STABILITY IN APPENDING RENEWABLE GENERATORS ON NATIONAL POWER GRID NETWORKS

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NOTE TO DR. RESTREPO: *This paper has likely been written at a lower level than it should have been if you are the only person reading it. We did this intentionally because it is possible that we may show the paper to research mentors, and if you approve of it, we would consider getting the paper published in a small student journal that CU offers. In that context we think it is best to assume that the reader is a scientist, but not one that is extremely familiar with network science or the power grid.*

Abstract

This study investigates the optimal approach to appending asynchronous (solar and wind) generators on a power system network using mathematics in network theory. Specifically, we consider an example network randomly generated from Chinese power system data as our experimental network to test our results. Before our significant findings the paper explores the fundamentals of power grid setup, necessary assumptions for modelling, and some mathematical formulation where we define our unique stability metric. Using our Chinese inspired experimental network, we were able to show a specific relationship between stability and connecting a asynchronous generator to the network. Other papers typically discuss appending a very general generation node, which ends up being equivalent to a synchronous generator and incorrectly modifies their optimal solution for appending renewable nodes. Our superior result incorporates the unpredictability of solar and wind resources when adding asynchronous generation to a national grid.

1 Introduction

Civilization in this day and age is entirely dependant on power. In the United States alone *one* average human consumes over 43,000 kWh of energy a year, which is equivalent to 200 fit workers, turning a generator 40 hours a week, all 365 days per year! To supply individuals with this unthinkable amount of power, national power grids around the world have established complex networks that are dependable enough for consumers to hardly ever have to consider how their power is even arriving. In recent years, modifications for these complex power networks are taking a turn towards integrating renewable generation technologies, such as photovoltaics (solar) or wind turbines, in order to conserve greenhouse gas emissions. As these renewables are appended onto well-established and continuously operating power grids, the resiliency of the networks become increasingly unstable. The purpose of this project is to give a mathematical outline of how these renewable technologies should be strategically connected to the networks using concepts of dynamic network stability and synchronization. Our findings are carried out through the application of a randomly generated graph based on centrality and connectivity parameters of the well-established Chinese power grid. After this model is completed it would be easy to complete further analysis on different national power grids that may have varying network arrangements. In this paper you will find a brief overview of the fundamental concepts that physically appear on a power grid, a theoretical and mathematical justification of our approach, results, and a final interpretation of the results and their implications.

2 Power System Fundamentals

Before considering the mathematics and main compositions of our project, it is important to first gain a physical understanding of how a national-scale power system is composed and physically behaves. This section contains a summary of important terminology used by electrical and power system engineers, and gives a physical explanation of grid stability that answers the question: Why can renewables threaten the overall synchronization of the grid?

2.1 Components of Power Grids

In its simplest form, a national power grid is a system that has the ability to harness energy from the environment around us and transport it across large distances for consumption. This process has three primary stages: *generation*, *transmission*, and *load consumers*. There are many different types of generators that exist on our grid. In this paper it will be important to distinguish between fossil (resources not easily renewed) generators–such as coal, natural gas, or nuclear plants– and renewable generators: solar photovoltaics, wind turbines, hydro, geothermal, etc. Power harnessed at these generators is transported through transmission lines, which will be considered as edges of our corresponding network model. Lastly, load consumers are the technology and devises that demand power from the grid. This could be everyday appliances like your computer or refrigerator, or it could be an entire office building. To avoid any confusion in modelling all the different types of consumer loads that may exist in an instant, a national scale model like ours usually considers large-scale load profiles that approximate the power required to run an entire city or neighborhood. With smaller scale generation types like rooftop solar, solar farms, and wind farms, it is becoming common for a "consumer" load profile to actually provide both ways into the *undirected* grid. You will find both *negative* and *positive* load consumption types in the models of this project.

The power grid is a perfect system to be modelled and simulated on a mathematical network. The intersection point connecting any two of the three stages mentioned above (or the intersection of two transmission lines) is called a *bus* and is analogous to a "node" in network theory. A bus can connect two or more edges of the network, making the grid a very complex system that expands quickly. Typically, data measurement like voltage difference, current, or frequency are physically measured at these electrical busses, which is perfect for including relevant data in the nodes of our power grid network.

2.2 Power Flow

Power is transferred across the system by alternating current (AC). Instead of flowing directly from generation to load as would be seen in direct current (DC), AC current flow is oscillatory and changes directions with a certain frequency (typically 60Hz on a power grid). Consider the simple system animated in Figure 1 below.

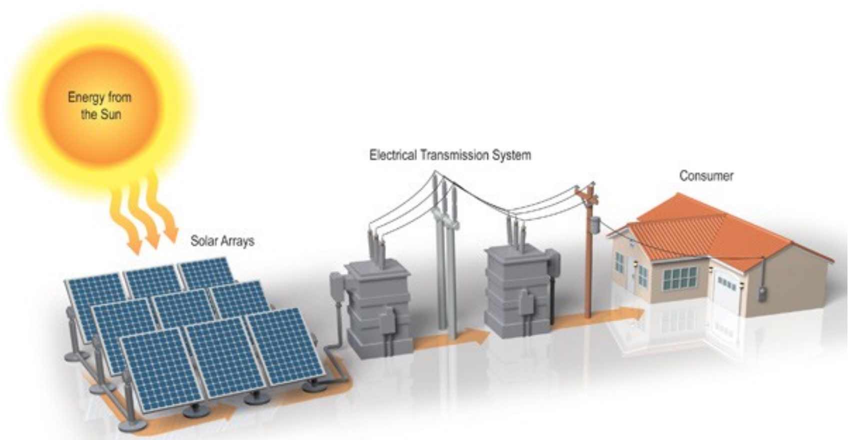


Figure 1: Simplification of the power delivery process. This figure renders a linear example of the three stages of power deliver: generation, transmission, and consumer loads. https://medium.com/@solar.dao/how-energy-travels-what-happens-with-pv-solar-power-16a047dbe87e

With AC oscillation for the direction of current, the resulting power at the consumer load will also oscillate. This would cause a pulsing power delivery, which would not be an optimal system for us to connect our many consumer loads to. The solution to this problem is described by the following analogy.

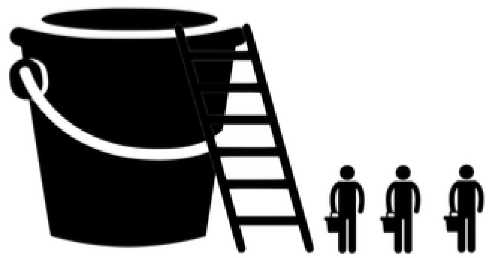


Figure 2: One man is tasked to fill up an extremely large bucket with water. He is given a ladder, a personal pale, and a river water source only a few steps away. To fill this large bucket with water, the man simply fills up his individual pale, climbs up the ladder, pours in the water, and repeats. This process is indeed getting the job done, however water is being poured into the large bucket in a "pulsing" fashion because the man is only pouring for a small fraction of his entire routine. If he wishes to deliver a constant flow of water from the nearby river into the large bucket, he will need two friends. With three, the team can stagger their oscillatory routines so at least one of them is pouring water into the bucket at any instant in time.

This analogy has demonstrated the importance of having three delivery lines in a "staggered" routine. This is the exact reason transmission lines typically consist of three separate wires as shown in the Electrical Transmission System stage of Figure 1. The angle difference in which the three lines "stagger" their AC power delivery (analogous to the angle difference of the three men's routines in Figure 2) is called the *phase angle*. On a full-size power grid, each bus will be operating at a particular phase angle that will be forced to synchronize with the rest of the grid for the most fluent power delivery.

Conveniently, power flow concepts of real and reactive power are hidden in the analogy of Figure 2. In a power system, *real power* is the actual power delivered to consumer loads and is analogous to the water delivered to the large bucket. This is the power that we pay for when we pay our electric bill. Reactive power on the other hand, is the power related to the oscillations occurring on the grid. In this analogy, the power (energy spent over time) it took for the three men to climb up and down the ladder is the *reactive power* of the system. This is the power that the system required to push and pull the real power to its final destination. Getting into the finite details, this phenomenon is caused from the inductive and capacitive charging on the transmission lines, but at this stage it is only important to understand that this power results in zero net-reactive power (the man in the analogy started and finished his day at the bottom of the ladder).

2.3 System Inertia and Stability

Now that the physical interpretation of phase angle has been established, it follows to define *frequency* as the rate at which these angles change (while keeping their same relative angle spacing to one another). As mentioned earlier, a typical national power system operates at a frequency of 60Hz. **The inertia** (aka stability) **of a power system is physically based on the kinetic energy conversion in synchronous generators** and the method they use to convert energy into an AC current.

Synchronous generators are generators that use the process of turning a turbine with heated steam to create a power output. Although these generators tend to be fossil (coal, nuclear, or natural gas), the category essentially only excludes solar photovoltaics and wind generation. Hydro, geothermal, biomass, and concentrated solar are all good examples of renewable synchronous generators. Through the strategic use of a rotating magnetic field, synchronous generators output power in three phases with a frequency near 60Hz as desired by the grid. This inherent rotational inertia of the spinning generator is typically very large, and is the physical basis of stabilization on the power system. With any sort of perturbation that might temporarily decouple the synchronous generator from the rest of the system, the frequency output of the energy conversion devise within the generator will essentially maintain the same exact frequency. This means that as long as the generator is reconnected to the system within a reasonable amount of time, the re-coupling process will be as easy as if the generator never left in the first place!

Now, the primary issue that many power system engineers and researchers are working to solve is the fact that solar photovoltaics and wind renewables have no mechanical inertia. The process of re-coupling these two asynchronous renewable generators will require the natural frequency of the generators to restart at 0Hz and "warm back up" to the 60Hz grid, as seen later in Figure 6a. Fundamentally, this means that with our current national grid systems, it is impossible to create a 100% renewable grid (from wind and solar) that is stable enough for our modern day power demands. Additional to this inherent instability of asynchronous generators, it is also important to consider that sun and wind are not always available resources, and wind in particular can be extremely hard to forecast. Therefore, there must exist some critical percentage value of how many solar and wind generators can supply the grid before it becomes too unstable to function. Optimizing a strategy for connecting these asynchronous renewable generators (solar and wind) to the power grid network will improve the tolerated critical percentage of renewable generation. This is what the rest of our project confronts.

3 Project Model

As power systems research and industries attempt to increase the theoretical limit of renewable generation, a strong intuition of optimal renewable integration methods–for large amounts of wind and solar–is imperative. The purpose of this study is to generate results that give clear direction and mathematical reasoning as to how renewable nodes should be added to the grid network. **Our measure of success for the addition of new renewable generators is *stability***. After reading several research papers we have found that there isn't a standardized way of calculating stability (see references [1] and [4] to see examples of ways other people have calculated stability) and thus we have come up with our own logical method for calculating the stability metric of a network in Equation 6. The final results consist of various appended renewable configurations that were created according to different attachment strategies, and their corresponding stabilities. Based on their affect on network stability, we are able to determine the attachment strategy which works best, and this serves as our recommendation for how we should proceed with adding renewable energy sources to an existing power grid (specifically one with similar parameters to those of the Chinese power grid) in the future.

3.1 Assumptions

As with any scientific investigation, a certain amount of assumptions and approximations were made throughout the duration of this project. Typically, these assumptions are embedded into the discussion of the study results (if they are mentioned at all), but after reading plenty of research investigations and papers ourselves, we thought it would be most informative for a reader to have an organized list of our assumptions. Without loss of generality for the final conclusion of our study, below are some key assumptions we made to simplify our model, reduce expensive computations, and emphasise the affect of the primary variables we intend to measure:

3.1.1 General Power System

1) Because line impedance is minimal, the distance and topology of transmission is also neglected. This means that our network is *not* a weighted graph and edges only determine which nodes are connected to each other.
2) Load consumer loads represent a larger scale profile approximation of a set of actual loads. For example, an entire neighborhood that includes a high number of connected loads–such as phones, computers, air conditioning, heating, electric vehicles, etc– would be approximated as a singular load consumer node with an approximated power demand.
3) Power associated with each node in our network is assigned a value uniformly distributed between 0.5 and -0.5 so that power generated is equal to power consumed.
4) Consumer load profiles are considered to have constant power demand. In reality some appliances, such as a refrigerator, have a more complicated method involving reactive power consumption from the grid. It is also true that load profiles will look different at different times of day (less people use appliances during the night when they are sleeping). These factors could affect stability, but this are not considered.
5) We have taken data from the Chinese power grid system to approximate values, such as generator power output and load consumption, on our randomly generated power grid model. Data based on other national grids may yield different results.

3.1.2 Building the Network

6) We are using data taken from the Chinese power grid system to establish parameters that are required for a national power grid network. Our results are then involving a randomly generated network that follows the same parameters from the Chinese. See the Future Work section of this paper to read about how considering other countries would improve this study.
7) Because we are following data from the Chinese power grid, our randomly generated network has a uniform distribution of connectivity. It is important to note that in most other national grid systems (including the power grid for the United States) the degree distribution is exponential instead of uniform.
8) Every node is connected to at least one other node, and no nodes are connected to themselves.
9) Our network model of the power system is unweighted and undirected.

3.1.3 Dynamics and Stability

10) The second-order Kuramoto model (Swing Equation) is implemented as a valid way to model dynamics on the power system network based an allotment of published grid network sources found in our references.
11) The purpose of our project investigates the process of appending nodes to a pre-established grid. Therefore our pre-established network, modeled again from Chinese grid data, is initialized to a stable 60Hz average frequency before any renewable nodes are added.
12) Appended renewables have a probability of 0.01 for losing power generation at each time

step (decoupling from the network) due to the lack of their respective resource. This will affect stability.
13) Nodes associated with our initial, randomly generated network are assumed to be synchronous generators, loads, or busses.
14) To focus our results on the affect of adding renewables we kept the probability of these initial nodes decoupling from the network at 0. Therefore our initial network is assumed to be completely stable as it is supposed to simulate a functioning grid.
15) Our reasoning behind our intuition of what are the best and worse strategies for appending a new node onto the network are entirely based on the results from paper REFERENCE.
16) For computational purposes frequencies are shifted by a value of 60Hz in order to express a stable network hovering around the 0 axis of our plots.

3.2 Approach

Before diving into the mathematical theory of our computation, it is necessary to give a brief review of how our project model will achieve the results that tells the story we are interested in. Our method begins with building an initial random network that follows parameters found in Chinese power grid data from [2]. This entails allocating a uniformly distributed network with 1800 edges and 1700 nodes. The nodes are then assigned a power output uniformly distributed from -0.5 to 0.5 (where a positive number indicates power generation and a negative number indicates power consumption) in order to maintain that the power produced is equal to the power consumed. The dynamics on this network maintain an average frequency around 60Hz.

Once our initial network is created, we consider the process of appending new asynchronous nodes. Thus, we strategically attach 200 renewable nodes to the network and run the dynamics on the updated network. The asynchronous renewables will attempt to synchronize with the rest of the network, and we evaluate their success by re-examining system stability. Then reflecting on our results we are able to make a conclusion about the most optimal node addition configuration for these renewable generators.

3.3 Mathematical Formulation

3.3.1 Building the Network

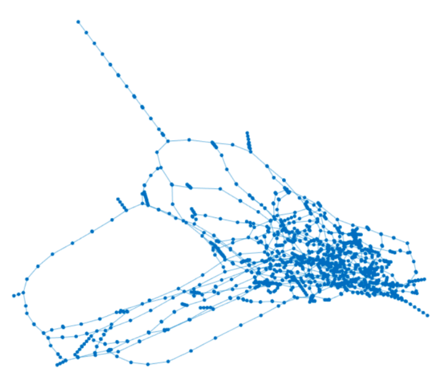
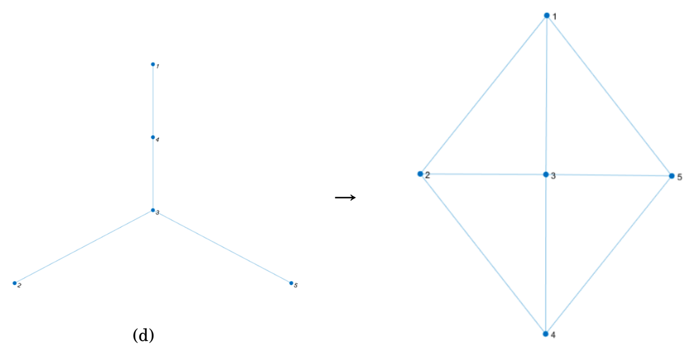


Figure 3: The randomly generated initial network built using parameters similar to the Chinese power grid. As the initial network, this power system is assumed to only contain synchronous generators, busses, and loads.

To construct our final network for our initial power system, we construct the random network seen in Figure 3 by an adjacency matrix as commonly done in network theory. To explain the basics of forming this adjacency matrix, we will show our process of building a random five node power system in Figure 4c.

$$\underset{\text{(a)}}{\begin{matrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{matrix}} \rightarrow \underset{\text{(b)}}{\begin{matrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{matrix}} \rightarrow \underset{\text{(c)}}{\begin{matrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{matrix}}$$



(d) (e)

Figure 4: Process of building an undirected adjacency matrix for a five node power system.

In this five node example, we begin building the power system network but following our assumption that all nodes are connected to at least one other network (and not to themselves) as shown in 4a. Then more edges are added according to the ratio we found in the Chinese grid data (4b). Because we are using China as our realistic inspiration of our random network, these three extra edges are added uniformly, but if we were modeling a different national grid we might add edges with an exponential distribution. Finally, the power grid network is undirected so we must turn the adjacency matrix from 4b into the symmetrical, final adjacency matrix in 4c. Figure 4d is the undirected visualization of adjacency matrix 4a, and Figure 4e is the undirected visualization of adjacency matrix 4c.

Adding nodes to an initialized power grid network follows a similar process that lead to 4c. To develop an updated network with two more nodes added onto the network described by 4c, our adjacency matrix undergoes a transformation to result with Figure 5c.

To finish at our updated seven node network, we first had to introduce two more nodes to the adjacency matrix as shown in 5b and 5e. Then we must symmetrically connect these nodes to the initial five node matrix as seen in 5c and 5f. The way in which this final connecting step is made is what our results investigate.

The final instance where the structure of the power system network changes is when an asynchronous, renewable node becomes decoupled from the frequency of the system due to a lack of natural resource (there's no sun or no wind). In this case our model temporarily disconnects the node from the adjacency matrix, and then reattaches it once the renewable generator is ready. In our adjacency matrix this would be the equivalent of toggling back and forth from decouple renewables in 5b to coupled in 5c.

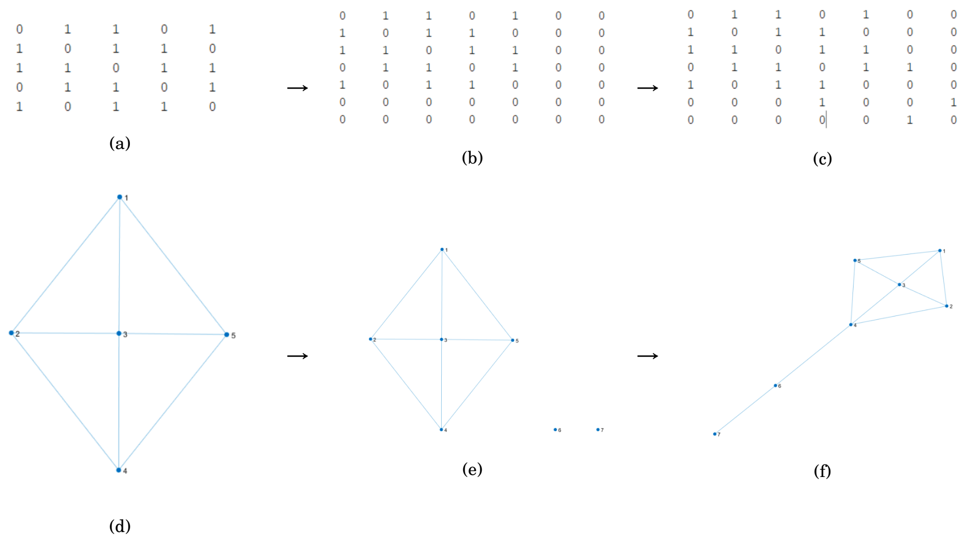


Figure 5: Process of adding two nodes to a five node power system.

3.3.2 Kuramoto's Dynamic Model

The Kuramoto model is a very similar equation to the power systems' *swing equation*, which are dynamic models that simulate frequency responses across an entire power systems network. When developing a dynamic solution for the power grid, we begin with the following power relationship of a node in (1). Here, P represents power at the node.

$$P_{mechanical} = P_{rotational} + P_{load} + P_{transmitted} + P_{damping} \tag{1}$$

$P_{rotational}$ is the power associated with the rotating turbine of a generator (if it has one), P_{load} is the power consumed, $P_{transmitted}$ is the external power the node must transmit from one edge to another, and $P_{damping}$ is power lost due to friction and resistance. Recalling some basic physics concepts, we can replace $P_{rotational} = \frac{d}{dt}\left[\frac{1}{2}I_n(\frac{d\phi_n}{dt})^2\right]$ with I_n as the moment of inertia. We also replace $P_{damping} = D_n\left(\frac{d\phi_n}{dt}\right)^2$ as done in plenty of papers in our references. This leaves us with

$$P_{mechanical} = I_n \frac{d\phi_n}{dt}\frac{d^2\phi_n}{dt^2} + P_{load} + P_{transmitted} + D_n\left(\frac{d\phi_n}{dt}\right) \tag{2}$$

If we define Ω as the natural frequency of the node, thus we can define

$$\frac{d\phi_n}{dt} = \Omega + \frac{d\theta_n}{dt} \tag{3}$$

Substituting (3) into (2) and solving for the $\frac{d^2\theta_n}{dt^2}$ term, we find the result

$$\begin{aligned} \frac{d^2\theta_n}{dt^2} &= \left(\frac{P_{mechanical} - P_{load} - D_n\Omega^2}{I_n\Omega}\right) - \left(\frac{2D_n}{I_n}\right)\frac{d\theta_n}{dt} - \frac{P_{transmitted}}{I_n\Omega} \\ &= P_n - \alpha_n\frac{d\theta_n}{dt} - \frac{P_{transmitted}}{I_n\Omega} \\ &= P_n - \alpha_n\frac{d\theta_n}{dt} + \sum_{m=1}^{N} K_{nm} sin(\theta_m - \theta_n) \end{aligned} \tag{4}$$

Finally, we have derived the second-order Kuramoto model in (5) by noticing $\omega_n = \frac{d\theta_n}{dt}$ with ω_n denoting the node frequency.

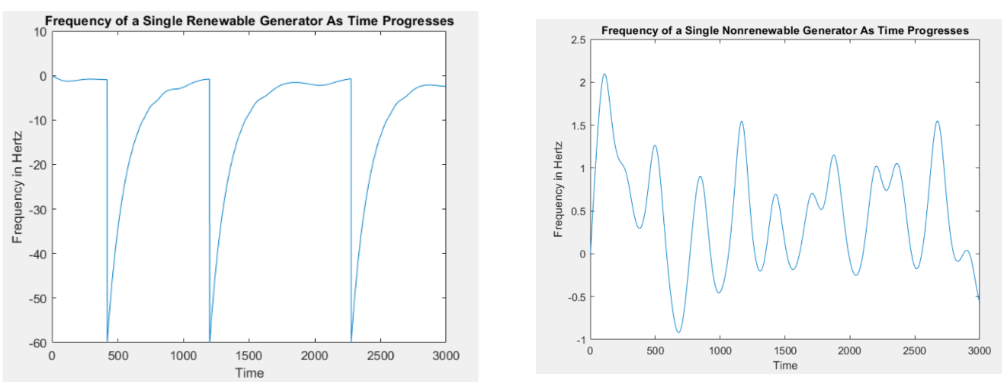
$$\boxed{\frac{d\omega_n}{dt} = P_n - \alpha_n\omega_n + \sum_{m=1}^{N} K_{nm} sin(\theta_m - \theta_n)} \tag{5}$$

In (5), P_n is the power output value associated with node n, α_n is a damping constant, and the term $\sum_{m=1}^{N} K_{nm} sin(\theta_m - \theta_n)$ keeps track of the edges connected to node n using the adjacency matrix and creates coupling by a factor of the sine of the difference in their phase angles. More details on this derivation can be found in our references and lecture notes.

Using (5), tracking the frequency of the entire system simply consists of tracking the dynamics on each individual node (n) with this equation.

3.3.3 Stability Metric

An integral part of our project is the creation of a stability metric that can measure system-wide functionality. To theorize this metric, it was crucial to first understand the frequency behavior of individual nodes (generators) on the network. The frequency behavior of individual generators are shown below:



(a)

(b)

Figure 6: For an updated grid with 200 renewables attached, these figures show the frequency response of synchronous and asynchronous generators on the network. (a) shows frequency vs time of a renewable generator with 0.01 probability of losing natural energy resource at each time step. (b) shows the frequency vs time of a non-renewable, synchronous generator over time. Note this generator is much more stable as the range in frequency smaller by orders of magnitude.

Figure 6 above shows (and explains in the caption) the behaviors of synchronous and asynchronous generators on an updated network with 200 appended solar or wind generators.

Given a network full of generators behaving in similar fashions as shown above, we establish our *stability metric* to be the average value of deviation from 60Hz for each generator in the system. Computationally, we have subtracted all frequency values by 60Hz so our desired average system frequency is 0Hz as seen in future plots.

$$\boxed{\text{Stability} = \frac{-\sum_{n=1}^{N} |P_n||\omega_n|}{N}} \tag{6}$$

This stability metric score can then be calculated at each time step to evaluate the stability of the system over a time progression. To show that (6) does indeed express how stable the system is, Figure 7 shows the stability metric measured on our original network (without any appended nodes). It is clear that our metric is showing a stable system and the stability metric score of this particular case is -0.99. As you will see in our results, -0.99 corresponds to a relatively stable system whereas a stability score that is much more negative may signify a very unstable system.

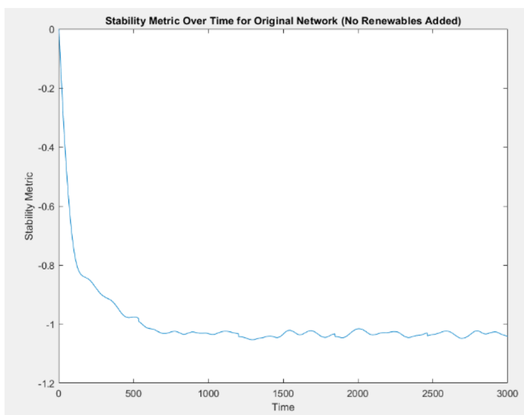
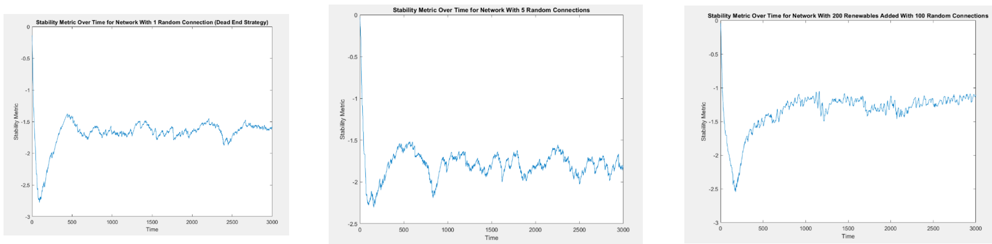


Figure 7: Example of our stability metric measuring the stability of the original network with no renewable nodes being attached yet. The average stability score over this time interval is -0.99.

Overall, the stability metric should be interpreted as: the more negative the score, the more unstable the network. A theoretically ideal network would have a stability metric of 0.

4 Results & Discussion

When initially finding results for this study, we tested three different configurations for appending 200 renewable nodes onto our network. These three configurations consisted of adding 1 edge per renewable node, 5 edges per renewable node, and 100 edges per renewable node.



(a) -1.68 (b) -1.79 (c) -1.34

Figure 8: Stability metric score over time of updated networks with 200 appended renewable generators. (a) appends each renewable generator using 1 edge per generator, (b) uses 5 edges per renewable generator, and (c) uses 100 edges per generator. Average stability score is stated in each sub-caption. To see these figures larger go to appendix.

After analyzing the updated networks using our stability metric scores, we found that the networks ranked from most stable to least stable as: 100 edges per renewable node, 1 edge per renewable node, and 5 edges per renewable node. This was a very curious result because we are lead to believe that there is not a linear relationship between stability and the number of edges you include when appending a renewable node.

To investigate this theory, we plotted the final average stability metric scores as a function of the number of edges added per node. This result is shown below:

Figure 9: Final average stability metric scores as a function of the number of edges added per asynchronous node. The plot includes multiple trials to confirm the trend in the data.

Figure 9 embodies the big discovery of our project. The trend illustrated in the plot shows that as one increases the edges associated with appending renewable nodes, the system becomes increasingly less stable up until about 20 edges. Then the relationship inverts and we can see that appending a renewable node with 70 edges creates a system that is significantly more stable than a "dead end" renewable node with one edge. It is important to make note here that no configuration of 200 appended renewable nodes shows a stability better than the initial network without any renewables. This is predicted in the System Inertia section describing the fundamentals of power systems.

The true significance of our finding in Figure 9 is shown when comparing the plot to other current research happening in the power systems research field today. We found that many papers, when anticipating the problem of renewable generation flooding onto the power system network, have done a lot of research in stability when adding a node to a power grid network. The general relationship between our final stability metric score and the number of edges involved in adding a **non-renewable** generator node is shown in the figure below:

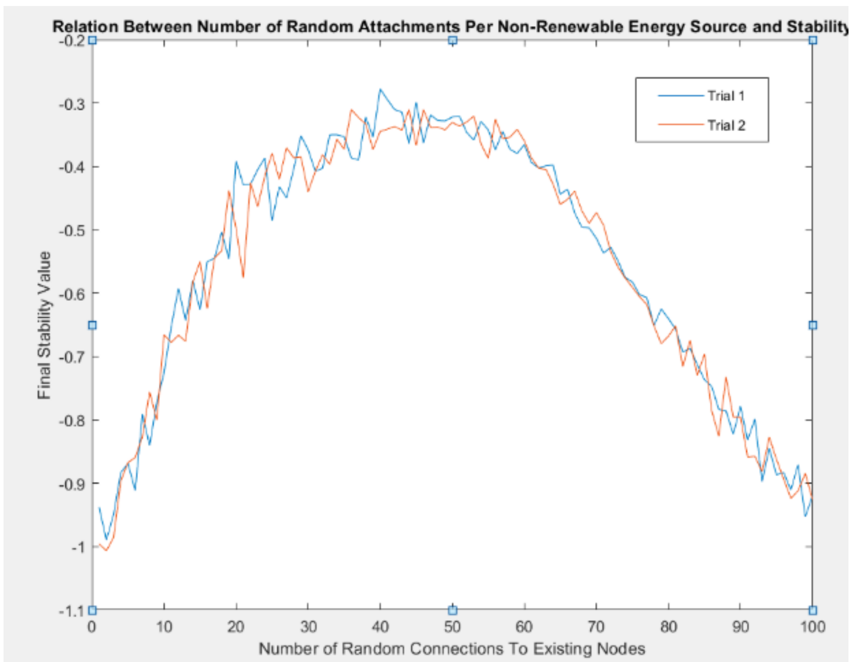
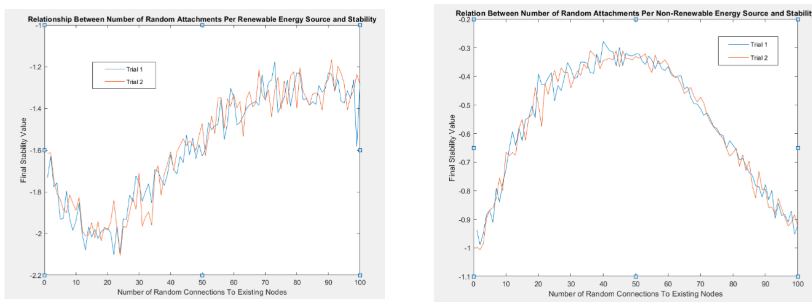


Figure 10: Final average stability metric scores as a function of the number of edges added per synchronous node. The plot includes multiple trials to confirm the trend in the data.

Figure 9 and Figure 10 are place side by side for comparison and emphasis of the finding of our study:



(a) Renewable

(b) Non-Renewable

Figure 11: Asynchronous, renewable generation nodes appended to a power system compared to general nodes that are equivalent to synchronous, non-renewable nodes. Keep into consideration the scale of each plot, as adding synchronous generation nodes with energy resources can improve the stability of the intial system.

The key difference between adding in renewable generation (as our project has considered) vs typical general nodes that are equivalent to non-renewable, synchronous generators (as many other papers in power systems research show) is that renewable nodes are assigned a probability chance of them losing their energy resource such as sunlight. This makes these asynchronous nodes much more unstable, and–as shown in Figure 9–changes the relationship of how to optimally connect them to a power system network! Again, both Figure 9 and Figure 10 are produced from our model and program, but only Figure 9 incorporates the unique instability of a renewable, asynchronous node. Figure 10 is produced as if we have considered all generation nodes as equivalent, and the relationship matches up perfectly with what is described in the results of [1] (which describe how dead ends are bad) and the results of [3] (which describes how networks that deviate more from the random topology are more fragile, and also how fragility increases as the elements of the Grid become more interconnected). This suggests a major issue in papers that study generally adding nodes to a power system network. An issue that our final result eliminates.

5 Future Work

- Create a real Machine Learning algorithm that can exactly optimise and predict how nodes should be added to a power system. In this project we randomly assigned a set amount of edges to connect to other nodes in the network, but the ML algorithm could improve this feature.
- Create a similar optimization model, but consider financial burden as well. Our results are suggesting connecting each renewable node with 70 edges, in reality this would be far too expensive for electric companies to follow.
- Involve an investigation into Solar Irradiance and Wind Forecasting to better model how the appended renewables will cut in and out of the grid system (Simon does research in this area right now). In our current model we have simply assigned a probability value of renewables cutting out, but with in depth research this is not random.
- Perform our project two more times on grid data from the USA and India to investigate how our results may change as the preexisting degree topology of the network is varied. This would allow our conclusions to be well-rounded for all types of power grid networks

6 Conclusion

The cumulative, and most substantial, result of this research investigation is the relationship we found between network stability and the number of edges used to connect a *renewable & asynchronous* generator to the power system network (shown in Figure 9). This is a great improvement from other papers released in the field that don't typically consider the unreliability of energy resources of solar and wind renewable generators. From our result, the final conclusion of our project is to connect an asynchronous generation node to a network using a high number of edges. In the case of a power grid network modeled with similar parameters to data from China, we recommend about 70 edges per appended renewable generator for optimal stability.

Appendix

Extra Plots

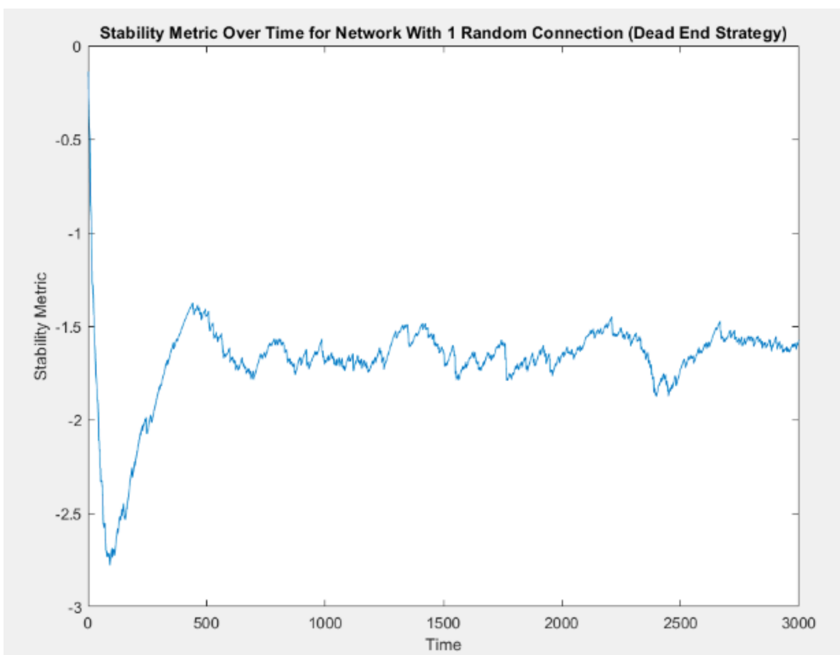


Figure 12: Larger version of Figure 16d

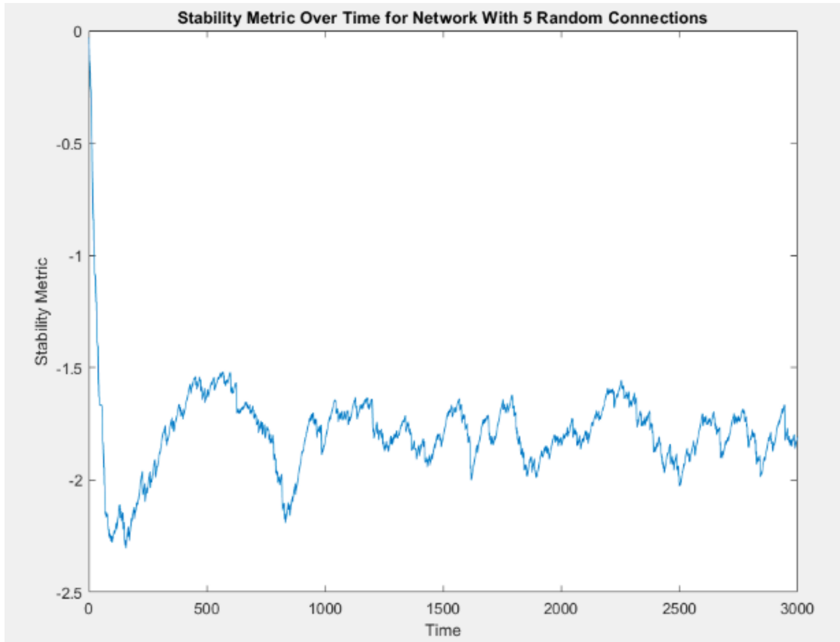


Figure 13: Larger version of Figure 16e

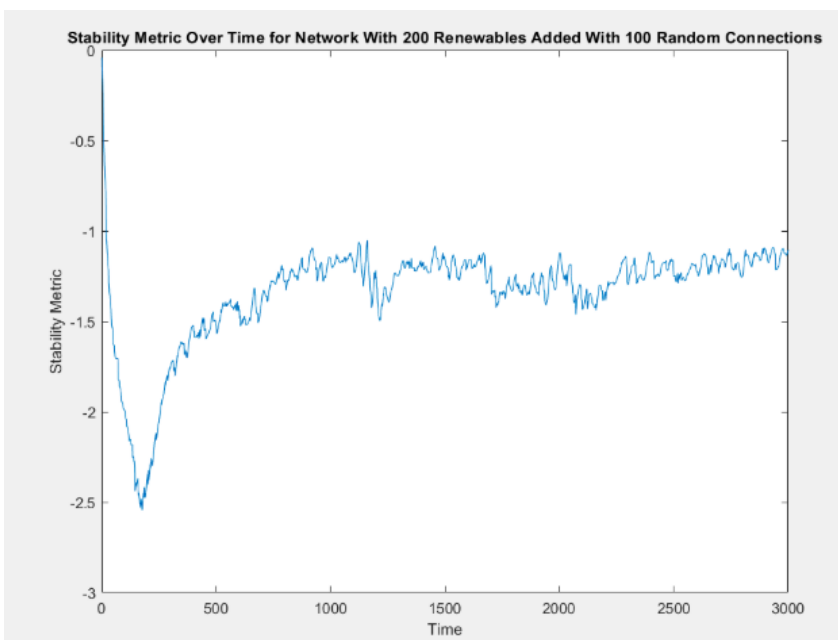


Figure 14: Larger version of Figure 16f

Program Code (MATLAB)

```
disp("going to generate you a base network")
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Variable Initialization
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
numOriginalNodes = 1700;
numAdditionalRenewableNodes = 0;     %this will be the number of new renewable energy
sources we are adding to existing grid
finalNumNodes = numOriginalNodes+numAdditionalRenewableNodes;   %total nodes when
network is fully created
numEdges = 1800;
numExtraEdges = 100;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%End Variable Initialization
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%We Assume every node in the network is connected to the network in some
%way, and none of the nodes are connected to themselves (no self loops)
%The Degrees will otherwise be randomly distributed yielding a degree
%distribution that is approximately uniform

connectionProbability = 1/numOriginalNodes;
flagForConnectionMade = 0;

A = zeros(numOriginalNodes);

%disp(A)

myVector = 1:numOriginalNodes;
%disp(myVector)

perm = myVector(randperm(length(myVector)));
%disp(perm)

for i = 1:numOriginalNodes
    while flagForConnectionMade == 0
        perm = myVector(randperm(length(myVector)));
        for j = 1:numOriginalNodes
            randomIndex = perm(j);
            if i ~= randomIndex         %doing this to avoid self loops dont want to connect to itself
                myRandNum = rand;
```

(a)

```
                if myRandNum < connectionProbability
                    A(i,randomIndex) = 1;
                    flagForConnectionMade =1;
                    break                   %break me out of inner for loop
                end
            end
        end
    end
    flagForConnectionMade = 0;     %resetting flag
end

%disp(A)
%AT THIS POINT EVERY NODE IN THE GRAPH HAS BEEN CONNECTED TO 1 OTHER
RANDOM
%NODE IN THE GRAPH
extraNodeCounter = 0;
while extraNodeCounter<=numExtraEdges           %while the number of nodes is less than
100 (to be consistent with 1800 total connections)
    indices = datasample(myVector,2);            %take a uniform random sample from all of the
nodes
    if indices(1) ~= indices(2)                  %if it is not about to connect to itself
        if A(indices(1),indices(2)) ~= 1          %if there is not already a connection
            A(indices(1),indices(2)) = 1;         %create a new connection between the nodes
            extraNodeCounter = extraNodeCounter+1;
        end
    end
end
%disp(A)
%AT THIS POINT THE CURRENT GRID FOR CHINA HAS BEEN CREATED AS
UNDIRECTED SO
%I MUST MAKE IT SYMMETRICAL AS POWER GRIDS ARE
for i = 1:numOriginalNodes
    for j = 1:numOriginalNodes
        if A(i,j) ==1
            A(j,i) =1;
        end
    end
end
%NOW THE AN UNDIRECTED GRID OF CHINA HAS BEEN CREATED
%disp(A)
firstSum = sum(A);
avgDegree = sum(firstSum)/numOriginalNodes;             %this is <k>
%/numNodes;
```

(b)

```
disp(sum(firstSum))
disp(avgDegree)

mySparseA = sparse(A);
%disp(mySparseA)

ChineseGraph = graph(A);
plot(ChineseGraph)
```

(c)

Figure 15: Generation of a network in a separate file so that base network stays constant

```
maxConnectionNum = 100;
stabilityResults = zeros(1,maxConnectionNum);

for iterateMe = 1:100
    numAdditionalRenewableNodes = 200;     %this will be the number of new renewable energy
sources we are adding to existing grid
    finalNumNodes = numOriginalNodes+numAdditionalRenewableNodes;   %total nodes when
network is fully created

    probabilityRenewableOn = 0.99;
    K = 3;      %coupling constant (this will only come into play when the nodes are connected on
the grid)
    dampingConstant = 0.7;
    numTimeSteps = 3000;          %this is how long the simulations will run
    changeTime = 0.01;
    stabilityTimeVector = zeros(1, numTimeSteps);      %this will hold the stability metric of the
network at each timestep (averaged at the end)
    singleOscillatorFrequency = zeros(1,numTimeSteps);
    avgFrequencyFrom0 = zeros(1,numTimeSteps);
    coupledComponent1Timestep = zeros(1,numTimeSteps);
    avgTheta = zeros(1,numTimeSteps);
    stableWn = zeros(finalNumNodes,1);     %this is what I subtract from the resulting frequencies
at each timestep when finding second norm

    thetaVector = zeros(finalNumNodes,1);
    dthetaVector = zeros(finalNumNodes,1);

    WnVector = zeros(finalNumNodes,1);             %this is wN WHICH IS WHAT WE ARE
CONCERNED ABOUT
    dWnVector = zeros(finalNumNodes,1);
```

(a)

```
    distFromTargetWn = zeros(finalNumNodes, 1);       %this will be used to keep track of how far
each generator is from target frequency

    powerGeneratedVector = randn(finalNumNodes,1);      %this is the power that each
oscillator/generator consumes or produces
    powerGeneratedVector = powerGeneratedVector-mean(powerGeneratedVector);      %this
enforces that power in=power out on grid

    dampingVector = dampingConstant*ones(finalNumNodes,1);

    numTrials = 1;

    strategy =3;


    for trial = 1:numTrials
        if strategy ==1           %here I connect the new renewables in the naive way with only 1
connection
            tempA = A;         %doing this so that I dont change the original network created which
affects results
            tempA(end+1:finalNumNodes,end+1:finalNumNodes) = 0;

            myFinalVector = 1:finalNumNodes;
            perm = myFinalVector(randperm(length(myFinalVector)));

            for i = numOriginalNodes+1:finalNumNodes   %THIS IS WHERE WE ATTACH THE
NEW NODES IN DIFFERENT WAYS FIRST HERE WE ILLUSTRATE A DEAD END
                while flagForConnectionMade == 0
                    perm = myFinalVector(randperm(length(myFinalVector)));
                    for j = 1:finalNumNodes
                        randomIndex = perm(j);
                        if i ~= randomIndex         %doing this to avoid self loops dont want to connect to
itself
                            myRandNum = rand;
                            if myRandNum < connectionProbability
                                tempA(i,randomIndex) = 1;
                                flagForConnectionMade =1;
                                break                         %break me out of inner for loop
                            end
                        end
                    end
                end
                flagForConnectionMade = 0;      %resetting flag
```

(b)

```
            end
            %MUST MAKE SYMMETRICAL AGAIN
            for i = 1:finalNumNodes
                for j = 1:finalNumNodes
                    if tempA(i,j) ==1
                        tempA(j,i)=1;
                    end
                end
            end
            mySparseA = sparse(tempA);             %Now A will have been created with the
additional renewables
        end

        if strategy == 2
            disp("This will be running the experiment with no renewables added")
            mySparseA = sparse(tempA);

        end

        if strategy == 3
            tempA = A;
            tempA(end+1:finalNumNodes,end+1:finalNumNodes) = 0;

            %%%%%%%%%%
            myFinalVector = 1:finalNumNodes;
            perm = myFinalVector(randperm(length(myFinalVector)));

            for i = numOriginalNodes+1:finalNumNodes   %THIS IS WHERE WE ATTACH THE
NEW NODES IN DIFFERENT WAYS FIRST HERE WE ILLUSTRATE A DEAD END
                connectionCount = 0;
                while connectionCount <iterateMe
                    perm = myFinalVector(randperm(length(myFinalVector)));
                    for j = 1:finalNumNodes
                        randomIndex = perm(j);
                        if i ~= randomIndex         %doing this to avoid self loops don't want to connect to
itself
                            if tempA(i,randomIndex)~=1
                                myRandNum = rand;
                                if myRandNum < connectionProbability
                                    tempA(i,randomIndex) = 1;
                                    connectionCount =connectionCount+1;
                                    break                         %break me out of inner for loop
                                end
```

(c)

```
                            end
                        end
                    end
                end
            end

            %%%%%%%%%%%%
            %MUST MAKE SYMMETRICAL AGAIN
            for i = 1:finalNumNodes
                for j = 1:finalNumNodes
                    if tempA(i,j) ==1
                        tempA(j,i)=1;
                    end
                end
            end
            %%%%%%%%%%%%
            mySparseA = sparse(tempA);             %Now A will have been created with the
additional renewables connected to 5 other things
        end

        if strategy ==4         %This is where future work could be explored with different connection
strategies
            tempA = A;
            tempA(end+1:finalNumNodes,end+1:finalNumNodes) = 0;

        end

        %must setup initial conditions of network (everything stable and good
        %initially at t=0

        WnVector = stableWn;                  %PUT THIS ONE BACK JACKSON
        %WnVector = 0.01*randn(finalNumNodes,1);
        %WnVector(1)=50;



        for i = 1:finalNumNodes          %here I setup the initial theta values of the generators
(assuming uniform dist)
            initialAngle = rand()*2*pi;
            thetaVector(i) = initialAngle;
        end
```

(d)

```
        for t = 1:numTimeSteps
            %Must first specify what A will be for this time iteration based on
            %which renewables are on
            currA = mySparseA;          %mySparseA will stay constant and this is because
generators get fixed at each timestep


            %for i = numOriginalNodes:finalNumNodes             %JACKSON I SWEAR IF YOU
DONT CHANGE BACK!!! :((((
                %myRandNum = rand;
                %if abs(WnVector(i))<1          %if the frequency of the renewable generator has
recovered to grid frequency
                    %if myRandNum>probabilityRenewableOn  %if this renewable generator is within
reasonable frequency of grid and suddenly goes off
                        %currA(i,:) = 0;
                        %currA(:,i) = 0;
                        %WnVector(i) = -60;
                    %end

                %end

            %end



            %%%%%%%%%%%%%%%%%%
            dthetaVector = WnVector;           %change in theta = Wn
            dWnVector = powerGeneratedVector-dampingVector.*WnVector +
imag(exp(-1i*thetaVector).*(currA*exp(1i*thetaVector)));      %change in frequency

            %omegap = P - alpha.*omega + imag(exp(-1i*theta).*(a*exp(1i*theta)));

            %ABOVE I NEED TO CALCULATE dWnVector for this timestep

            singleOscillatorFrequency(t) = WnVector(1550);

            avgFrequencyFrom0(t) = -mean(abs(WnVector).*abs(powerGeneratedVector));


            WnVector = WnVector+changeTime*dWnVector;
            thetaVector = thetaVector+changeTime*dthetaVector;
            %disp("wankers")
        end
```

(e)

```
    end



    finalStabilityValue = mean(avgFrequencyFrom0);
    stabilityResults(iterateMe) = finalStabilityValue;

    %ChineseGraph = graph(tempA);
    %plot(ChineseGraph)
end

plot(stabilityResults)
```

(f)

Figure 16: Running Dynamics on Network, adding Renewables According to Different Strategies, and calculating resulting stabilities

References

[1] Peter J. Menck. *How dead ends undermine power grid stability*.

[2] Giuliano Andrea Pagani. *The Power Grid as a Complex Network: a Survey*.

[3] M. Rosas-Casals. *Assessing European power grid reliability by means of topological measures*.

[4] Thanh Long Vu. *A Framework for Robust Assessment of Power Grid Stability and Resiliency*.