

Financial Instrument.

Bond Market:

- Market rate:

Common market rate:

① Treasury rates:

→ risk-free rates

→ artificially low.

- no retain capital requirement.

- preferential tax

② LIBOR: 信用

→ unsecured borrowing cost

→ highly rated global bank

③ Repo rates: (repurchase agreement)

→ selling price - repurchased price.

④ SOFR

(repo-based secured overnight financing rate)

→ actual transaction

→ 规模大.

⑤ Risk-free rate:

- derivatives are priced from OIS rate

- Overnight indexed swaps (固定利率)

二. Compounding:

1) discrete compounding

→ Bond, forward, futures, swap

$$FV = PV \left(1 + \frac{R_m}{m}\right)^{mT}$$

$$PV = \frac{FV}{\left[1 + \frac{R_m}{m}\right]^{mT}}$$

2) continuous compounding

→ option

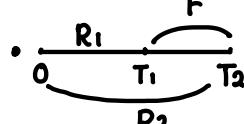
$$FV = PV \cdot e^{R_c \cdot T}$$

$$PV \left(1 + \frac{R_1}{m_1}\right)^{m_1 T} = PV \left(1 + \frac{R_2}{m_2}\right)^{m_2 T}$$

$$PV \left(1 + \frac{R_m}{m}\right)^{mT} = PV e^{R_c \cdot T}$$

三. Spot rate and Forward rate.

zero rate



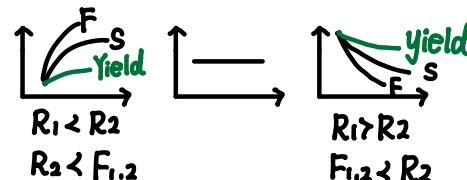
$$\left(1 + R_1\right)^{T_1} \left(1 + F\right)^{(T_2 - T_1)} = \left(1 + R_2\right)^{T_2}$$

$$e^{R_1 T_1} \cdot e^{F(T_2 - T_1)} = e^{R_2 T_2}$$

$$\rightarrow F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

• Term structure

(spot rate 与 T 的关系)



$$\left(1 + R_1\right)^{T_1} \left(1 + F\right)^{T_2 - T_1} = \left(1 + R_2\right)^{T_2} \left(1 + R_2\right)^{T_2 - T_1}$$

→ yield curve: 平均的利率水平

$$P = \frac{CF_1}{(1+R_1)^{T_1}} = \frac{CF_1}{(1+y)^{T_1}} \quad R_1 = y$$

2年按年复利:

$$P = \frac{CF_1}{1+R_1} + \frac{CF_2}{(1+R_2)^2} = \frac{CF_1}{(1+y)} + \frac{CF_2}{(1+y)^2}$$

if upward: $R_1 < R_2$, $y < R_2$

downward: $R_1 > R_2$, $y > R_2$

四. Treasury instrument:

① Treasury Bills < 1yr

→ zero coupon

→ discount basis

→ 360 days./act

→ cash price:

face value = 100, discount rate = 5%, $T = 182$

$$\text{cash price} = 100 (1 - 5\%) \times \frac{182}{360}$$

② Treasury notes and treasury Bonds:

→ coupon bearing (semi-annually)

→ Quoted price:

• one dollar and thirty-second

$$90.03 = 90 + \frac{5}{32}$$

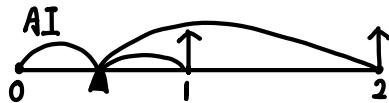
• 报 clean price.

→ Treasury STRIPS: { C-strips (coupon)
P-strips (principal) }

③ Clean price and dirty price:

1) clean price 报价: flat price / quoted price.

2) Dirty price 交易价: full price / invoice price.



$$DP = \sum PVCF_t$$

$$CP = DP - AI$$

• Dirty price = Clean price + Accrued Interest.

3) 已知 CF → DP → CP:

上一期
 付息日 → 下一期付息日
 ~~~~~~  
 •  $AI = \text{Coupon} \times \frac{\text{AI 天数}}{\text{-期付息天数}}$

4) 已知 CP → DP (交易价)

$$DP = CP + AI$$

5) Accrued Interest:

→ Treasury Bond:  $\frac{\text{Actual}}{\text{Actual}}$

→ Money market instruments:

(Treasury bill:  $\frac{\text{Actual}}{360}$ )

→ Corporate and municipal:  $\frac{30}{360}$

(算头不算尾)

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## Corporate Bond.

- 1. Bond Indentures.
- 2. Classification of Bonds
- 3. Bond Risk

### 一. Bond Indentures: (contract)

▷ corporate trustee.

• 由 Issuer 雇佣的.

• 代表 bondholder's interests

• ensure that issuer complies with the indentures.

• in the indentures.

(no obligation to exceed those duties)

### 二. Classification of Bonds:

① Interest rate:

• fixed-rate bond. } 付息 → 本金

• floating-rate bond. }

• zero-coupon bond. → 零息 → 初始价 + AI

### ② Collateral

• Mortgage Bonds (固定资产)

• collateral trust bonds (金融资产)

• Debenture Bonds (信用)

• Guaranteed Bond (有保证的)

• ETCs : Equipment Trust Certificates

A  $\xrightarrow[\text{租金}]{\text{设备}}$  Trust  $\xrightarrow[\$]{\text{ETC}}$  Investor

→ risk all

### ③ High-yield Bond.

### 三. Bond Risk:

▷ Event risk:

• natural disasters

• risk of a large increase in leverage.

- 2) Credit risk
  - credit default risk
  - credit spread risk:  
changes in the level of credit spreads.

- OTC:
- 1° standard dealer, CCP (central counterparties)
  - 2° bilateral margin.

## Bond. Summary

- 1° Interest rate:
  - 1) Market rate
  - 2) Compounding
  - 3) Spot rate & forward rate.

## 2° Treasury Market:

- 1) Treasury bill (cash price)
- 2) Treasury Bond - 半年付息

↳ 报价: — 100

—  $\frac{1}{32}$

— clean price & dirty price

## 3° Corporate Bond

- 1) Trustee
- 2) ETC

## Derivatives Market.

### 1. Introduction of derivative

- swap
  - IRS interest rate swap
  - Currency Swap

### ② Linear and non-linear derivatives:

- linear derivatives: forward payoff =  $S_T - k$  (Delta)
- non-linear: option calls:

$$\text{payoff} = \max(S_T - k, 0) \quad (\text{Delta, Gamma})$$

### ③ OTC and Exchange Traded: (over-the-counter)

- Exchange: standardized

流动性优, 对冲效果差。

## Exchange market:

- 1° membership. → default fund.
- 2° easier for exchange members to close out position.
- 3° Netting.
- 4° Margin. (cash, asset)  
↓ haircut.

→ margin accounts between CCPs and their member:

- 1) Initial margin.
- 2) Variation margin.

## 非清算会员与清算会员之间:

- 3) Maintenance margin. (注意盈亏方向)
  - to the initial margin level.
  - margin call.

## ④ Central Counterparty: CCP

• 会员制 → default fund

建仓 → initial margin.

持仓 → variable margin.

• CCP 违约处置

平仓 { • 平仓拍卖 auction

• 多边净额终止 (tear-up)

• 损失分摊:

→ defaulter → initial margin.

→ default fund.

→ 损失共担 → non-defaulter's default fund.

• advantages / disadvantages.

Summary:

▲ Derivative markets:

- 1° exchange market - vs OTC.
  - margin.
- 2° OTC CCP - risk management.
  - advan & disadvan.

Topic 2 : Forward and Futures:

long 锁定买价 payoff =  $S_T - k$ .

short 锁定卖价 payoff =  $k - S_T$ .

- Forward Rate Agreement:

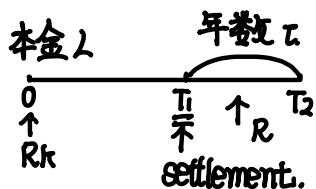
→ underlying: interest rate.

→ an agreement that a certain rate will apply to a certain principal during a certain future time period.

→ Buyer locks a borrowing rate.

Seller locks a lending rate.

→ Settlement.



• long :  $\frac{(R - R_k)t \cdot L}{(1 + R_t)}$

• short :  $\frac{(R_k - R)t \cdot L}{(1 + R_t)}$

→ valuation:

• long :  $PV \left[ \frac{(R_F - R_k) \cdot t \cdot L}{(1 + R_F \cdot t)} \right]$

• short :  $PV \left[ \frac{(R_k - R_F) \cdot t \cdot L}{(1 + R_F \cdot t)} \right]$

$R_F$  - forward rate.

二. Operation of exchange:

① open interest & Trading volume.  
(未平仓合约数) (交易量)

② 约定期  $F_0$  价格特征:

1° 到期

future price converges to the spot price.

2° 到期前:

→ Normal / Contango :  $F \uparrow$  远月 > 近月.  
 $F > S$

→ Inverted / Backwardation:  $F \downarrow$  远月 < 近月.  
 $F < S$ .

3° Mixed: partly normal, partly inverted.

③ Trading order types:

• Market order:

(Best price available in the market)

• Limit order:

(limit price) or at one more favourable.

• 止损指令: (stop order/ Stop-Loss order)

→ stop-limit order.

→ Board order / Market-if-touch order:

(make profit)

→ fill-or-kill order.

三. Forward and Futures Prices:

→ no arbitrage principle.

① 无成本, 无收益:

$$F_0 = S_0 (1 + R)^T$$

• if  $F_0 > \underline{S_0 (1 + R)^T}$  低买高卖.

long spot, short futures.

借  $S_0$  → 还  $S_0 (1 + R)^T$

收  $F_0$

$$\text{profit} = F_0 - S_0 (1 + R)^T$$

• if  $\underline{F_0 < S_0 (1 + R)^T}$  short spot.

long future.

$$\text{profit} = S_0 (1 + R)^T - F_0 > 0$$

• underlying

↳ financial → 收益  $\begin{cases} \text{离散收益 I 现值.} \\ \text{收益率: } Q \end{cases}$

commodity → 成本 → 离散成本 II 现值.

→ 收益率: → lease rate I.

→ convenience yield Y

⑥ Foreign Exchange Forward / Futures

→ underlying: Currency

→ 报价:  $xxxYYY; xxx/YYY$

$\Rightarrow \underline{xxx} = ? \underline{YYY}$

标的资产: 货币 (货币收益)  $R_{xxx}$

$$F_0 = S_0 \left( \frac{1+R}{1+Q} \right)^T \Rightarrow$$

$$= S_0 \left( \frac{1+R_{YYY}}{1+R_{xxx}} \right)^T$$

① 有离散成本 (现值 II), 无收益:

$$\frac{S_0}{0} \xrightarrow{T} \frac{(S_0 + II)(1+R)^T = F_0}{\text{现值.}}$$

• if 市场  $F_0 > (S_0 + II)(1+R)^T$  套利. (套利: 低买高卖)

long spot, short futures

$$\text{profit} = F_0 - (S_0 + II)(1+R)^T$$

⑥ Forward Price for a commodity Asset with a Lease Rate

$$S_0(1+R)^T = F_0(1+I)^T$$

I (income)

$$\Rightarrow F_0 = S_0 \left( \frac{1+R}{1+I} \right)^T$$

② 有离散收益 (现值 I):

$$\frac{S_0}{0} \xrightarrow{T} \frac{(S_0 - I)(1+R)^T = F_0}{\text{现值.}}$$

if  $F_0 > (S_0 - I)(1+R)^T$

short futures long spot

$$\text{profit: } F_0 - (S_0 - I)(1+R)^T > 0$$

① Forward Price for a commodity with

Storage Cost & Convenience yield.

$$\frac{S_0 + II}{U} \xrightarrow{T} \frac{F_0}{Y}$$

$$(S_0 + II)(1+R)^T = F_0(1+Y)^T$$

$$\Rightarrow F_0 = (S_0 + II) \left[ \frac{1+R}{1+Y} \right]^T \xrightarrow{\text{yield storage cost 现值}}$$

③ 有收益率 (Q, Y, I)

↑ 按年复利.

$$S_0(1+R)^T = F_0(1+Q)^T$$

$$\Rightarrow F_0 = S_0 \left( \frac{1+R}{1+Q} \right)^T$$

• if 市场  $F_0 > S_0 \left( \frac{1+R}{1+Q} \right)^T$

short future. long spot.

$$\text{profit} = F_0 - S_0 \left( \frac{1+R}{1+Q} \right)^T > 0$$

④ Value of Forward Contract:

→  $t=0$ , value = 0

$$\rightarrow \frac{k \xrightarrow{\text{签订 T}} F_0}{\frac{-t}{0} \xrightarrow{T} (F_0 - k)}$$

$$V_{\text{long}} = \frac{F_0 - k}{(1+R)^T}$$

▲ if  $F_0 = S_0(1+R)^T$

$$\Rightarrow V = S_0 - \frac{k}{(1+R)^T} = S_0 - PV(k)$$

▲ if  $F_0 = (S_0 - I)(1+R)^T$

$$\Rightarrow V = (S_0 - I) - PV(k)$$

⑤ Forward price for stock Index:

→ dividend: Q

$$F_0(1+Q)^T = S_0(1+R)^T$$

$$\Rightarrow F_0 = S_0 \cdot \left[ \frac{1+R}{1+Q} \right]^T$$

#### 四、Interest Rate Futures (利率期货)

• 对冲利率风险.

##### ① T-Bond Futures

underlying asset: 虚拟券 (可选择)

→ choose which particular bond with a maturity more than 15 yrs on the first day of the delivery month and is not callable within 15 yrs from that day to deliver.

→ conversion factor: defines the price received.

• (交割)月第 - 天:)

• 假设期限3个月整数倍

•  $y = 6\%$

• face value = 1

⇒ clean price.

→ 报价 QFP Quoted future price

→ short position (cash inflow):

$$\text{Cash received} = \frac{(\text{QFP} \times \text{CF}) + \text{AI}}{\text{conversion factor.}}$$

→ 交割成本最低的 bond:

CTD: cheapest-to-deliver bond

$$\text{net cost} = \begin{cases} \text{买 Bond QBP} + \text{AI} \\ (\text{quoted bond price}) \\ \text{收到 QFP} \times \text{CF} + \text{AI} \end{cases}$$

$$\Rightarrow \text{net cost} = \text{QBP} - (\text{QFP} \times \text{CF}) \quad (\text{选 cost 低的})$$

→ 交割结算价: last settlement price.  
QFP

##### ② Eurodollar Futures: (3 month)

欧洲美元期货.

• underlying: long lending rate (存款利率)  
收固定.  
 $r \uparrow$ , 收益  $\downarrow$

• 本金 1 million, 3月

$$\text{报价 Final settlement price: } FQ = 100(1 - F_t)$$

$$V = 1 \text{ million} \times (1 - F_t \times \frac{1}{4})$$

$$|DV0| = \$25$$

(1 basis point move  $\Rightarrow$  gain/loss)

$$F_t \uparrow 1 \text{ bps} \rightarrow V \downarrow \$25$$

##### • Eurodollar Futures vs. FRA

marking to market.

→ if  $F_t \uparrow$   $V \downarrow$ , 补 margin, 不利, Futures 偏低  
(利率)

→ if  $F_t \downarrow$ ,  $V \uparrow$  投资, 不利, Futures 偏低

$\Rightarrow$  Futures Rate  $\uparrow$

→ Forward Rate

= Futures rate - convexity adjusted.

$$\frac{1}{2} 6^2 T (T + 0.25)$$

① Act / Act

② continuous compounding } Forward rate

} Future rate.

#### 五、Hedging strategies using Futures.

##### ① Basis risk: (uncertainty in the basis)

$$\text{Basis} = \text{spot price} - \text{futures price.}$$

① 担心价格下跌 : long the Basis

$$\rightarrow \text{long cash} + \text{short futures} : \frac{S_T - (F_0 - F_t)}{T} \Rightarrow F_0 + \frac{(S_T - F_t)}{Basis}$$

→ Basis 上升有利

② 担心价格上涨 : short the basis.

short + long futures

$$-S_T + [F_t - F_0] \Rightarrow -F_0 - \frac{(S_T - F_t)}{Basis}$$

④ Basis risk { • different asset.  
• different maturity.

⑤ Cross hedging : { underlying asset A

Future  
underlying asset B)

## ⑥ Short Hedge and Long Hedge:

1) long hedge: long position in a future contract.

Short hedge: short position in futures contracts.

对冲系统性风险:

→ hedging with stock Index futures:

$$N = \frac{\beta^{\text{Target}} - \beta_S}{\beta_F} \cdot \frac{V_S}{V_F}$$

股指期货与市场组合  $\beta_F = 1$

$$\Rightarrow N = (\beta^T - \beta_S) \cdot \left( \frac{V_S}{V_F} \right)$$

↓  
index × 250

→ return:  $\delta_S, \delta_F, \beta_{S,F}$

$$\beta = \rho_{S,F} \cdot \frac{\delta_S}{\delta_F}$$

## ⑦ Hedging with Futures Contract:

1) hedge 总价格风险

2) hedge 系统性风险

3) hedge 利率风险

对冲总价格风险:

• Minimum variance hedge ratio

(changes in the spot price, changes in the futures price)

$$\Delta S \qquad \qquad \Delta F$$

$$\rightarrow \min \sigma^2[\Delta S + h \Delta F]$$

$$\Rightarrow h^* = \rho_{\Delta S, \Delta F} \cdot \left( \frac{\delta_S}{\delta_F} \right)$$

↓  
h份 futures 去抵冲  $\Delta S$  变化

→ 实际单位调整:

$$N^* = h \cdot \frac{Q_S}{Q_F}$$

→ hedging with futures contract:

• daily settlement and series of one-day hedge

• Trailing the hedge

$$\rightarrow N^* = \frac{h^* \cdot V_S}{V_F}$$

Summary:

• 对冲总价格风险: ①  $\Delta S$  与  $\Delta F$

$$h = \rho_{\Delta S, \Delta F} \cdot \left( \frac{\delta_{\Delta S}}{\delta_{\Delta F}} \right)$$

②  $Q_S$  与  $Q_F$

$$N^* = h \cdot \left( \frac{Q_S}{Q_F} \right)$$

⇒ one-day hedge:

$$N = h^* \cdot \frac{S}{F} \cdot \frac{Q_S}{Q_F} = h^* \cdot \frac{V_S}{V_F}$$

→ Hedging with Interest Rate Futures (利率风险)

• D duration { T-Bond  
Eurodollar

• Duration Neutral: →  $D^T=0$ . (完全不受利率影响)

$$N = \frac{D^T - D_S}{D_F} \cdot \frac{V_S}{V_F}$$

$$\rightarrow \text{T-Bond: } N = -\frac{D_S}{D_F} \cdot \frac{V_S}{V_F} \quad (D^T=0)$$

$$\rightarrow \text{Eurodollar: } N = \frac{D_{Vol}}{D_{Vol,F}} \quad (\text{Eurodollar } DVol \approx 25)$$

→ Creating Long-Term Hedges: (use short-term futures)

roll yield: (solve lack of liquid)

1. long + short futures

→ contango  $F > S$ : roll yield > 0

→ Backwardation  $F < S$ : roll yield < 0

2. short + long futures:

→ contango  $F > S$ : roll yield < 0

→ Backwardation  $F < S$ : roll yield > 0

## 六. Foreign Exchange Quotes:

### ① Quotes:

$\underbrace{xxx}_{\text{base currency}} \text{ YYY}, \quad xxx / YYY$

xxx base currency

YYY quoted currency

(how much quoted currency is required to buy a unit of base currency)

example:

现货: 1.2 EUR USD

远期: 20

$$\Rightarrow 1.2 + \frac{20}{1000}$$

### ② Bid-ask spread: Ask price - bid price.

dealer 买价

### ③ 产品:

- Outright transaction. / forward outright transaction.  
→ two parties agree on an exchange at some future date
- FX swap

期初 A  $\xrightarrow{\text{转}}$  B, 期末 B  $\xrightarrow{\text{转}}$  A

## 七. FX risk:

### ① Transaction risk

- related to Receivable, Payable.
- hedged with outright forward transaction.
- FX swaps.

### ② Translation Risk: 外币折算风险.

- from assets and liability  
(dominated in a foreign currency)

- have a big effect on its reported earnings.

### ③ Economic risk:

(美国 company, 在英国收美元)

## ④ Factors that determine exchange rates:

- Trade flows, balance of payments.
- Inflation. • Monetary policy

→ 当前汇率  $S_0 \text{xxx} \text{ YYY}$

$$(1 \text{xxx} = S_0 \text{ YYY})$$

X国用1买一篮子货币  $\xrightarrow{\text{inflation } X} 1 + \text{Inflation } X$

Y国用  $S_0$  买一篮子货币  $\xrightarrow{\text{inflation } Y} S_0 (1 + \text{Inflation } Y)$

$$S' = \frac{S_0 (1 + \text{Inflation } Y)}{1 + \text{Inflation } X}$$

$$\frac{\Delta S}{S} = \frac{S' - S_0}{S_0} = \frac{1 + \text{Inflation } Y}{1 + \text{Inflation } X} - 1$$

$$= \text{Inflation } Y - \text{Inflation } X$$

## ⑥ Nominal and Real Rates

→ nominal interest rate: quoted in the market

→ Real interest rate: adjusted for inflation.

$$R_{\text{nom}} = R_{\text{real}} + R_{\text{infla}}$$

$$(1 + R_{\text{nom}}) = (1 + R_{\text{real}})[1 + R_{\text{infla}}]$$

### ⑦ Covered Interest Parity:

$$F = S \left[ \frac{1 + R_{\text{xxx}}}{1 + R_{\text{yyy}}} \right]^T$$

## Forward & Futures

- ① FRA - long, short,
  - settlement
  - valuation

## 2° Futures market

→ open interest vs trade volume.

→ normal vs. Inverted.

→ Trading order

## 3° Forward / Futures price.

$$\textcircled{1} \text{ 标准式: } F_0 = S_0 (1+R)^T$$

$$\textcircled{2} \text{ 有成本: } F = (S_0 + M) (1+R)^T$$

$$\textcircled{3} \text{ 有 discrete 收益: } F = (S_0 - I) (1+R)^T$$

$$\text{有 continuous 收益: } F = S_0 \left[ \frac{1+R}{1+Q} \right]^T$$

⇒ arbitrage 低买高卖, profit.

⇒ forward valuation.

## 4° Interest Rate futures:

① T-Bond futures, CTD 选择

② Eurodollar futures, DV01=25

• 与 FRA 比较

• 凸性调整, convexity adjustment.

## 5° Hedging:

① Basis risk

② 建交易策略 Strategies

{ 方向

{ 风险 - 总价格 risk (h.N)

- 系统性 risk ( $\beta$ )

- 利率 risk. ( $D$ )

③ long-term 流动性要求 ⇒ stack & roll  
优缺点.

## 6° Foreign Exchange Market

① 报价 现货, 远期, 期货 bid-ask.

② 风险

③ 汇率影响因素 - inflation

## Topic 3: Swap

{ Interest rate swap

{ Currency swap.

△ Swaption. (基于 swap 的 option)

### 一. Interest rate swaps:

• fixed ⇌ floating 无本金交换

• long 支固定 (bid) 买价

short 收固定. (ask) 卖价)

• Swap rate =  $\frac{1}{2}(\text{bid} + \text{ask})$

• 允许两者都照比较优势去借.

### ② Valuation:

{ 原 swap 收  $x\%$  支  $2$

现 swap 支  $y\%$  收  $2$

•  $V_{\text{组}} = \sum PV(x\% - y\%)$

•  $t=0 \Rightarrow V=0$

• 先算 swap rate =  $\frac{1}{2}(\text{bid} + \text{ask})$

• if remaining life = 2.5 yrs ⇒ interpolate between quotes.

### 二. Currency Swaps:

{ fixed - fixed currency swap), { 期间交換利息 )

到期交換本金

#### ① Bond valuation

• 收 USD & GBP:



$$\Rightarrow \text{Swap} = \text{Bond}_{\text{USD}} - \text{Bond}_{\text{GBP}} * S_0$$

$$S_0 \text{ GBP } \text{USD} \Rightarrow 1 \text{ GBP} = S_0 \cdot \text{USD}$$

#### ② Valuation using Forward Exchange Rate

→ Assume the forward Libor rates will be realized.

→ calculate net cash flow.

→ discount the net cash flow.

$$F_0 = S_0 \left[ \frac{1+R_{\text{XXX}}} {1+R_{\text{YYY}}} \right]^T$$

$$S_0 \text{ XXX } \text{YYY}$$

## Summary: Swap

### 1. IRS (interest rateswap)

① Bid, ask, swap rate.

② 比较优势分析

### ③ Valuation { · 组合 (swap + interest swap)

· 假设 forward rate Realised)

### 2. Currency Swap:

· valuation

{ ① Bond

② 假设 forward exchange rate Realized.

### ④ Moneyness 1) 价值状态:

· in the money: positive payoff

· at the money: payoff = 0

· out of the money: payoff < 0

### ⑤ Intrinsic value and time value:

→ Intrinsic value:

$$\text{call: } C = \max[S - X, 0]$$

$$\text{put: } P = \max[X - S, 0]$$

→ Time value = option price - time value.

### ⑥ Properties of options:

① Call option.

European (no dividend) { up:  $S_0$   
down:  $\max[S_0 - PV(k), 0]$

European (dividend) {  $S_0$   
 $\max[S_0 - PV(k) - PV(\text{Dis}), 0]$

American (no dividend) {  $S_0$   
 $\max[S_0 - PV(k), 0]$

American (dividend) {  $S_0$   
视红利情况而定

$S_t \rightarrow S_0 \rightarrow S_0 - PV(\text{Divs})$

$$\hookrightarrow \frac{S_0}{(1+Q)^T}$$

$$\hookrightarrow S_0 e^{-\alpha T}$$

② put options:

European (no dividend) { up:  $PV(k)$   
down:  $\max[PV(k) - S_0, 0]$

European (dividend) { up:  $PV(k)$   
down:  $\max[PV(k) - S_0 + PV(\text{Divs}), 0]$

American (no dividend) { up:  $k$

会提前行权 [down:  $\max[k - S_0, 0]$

American (dividend) { up:  $k$

视红利情况而定 [提前行权视红利而定]

## Topic 4: Options Market.

- Properties of stock options
- Trading strategies
- Exotic options

- properties of stock options:

① 分类: call put

European and American option

不分红的美式看涨期权不会提前行权)

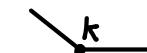
② payoff:

③ long call



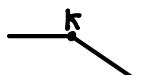
$$\text{payoff} = \max(S_T - k, 0)$$

④ long put



$$\text{payoff} = \max(k - S_T, 0)$$

⑤ short call



$$\text{payoff} = -\max(k - S_T, 0)$$

⑥ short put



$$\text{payoff} = \max(k - S_T, 0)$$

### ⑥ Put-call parity: European.

$$C + \underline{pv(k)} = P + S_0$$

$$\Rightarrow S_0 - PV(D_{i+1})$$

$$\Rightarrow S_0 / (1+Q)^T$$

$$\Rightarrow S_0 e^{-QT}$$

$$\Rightarrow Call + PV(k) + PV(D_{i+1}) = Put + S$$

### ⑦ Bear spread (预期 P↓)

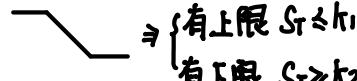
高买低卖

Bear call spread ( $k_1 > k_2$ )

→ long  $k_2$  call, short  $k_1$  call.

$$\rightarrow profit = \max[S_T - k_2, 0] - C_2$$

$$- \max[S_T - k_1, 0] + C_1$$

  $\Rightarrow \begin{cases} \text{有上限 } S_T \leq k_1 \\ \text{有下限 } S_T \geq k_2 \end{cases}$

### ⑧ Box Spread:

→ Combination of a bull call spread

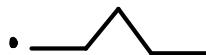
and a Bear put spread

→ payoff is always  $k_2 - k_1$

### ⑨ Butterfly spread:

• 4 options, 3 ↑ strike price

$$\left( \frac{k_1, k_2, \frac{1}{2}(k_1+k_2)}{\downarrow \text{long} \quad \downarrow \text{sell} \times 2} \right)$$



• Expects low volatility, capped risk

• Butterfly call spread:

→ long  $k_1$  call, long  $k_2$  call, short  $2 \times (\frac{k_1+k_2}{2})$  call

$$\rightarrow profit = \max[S_T - k_1, 0] - C_1 + \max[S_T - k_2, 0] - C_2$$

$$- 2 \max[S_T - (\frac{k_1+k_2}{2}), 0] + 2C_3$$

### ⑩ principal protected notes (PPN)

$$\underline{pv(k)} + C \leq k$$

无风险投资

$$\max(S_T, k)$$

$$C + \underline{pv(k)} \leq S_0$$

### 三. Spread Strategies 同种期权

{ 不同 strike price: vertical  
不同 T: horizontal calendar  
不同大. 不同 T:

→ 结构, profit

#### ① Bull Spread: (预期 P↑)

低买高卖

$k_1$  低  $k_2$  高

• long  $k_1$  call, short  $k_2$  call ( $k_1 < k_2$ )

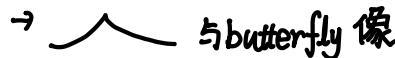
$$profit = \max[S_T - k_1, 0] - C_2$$

$$- \max[S_T - k_2, 0] + C_1$$

→ 有下限 ( $S_T \leq k_1$ ) 有上限 ( $S_T \geq k_2$ )

#### ② Calendar spread (不同 T)

→ long 长期, short 短期

→  与 butterfly 像

### 四. Combination strategies [不同种 option]

#### ① Straddle and Strangle



• 赌波动

• Direction neutral.

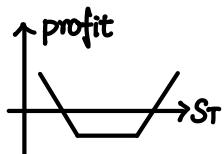
• a call and a put (same strike price)

• wants volatility

### ▲ Straddle:

- long  $k_1$  call, short  $k_1$  put
- profit =  $\max(S_T - k_1, 0) - C + \max(S_T + k_1, 0) - P$

### ▲ Strangle:



Strangle: a call and a put  
高 k      低 k

- different strike price
- like straddle, but cheaper.
- wants volatility.

### Inclusion

| Simple Strategy      | A share and an option          |
|----------------------|--------------------------------|
| Spread Strategy      | Both are call, or both are put |
| Bull                 | 2 different K                  |
| Bear                 | 2 different K                  |
| butterfly            | 3 different K                  |
| Calendar             | 2 different T                  |
| Combination Strategy | Call and put                   |
| Straddle & Strangle  | Wants volatility               |

## 五. Exotic options (特殊性质)

### ① Gap option. (判断, 执行不一样)

• The payoff from a call option:  $S_T - k_1$  (if  $S_T \geq k_2$ )

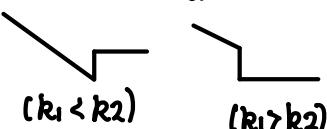
• the payoff from a put option:  $k_1 - S_T$  (if  $S_T \leq k_2$ )



1) negative  
2) discontinuous

Gap put:  $S_T \leq k_2 \rightarrow k_1 - S_T$

$S_T > k_2 \rightarrow 0$



### ② Forward start options:

(远期生效期权)

• will be active at some specified future time.

### ③ Compound options (复合期权)

• options on options. (2个K, 2个T)

1) a call on a call.

2) a put on a call.

3) a call on a put.

4) a put on a put.

• 支付2个premium 期权费.

#### ④ Chooser Option:

→ after a specified period of time, the holder can choose whether the option is a call or put.

$$\max(C, P) \quad \text{K.T. 欧式}$$

→ 复制] : payoff =  $\max[C, P]$

$$C = P + S_T - PV(k)$$

$$\Rightarrow \text{payoff} = [P + S_T - PV(k), 0]$$

$$\Rightarrow \text{payoff} = [C, C + PV(k) - S_0]$$

$$= C + \max(PV(k) - S_0, 0)$$

#### ⑤ Barrier options: (便宜)

→ knock-in 生效

→ knock-out 失效.

1°  $S_0 < \text{barrier up}$

2°  $S_0 > \text{barrier down.}$

- down-and-out call

- down-and-in call.

- up-and-out call.

- up-and-in call.

→ path dependency 路径依赖

#### ⑥ Binary Options / Digital options

- cash-or-nothing { call  $S_T > k \rightarrow \text{cash } k'$   
 $S_T < k \rightarrow 0$

$$\left\{ \begin{array}{l} \text{put: } S_T \leq k \rightarrow k' \\ S_T > k \rightarrow 0. \end{array} \right.$$

- asset-or-nothing: call {  $S_T \geq k \rightarrow S_T$  asset  
 $S_T < k \rightarrow 0$

$$\left\{ \begin{array}{l} \text{put: } S_T \leq k \rightarrow S_T \\ S_T > k \rightarrow 0. \end{array} \right.$$

- regular European call option

- regular European put option.

#### 3° exotic option.

① Gap option.

④ Lookback option

② chooser option.

③ Asian option.

⑤ Barrier option.

#### ① Lookback options:

→ payoff depend on maximum or minimum of the underlying asset.

- call  $\max[S_T - k, 0]$

floating strike:  $\max[S_T - S_{\min}, 0]$   
 $= S_T - S_{\min}$ .

fixed strike:  $\max[S_{\max} - k, 0]$   
 $=$

- put  $\max[k - S_T, 0]$

floating strike  $\max[S_{\max} - S_T, 0] = S_{\max} - S_T$

fixed strike:  $\max[k - S_{\min}, 0]$

#### ② Asian Option 便宜.

→ payoff depends on arithmetic average of the underlying asset price.

- average price option:

$S_T \rightarrow S_{avg}$

- Average strike option:

$k \rightarrow S_{avg}$

#### ③ Volatility and Variance Swap:

- Volatility swap.

→ exchanging of volatility based on a national principal

- Variance swap

#### ④ Static options Replication: 静态期权复制

Summary:

1° Stock option properties

① American 不分红的美式看涨期权不会提前行权

② 价值的上下限 (upper & lower bounds)

③ put-call parity.

2° Strategies

① simple strategies: covered call.

② spread: bull & Bear.

Butterfly.

Calender spread.

③ Combination: Straddle & Strangle.

## Topic 1: MBS

- { • Mortgage
- Mortgage-based Security

### - Mortgages:

① 月供，等额本息，完全摊销

[2ND] [PV] → AMORT

P<sub>1</sub> 起始月

P<sub>2</sub> 截止月

for example P<sub>1</sub>=1, P<sub>2</sub>=1

② Principal / Balance =  $\sum PVCF_t$

(assumption: R不变)

③ Mortgage portfolios / (mortgage pools)

→ WAC weighted average coupon:

→ 本金占比的权重

→ WAM: weighted average maturity:

→ 期限的加权平均；

④ 提前偿付风险: (prepayment)

→ SMM:

单月提前偿付率:

one-month mortality (SMM)

$$SMM = \frac{\text{prepayment}}{\text{Balance} - \text{scheduled Principal Payment}}$$

→ CPR: annualized SMM:

假设每月的都是SMM

→ constant prepayment rate:

$$\rightarrow CPR = 1 - (1 - SMM)^{12}$$

### 二. Mortgage-backed Securities:

① MBSs:

Agency Mortgage-backed securities.

→ pass-through securities: (MPS)

(all investors in a pool receive same return)

→ specified pools,

→ to be announced (TBAs).

CTD 选择

• dollar roll.

→ sell TBA (to-be-announced) in one settlement month and buying a similar TBA in the next.

→ originator losing one month's interest payment

② Value of the roll:

卖出 ≈ 买入

A — sale price of first month.

(30 days/month,

B — second month purchase pool price (AI)

C — interest's on one month's sales.

D — coupon and principal repayment.  
sold in the first month

A-B+C-D

value of a roll.

① calculate accrued interest.

② calculate A, B (1million).

③ C-interest.

④ D - 放弃的

### ③ Collateralized Mortgage Obligation: (CMO)

- tranches A, B, C.
- take on different amount of prepayment risk.

### 3. MBS

#### ① products

- pass-through
- CMO
- IOs vs POs

### ④ IOs (interest-only) and principal-only securities (POs)

- Stripped MBSs
- as prepayments increase, a POs become more valuable  
IOs become less valuable.

### ② Valuation.

① 绝对价值估计

② 相对价值估计

{ simulation → v.  
OAS }

### ⑤ Evaluating the MBS

- 绝对价值
- 相对价值

$$\text{Bond: Market price} = \sum \frac{C_{Ft}}{(1+R_f + R_p)^t} \rightarrow \begin{matrix} \text{2-spread} \\ (\text{risk-premium}) \end{matrix}$$

$$\text{MBS: Market price} = E \left[ \sum \frac{C_{Ft}}{(1+R_f + R_p)^t} \right] \xrightarrow{\text{OAS}}$$

→ 因为 prepayment 导致 cash flow 不稳定  
用 Monte Carlo simulation.

- 1° 对利率 (mortgage rate, R\_f) 房价进行建模
- 2° 结合 prepayment model → prepayment
- 3° prepayment → 确定 CF
- 4°  $\sum PVCF_t$ . 每个 path.
- 5° Repeat 1→4, 求出 N↑ path
- 6° 求平均 → v

### → Option-adjusted Spread (OAS)

- 去除 pre-payment 的 premium.
- 用试错法测 OAS

### Summary:

1° Mortgage : cash flow, PMT, INT, PRN, BAL

2° Mortgage pool 特征:

1) WAM, WAC } 计算

2) SMM, CPR }

### 4. specified pool vs TBA.

TBA 运用 Dollar Roll:

→ 与 Repo 比较

→ dollar roll 的 value 如何计算

### Topic I: Financial Institution

- Bank
- Insurance Companies
- Fund Management

#### 1. Bank.

##### ① Originate-to-Distribute Model

##### • Securitization

• it: get them off the balance sheet.

释放 funds. capital  
future fee.

缺: relax their mortgage lending standard

### ② Investment Banking

#### • Private placement (私募)

#### • Public offering

→ Best offers 代销

→ Firm commitment 包销

#### • IPO: initial public offering

– Dutch auction approach.

– Issues shares is note publicly traded.

### ③ Conflicts of Interest problem.

- Internal Barriers (Chinese walls)

### ④ Three main types of risk facing bank.

- market risk (与价格变动相关)
- credit risk
- operational risk (人、系统、流程、外部事件)

### ⑤ Banking Book and Trading Book

- Banking Book: loans
  - credit risk capital requirement.
- Trading Book: asset & liability
  - market risk capital.

### ⑥ Capital management:

- regulatory capital
- Economic capital

→ Deposit Insurance { risk-adjusted premium.  
capital

- produced by government regulators
- moral hazard problem.
- up to a certain level

### 二. Insurance companies:

- 1) Life
- 2) property
- 3) pension plan.

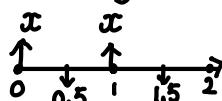
#### ① Life insurance:

- Break-even premium:

$$\sum PVCF(\text{expected premium}) = \sum PVCF(\text{expected payout})$$

- mortality tables

(假设 payout 年中)



流入  $x + (1 - 0.017257) \frac{x}{1+3\%}$

流出:  $0.017257 \times 1m \times \frac{1}{(1+3\%)^{0.5}}$

$+ (1 - 0.017257) 0.01904 \times 1m \times \frac{1}{(1+3\%)^{1.5}}$

### ② Basic risks:

- Mortality risk: 死得早
  - annuity contract 好
  - life insurance contract 不好.
- Longevity risk: live longer.
- Hedging: reinsurance,  
longevity derivative contract.  
forward / swap.

### ⑦ Property - Casualty Insurance:

财产险, 盈亏情况:

$$\textcircled{1} \text{ Loss ratio} = \frac{\text{Payouts}}{\text{premiums}}$$

(支付给受益人的费用)

$$\textcircled{2} \text{ Expense ratio} = \frac{\text{Expenses}}{\text{premiums}}$$

(给第3方) {理赔  
营销.

$$\textcircled{3} \text{ Combined Ratio} = \text{Loss ratio} + \text{Expense Ratio}$$

④ Combined ratio after dividends.

⑤ Operating ratio

$$= \text{combined ratio after dividends} - \text{investment income.}$$

(每年每单位保费对应的净支出)

> 1 (亏); < 1 (盈)

### ③ Pension plan

- Defined Benefit Plan.

→ 收益确定好的

→ significant risks on employers.

→ all contributions are pooled.

- Defined Contribution plan  
→ 收益不确定的
- contributions are invested on behalf of the employee.
- employee 承担风险.

#### ④ Risks facing Insurance company:

##### ① moral hazard: deductible

co-insurance provision  
policy limit.

##### ② Adverse selection:

cannot distinguish between good and bad risks.

→ 多了解投保人

##### • 欧洲: 文件 Solvency II

→ 要求资本. capital. requirement

solvency capital requirement SCR → risk plan.

Minimum capital requirement MCR.

不可交易

1° investment risk (market, credit)

2° underwriting risk (life property)

3° operational risk.

### 三. Fund Management (基金公司)

#### ① Mutual fund.

#### ② ETFs

#### ③ Hedge fund.

##### ① Mutual fund (中水投资者)

• open-end fund: 随时可以申购赎回.

→ net asset value. NAV.  $\Rightarrow$  Asset - Liability

→ 未知价法

→  $\frac{\text{Market value of the fund portfolio}}{\text{Number of shares outstanding}}$

• close-end funds:

Have a fixed number of shares outstanding

→ NAV

→  $\frac{\text{Market value of the fund portfolio}}{\text{Number of shares outstanding}}$  (fair market value)

#### ② ETF: exchange-traded funds:

→ 可以随时申购, 赎回.

→ 也可在交易所

→ 实物申购, 实物赎回.

→ No appreciable difference between  
trading price and fair market value.

#### ③ Undesirable Trading behaviour in Mutual Fund and ETF market.

→ late trading (not permitted by SEC)

→ 捕时交易: market timing. (not illegal)

→ Front Running (illegal) 强跑.

→ Directed Brokerage (from upon by regulators)  
定向经纪.

#### ④ Mutual fund vs. Hedge fund.

• relative small investors

• stringent

(监 管 要 求 严)

• wealthy individuals

• Large investors

• Regulatory less.

|                       | Mutual Fund                 | Hedge Fund                                         |
|-----------------------|-----------------------------|----------------------------------------------------|
| Client Group          | Relatively Small Investors  | Wealthy Individuals/Large Investors                |
| Redemption Provisions | Can Redeem on Any Given Day | Lock-Up Period                                     |
| Regulatory            | Stringent                   | Less                                               |
| NAV                   | At Least once a Day         | No Such Requirement                                |
| Strategies            | Must Disclose               | Don't Disclose Everything                          |
| Leverage              | Constrained                 | Only Limited by How Much Banks are Willing to Lend |
| Fee Structure         | Management                  | Management + Incentive profit.                     |

2 and 20. → (incentive fee)  
↓  
management fee.

- hurdle rate.
- high water mark clause.
- clawback clause.

## ⑤ Hedge fund Strategy

- Long / Short Equity

long undervalued short overvalued.

$\beta=0$

- Dedicated short.

look exclusively for overvalued and sell them short.

- Fixed Income Arbitrage :

Buy bonds that seem relatively cheap

while shorting those that are relatively expensive

- Convertible Arbitrage :

convertible bond  $\rightarrow$  stock

- Distressed securities (ccc)

undervalued

- Merger Arbitrage  $\rightarrow$  deal risk

long [被收购方], short [收购方]

- Global Macro

- Managed Futures