→ (建模)

→ cut off score

→ 验证.

◆ Mortgage Credit Assessment

➤ Key Variables in Mortgage Credit Assessment

① Documentation Type asset, income \rightarrow DTI.

✓ **Full doc:** A mortgage loan that requires proof of income and assets.

Debt-to-income ratios are calculated. (asset, income 可验证)

✓ **Stated income:** Specialized mortgage loan in which the mortgage lender verifies employment but not income. 只能 verify 雇佣关系.

✓ **No income/No asset:** allows the borrower to state income and assets on the loan application without verification by the lender; however, the source of the income is still verified.

✓ **No ratio:** documents employment but not income. Income is not listed on the application, and no debt-to-income ratios are calculated.

✓ **No doc:** A mortgage loan requires no income or asset documentation.

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专业·创新·增值

◆ Mortgage Credit Assessment

→ Prime borrower (越高越好.)

➤ Key Variables in Mortgage Credit Assessment (cont'd)

② • **FICO:** Number score of the default risk associated with a borrower's credit history. > 660 (好) prime < 660 (sub)

③ • **DTI:** Debt-to-income ratio is used to qualify mortgage payment and other monthly debt payments versus income. Debt / Income.

④ • **LTV:** Expresses the amount of a first mortgage lien as a percentage of the total appraised value of the property – i.e., the loan-to-value ratio. (越小越好).

⑤ • **Payment Type (Pmt)** – e.g., adjustable rate mortgage, monthly treasury average.

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专业·创新·增值

◆ Credit Scoring Model

➤ The model uses a statistical procedure to convert information about a credit applicant or an existing account holder into numbers that are then combined to form a score.

➤ Three Types of Models:

① • **Credit Bureau Scores (FICO scores),** because the methodology for producing them was developed by Fair Isaac Corporation.

② • **Pooled Models.** These models are built by outside vendors, such as Fair Isaac, using data collected from a wide range of lenders with similar credit portfolios. (外部定制) 外部第三方建模型.

③ • **Custom Models.** These models are usually developed in-house using data collected from the lender's own unique population of credit applications. (内部定制)

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专业·创新·增值

Cutoff Scores

主观的。

- Cutoff Scores: the point at which applicant were accepted, based on subjective criteria. (风险偏好).

在所有贷款申请中占比。

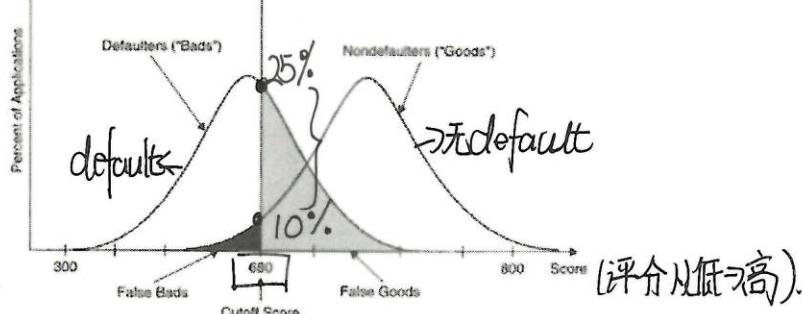


FIGURE 17-2 Distributions of "goods" and "bads"

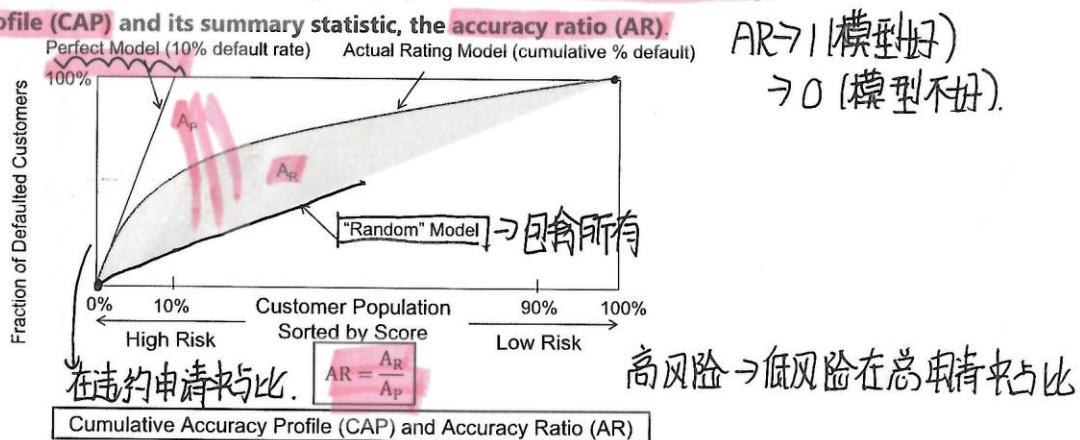
103-106

专业·创新·增值

Cutoff Scores 验证

- Scorecard Performance: When measuring a scorecard's performance, the validation technique traditionally employed is the cumulative accuracy profile (CAP) and its summary statistic, the accuracy ratio (AR).

图表。



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It's not the end but just beginning.

Always believe that good things are possible, and remember that mistakes can be lessons that lead to discoveries. Take your fear and transform it into trust; learn to rise above anxiety and doubt. Turn your "worry hours" into "productive hours". Take the energy that you have wasted and direct it toward every worthwhile effort that you can be involved in. You will see beautiful things happen when you allow yourself to experience the joys of life. You will find happiness when you adopt positive thinking into your daily routine and make it an important part of your world.

请坚信，美好的降临并非不可能，失误也许是成功的前奏。将惶恐化作信任，学会超越担忧和疑虑。让“诚惶诚恐”的时光变得“富有成效”。不要挥霍浪费精力，将它投到有意义的事情中去。当你下意识品尝生命的欢愉时，美好就会出现。当你积极地看待生活，并以此作为你的日常准则时，你就会找到快乐的真谛。

