# 强化段：Valuation and Risk model

- Valuation: { Bond / Options
- Risk models: { Market risk models / credit risk models / Operational risk models
- Stress Testing.

## Topic 1: Bond valuation. pricing.

一、Bond valuation.

- method 1: $P=\sum PV(CF_t)$
- method 2: Replication approach.

→ Bond return: YTM

→ P&L Components: (定性拆分收益)

- Price appreciation
- cash-carry.

二、未来现金流折现：

$$P=\sum\frac{CF_t}{(1+y)^t}$$

$CF_t$ → { coupon / par

$(1+y)^t$ ↓ discount factor { spot rate. / forward rate.

- spot rate:

$$P=\frac{CF_1}{1+S_1}+\frac{CF_2}{(1+S_2)^2}+\cdots+\frac{CF_t}{(1+S_T)^T}$$

- forward rate:

一期一期往前折现.

$$P=\frac{CF_1}{1+f(1)}+\frac{CF_2}{[1+f(1)][1+f(2)]}+\cdots+\frac{CF_T}{[1+f(1)][1+f(2)]\cdots[1+f(T)]}$$

- Discount factor: $P=\sum CF_t\times d_t$

(折现因子)

未来的1元在今天的现值.

$$d(1)=\frac{1}{[1+\frac{S_1}{2}]^2}\qquad d(t)=\frac{1}{[1+\frac{S_t}{2}]^{2t}}$$

$$d(t)=1\cdot e^{-S_t\times t}.$$

三、复制法：replication approach.

① Law of one price:

Absent confounding factors (liquidity, financing, taxes, credit risk), identical sets of cash flows should sell for same price.

② arbitrage

example:

$X_1$: 95.238 —— \$100

$X_2$: 89. —— \$100

$X_3$: 100. —— \$6 —— \$6+\$100

$$\begin{cases}100x_1=6\\100x_2=6+100\end{cases}\rightarrow\begin{cases}x_1=\\x_2=\end{cases}$$

$$x_1\cdot 95.238+x_2\cdot 89\overset{?}{=}100$$

{ 买 undervalued. / 卖 overvalued

四、Bond Return:

① Gross Realized Returns: $R_{t,t+1}=\dfrac{P_{t+1}+C-P_t}{P_t}.$

② Net Realized Returns: (考虑融资成本)

$$R_{t,t+1}=\frac{P_{t+1}+C-B_{\text{funded price}}}{P_t}$$

$B_{\text{funded price}}$ → $P_t\times(1+\underset{\text{funding cost}}{r})$

$$=\frac{P_{t+1}+C-P_t}{P_t}-r.$$

③ Spread = $R_p-R_f$

(relative value)

risk↑ → spread↑, $R_p=R_f+\text{Spread}$↑

④ YTM (yield to maturity) ⇒ reinvestment risk.

- Single discount rate:

$$P=\frac{CF_1}{1+YTM}+\frac{CF_t}{(1+YTM)^2}+\cdots+\frac{CF_n}{[1+YTM]^n}$$

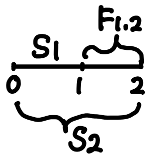
→ assume cash flows are reinvested at the YTM

→ bond is held to maturity.

• YTM取值接近最后一期 $R_n$.

⑤ Relationship between spot rates and forward rates:

• spot rates are an $\boxed{\text{average}}$ of forward rates.




$(1+S_1)(1+F_{1\to2})=[1+S_2]^2$

$S_2=\sqrt{(1+S_1)(1+F_{1\to2})}-1$

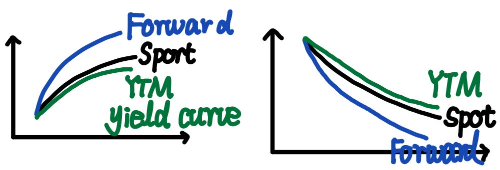
⑥ Coupon Effect

• short term rate 对 YTM 起主导作用

• 改变 coupon → $CF_t$ → 改变 spot rate weight

• upward-sloping trend, YTM↓ as coupon↑

downward-sloping trend, YTM↑ as coupon↑




## 五. Profit & Loss Components

债券的损益.

① price appreciation (or depreciation)

$P_1-P_0$

② cash-carry: cash flows such as coupon payments

① price appreciation (or depreciation)

decomposed → three components:

1) carry-roll-down.

→ passage of time.

1. 收到 coupon

2. 债券价格在自然发生变化.

→ assumption: forward rate are realized.

2) 利率改变 rate changing

3) spread from $S(t) \to S(t+1)$

# Topic 2: Bond Market Risk

利率敏感

## 一. 利率期限结构的变化:

① parallel term structure shifts:

• plain bond

采用 Yield Shifts

→ Macaulay Duration & Modified Duration

Dollar duration & DV01

→ Convexity. (大幅变化)

• Embedded option:

→ effective duration & DVDZ & DVOF

→ effective convexity.

② non-parallel term structure shifts:

→ PCA

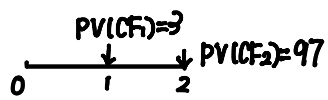
→ key-rate expouse

→ Forward Bucket Shift.

## 二. Macaulay Duration:

一张债券未来现金流的平均回流时间.

$$Mac.\ D=\sum_{t=1}^{T}\left[\left(\frac{PV(CF_t)}{P}\times t\right)\right]=\sum_{t=1}^{T}(W_t\times t)$$




$$Mac.D=\sum W_t\times t=1\times\frac{3}{3+97}+2\times\frac{97}{97+3}$$

Plain bond:

• the Macaulay duration is less than or equal to its maturity.

• zero-coupon bond,

the Macaulay duration equals to its maturity.

## 三、Modified Duration: ($\Delta y \to \Delta\% P$)

- Modified Duration (MD) $= -\dfrac{\Delta P/P}{\Delta y} = \dfrac{\text{Macaulay Duration}}{1+y/m}$
  $= \dfrac{1}{1+\frac{y}{m}}$ [Mac.D]
  $\Delta y$: 利率变化.

- if yield continuous compounding:
  Mac. duration = Modified duration.

- The approximate duration relationship of Bond is:
  $\Delta P = -MD \times P \times \Delta y$.

- Dollar Duration:
  $DD = MD \times P = \dfrac{\Delta P}{\Delta y}$ → first derivative / slope

- $DV01 = MD \times P \times 0.0001 = DD \times 0.0001$.
  $0.0001$: 0.01%

→ DV01 hedge:
(敏感性变动的话,价值不受影响)

$\Delta P_p = 0$ 当 1bps 变动 $\Delta DV01_p = 0$ — A: $F^A$, $DV01A$; B: $FB$, $DV01B$

$F^B \cdot DV01^B = F^A \cdot DV01^A$

$\Rightarrow F^B = \dfrac{F^A \times DV01^A}{DV01^B}$ → $F^A \times DV01^A$: 被对冲的; $DV01^B$: 对冲工具.

$F^B$: face value

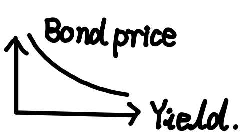
→ Duration Hedge:
$\Delta P_{portfolio} = 0$ × bps ($\Delta y$)
A, B (n份)

$-MD_A \times P_A \times \Delta y - MD_B \times P_B \times \Delta y \times n = 0$

$n = -\dfrac{MD_A \times P_A}{MD_B \times P_B} = \dfrac{DD_A}{DD_B}$

n: 份数

## 四、Convexity: (non-linear relationship)



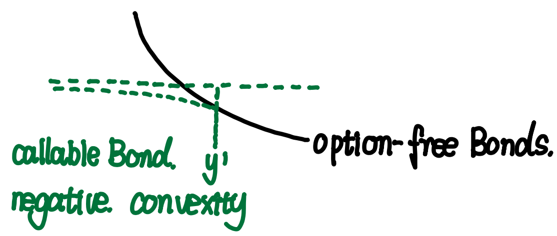

$\Delta P = -DP\Delta y + \frac{1}{2} C \cdot P \cdot (\Delta y)^2$ ★ $\begin{cases} y\downarrow, \Delta y < 0, \frac{1}{2}CP\Delta y^2 > 0, P\uparrow \text{ more.} \\ y\uparrow, \Delta y > 0, \frac{1}{2}\cdot C \cdot P \cdot \Delta y^2 > 0, P\downarrow \text{ less.} \end{cases}$

D: Modified D; C: Curvature

涨多跌少.
正凸性 ∨ benefit to investor.

- percentage bond price change = Duration effect + Convexity effect.
  $\Delta B$

- Negative Convexity:
  ① most mortgage bonds are negatively convex,
  ② callable bonds usually exhibit negative convexity at lower yields.




## 五. Effective Duration / and Effective Convexity:

MD / C

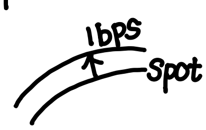
⇒ embedded options effective duration:

① $D^E = \dfrac{\Delta P/P}{\Delta y} = \dfrac{P_- - P_+}{2P_0 \Delta y}$.

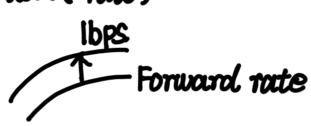
⇒ effective convexity:

$C^E = \dfrac{D_- - D_+}{\Delta y} = \dfrac{P_- + P_+ - 2P_0}{P_0 \cdot \Delta y^2}$

- **DVDZ / DPDZ**: the change in price from a one-basis-point increase in all spot (i.e. zero) rate.




- **DVDF / DPDF**: the change in price from a one-basis-point increase in forward rate;



⇒ Portfolio Duration and Convexity:

(加权平均)

weighted sum of individual

its value as a percentage of portfolio value.

$D_p = \sum W_i \cdot D_i$ (%)

$C_p = \sum W_i \cdot C_i$

$DV01_p = \sum DV01_i \to \$$.

⇒ Bullet versus Barbell Portfolio:

- Bullet 子弹式: medium term.
- Barbell portfolio 杠铃式 $\begin{cases} \text{long term} \\ \text{short term.} \end{cases}$
- 在Duration相同的情况下, $C_{barbell}$ 大

  convexity 涨多跌少

  (benefit to investor)

Large parallel shift, barbell outperforms.

③. KR01, 1bps, $\Delta KP01_p = 0$

$F^A \cdot KR01_A$ $\quad F^B \times KR01_B$.

$F^A \times KR01_A + F^B \times KR01_B = 0$.

$$F^B = -\frac{F^A \times KR01_A}{KR01_B}$$

## 六、Non-parallel Term Structure Shifts:

### ① Principal Component Analysis: (PCA)

- factors are uncorrelated.
- daily changes in term structure are linear combination of the factors.
- the first two or three factors account for the majority of the observed daily movements.

### ② Key Rate Exposure:

- short - $S_2$
- medium - $S_5$
- large - $S_{10}$

→ Shifts in the key-rates are declined linearly.

→ the rate of a given maturity is affected solely by its closest key-rate.

## Topic 3: Option Valuation

- Binomial Trees.
- The Black-Scholes-Merton Model.
- Option Sensitivity Measures

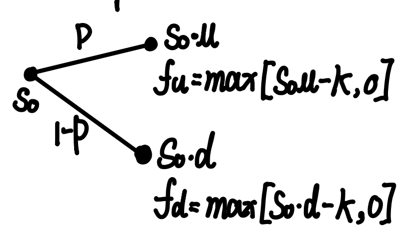
### 一、Binomial Tree

- European options
- American options.

① Risk-neutral valuation:

·对风险无所谓默不关心)

② One-step binomial model:



$S_0=(S_0\cdot u\cdot p+S_0\cdot d\cdot(1-p))e^{-r\cdot\Delta t}.$

•价格上升下降的概率: one-step binomial tree:

$0 \longrightarrow \Delta t$

$S_0 \xrightarrow{rf} S_0\cdot u\cdot p+S_0\cdot d\cdot(1-p)$

$S_0\cdot e^{r\Delta t}=[S_0\cdot u\cdot p+S_0\cdot d\cdot(1-p)]$

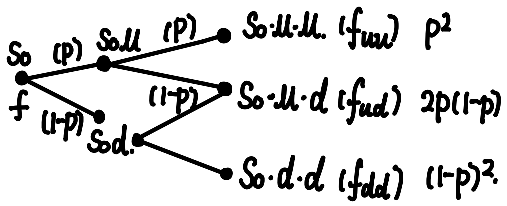
- $p=\dfrac{e^{r\cdot\Delta t}-d}{u-d}.$
- $ud=1$

→ $u\cdot d$

已知:
- \$ $\quad \underset{(40)}{S_0} < \begin{matrix} \overset{(48)}{S_0\cdot u} \\ \underset{(32)}{S_0\cdot d} \end{matrix}$ $\quad \begin{cases} u=\dfrac{S_0\cdot u}{S_0}=\dfrac{48}{40} \\ d=\dfrac{S_0\cdot d}{S_0}=\dfrac{32}{40} \end{cases}$
- % $\pm 10\%$ $\begin{cases} u=1.1 \\ d=0.9. \end{cases}$

未知: $\begin{cases} u\cdot d=1 \text{ (assume)} \\ u=e^{\sigma\sqrt{\Delta t}} \\ d=e^{-\sigma\sqrt{\Delta t}} \end{cases}$

⇒ Two-steps binomial tree:




•如果有分红:

$p=\dfrac{e^{r\Delta t}-d}{(u-d)}$ or 有分红: $p=\dfrac{e^{(r-q)\Delta t}-d}{(u-d)}$

$f=e^{-2r\Delta t}[p^2 f_{uu}+2p(1-p)f_{ud}+(1-p)^2 f_{dd}]$

•American Options:

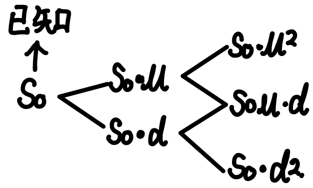
判断是否提前行权

⇒ step 1: $\Delta t$ 1yr $<$ 1 step $\Delta t=1$; 2 step $\Delta t=0.5$

⇒ Step 2: P, u, d, $\begin{cases} u=e^{\sigma\sqrt{\Delta t}} \\ d=e^{-\sigma\sqrt{\Delta t}}=\frac{1}{u}. \end{cases}$

$P=\dfrac{e^{(r-q)\Delta t}-d}{(u-d)}$

• step 3: each node → stop price.




• step 4: option payoff

$\begin{cases} \text{call}: \max[S_T-k,0] \\ \text{put}: \max[k-S_T,0] \end{cases}$

• Step 5: discount

- European $f=e^{-r\cdot\Delta t}(f_u\cdot p+f_d(1-p)).$
- American $\begin{cases} \text{一步一步往前折现.} \\ \text{判断}: \max[\text{early exercise}, PV(CF_t)] \end{cases}$

二、Other Assets:

① Options on stocks with Dividends:

$p=\dfrac{e^{(r-q)\Delta t}-d}{(u-d)}$

② Options on stock Indices:

→ provide a dividend yield.

③ Options on Currencies:

→ asset providing a yield.

④ Options on Futures:

$q=rf$

$p=\dfrac{e^{(r-r)\Delta t}-d}{(u-d)}=\dfrac{1-d}{u-d}$

二. Black-Scholes-Merton Model:

① Assumption: 几何布朗运动

- $\mu$ and $\sigma$ constant.
- the risk-free rate of interest, r, is constant and all maturities.

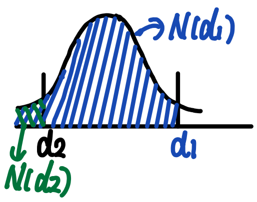
• $P_t \sim$ lognormal, $R \sim$ normal.

② Valuation:

→ $call = S_0 \cdot N(d_1) - ke^{-rT}\underbrace{N(d_2)}_{\substack{prob\,(S_T>k)\\ \text{行权概率}}}$

→ $put = k\cdot e^{-rT}N(-d_2) - S_0N(-d_1)$

→ $d_{1,2} = \dfrac{\ln(\frac{S_0}{k}) + (r\pm\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$ $\begin{cases} r+\frac{1}{2}\sigma^2 & d_1 \quad (d_1>d_2) \\ r-\frac{1}{2}\sigma^2 & d_2 \quad (d_1=d_2+\sigma\sqrt{T}) \end{cases}$




$N(d_1)+N(-d_1)=100\%$

$N(d_2)+N(-d_2)=100\%$

③ The early exercise of American options: $\begin{cases} AC \\ AP. \end{cases}$

$\begin{cases} AC: D\text{大} \rightarrow \text{行权.} \\ AP: D\text{小} \rightarrow \text{行权} \end{cases}$

• American call:

$S_t\cdot D_t$ $S_T\cdot D_T$

0 t T

→ early exercise: $S_t\cdot D_t$.

→ Maturity: $S_t - k > S_t - D_t - ke^{-r(T-t)}$

(①>②) $D_t > k[1-e^{-r(T-t)}]$

④ Options on stocks with Dividends:

→ $c = S_0\cdot e^{-qT}N(d_1) - ke^{-rT}N(d_2)$

→ $p = k\cdot e^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1)$

→ $d_{1,2} = \dfrac{\ln(S_0/k) + (r-q\pm\sigma^2/2)T}{\sigma\sqrt{T}}$.

⑤ Warrant: (权证)

• the payoff to an option holder if the option is exercised is;

• 上市公司 → 一级市场 (增发)

• $\dfrac{NS_T+Mk}{N+M} - k = \dfrac{N}{N+M}(S_T-k) = \dfrac{N}{N+M}\cdot c$

• 0时刻 $\begin{cases} N\text{份}: S_0 \quad \text{Total Equity } (N\cdot S_0) \\ \text{warrants}: M\text{份} \quad k\$ \end{cases}$

• T时刻: $\left.\begin{matrix} N\text{份} \quad S_T \quad \text{equity.} \quad N\cdot S_T \\ \text{增发}M\text{份} \quad k \text{ equity } Mk \end{matrix}\right\}$ Total equity $\dfrac{N\cdot S_T+Mk}{\#(N+M)}$

⇒ payoff: $S_T' - k = \dfrac{N\cdot S_T+Mk}{N+M} - k$

$= \dfrac{N\cdot S_T+Mk-Nk-Mk}{N+M} = \dfrac{N}{(N+M)}(S_T-k)$

⇒ payoff $= \left(\dfrac{N}{N+M}\right)\underbrace{(S_T-k)}_{\text{call payoff}}$

$V_{warrant} = \dfrac{N}{N+M}\cdot$ call value.

## Topic 4: Greeks
## (期权的风险)

一、Impact of underlying asset price - Delta.

$C = S\cdot N(d_1) - ke^{-rt}N(d_2)$

$d_{1,2} = \dfrac{\ln(\frac{S}{k}) + (rf\pm\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$

$C = f(\underset{\substack{\downarrow \\ \text{Delta, Gamma,}}}{S}, k, T, Rf, \sigma)$

• S - stock price: Delta, Gamma.

• T − Theta

• Rf − Rho

• σ − vega

## 二. Delta

① $\Delta$ Delta $= \dfrac{\Delta \text{Option}}{\Delta \text{Stock}}$ (first derivative)/slope

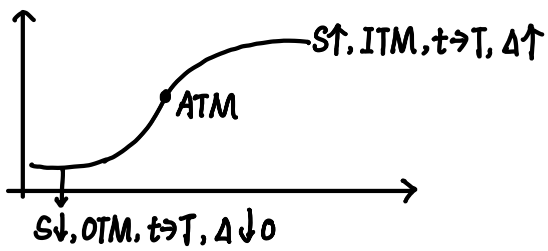
- $C = SN(d_1) - K\cdot e^{-rt}N(d_2)$

  $\Delta C' = N(d_1)$

- $P = Ke^{-rt}N(-d_2) - SN(-d_1)$

  $\Delta P' = -N(d_1)$
  $= N(d_1) - 1$

$\Rightarrow$ Call: option $\Delta$ range from 0 to 1 $\Rightarrow N(d_1)$
put: option $\Delta$ range from to $-1$ to 0. $\Rightarrow N(d_1)-1$




$\Rightarrow$ when $t\to T$, Delta is unstable.

② According to the BSM model, (Option delta)
Option delta:

- $C = S\cdot e^{-qT}N(d_1) - Ke^{-rT}N(d_2)$

  $\Delta C' = e^{-qT}N(d_1)$.

- $P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$

  $\Delta P' = e^{-qT}[N(d_1)-1]$

③ Portfolio Delta:

- Summation of product of each position and its delta
- Forward Delta: 1 or $\underbrace{e^{-qT}}_{\text{yield}}$.

  Futures Delta: $e^{rT}$ or $e^{(r-q)T}$

- Stock: Delta原 $+ N\cdot\underbrace{\Delta S}_{1} = 0$.

$\underbrace{N = -\text{Delta原}}_{\text{反向}}$ $\begin{cases} \text{Delta} > 0. & \text{short stock.} \\ \text{Delta} < 0 & \text{long stock} \end{cases}$ Delta仿

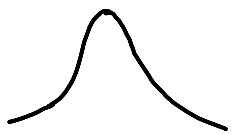
$\Rightarrow$

④ Delta Hedge:

- A position with a delta of zero is called a <u>delta neural position</u>.
- Hedge against small changes in asset price.

## 三. Gamma: $= \dfrac{\Delta \text{Delta}}{\Delta \text{Stock}}$

- Gamma: second derivative - option.
  (curvature)
- Rate of delta change with respect to price change of underlying asset.
- same for call and put.
- $t\to T$, Gamma $\uparrow$



① 将 Gamma 对冲致 0

→ Gamma hedge option

$\begin{cases} \text{Gramma} > 0. & [\text{short option}] \\ \text{Gramma} < 0. & [\text{long option}] \end{cases}$

→ Delta hedge stock:

$\begin{cases} \text{Delta} > 0 & \text{short stock} \\ \text{Delta} < 0 & \text{long stock.} \end{cases}$ Delta仿

② Delta and Gamma Hedge:
→ create a gamma-neutral position

## 四、Vega:

- Rate of change of the value with respect to the $\boxed{\text{volatility}}$ of the underlying asset.

隐含波动率倒求出来的 volatility.

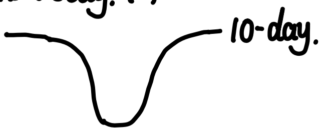
→ at the money
→ same for call and put options
→ $t\to T$, Vega $\downarrow$
long $\Lambda > 0$; short $\Lambda < 0$.

- Vega > 0, short option.
  Vega < 0, long option.

## 五 Theta:

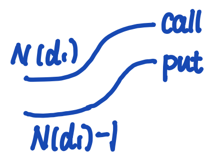
- Rate of change of the value of option with respect to the passage of time.

① Time decay. (-)

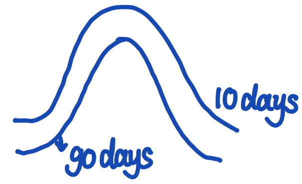



② $t \to T$, Theta↑ → in-the-money 深度实值. ($k >> S_t$)
③ Deep $\boxed{ITM}$ European put, $\theta > 0$; 极端赚钱.
④ Largest when is at-the-money;
⑤ $\theta$ is not a risk factor.

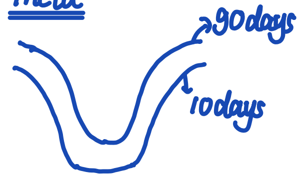
**Delta** $N(d_1)$



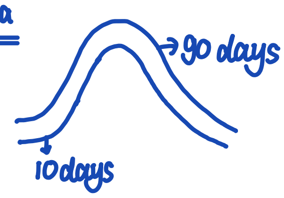

**Gamma**



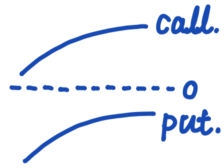

**Theta**




**Vega**




**Rho**




## 六、Rho:

- sensitivity to the interest rate.
- in the money calls and puts are more sensitive to. changes

## Summary: 五个 Greek Letters:

时间的流逝对期权影响.

| | $\Delta$ (Delta) | $\Gamma$ (Gamma) | $\Lambda$ (vega) | $\theta$ (theta) | $R$ (rho) |
|---|---|---|---|---|---|
| Long call: | + | + | + | − | + |
| Short call: | − | + | + | + | − |
| Long put: | − | − | − | − | − |
| Short put: | + | − | − | + | + |

In-the money　OTM

| | $\lvert\Delta\rvert$ (Delta) | $\lvert\Gamma\rvert$ (Gamma) | $\lvert\Lambda\rvert$ (Vega) | $\lvert\theta\rvert$ (theta) | $\lvert R\rvert$ (利率) (rho) |
|---|---|---|---|---|---|
| Long term | | 小 | 大 | 小 | 小 |
| Short term | | 大 | 小 | 大 | 大 |

## Topic 5: Market Risk models (市场风险)

① Measures of financial risk:
1) Coherent risk measures
2) VaR
3) ES

② Measure and monitor Valatility:
1) EWMA
2) GARCH.

③ Calculate and Apply VaR:
- Delta-normal method.
- Delta-gamma method.

### 一、好的风险指标:

① coherent risk measures:

1) Monotonicity 单调性 $R(A) > R(B) \Rightarrow \rho(A) < \rho(B)$

2) Subadditivity 次可加性 $P = A+B \Rightarrow \rho(A+B) \leq \rho(A) + \rho(B)$
diversification effect.

3) Homogeneity: $\rho(\lambda R) = \lambda \rho(R)$
(正齐次性) 风险

4) Translation Invariance 转移不变性. $\rho(R+k) = \rho(R) - k$
cash $k \rightarrow$ risk measure should decrease by $k$.

② Mean-Variance Framework:
- mean – P/L (or returns)
variance — standard deviation.
- daily P/L / return – normal distribution.

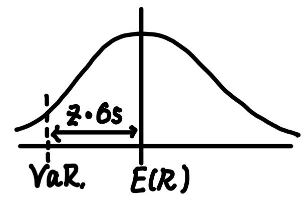
### 二、VaR:
- VaR is the maximum loss over a target horizon and for a given confidence level.
- disadvantage:
→ did not contain worst conditions, did not describe tail loss.
→ Not sub-additive $VaR_p > VaR_{(R_1)} + VaR_{(R_2)}$

③ Conditional VaR / Expected Shortfall / Tail loss:
- Average of $100 \times (1-\alpha\%)\%$
- sub-additive: $ES_p \leq ES_1 + ES_2$

### 三、VaR 的计算:
① Delta-Normal Model:
② Historical Simulation: $\begin{cases} \text{historical simulation.} \\ \text{Monte Carlo Simulation.} \end{cases}$
(不依赖过去) (model risk) (cost)

① Delta-normal Model: (bond, options)
Delta – first-derivative
normal – normal distribution.

1) underlying ~ normal → VaR(dy) / VaR(ds)
2) 转化为 derivative VaR. $\xleftrightarrow{\text{Delta}}$ underlying VaR.




$$\begin{cases} VaR(X\%) = |E(R) - Z_{X\%} \times \sigma| \\ VaR(\$) = |E(R) - Z_{X\%} \times \sigma| \times V \end{cases}$$

② Square Root Rule:
- 一天VaR $\xrightarrow[\text{转换}]{}$ T天. $\begin{cases} 1\text{ yr} = 250/252\text{ day} \\ 1\text{ month} = 20\text{ day} \\ 1\text{ week} = 5\text{ day} \end{cases}$
- $VaR(T, X) = \sqrt{T} \times VaR(1, X)$
- assumptions: ① $E(R) = 0$
② iid $\rho = 0$, $\sigma$ constant.

③ Delta-Normal Approximation:

- not good for derivatives with extreme nonlinearity.
- $VaR(dP) = |-D \times P| \times VaR(dy)$
  (bond)
  $VaR(df) = |\Delta| \times VaR(dS)$ (derivative)
- Delta-Gamma Approximation:
  $VaR(P) = |-D \times P| \times VaR(dy) - \frac{1}{2} \times \underbrace{C}_{\text{convexity}} \times P \times [VaR(dy)]^2$
  $VaR(f) = |\Delta| \times VaR(dS) - \frac{1}{2} \times \Gamma \times VaR(dS)^2$

④ Historical Simulation:

- Percentage change: stock prices / exchange rates.
- Actual change: interest rates / credit spreads.

## 四、Quantifying Volatility in VaR Models:

① Unconditional normality:

- same normal distribution.
- same standard deviation.
- fatter tail

② Conditional Normality:

- normal conditioned on the volatility.
- Regime switching model.

③ Parametric Volatility:

- historical returns (No)
  Implied volatility: (隐含波动率)
  ⇒根据BSM倒推出来的波动率
  - current data
  - forward-looking.
- Historical returns (Yes)
  ⇒ How do we weight the data?
  i) Equally:
  historical standard deviation.
  2) More weight to more recent:
  - EWMA 指数加权移动平均模型
  - GARCH 广义自回归条件异方差模型

① Historical Standard Deviation Approach:

- Equally weighted
- $\sigma_n^2 = \left(\frac{1}{M}\right) \sum_{i=1}^{M} u_{n-i}^2$
  (expected mean is assumed zero)

② EWMA: $(0<\lambda<1)$

$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) u_{n-1}^2$

$cov_n = \lambda cov_{n-1} + (1-\lambda) x_{n-1} y_{n-1}$

$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) r_{n-1}^2$

$\sigma_n^2 = \lambda^2 \sigma_{n-2}^2 + \lambda(1-\lambda) r_{n-2}^2 + (1-\lambda) \cdot r_{n-1}^2$

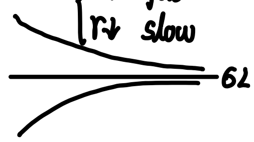
| | $r$ (收益率) | $\sigma^2$ (方差项) |
|---|---|---|
| 1 | $(1-\lambda)\cdot\lambda^0$ | $\lambda^1$ |
| 2 | $(1-\lambda)\cdot\lambda^1$ | $\lambda^2$ |
| 3 | $(1-\lambda)\cdot\lambda^2$ | $\lambda^3$ |
| 4 | ⋮ | ⋮ |
| k | $(1-\lambda)\cdot\lambda^{k-1}$ | $\lambda^k$. |

③ GARCH:

$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$

$V_L = \frac{\omega}{1-\alpha-\beta} = \frac{\omega}{\gamma}$ $(\alpha+\beta+\gamma=1)$

$\begin{cases} \gamma\uparrow & \text{fast} \\ \gamma\downarrow & \text{slow} \end{cases}$




$\underbrace{cov_n}_{\text{协方差}} = \omega + \alpha x_{n-1} \cdot y_{n-1} + \beta \cdot cov_{n-1}$

- alpha ($\alpha$) and beta ($\beta$) ⇒ persistence.
  持续因子.

- 对比: EWMA: $\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) r_{n-1}^2$
  GARCH: $\sigma_n^2 = \omega + \beta \sigma_{n-1}^2 + \alpha \cdot r_{n-1}^2$
  $\omega = \gamma \cdot V_L$
  if $\gamma=0$, $\beta=\lambda$, $\alpha=1-\lambda$ EWMA ⇔ GARCH

## Market Risk models:

- Coherent risk measures (4种)
- VaR ①confidence level ②horizon ③max loss
- ES ⇒ VaR { tail loss ×; 次可加性 ?

avg(loss > VaR) { tail loss ✓; 次可加性 ✓

## Calculate and Applying VaR

- Delta-normal method ①
- Delta-gamma method

① underlying: VaR $VaR(ds)=|E(R)-z\cdot\sigma|\times V_s$

② derivative VaR { bond: Dollar duration + Dollar convexity; derivative: Delta + Gamma

- bond: $|\Delta|\cdot VaR(ds)-\frac{1}{2}\cdot\Gamma\cdot VaR^2(ds)$
- option: $|-MD\times P|\times VaR(dy)-\frac{1}{2}\times C\times P\times VaR^2(dy)$

## Measuring and monitoring Volatility:

- EWMA: $\sigma_n^2=\lambda\sigma_{n-1}^2+(1-\lambda)\cdot r_{n-1}^2$
- GARCH: $\sigma_n^2=\omega+\beta\cdot\sigma_{n-1}^2+\alpha\cdot r_{n-1}^2$

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# Topic 6: Credit Risk Models

▲ Ratings:
- External Ratings BBB⁻/Baa3
  - Transition matrics → cummulative default rate
- Internal Ratings (back-test) Z-score. (>3 PD↓, <1.8, PD↑)
- Hazard Rates (λ) → $\bar{\lambda}$, $C_t=1-e^{-\bar{\lambda}\cdot t}$

▲ Country Risk: • credit spred ($R_p-R_f$), 优缺点

▲ Capital Model:
- EL/UL → capital (pricing)
- Mean and standard Deviation of credit loss

loan: $\begin{cases} E(x)=EL=PD\times EAD\times LGD \\ \sigma(x)=\sqrt{PD\times(1-PD)}\times EAD\times LGD \end{cases}$

portfolio: $\begin{cases} EL_p=n\,EL_i \\ \sigma_p^2=n\sigma^2+n(n-1)\rho\cdot\sigma^2 \end{cases}$

- Measure credit Risk. (Vasicek model) $UL=(WCDR-PD)\times LGD\times EAD$

CreditMetrics: default & Rating change → economic capital

## 一. Rating:

- Rating for bonds: long-term ratings:
- Speculative: $\frac{BBB}{BB}$ (BBB⁺, BBB, BBB⁻) → 质量差, junk bond, high yield bond;
- Rating for money market: short-term rating

## 二. Transition matrices

- Agency publish cumulative default rates. categorized by rating and transition matrices.

→ 期末评级

期初评级

- D → default

## 三. 评级方法:

Point-in-time and Through-the-cycle:

Point-in-time → 短期
- current rating
- unstable.

Through-the-cycle → 长期
- "filter out" cycle fluctuation
- incorporate an average. (平均)
- stable
- bias (over- and under-estimate credit quality)

## 四 KMV/Merton model:

B/S

| Asset | Debt |
| --- | --- |
| | Equity |

$A = D + E$

if $A < D$, $E = 0$, ⇒ default

$E$: $\max(A-D, 0)$

call: $\max(S_T - k, 0)$

- call: $S \cdot N(d_1) - k \cdot e^{-rT} N(d_2)$

  $N(d_2)$ 行权概率 prob $(S_T > k)$

⇒ value of equity: = call on firm

$= V \cdot N(d_1) - D \cdot e^{-rT} N(d_2)$

$N(d_2)$: prob (Asset > Debt)

not default.

$1 - N(d_2)$ – not exercise default

## 五. Internal Ratings:

- several factors → scored → a weighted average score → overall final rating → back-test. 回测
- Altman's Z-score

  ⇒ distinguish that default or not.

  ⇒ Z score > 3: unlikely to default.

  Z score < 1.8: very high probability of default.

## 六. Harzard Rates:

- Harzard rate ($\lambda$) is the rate at which default are happening.

  $0 \to t$ 累积违约概率:

  $C_t = 1 - e^{-\bar{\lambda} t}$

  → the probability of default during t.

  → $PD_{2\to3} = C_3 - C_2$

## 七. Country Risk

① Sources of country Risk:

- GDP growth Rates (GDP growth ↑, risk ↓)
- political risk 政治风险 government, corruption, violence

  expropriation (强制征收)

- legal risk (property right, contract enforcement)
- the Economy (diversity)

② Composite measures of country risk

(评估国家风险的机构)

- PRS (Political Risk Services) 22指标, 0~100, score ↑, risk ↓
- Euromoney 0~100, Score ↑, risk ↓ 400 economist.
- the Economist. 0~100, score ↑, risk ↑

  (banking, currency, sovereign)

- the World Bank (打分0附近, "–" risk ↑)

③ Sovereign Default (主权债务是否违约) lag

- Foreign currency default (risk ↑)
- Local currency defaults → (inflation)

④ Sovereign default spread.

- interest rate on the bond on a foreign currency compared to a rate on a riskless investment.
- Advantage:

  → more granular (细致) than rating agencies.

  → dynamic

- Disadvantage:

  → more volatile. (unstable)

  → can be affected by variables that have nothing to do with default.

  (Liquidity and investor demand 导致变化)

## 八. Credit risk 建模

default – related event

credit migration 信用质量变化 → rating change.

## ① Capital model:

- Expected losses (take into account)
- expected loss (EL)

EL = Probability of default × loss given Default.

$EL = PD \times LGD$

$LGD + RR = 1$

$\Rightarrow EL = PD \times (1-RR)$

RR (Recovery rate $= \frac{recovery}{exposure} = 1 - \frac{LGD}{exposure}$

$EL = PD \times (1-RR) \times \underbrace{EAD}_{贷款总金额}$

## ③ Model for determing Capital:

- Bank's capital is a buffer (缓冲垫) against unexpected loss

capital $\to$ $\begin{cases} \text{economic capital} \\ \text{regulatory capital} \end{cases}$

- $UL = \underbrace{WCL}_{\text{worst case loss}} - EL$

$EL = PD \times EAD \times LGD$

$WCL = \underbrace{PD}_{(99.9\%)} \times EAD \times LGD$

$\Rightarrow WCL = \underbrace{WCDR}_{\text{(worst case default case)}} \times EAD \times LGD$

$\Rightarrow UL = (WCDR - PD) \times LGD \times EAD.$

## ② Mean and Standard deviation of credit losses

| | | | |
|---|---|---|---|
| Loan → loss | PD: default | 1 | EAD×LGD |
| | 1-PD: not default | 0 | 0 |

伯努利变量: $E(x) = P$

$\sigma^2(x) = P(1-P)$

$\sigma(x) = \sqrt{P(1-P)}$

▲ 单笔 loan:

loss: $E(x) = EL = PD \times EAD \times LGD$

$\sigma^2(x) = PD \cdot (1-PD) \times EAD^2 \times LGD^2$

$\sigma(x) = \sqrt{PD \times (1-PD)} \times EAD \times LGD$

▲ n 笔 loan: $\rho$ (违约相关性) 一样

loan portfolio: $EL_p = n \cdot EL_i$

$\sigma_p^2 = n\sigma^2 + n(n-1)\rho \cdot \sigma^2$

$EL_p = EL_1 + EL_2 + \cdots + EL_n$

$= n \cdot EL_i$

$\sigma_p^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2 + 2\rho_{1,2}\sigma_1 \cdot \sigma_2 + \cdots$

$+ 2\rho_{i,j} \cdot \sigma_i \cdot \sigma_j$

$= n\sigma^2 + 2\rho\sigma^2 \times \frac{n(n-1)}{2}$

$\sigma_p^2 = n\sigma^2 + n(n-1)\rho \cdot \sigma^2$

## ④ Vasicek Model and CreditMetrics:

- Vasicek model ($\underbrace{\text{Regulatory capital}}_{\text{Basel II}}$)

$UL = (WCDR - PD) \times LGD \times EAD$

PD - 一般情况下违约概率

WCDR - worst case default rate (99.9%) of the default rate distribution.

- Gaussian copula → define the correlation between defaults.
- Credit Metrics (Economic capital)

→ Monte Carlo simulation.

(模拟 rating 变化) $A \to A'$, $\vdots$, $A^n$

→ it takes into account the impact of rating changes as well as defaults.

# Topic 7: Operational risk models

- Definition (4个要素)
- Categories (七种) Basel's
- Regulatory capital requirement

1) BIA: $\frac{\Sigma GI_{1-3}}{n} \times 15\%$

2) SA: $\Sigma_{i=1}^{3} \max[(GI_{1-8} \times \beta_{1-8}), 0] / 3$

3) AMA → LDA → frequency: poisson.
→ severity: lognormal.

4) SMA { loss component → loss data
BI component. → Gross income.

- Reducing operational risk.
(操作风险缓释方法)

## 一、Operational Risk.

- the risk of loss resulting from:
1) inadequate or failed internal processes
2) people
3) systems.
4) from external events.

- three large operational risks:
1) cyber risks
2) compliance risks
3) Rogue trader risk.
(流氓交易员)

- Basel's seven categories of operational risk

## 二、Regulatory capital requirement:
计算操作风险资本金

① Basic Indicator approach: (BIA)

$$ORC^{(BIA)} = [(GI_{1,\dots,n} \times \alpha)] / n \quad (\alpha = 15\%)$$

n: GI>0的年份数.
GI<0, 剔除, n↓

- average gross revenue for past 3 yrs.
- 粗糙, simple.

② Standardized approach:

$$ORC^{TSA} = \frac{\Sigma_{year(1-3)} \max[(GI_{1-8} \times \beta_{1-8}), 0]}{3}$$

- if GI<0, GI>0.
- three years average of simple summation.
(across each of the business lines in each year)

Advanced measurement approach. (AMA)

- allow a bank to design its own model for calculating risk capital.
- 1-year horizon, 99.9% confidence level.

$ORC^{AMA} = UL$ (1-year, 99.9% confidence)

代表 model:

Loss Distribution approach: < HFLS / LFHS.

③ 1) Loss frequency distribution
number of losses
Poisson distribution.

2) Loss Severity Distribution:
size of a loss
Lognormal distribution.

3) Loss Distribution:
- loss severity, loss frequency ⇒ independent.
- Monte Carlo.

④ Standardized measurement approach:

- bank regulators unsatisfy to the high degree of variation.
- Business Indicator (BI) $\xrightarrow{\text{similar}}$ gross income.
- calculate the required capital from loss component and the BI component.

## 三、Reducing operational risk

- causes of losses → manageable factors.
- Education.
- Risk control and self assessment.
- key risk indicator.

① Insurance:

- Moral Hazard
  - → deductibles 免赔额
  - → coinsurance provisions 共同赔付
  - → policy limits.
- Adverse selection.
  - → research potential customers.

③ Choosing Scenarios:

↓
stress test
↓
output
↓
decision.

1) Historical Scenarios
2) Stress key variables
3) Ad Hoc Stress Test. (特定情景)
4) Using the Results { more capital. / liquidity.

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Topic 8: Stress Testing (压力测试) 特例.

≠ scenario analysis

- stree. testing versus VaR. and ES.
- Governance over Stress Testing
- Principles for Sound Stress Testing.

① Stress Testing versus VaR and ES

VaR and ES { market (daily) / credit / operational (year)

| | Stress Testing | VaR and ES |
|---|---|---|
| Analysis: | forward-looking | backwood-looking |
| Scenarios: | few scenarios (all negative) | a wide range of scenario. (both good and bad) |
| Horizons: | long period. | short time. |

② Governance over Stress Testing

- Board and Senior Management
  - Board of director 指点江山.
  - Senior management 具体执行.
  - covers all business lines and exposures.
- Policy and Procedures:
- Validation and Independent Review.
- Internal Audit.

④ Principals for Sound Stress Testing.
稳健的

- stress testing principles for Banks.