Singular Value Decomposition (SVD) in Image Processing

Applications, Techniques, and Findings

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November 2024

Objective, Methodology and Approach

- Objective: To explore Singular Value Decomposition (SVD) applications in image processing.
- Methodology :
  - Replication of Sadek's Works :
    - Reproduce key findings and techniques from Sadek's research on SVD in image processing.
    - Conduct mathematical experiments to evaluate and validate SVD-based image compression, denoising, and watermarking techniques.

Introduction

- **Singular Value Decomposition (SVD)** is a powerful matrix factorization tool widely used in image processing.
- **Applications** include:
  - Image Compression
  - Image Denoising
  - Digital Watermarking for forensics
- **Objective**: Validate SVD's effectiveness and explore enhancements across these applications.

SVD Fundamentals

- SVD decomposes a matrix X into:

$$X = U\Sigma V^T$$

- Properties:
  - **Rank Approximation**: Reduces dimensionality by focusing on dominant singular values.
  - **Energy Compaction**: Most image energy is captured in the largest singular values, allowing effective compression [2, 3].

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Image Compression using SVD

- **Goal**: Minimize storage while preserving key image details.
- **Method**: Retain only top-k singular values to approximate the original image.

SVD reconstruction formula

$$X \approx X_{k=40} = \sum_{i=1}^{40} \sigma_i \cdot u_i \cdot v_i^T$$

- where:
  - σ_i is the i-th singular value,
  - u_i is the i-th left singular vector (column of U),
  - v_i is the i-th right singular vector (column of V).

Visual Comparison

Figure: Reconstructed image using SVD with low-rank approximation (k=40).

Compression Method Comparison

Method	MSE	PSNR (dB)	SSIM
SVD	36.18	32.55	0.8247
Discrete Cosine Transform	107.66	27.81	0.8217
Wavelet Transform (Haar)	32.94	32.95	0.9582
Fractal Compression	20.47	35.02	0.9320
Run-Length Encoding	0.00	∞	1.000
Predictive Coding	107.25	27.83	0.5477

Table: Comparison of Image Compression Methods

Correlation between the truncation factor and PSNR and SSIM metrics

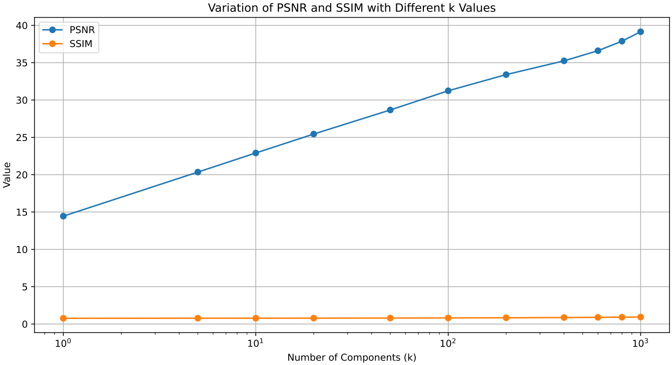

Figure: Variation of PSNR and SSIM with respect to the truncation factor k.

Image Denoising with SVD

- **Goal**: Suppress noise without significant loss in image content.
- **Method**: Filter smaller singular values that represent noise.
- **Dynamic Thresholding**: $0.618 \times \text{mean}(S)$.
- **Results**:
  - PSNR Improvement: 12.42 (noisy) to 20.31 (denoised)
  - SSIM Improvement: 0.0324 (noisy) to 0.4374 (denoised)

Visual Comparison

(a) Original Image in JPEG format.

(b) Noisy Image (PSNR: 12.42, SSIM: 0.0324.)

(c) Denoised (PSNR:20.31, SSIM: 0.437.)

Figure: Comparison of Original, Noisy, and Denoised Images using SVD.

SVD Denoising on BSD400 Dataset

(a) Original Image from the BSD400 dataset.

(b) Noisy Image (PSNR: 30.27, SSIM: 0.7794).

(c) Denoised (PSNR: 32.27, SSIM: 0.8636).

Figure: Comparison of Original, Noisy, and Denoised images using SVD on BSD400 sample image.

Image Forensics - Watermarking with SVD

- **Purpose**: Embed secure watermarks for authenticity verification.
- **Techniques**:
  - Scaled Additive Approach: Adds scaled watermark data to singular values [1].

$$SV_{\text{modified}} = SV_{\text{original}} + \alpha \cdot \text{Watermark}$$

  - Adaptive Scaled Additive (ASA): Fine-tunes watermark strength for resilience.
- **Formula**:

$$\text{SV}_{\text{mod}} = (1 - \alpha) \cdot \text{SV}_{\text{img}} + \alpha \cdot \text{Watermark}$$

Image Forensic Workflow

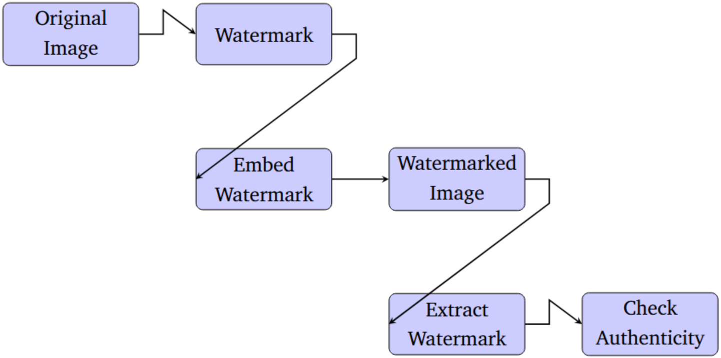

Figure: General image forensic workflow.

Watermarking Results and Comparison

Table: Peak Signal to Noise Ratio of various watermarked versions of `test_077` image from BSD400 dataset under scaled additive (SA) and adaptive scaled additive (ASA) approaches.

Image type	$\alpha = 0.01$		$\alpha = 0.1$		$\alpha = 0.2$		$\alpha = 0.3$	
	SA	ASA	SA	ASA	SA	ASA	SA	ASA
Watermarked	61.84	46.41	38.83	26.56	30.82	20.68	25.16	17.35
Noised after watermarked	20.70	20.66	20.66	19.74	20.48	17.86	19.91	16.06
Watermarked & Compressed	49.32	44.56	38.60	26.54	31.07	20.70	26.03	17.49

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Adaptive Scaled Aditive approach- a compromise

Table: Peak Signal to Noise Ratio of various watermarked versions of `test_077` image from BSD400 dataset under scaled additive (SA), adaptive scaled additive (ASA) and perceptual forensic (PF) approaches [4].

Image type	$\alpha = 0.01$		
	SA	ASA	PF
Watermarked	61.84	46.41	75.17
Noised after watermarked	20.70	20.66	20.68
Watermarked & Compressed	49.32	44.56	38.87

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Visual Comparison

(a) Original Image (test_077).

(b) Chandra's method output [1].

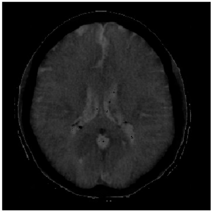
(c) Adaptive method output.

(d) Perceptive Method Output with $k = 5$.

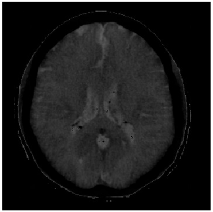
(e) Perceptive Method with GN ($k = 5$).

(f) Chandra's method with GN.

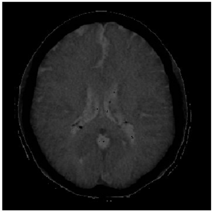
SVD based image forensic on Medical Image



(a) Original Brain CT Image from radiopedia.



(b) Scaled Additive Watermarked Image



(c) Perceptual Forensic Watermarked Image

Figure: Comparison of Brain CT images: (a) Original Brain CT Image, (b) Watermarked with scaled additive approach, (c) Watermarked with perceptual forensic approach.

Experimental Analysis - Quality Metrics

- **Metrics**:
  - **MSE**: Mean squared pixel difference.
  - **PSNR**: Higher values indicate better signal quality.
  - **SSIM**: Structural similarity closer to 1 represents better quality.
- **Optimization**:
  - Dynamic Thresholding: Balances noise reduction and detail preservation.
  - Adaptive Truncation: Tailors k to desired PSNR/SSIM levels.

Challenges and Future Directions

- **Challenges**:
  - High computational load for large images.
  - Optimizing truncation across applications.
- **Future Directions**:
  - Hybrid SVD and alternative denoising methods.
  - Expanding SVD applications to real-time digital forensics.

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Conclusion

- **SVD as a versatile tool**: Effective in image compression, denoising, and watermarking.
- **Key Outcomes**:
  - Reliable quality retention across image applications.
  - Noise reduction with high structural fidelity.
- **Future Potential**: Explore adaptive, hybrid SVD methods for advanced applications.

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Thank you

Thank you very much for your patient listening