Session 2: Credit Risk Management

{ (1) Credit risk identification.
(2) Credit risk measurement.
(3) Credit risk management.

R1: Credit risk identification.

1. Classification: default risk, Recovery risk, Exposure risk.

2. key indicators:

① Expected loss: (EL) individual: $EL_i = PD \times EL \times LR = PD \times EL \times (1-RR)$
portfolio: $EL_p = \Sigma EL_i$

② Unexpected loss (Credit VaR): WCL(x%) - EL (highly skewed to left)

- individual: $UL = \underbrace{WCL}_{\text{损失分布}} - EL$
- portfolio: → default correlation ρ:

① ρ default correlation:

$$\rho_{x_1,x_2} = \frac{cov(x_1,x_2)}{\sigma_{x_1}\cdot\sigma_{x_2}} = \frac{\pi_{12}-\pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

→ ρ impact Valatility, WCL not EL

→ $\rho=1$, one credit, no diversification { default 2% (1000.000) / no default 98% (0)

$\rho=0$, binomial distribution. have diversification.

3. Capital Structure:

① steps to derive Economic Capital:

EL → UL(standalone) → ULC → EC

- individual: $EL = PD \times EA \times LR$

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \underbrace{\sigma_{PD}^2}_{\sigma_{PD}^2 = PD\times(1-PD)}}$$

- portfolio: $UL_p^2 = UL_1^2 + UL_2^2 + 2\rho\cdot UL_1\cdot UL_2$

② ULC: unexpected loss contribution: $\underbrace{ULMC}_{\frac{\partial UL_p}{\partial UL_i}} \rightarrow \underbrace{ULC}_{ULMC_i \times UL_i} \rightarrow EC = \Sigma ULC \times CM$

- ULMC (marginal)

$$ULMC = \frac{\partial UL_p}{\partial UL_1} = \frac{UL_1 + \rho\cdot UL_2}{UL_p}$$

- $ULC = ULMC_1 \times UL_1 = \frac{UL_1^2 + \rho\cdot UL_1\cdot UL_2}{UL_p}$ $(UL_p = UL_1 + UL_2)$
- $EC = ULC_i \times CM$ (capital multiplier)

4. Credit decision and analyst:

① individuals: (credit score) income + asset

② Non-financial company: liquidity, cash flow, earning capacity, profitability solvency, capital position.

③ financial firms: cash flow不重要
→ Capital adequacy
→ Asset quality
→ qualitative analysis. important

④ Sovereigns; subjective.

Summary:

1. Credit VaR (UL) 计算:

① single asset: $UL = WCL(\%) - \underline{EL}$,
$EL = PD \times LR \times ED$

② Portfolio Credit VaR:

• default correlation: $\rho(2\ asset) = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$

• $\rho=1$ 时, one asset, loss 从小→大, 大于 confidence interval: { no default 98% (0); default 2% (1,000,000) } ↓ 99%
↑ WCL = 1,000,000.

• $\rho=0$ 时, binomial distribution.
loss 从小→大, 超过 confidence level:
loss小, default 从0开始
→ 0 default $C_{50}^{0}(98\%)^{50} = 0.364$
→ 1 default $C_{50}^{1}(98\%)^{49}2\% = 0.372$
→ 2 default $C_{50}^{2}(98\%)^{48}(2\%)^2 = 0.186$ → 92.18%
→ 3 default $C_{50}^{3}(98\%)^{47}(2\%)^3 = 0.0607$ → 98.27%

2. Economic Capital: → UL 6v

① $UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$
(single asset) $\sigma_{PD}^2 = PD \times (1-PD)$

② 2 asset $UL_P^2 = UL_1^2 + UL_2^2 + 2\rho_{1,2} \cdot UL_1 \cdot UL_2$

③ $ULMC_1 = \frac{UL_1 + \rho UL_2}{UL_P}$, $ULMC_2 = \frac{UL_2 + \rho UL_1}{UL_P}$

④ $ULC_1 = ULMC_1 \times UL_1$, $ULC_2 = ULMC_2 \times UL_2$

⑤ $EC = \Sigma ULC_i \times CM_i$

3. 分析 4个对象:

• individual (credit score) asset, income.

• non-financial: liquidity, cash flow, earning capacity & Profitiability, solvency, capital position

• financial: qualitative analysis, asset quality, capital position.

• soveneign: 主观 subjective.

Credit Risk measure: Probability of default

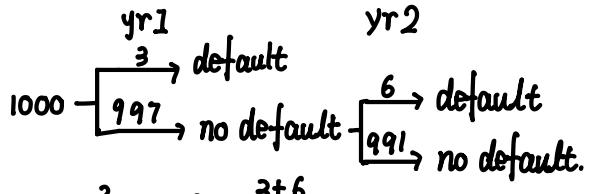
1. Basic approach used to predict default.

- Expert-rating
- statistic-based. { Structural approach.: financial theory assumption.
  Reduced-form: variables

2. Rating system:

- features: measurability (直接使用) objectivity (只反应 credit risk)
  verifiability (借鉴) homogeneity Specificity (特异性)

- Moody's: issues' rating 产品
  S&P: issuer 发行人
  FITCH: issuer rating on publicly listed Bond. BBB- ✓ / BB+ ✗
- Measurement rating: → 直接法: transition matrix Baa3 ✓ / Ba1 ✗
  → 间接 5 probability.
  - C Cumulative default Probability: C
  - M (MDPt,t+k): marginal default. probability.
  - F (d): Forward Probability. (当期存活率)
  - Survival Rate (over multipal year)
  - Annualized default Rate (ADR)



(1) $C_1 = \frac{3}{1000}$, $C_2 = \frac{3+6}{1000}$

(2) $MDP_{0,1} = \frac{3}{1000}$, $MDP_{1,2} = \frac{6}{1000}$

(3) d (forward) $= \frac{3}{1000}$, $d_2 = \frac{6}{1000-3}$

(4) $S_1 = (1-d_1)$, $S_2 = (1-d_1)(1-d_2)$

(5) ADR: 年 $(1-d_1)(1-d_2) = (1-ADR)^2$

秒 $(1-d_1)(1-d_2) = 1-e^{-ADR\times 2}$

3. Bond 法 from market price: measure PD (3个公式)

① $P = \frac{\$1}{YTM} = \frac{\$1 \times PD \times RR + \$1 \times (1-PD)}{(1+R_f)}$

2年: $\frac{1}{(YTM)^2} = \frac{1 \times PD \times RR + 1 \times (1-PD)}{(1+R_f)^2}$.

② $PD = \frac{1}{2R} \times (1 - \frac{1+R_f}{1+YTM})$ face value ≠ 1

③ YTM - R_f = credit spread ≈ PD × 2R (付息)

- T ≠ 1 时，用 ①
- face value ≠ 1 时，用 ①②
- 付息，用 ③
- YTM 未知，market Price 已知 用 ①
- credit spread 已知，用 ③，用 ③ 式期限调整. (T ≠ 1 时，不是年化)

  计 T = 0.5，$\frac{PD}{\downarrow} \times 2R = CS \times \frac{1}{2}$

  PD 算出来是半年的 PD

  计 T = 2，$PD \times 2R = CS \times 2$.

4. Equity 法 用 market Price → measure PD

I. Merton model

① Equity: shareholder $E = VN(d_1) - ke^{-rT}N(d_2)$ V - firm value.

call option on firm. $d_{1,2} = \frac{\ln(\frac{V}{ke^{-rT}})}{\sigma\sqrt{T}} \pm \frac{\sigma\sqrt{T}}{2}$ k - strike price: / face value of debt.

② Bondholder: $D = ke^{-rT}N(d_2) + VN(-d_1)$

$= ke^{-rT}$ - put

= risk-free debt ke^{-rT} − put option on firm $-VN(-d_2)$

= V - Equity

③ BSM assumption: 1) $V \sim$ lognormal

2) European option (不可以分析提前违约)

3) continuous (不可以解释突发违约)

④ Shareholder: $PD = P(V \le k) = 1 - N(d_2)$

$PD = N(-d_2)$

- risk-Neutral PD: 用 R_f → $N(-d_2)$ $d_2 = \frac{\ln(\frac{V}{ke^{-rT}})}{\sigma\sqrt{T}} - \frac{\sigma\sqrt{T}}{2}$.

  physical PD: 用 asset return → $N(-d_2)$
- d_2 ↑ → $N(-d_2)$ ↓ → PD ↓
- $T = 1 \Rightarrow d_2 \approx \frac{\ln V - \ln T}{\sigma_V}$
- d_2: distance to default. k: default point.

⑤ Credit spread: D: debt value
F: face value

$$D = F \cdot e^{-(CS+Rf)(T-t)}$$

$$\Rightarrow CS = \frac{1}{T-t} \ln\left(\frac{D}{F}\right) - Rf$$

II: KMV model: (Merton model 的补充)

- default point (Merton 单笔)
- KMV { short-term / long-term
- No lognormal assumption
- 先计算 distance to default → DD 结合 historical data.

$$DD = \frac{V-K}{\sigma_V * V} \qquad K = \begin{cases} 0.7LT + 0.7ST & (LT/ST \geq 1.5) \text{ debt} \\ 0.5LT + 1.0ST & (LT/ST < 1.5) \text{ debt} \end{cases}$$

5. Exponential distribution: (下一次 default 所用时间的分布:

- Poisson distribution: 违约个数的 distribution
- hazard rate (default density): $\lambda = \frac{\text{Spread}}{1-RR.}$
- cummulated default time:

$$P(t^* < t) = F(t) = 1 - e^{-\lambda t}$$

- survival distribution: $P(t^* > t) = 1 - F(t) = e^{-\lambda t}$

6. Single factor model:

① asset return $\alpha \sim$ normal distribution:

$$\alpha = \beta m + \sqrt{1-\beta^2}\,\varepsilon$$
$$E(\alpha) = 0, \quad Var(\alpha) = 1$$

② $\rho_{1,2} = \beta_1 \times \beta_2$

③ m 未知: unconditional $PD = P(\alpha < K')$

m 已知: conditional PD: $\alpha = \bar{m}\beta + \sqrt{1-\beta^2}\,\varepsilon$

$$E(\alpha) = \beta\bar{m}, \quad Var(\alpha) = \sqrt{1-\beta^2}$$

$$PD = P(\alpha < K') = P\left(\alpha < \frac{K' - \beta\bar{m}}{\sqrt{1-\beta^2}}\right)$$

Summary:

1° Rating system { 直接 transfer matrics / 借鉴 (5) C, MDP, d, S, ADR.

2. Bond 法 → PD：①-⑤式

3. Stock 法：1) Merton： Equity value

Debt value

PD = $N(-d_2)$. T=1时 d_2 近似

CS

assumption

与 KMV 比较

2) KMV：与 Merton 差异.

Moody's DD

5. 指数分布：1) 入估计

2) cummulated. MDP, S, 条件DD

6. Single factor model: PD：用 α 建模：$\alpha = \beta m + \sqrt{1-\beta^2}\,\varepsilon$

(1) m 未知, N

(2) m 已知, 标准化.

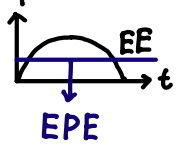
(3) asset correlation $\rho_{i,j} = \beta_i \cdot \beta_j$

Credit risk measure : Counterparty Exposures

1. Exposure metrics:

① Current Exposure : max(V, 0)

② Expected Exposure (EE)



1) EE可个可↓

2) EE到期为0

③ EPE Expected Positive Exposure

(weighted average over time)

④ Negative Exposure.：站在我的角度，对手所面对的 Exposure.

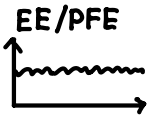
⑤ Peak Exposure / Potential Future Exposure. (a high percentile 95%, 99%)

⑥ Maximum PFE.

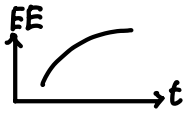
→ Effective Expected Positive Eexposure (roll-over risk)

2. Exposure Profiles

① Bond : fixed rate:

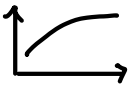


② loans:

③ Forward: 

④ Interest rate swap: 

peaked shape.

⑤ Cross-currency swap: (货币互换.有本金交换) 

⑥ Credit derivatives: hard to characterize.

a. WWR & RWR:

① Wrong-way risk: PD↑ 时, collateral ↓, exposure↑

right-way risk: PD↑时, collateral↑, exposure↓

② option: (RWR)

A —long call on B→ B

Exposure↓ PDB↑→ stock B↓→ call↓

③ Option: (WWR)

A —long put on B→ B

Exposure↑ PDB↑ → Equity B↓ → Put↑

④ Commodity forward/swap: (RWR)

A —long oil forward.→ oil company

Exposure↓ (锁定买入价) oil$↓→ PD↑

合约不值钱了.

(WWR)

A —long oil forward.→ airplane corp.

Exposure↑ oil$↑ → air Equity↓→ PD↑

⑤ CDS (WWR)

A —long CDS on C→ B (B与C ρ>0)

PDB↑→ PDC↑

Exposure↑ CDS↑

(RWR)

A —short CDS on C→ B (PDB↑→ PDC↑)

Exposure↓

b. Credit Senario analysis

① Correlation ρ不变, PD↑: Value: V_E↓, V_{senior}↓

VaR: VaR_E↓, VaR_{senior}↑

② PD不变, Correlation ρ↑: Value: V_E↑, V_{senior}↓

VaR: VaR_E↑, VaR_{senior}↑

c. Cutoff Score: Accuracy ratio AR= $\frac{A_R}{A_P}$ → random model / → perfect model

AR→1 好.

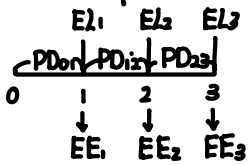
CAP: cumulative accuracy profile.

d. Value adjustment：直接在value上调整；
market-implied default probability

① Credit Value adjustment CVA：

→ assumption：自身不违约，no wrong-way-risk.

→ standard equation：



$CVA = \sum PVEE_i$

$CVA = LR \sum_{i=1}^{m} EE(t_i) \times PD(t_{i-1}, t_i)$

- PD采用 marginal default probability MDP
- 有collateral，EE↓

→ approximation：(将CVA看为每一期费用调整；

$CVA = \underset{\downarrow}{EPE} \times$ Credit Spread.

expected Positive exposure.

→ Portfolio：1) Incremental CVA：change in CVA in a new trade.
2) marginal CVA：(每笔交易的 contribution)

② Bilateral CVA (BCVA)

- DVA：debt value adjustment (自身default)
- BCVA (双边交易中，双方都有可能 default 的情况下的价值调整)

Party ⇌ counterparty

标准：LR_C, PD_C, EE　　LR_P, PD_P, NEE, S_C.
Survival (自身) S_P

近似：EPE, CS_C　　ENE, CS_P

- 标准：$BCVA = LR_C \sum EE(t_i) \times PD_C(t_{i-1}, t_i) \times S_P(t_{i-1}) - LR_D \sum NEE(t_i) \times PD_P(t_{i-1}, t_i) \times S_C(t_{i-1})$
- approximate：$BCVA = EPE \times Spread_C - ENE \times Spread_P$

③ Impact on CVA

1) creadspread：$CS = PD \times LR$，PD↑，CS↑，CVA↑
- CS 极大时，smallest CVA.

2) Recovery rate：RR↑ → LR↓：cancellation effect：LR has small impact on CVA.

3) margin / collateral：→ Exposure↓ → CVA↓
↑
→ threshold↑ → Exposure↑ → CVA↑
→ initial margin↑ → Exposure↓ → CVA↓

1. Netting: Payment netting: on the same day

close-out netting: default event.

2. Netting factor = $\frac{EE(netting)}{EE(no\ netting)} = \frac{\sqrt{n+n(n-1)\bar{\rho}}}{n}$

3. Funding exposure = value - margin. (多收少收 collateral 带来好处)

→ difference between funding and credit exposure

①
②
③
④
⑤

3. Bilateral margin requirement

① variation margin.

② Initial margin.

→ eligible asset.

4. Termination.

① ATE: addition termination event

② Break Clauses, mutual Puts

③ walkaway feature.

(对手方 default 时, 中止合约)

5. CDS settlement