Random Errors for Pointing

Some notes on my understanding of random errors for pointing in relation to the pointing engineering handbook.

At any point in time, a random pointing error $\mathcal{E}(t)$, corresponding to a particular axis of ratation, could be defined as a stochastic process. As a stochastic process, at time instant, t, $\mathcal{E}(t)$ would represent a random variable with some probability distribution function or, more generally, probability measure.

Payload parameters

Oftentimes, the definition of a probability distrubution for $\mathcal{E}(t)$ depends upon a set of parameters on the payload/satellite. Let these parameters be grouped into the vector $\vec{\theta}$. To be specific, these parameters could include entries, such as

$$\vec{\theta} = \begin{bmatrix} \text{x-coordinate of momentum wheel 1} \\ \text{y-coordinate of momentum wheel 1} \\ \text{z-coordinate of momentum wheel 1} \\ \text{x-coordinate of momentum wheel 2} \\ \text{y-coordinate of momentum wheel 2} \\ \text{z-coordinate of momentum wheel 2} \\ \text{x-coordinate of the center of mass} \\ \text{y-coordinate of the center of mass} \\ \text{z-coordinate of the center of mass} \\ \vdots \end{bmatrix}$$

One could then, theoretically, define the probability distribution $\mathrm{d}F_{\mathcal{E}(t)|\vec{\theta}}\left(e(t)|\vec{\theta}\right)$. The mean value of $\mathcal{E}(t)$ may, for instance, be a function of these parameters.

Random parameters and a compound distribution

For any actually manufactured (realized) satellite, $\vec{\theta}$ will assume certain values. In general, these values will be a particular realization of some random vector $\vec{\Theta}$.

As an example, the x-coordinate of any of a particular momentum wheel will conform to some probability distribution, hopefully some distribution with very low variance, but never-the-less random at some level. So if this x-coordinate of many identically manufactured satellites were to be plotted in a histogram, the histogram would mimic the shape of the probability distribution of this x-coordinate.

The random nature of $\vec{\Theta}$ is what is meant by **random in ensemble**.

The random vector $\vec{\Theta}$ may contain elements for which a set of measurements, $\vec{m}$, can be made, thereby reducing the variance of their individual distributions.

That is, the distribution for $\vec{\Theta}$ would become $\mathrm{d}F_{\vec{\Theta}|\vec{m}}(\vec{\theta}|\vec{m})$. Indeed, if a parameter could be measured with infinite accuracy, the conditional measure would become a delta-function probability measure.

Conditional Probability and Marginalization

The conditional probability distribution for $\mathcal{E}(t)$ is defined as

$$\mathrm{d}F_{\mathcal{E}(t)|\vec{\Theta}}\left[e(t)|\vec{\theta}\right]$$

With some model for the probility distribution of $\vec{\Theta}$ given also some set of measurements, $\vec{m}$, the probability distribution for $\mathcal{E}(t)$ is given by

$$\mathrm{d}F_{\mathcal{E}(t)}\left(e(t)\right) = \int \mathrm{d}F_{\mathcal{E}(t)|\vec{\Theta}}\left[e(t)|\vec{\theta}\right] \mathrm{d}F_{\vec{\Theta}|\vec{m}}(\vec{\theta}|\vec{m})$$

Time dependence of parameters

No time dependence has been assigned to $\vec{\theta}$.