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Abstract Farmers frequently decide where to locate the colonies of their domesticated eusocial bees, especially given the following mutually exclusive scenarios: (i) there are limited nectar and pollen sources within the vicinity of the apiary that cause competition among foragers; and (ii) there are fewer pollinators compared to the number of inflorescence that may lead to suboptimal pollination of crops. We hypothesize that optimally distributing the beehives in the apiary can help address the two scenarios stated above. In this paper, we develop quantitative models (specifically using linear programming) for addressing the two given scenarios. We formulate models involving the following factors: (i) fuzzy preference of the beekeeper; (ii) number of available colonies; (iii) unknown-but-bounded strength of colonies; (iv) probabilistic carrying capacity of the plant clusters; and (v) spatial orientation of the apiary.

Keywords Beekeeping · Pollination management · *Apis mellifera* · Stingless bees · Bumblebees · Crop pollination

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Mathematical Programming Models for Determining the Optimal Location of Beehives

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Abstract. Farmers frequently decide where to locate the colonies of their domesticated eusocial bees, especially given the following mutually exclusive scenarios: (1) there are limited nectar and pollen sources within the vicinity of the apiary that cause competition among foragers; and (2) there are fewer pollinators compared to the number of inflorescence that may lead to suboptimal pollination of crops. We hypothesize that optimally distributing the beehives in the apiary can help address the two scenarios stated above. In this paper, we develop quantitative models (specifically using linear programming) for addressing the two given scenarios. We formulate models involving the following factors: (1) fuzzy preference of the beekeeper; (2) number of available colonies; (3) unknown-but-bounded strength of colonies; (4) probabilistic carrying capacity of the plant clusters; and (5) spatial orientation of the apiary.

Keywords: beekeeping, pollination management, *Apis mellifera*, stingless bees, bumblebees, crop

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pollination

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1. Introduction

Pollinators, such as bees, contribute to biodiversity maintenance, especially of plants requiring outcrossing [28, 38, 60]. Bees can serve as indicator of environmental stress [36], and are important in food production [11, 29, 49, 54]. Hived bee colonies have been used to pollinate orchard plants, such as mangoes and apples [26, 52], coffee [37, 46], almonds [13, 49], cucurbits [15, 22], Chinese cabbage and radish [16], and mangroves [4, 56]. Bee colonies are also managed for the production of honey, pollen and beeswax.

By looking at the current trend, populations of some bee species are inadequate to sustain global demands [1, 2, 31, 70]. The decline in the number of bee pollinators is due to various factors, such as prevalence of diseases [7, 33, 40] and pesticides [23, 45, 53], and starvation [69]. Minimizing the exposure to these factors can reduce bee stress and can avoid colony collapse [20, 30, 50, 51, 72].

Best practices in beekeeping and pollination management help increase agricultural yield [3, 11, 44, 73]. Placement of hives in farmlands is crucial in optimizing the efficiency of bees in pollination and production of hive products [6, 12, 14, 35, 43]. It is better to have a systematic way of locating new beehives or relocating existing beehives than haphazardly placing them in any area. The hives should be positioned in an apiary in such a way that competition for forage is minimized and access of bees to target flowers are facilitated. Various factors need to be considered, such as strength of available colonies [21, 59] and carrying capacity of the foraging sites (plant clusters or food patches) [55, 59].

Operations Research (OR) techniques have been employed to solve combinatorial problems

[5, 10, 27, 67], such as choice selection and facility location. Determining the optimal placement of bee colonies in a large apiary can be regarded as a combinatorial problem solvable using mathematical programming tools. In this study, we formulate mixed-integer linear programs that can be used in determining the best location of beehives taking into account the preferred location of the beekeeper, number and strength of available colonies, carrying capacity of the plant clusters, maximum flight distance that the bees can travel, and spatial orientation of the apiary. We present deterministic models and then modify them by including fuzzy preference of the beekeeper, unknown-but-bounded colony strength, and probabilistic carrying capacity of the plant clusters.

In this paper, colony strength denotes the number of foragers, and the carrying capacity of the plant cluster denotes the number of foragers it can support or sustain (in view of the abundance of pollen and nectar sources). The beekeeper's preferred location may depend on the proximity of the hives to his house or workplace, water sources, incidence of predation, insecticides or diseases, and target plants to be pollinated. Environmental factors, type of bee species and floral choice of bees should be considered in the modeling process [24, 48, 57, 58, 61]. The models are applicable to a variety of social insects (such as honeybees, stingless bees and bumblebees) and to any farming designs (such as forest farming, monoculture and permaculture) that fit our model assumptions.

The mathematical models are grouped into two sections considering the following scenarios (1) minimizing competition among foragers due to limited food sources, and (2) maximizing the use of the species as pollinators given small number of foragers. Note that these two scenarios are mutually exclusive, that is, the first scenario happens when food from plant patches are limited, and the latter scenario happens when there are more plant patches to pollinate compared to the available number of bee foragers. The latter scenario is intended for crop pollination management. In addition, the model can be used in determining the minimum number of hives to support optimal pollination of crops.

2. Minimization of Competition for Limited Resources

In this section, we present mathematical models for identifying the optimal spatial distribution of beehives in an apiary for minimizing competition among colonies. In determining the best location site for a beehive, we need to consider (1) the distance of the hive from the food sources and (2) the maximum flight distance that the bee species can travel [8, 39, 64, 68]. These two considerations can be incorporated in the network characterizing the spatial orientation of the apiary (see Figure 1). We assume that a forager can only visit a plant cluster within the maximum flight distance. We can make the latter assumption stricter if we want to minimize the flight stress of the bees.

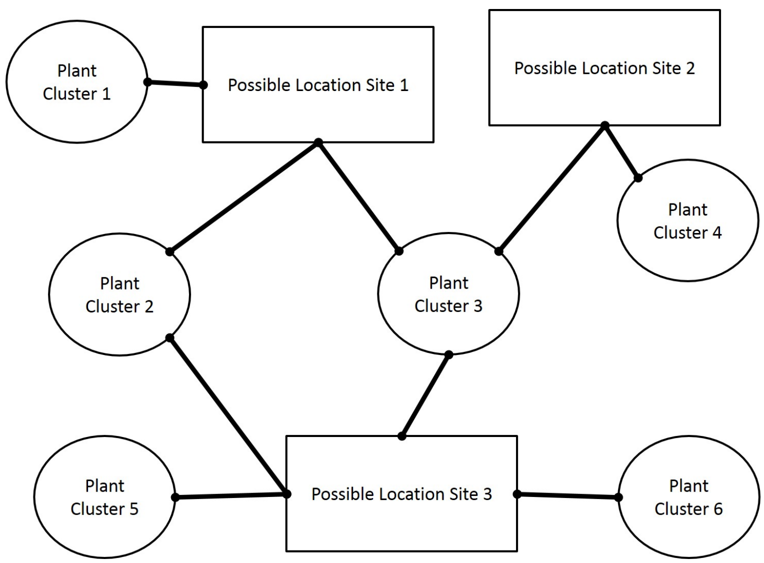

Figure 1: A sample network characterizing the spatial orientation of the apiary: The rectangular nodes represent the possible location sites; the circular nodes represent the plant clusters; and an edge denotes that the connected plant cluster is contained in the area that the foragers from the connected location site can visit (i.e., within the maximum flight distance of the bees).

Our goal is to minimize the "overpopulation" of bees in a locale. In this paper, we define "overpopulation" as the number of foragers that cannot anymore be accommodated by any plant cluster in the foraging area. If the total number of bees foraging in nearby plant clusters exceeds the total carrying capacity of the plant clusters then overpopulation arises. This overpopulation is caused by the mismatch in the capacity of the habitat and the population size. Likewise, overpopulation stimulates competition among foragers. We hypothesize that minimizing overpopulation by optimally distributing the beehives will lessen this competition.

In this paper, we use the following notations for the decision variables and parameters. Strength of a colony denotes the number of foragers in the colony, and the carrying capacity of a plant cluster denotes the number of foragers that the plant cluster can accommodate.

Decision Variables:

x_{ij} : portion of foragers in location site i that can be accommodated by plant cluster j, $x_{ij} \in \mathbb{R}^{\oplus}$

$$Z_{ki} = \begin{cases} 1 & \text{if colony } k \text{ is located to site } i \\ 0 & \text{otherwise} \end{cases}$$

E_i : portion of foragers in location site i that cannot be accommodated by nearby plant clusters, $E_i \in \mathbb{R}^{\oplus}$

Specified Parameters:

m : number of location sites

p : number of plant clusters

n : number of colonies

b_k : strength of colony k, $b_k \in \mathbb{R}^+$

y_j : carrying capacity of plant cluster j, $y_j \in \mathbb{R}^{\oplus}$

μ_j : mean of y_j

σ_j : standard deviation of y_j

w_i : priority weight given to site i, $w_i \in \mathbb{R}$

M : a relatively large positive real number used as penalty weight to minimize overpopulation

$j \in C_i$ denotes that plant cluster j is connected to site i

$i \in D_j$ denotes that site i is connected to plant cluster j

2.1. Crisp and deterministic mixed-integer program

The following mathematical program is a crisp deterministic model for determining the optimal beehive location sites such that overpopulation is minimized. This model is presented and discussed in [25]. Let us describe the term $\sum_{j \in C_i} x_{ij}$ as the total strength of all colonies to be located at site i that can be accommodated by nearby plant clusters, and let the variable E_i represent the amount of overpopulation at site i.

$$Objective\ function:\ Maximize \sum_{i=1}^{m} \left(w_i \sum_{j \in C_i} x_{ij} - ME_i \right) \tag{2.1}$$

$subject\ to$

$$Constraint\ 1:\ \sum_{i \in D_j} x_{ij} \leq y_j,\ j = 1, 2, ..., p \tag{2.2}$$

$$Constraint\ 2:\ \sum_{i=1}^{m} Z_{ki} = 1,\ k = 1, 2, ..., n \tag{2.3}$$

$$Constraint\ 3:\ \sum_{k=1}^{n} b_k Z_{ki} - \sum_{j \in C_i} x_{ij} - E_i \leq 0,\ \forall i = 1, 2, ..., m. \tag{2.4}$$

In the objective function, the penalty weight M could be very large and could be greater (in absolute value) than the beekeeper's preference weights w_i; hence, the minimization of the overpopulation E_i is prioritized compared to the maximization of the total strength of the colonies to be located at beekeeper's preferred sites. The number of bee colonies to be located (represented by the number of foragers $\sum_{j \in C_i} x_{ij}$) at the site with the higher beekeeper's preference weight is maximized first before maximizing those with lower preference weights. Note that a negative w_i denotes that the beekeeper avoids site i, e.g., due to existence of harmful elements in the nearby areas of site i. One technique for choosing values for w_i, $i = 1, 2, ..., m$, is by using the weights derived from Analytic Hierarchy Process (AHP) [63, 66].

Constraint 1 requires that the total number of foragers accommodated by plant cluster j does not exceed the carrying capacity of plant cluster j. Constraint 2 assures that each beehive is located at exactly one site.

In Constraint 3, if $Z_{ki} = 1$ for some k then $\sum_{k=1}^{n} b_k Z_{ki} > 0$, which forces $\sum_{j \in C_i} x_{ij} + E_i > 0$. This implies that if colonies are located at site i then the plant clusters connected to site i are compelled to accommodate all the strengths of these colonies. However, the value of $\sum_{j \in C_i} x_{ij}$ is restrained by Constraint 1; thus, if the plant clusters connected to site i cannot accommodate all the strengths then overpopulation arises (i.e., E_i is forced to be greater than zero). On the other hand, if $Z_{ki} = 0$ for all k then $\sum_{j \in C_i} x_{ij}$ can be equal to zero.

Note that by defining $E_i = \sum_{k=1}^{n} b_k Z_{ki} - \sum_{j \in C_i} x_{ij}$, we can have the equivalent model presented below. However, we prefer to use the model presented above to avoid a long objective function (with all the decision variables written in one mathematical expression) and to avoid recomputation for the value of E_i.

$$Objective\ function:\ Maximize \sum_{i=1}^{m} \left((M + w_i) \sum_{j \in C_i} x_{ij} - M \sum_{k=1}^{n} b_k Z_{ki} \right) \tag{2.5}$$

$subject\ to$

$$Constraint\ 1: \sum_{i \in D_j} x_{ij} \leq y_j,\ j = 1, 2, ..., p \tag{2.6}$$

$$Constraint\ 2: \sum_{i=1}^{m} Z_{ki} = 1,\ k = 1, 2, ..., n. \tag{2.7}$$

In the following subsections, we consider nondeterministic parameters. We transform a linear program with fuzzy and probabilistic parameters to an auxiliary crisp and deterministic mathematical program. The derived auxiliary model is an approximation of the fuzzy and probabilistic model.

2.2. Mixed-integer program with fuzzy coefficient in the objective function

Suppose the preference of the beekeeper is represented by a triangular fuzzy number, that is, we have an imprecise coefficient $\tilde{w}_i = (L_i, w_i, R_i)$ in the objective function. L_i represents a preference value using low assumption, w_i represents a preference value using medium assumption and R_i represents a preference value using high assumption. Let the membership function be defined by

$$\mu_{w_i}(u_i) = \begin{cases} \frac{u_i - L_i}{w_i - L_i} & \text{if } L_i \leq u_i \leq w_i, \\ \frac{R_i - u_i}{R_i - w_i} & \text{if } w_i \leq u_i \leq R_i, \\ 0 & \text{otherwise} \end{cases} \quad \forall u_i \in \mathbb{R}. \tag{2.8}$$

Let $\varphi_i = (R_i - w_i) - (w_i - L_i) = L_i + R_i - 2w_i$, where $R_i - w_i$ and $w_i - L_i$ are respectively the right and left lateral margins of $\tilde{w}_i$. The fuzzy number $\tilde{w}_i$ is not necessarily symmetric, that is, it is possible that $R_i - w_i \neq w_i - L_i$. The sign of φ_i affects positively or negatively the value of w_i when we convert the fuzzy objective function to an auxiliary crisp objective function.

Using the first index of Yager [34] (a technique for ranking fuzzy numbers), we can approximate $\tilde{w}_i$ as $w_i + \varphi_i/3$. The first index of Yager for triangular fuzzy numbers can be derived by obtaining the first coordinate of the centroid of the region bounded by the membership function (2.8) [19, 34, 71],

$$\frac{\int_{L_i}^{w_i} u_i \frac{u_i - L_i}{w_i - L_i} du_i + \int_{w_i}^{R_i} u_i \frac{R_i - u_i}{R_i - w_i} du_i}{\int_{L_i}^{w_i} \frac{u_i - L_i}{w_i - L_i} du_i + \int_{w_i}^{R_i} \frac{R_i - u_i}{R_i - w_i} du_i} = w_i + \frac{L_i + R_i - 2w_i}{3} = \frac{w_i + L_i + R_i}{3}. \tag{2.9}$$

Then, we have the following representation of the objective function:

$$Maximize \sum_{i=1}^{m} \left(\left(w_i + \frac{\varphi_i}{3} \right) \sum_{j \in C_i} x_{ij} - ME_i \right). \tag{2.10}$$

Note that there are other methods for converting the fuzzy objective function to an auxiliary crisp objective function, such as using multiobjective parametric model [34].

2.3. Mixed-integer program with nondeterministic constraints

2.3.1. A constraint with unknown-but-bounded coefficient

Suppose we have a constraint $bZ \leq RHS$, where $Z \geq 0$ and RHS is the right-hand side of the constraint, with uncertain value of b but we know the interval where b belongs. Let $b \in [\hat{b} - \Delta\hat{b}, \hat{b} + \Delta\hat{b}]$, $\hat{b}$ is the midpoint of the interval. By considering the worst-case situation, we can convert the nondeterministic constraint $bZ \leq RHS$ to $(\hat{b} + \Delta\hat{b})Z \leq RHS$ [9]. Using the worst-case situation assures that the constraint holds for all values of $b \in [\hat{b} - \Delta\hat{b}, \hat{b} + \Delta\hat{b}]$. On the other hand, note that when we have a nondeterministic constraint of the form $bZ \geq RHS$ where $Z \geq 0$, we convert it to $(\hat{b} - \Delta\hat{b})Z \geq RHS$ to represent the worst-case scenario [9].

If the strength of each colony is uncertain but the range of its possible values can be estimated, then we can follow the method discussed above. If constraint (2.4) has unknown-but-bounded coefficients $b_k \in [\hat{b}_k - \Delta\hat{b}_k, \hat{b}_k + \Delta\hat{b}_k]$ where $\hat{b}_k$ is the midpoint of the interval, then we transform

$$\sum_{k=1}^{n} b_k Z_{ki} - \sum_{j \in C_i} x_{ij} - E_i \leq 0, \ \forall i = 1, 2, ..., m \tag{2.11}$$

to

$$\sum_{k=1}^{n} (\hat{b}_k + \Delta\hat{b}_k) Z_{ki} - \sum_{j \in C_i} x_{ij} - E_i \leq 0, \ \forall i = 1, 2, ..., m. \tag{2.12}$$

2.3.2. A constraint with stochastic right-hand side

Here, we discuss one of the techniques in chance-constrained programming [9, 32, 41, 42]. Consider a constraint with a normally distributed random variable y at the right-hand side of the constraint, where y can take on any value in an unbounded set. In most cases, it is not reasonable to use the worst-case situation in transforming this chance-constraint into an auxiliary deterministic constraint. However, we can rather replace y by $\Phi^{-1}(\alpha)$ [9, 42].

We can obtain $\Phi^{-1}(\alpha) = U$ by taking the inverse of the cumulative distribution Φ, where

$$\Phi(U) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{U} \exp\left(\frac{-(\tau - \mu)^2}{2\sigma^2} \right) d\tau = \alpha. \tag{2.13}$$

In Microsoft Excel 2013 and earlier versions, $\Phi^{-1}(\alpha) = NormInv(\alpha, \mu, \sigma)$. The parameter $\alpha \in [0, 1]$ denotes the maximum error in the linear probabilistic constraint

$$Probability\,(LHS \leq y) \geq 1 - \alpha. \tag{2.14}$$

Inequality (2.14) suggests that the probability of satisfying the constraint ($LHS \leq y$, where LHS is the left-hand side of the constraint) should be greater than or equal to $1 - \alpha$. The parameters μ and σ are the mean and standard deviation of y, respectively.

Now, we consider the carrying capacity y_j of plant cluster j which is normally distributed with mean μ_j and standard deviation σ_j. If the data set used for obtaining the estimates for the carrying capacity is acceptably normally distributed then the technique discussed above is applicable. Although, it is possible to extend our model when other probability distributions are involved [9, 32, 41, 42]. Several methods are available for testing normality of data, e.g., normal probability plot, Shapiro-Wilk, Kolmogorov-Smirnov and Chi-square goodness-of-fit tests [65].

Suppose we have data set $\{W_{1j}, W_{2j}, ..., W_{tj}\}$ where each element represents the value of y_j at a certain period with $\overline{\mu}_j$ and $\overline{\sigma}_j$ as the sample mean and standard deviation, respectively. We can replace y_j in constraint (2.2) by $\psi_j = max\left\{min\{W_{1j}, W_{2j}, ..., W_{tj}\}, \Phi_j^{-1}(\alpha)\right\}$ where $\Phi_j^{-1}(\alpha) = NormInv(\alpha, \overline{\mu}_j, \overline{\sigma}_j)$. Using the value of $max\left\{min\{W_{1j}, W_{2j}, ..., W_{tj}\}, \Phi_j^{-1}(\alpha)\right\}$ (instead of just using $\Phi_j^{-1}(\alpha)$) relaxes the conservative value of $\Phi_j^{-1}(\alpha)$ and assures that y_j (carrying capacity) is always nonnegative, especially when the standard deviation is large. In summary, we have

$$\sum_{i \in D_j} x_{ij} \leq \psi_j, \ \forall j = 1, 2, ..., p \tag{2.15}$$

as replacement for constraint (2.2).

Beekeepers should choose the value of α in such a way that colonies have the opportunity to replenish the consumed stored food. A large α can be chosen to take advantage of the periods with abundant food sources. However, increasing the value of α may raise the risk of overpopulation during scarce periods.

2.4. Model with fuzzy objective function and nondeterministic constraints

Integrating Subsections 2.2. and 2.3., we have the following auxiliary crisp-and-deterministic model:

$$Objective\ function:\ Maximize \sum_{i=1}^{m}\left(\left(w_i+\frac{\varphi_i}{3}\right)\sum_{j\in C_i}x_{ij}-ME_i\right) \tag{2.16}$$

$subject\ to$

$$Constraint\ 1:\ \sum_{i\in D_j}x_{ij}\le \psi_j,\ \forall j=1,2,...,p \tag{2.17}$$

$$Constraint\ 2:\ \sum_{i=1}^{m}Z_{ki}=1,\ k=1,2,...,n \tag{2.18}$$

$$Constraint\ 3:\ \sum_{k=1}^{n}(\hat{b}_k+\Delta\hat{b}_k)Z_{ki}-\sum_{j\in C_i}x_{ij}-E_i\le 0,\ \forall i=1,2,...,m. \tag{2.19}$$

Illustrative example 1:

Consider the network shown in Figure 1 to characterize the spatial orientation of the apiary with three possible location sites, six plant clusters (patches) and six available colonies of stingless bees (*Tetragonula biroi*). Suppose a forager can travel a maximum distance of 500 meters. Thus, in the network, if two nodes are connected then it means that the distance between the nodes is less than or equal to 500 meters.

Assume that the beekeeper desires to give a higher priority weight to location site 1 because it is near to his house and to a natural water source. Let $\tilde{w}_1 = (2,4,6)$, $\tilde{w}_2 = (0,1,2)$ and $\tilde{w}_3 = (0,1,2)$. Tables 1 and 2 present the strength of each colony and carrying capacity of each plant cluster.

The derived mathematical model and the optimal solution (shown in Table 3) are as follows:

	Strength ($\times 10,000$)
Colony 1	$[0.5, 1.5]$
Colony 2	$[1.5, 2.5]$
Colony 3	$[0.01, 1]$
Colony 4	$[2.5, 3.5]$
Colony 5	$[0.01, 0.75]$
Colony 6	$[3.5, 4.5]$

Table 1: Range of the possible values of the strength of each colony.

	W_{1j}	W_{2j}	W_{3j}	W_{4j}	W_{5j}	W_{6j}	W_{7j}	W_{8j}	W_{9j}	W_{10j}	W_{11j}	W_{12j}	$\overline{\mu}_j$	$\overline{\sigma}_j$	$\Phi_j^{-1}(0.1)$
PC 1	0.06	1.59	1.70	0.84	1.65	1.80	1.40	0.99	1.85	0.54	1.08	0.84	1.20	0.56	0.47
PC 2	2.87	2.16	0.62	1.38	2.95	2.69	1.33	0.74	1.60	0.74	1.47	2.76	1.78	0.88	0.65
PC 3	1.65	2.21	1.18	1.96	2.73	2.92	2.62	3.83	0.49	2.64	2.30	2.39	2.24	0.86	1.14
PC 4	0.74	0.52	0.74	0.45	0.58	0.16	0.53	0.61	0.63	0.36	0.41	0.64	0.53	0.17	0.32
PC 5	2.71	4.48	4.53	3.38	5.66	7.94	4.25	7.78	4.47	4.33	6.49	5.24	5.11	1.61	3.04
PC 6	1.32	0.99	1.69	1.29	1.15	1.36	1.41	0.71	0.82	1.21	1.43	0.78	1.18	0.30	0.80

Table 2: Carrying capacity of plant clusters ($\times 10,000$). Values are derived by determining the number of flowers and the number of foragers that can be accommodated by each flower. Suppose a survey of carrying capacity is done for 12 weeks. "$PC\ j$" denotes Plant Cluster j and "W_{tj}" denotes Week t corresponding to $PC\ j$.

$$Maximize$$

$$4(x_{11} + x_{12} + x_{13}) + x_{23} + x_{24} + x_{32} + x_{33} + x_{35} + x_{36} - 1000\,(E_1 + E_2 + E_3)$$

$$subject\ to$$

$$x_{11} \leq 0.47$$

$$x_{12} + x_{32} \leq 0.65$$

$$x_{13} + x_{23} + x_{33} \leq 1.14$$

$$x_{24} \leq 0.32$$

$$x_{35} \leq 3.04$$

$$x_{36} \leq 0.80$$

$$Z_{11} + Z_{12} + Z_{13} = 1$$

$$Z_{21} + Z_{22} + Z_{23} = 1$$

$$Z_{31} + Z_{32} + Z_{33} = 1$$

$$Z_{41} + Z_{42} + Z_{43} = 1$$

$Z_{51} + Z_{52} + Z_{53} = 1$

$Z_{61} + Z_{62} + Z_{63} = 1$

$1.5Z_{11} + 2.5Z_{21} + Z_{31} + 3.5Z_{41} + 0.75Z_{51} + 4.5Z_{61} - x_{11} - x_{12} - x_{13} - E_1 \leq 0$

$1.5Z_{12} + 2.5Z_{22} + Z_{32} + 3.5Z_{42} + 0.75Z_{52} + 4.5Z_{62} - x_{23} - x_{24} - E_2 \leq 0$

$1.5Z_{13} + 2.5Z_{23} + Z_{33} + 3.5Z_{43} + 0.75Z_{53} + 4.5Z_{63} - x_{32} - x_{33} - x_{35} - x_{36} - E_3 \leq 0.$

Decision variable	Optimal value
x_{11}	0.47
x_{12}	0.65
x_{13}	1.14
x_{24}	0.32
x_{35}	3.04
x_{36}	0.80
x_{23},x_{32},x_{33}	0
Z_{31},Z_{41},Z_{61},Z_{52},Z_{13},Z_{23}	1
Z_{11},Z_{21},Z_{51},Z_{12},Z_{22},Z_{32},Z_{42},Z_{62},Z_{33},Z_{43},Z_{53},Z_{63}	0
E_1	6.74
E_2	0.43
E_3	0.16

Table 3: Optimal solution to the derived mathematical program. x_{ij} and E_i $\times 10,000$.

The solution to the above example shows that colonies 3, 4 and 6 should be located at site 1; colony 5 at site 2; and colonies 1 and 2 at site 3 (see Table 3). The estimated total amount of overpopulation is $E_1 + E_2 + E_3 = 7.33$ ($\times 10,000$ foragers). If overpopulation is inevitable, artificial sucrose feeders can be provided by the beekeeper as temporary replacement for natural food source.

For comparison, let us change some parameter values and see the effect to the solution. Suppose we change the given $\tilde{w}_2 = (0, 1, 2)$ to $\tilde{w}_2 = (3, 4, 7)$ (or $w_2 + \varphi_2/3 = 4.67$), then the new solution shows that colonies 2, 3 and 6 should be located at site 1; colonies 1 and 5 should be located at site 2; and colonies 4 and 5 should be located at site 3. The estimated total amount of overpopulation is still $E_1 + E_2 + E_3 = 7.33$ ($\times 10,000$ foragers) but $E_1 = 6.88$, $E_2 = 0.04$ and $E_3 = 0.41$. There is rearrangement of colony assignment to accommodate the increased preference of the beekeeper towards site 2.

Now, let us observe the solution using the model with deterministic constraints. Consider again the original example (with $\tilde{w}_2 = (0, 1, 2)$), but let us use the mean value of the surveyed carrying capacity of each plant cluster and the midpoint of the range of the strength of each colony. The solution to this problem proposes that colonies 2 and 4 should be located at site 1; colony 5 at site 2; and colonies 1, 3 and 6 at site 3. This deterministic model results in a solution indicating that there is no expected overpopulation ($E_1 + E_2 + E_3 = 0$). Consequently, this suggests that the deterministic model may not be able to predict the possible overpopulation that arises from the uncertainties in the carrying capacity of the plant clusters and in the strength of the colonies.

3. Maximization of Bee Pollination of Crops

We present models for identifying optimal spatial distribution of beehives to maximize utilization of bees as pollinators of crops. These models are variants of the maximal coverage location problem (MCLP) [17, 18, 47]. Here, we also consider the distance of the hive from the food sources and the maximum flight distance of foragers. We use the same notation for the variables and parameters as in Section 2., but we define two additional decision variables:

P_j : the portion of the carrying capacity of plant cluster j not utilized by foragers, $P_j \in \mathbb{R}^{\oplus}$; and

$$g_j = \begin{cases} 1 & \text{if one or more colonies are located in a site connected to plant cluster } j \\ 0 & \text{otherwise.} \end{cases}$$

The concept of bee pollination has a complicated mechanism, e.g., successful pollination usually requires allogamy (cross-pollination). However, in this paper, we simply assume that when foragers exploit the carrying capacity of a plant cluster, pollination occurs. The variable P_j approximates the quantity of poorly pollinated flowers measured in terms of the carrying capacity of the plant cluster.

Let us redefine w_j as the priority weight given to plant cluster $j,\ w_j \in \mathbb{R}$. Let $S_+ = \{j|w_j \geq 0\}$. A negative priority weight is assigned to plant clusters located in a hazardous area (e.g., trees with insecticides). To prevent foraging of bees in hazardous locations, all plant clusters with $w_j < 0$ and all location sites connected to these plant clusters are ignored in the models. That is, we force $Z_{ki} = 0\ \forall i \in D_j$ where $j \notin S_+$, $k = 1, 2, ..., n$.

3.1. Crisp and deterministic mixed-integer program

We present here the crisp deterministic model for determining the optimal beehive location sites such that P_j is minimized. We also consider in the objective function the preference of the beekeeper regarding the plant clusters with high priority for pollination.

Let $\gamma_{ij} = y_j / \sum_{l \in C_i} y_l$ where $l \in C_i$ denotes that plant cluster l is connected to location site i. The parameter γ_{ij} is a naive estimate of the proportion of the total number of foragers coming from site i that are expected to forage on plant cluster j. This estimate is based on the concept of Ideal Free Distribution (IFD) [62] in terms of the amount of available resources (although, this estimate can be adjusted to consider quality of food). For example, consider Figure 1 and the corresponding mean carrying capacities in Table 2. Suppose there are 100 foragers from location site 1. Since site 1 is connected to plant cluster 1, 2 and 3 then $\frac{1.20}{1.20+1.78+2.24} = 22.99\%$ of the 100 foragers are estimated to forage on plant cluster 1.

$$Objective\ function:\ Maximize \sum_{j \in S_+} w_j \left(Mg_j - P_j\right) \tag{3.1}$$

$subject\ to$

$$Constraint\ 1: \sum_{i \in D_j} \sum_{k=1}^{n} \gamma_{ij} b_k Z_{ki} + P_j \geq y_j g_j,\ \forall j \in S_+ \tag{3.2}$$

$$Constraint\ 2: \sum_{i \in D_j,\ j \in S_+} Z_{ki} = 1,\ \forall k = 1, 2, ..., n. \tag{3.3}$$

In the objective function, we want more $g_j = 1$ except for those with negative w_j while minimizing P_j. In Constraint 1, if $g_j = 1$ then $\sum_{i \in D_j} \sum_{k=1}^{n} \gamma_{ij} b_k Z_{ki} + P_j > 0$. This implies that if we want to pollinate the flowers in plant cluster j then we need to locate suitable colony k (for some k) to site i (for some i where $i \in D_j$); otherwise, $P_j > 0$.

If $P_j > 0$ is inevitable in the solution, then one strategy is to add more colonies. Moreover, as a caution, if the solution to the model suggests that the portion of the strength of the colonies to be located in a certain site is larger than the total carrying capacity of the nearby plant clusters, then the solution to this model and the solution to the model discussed in Section 2. should be compared to determine the more appropriate scheme.

3.2. Model with fuzzy objective function and nondeterministic constraints

This model represents the auxiliary program for the fuzzy and probabilistic version of the preceding model. Suppose the priority weight assigned to each plant cluster is fuzzy, the strength of each colony is unknown but bounded, and the carrying capacity of each plant cluster is normally distributed. Let $\tilde{S}_+ = \{j | w_j + \varphi_j/3 > 0\}$, $\Phi_j^{-1}(\beta) = NormInv(\beta, \overline{\mu}_j, \overline{\sigma}_j)$ and $\widehat{\psi}_j = min\left\{max\{W_{1j}, W_{2j}, ..., W_{tj}\}, max\{0, \Phi_j^{-1}(\beta)\}\right\}$. Note that in this model, we use $\hat{b}_k - \Delta\hat{b}_k$ to replace b_k, as explained in Section 2.3.1..

$$Objective\ function:\ Maximize \sum_{j \in S_+} \left(w_j + \frac{\varphi_j}{3} \right) (Mg_j - P_j) \tag{3.4}$$

$subject\ to$

$$Constraint\ 1:\ \sum_{i \in D_j} \sum_{k=1}^{n} \gamma_{ij} (\hat{b}_k - \Delta \hat{b}_k) Z_{ki} + P_j \geq \widehat{\psi}_j g_j,\ \forall j \in \tilde{S}_+ \tag{3.5}$$

$$Constraint\ 2:\ \sum_{i \in D_j,\ j \in \tilde{S}_+} Z_{ki} = 1,\ \forall k = 1, 2, ..., n. \tag{3.6}$$

In this model, it is often appropriate to have a larger $\beta \in [0, 1]$ (e.g., $\beta = 0.9$) because we want to pollinate more flowers particularly during periods of abundant food source. Actually, $\beta = 1 - \alpha$, where α is the error. We use $min \left\{ max\{W_{1j}, W_{2j}, ..., W_{tj}\}, max\{0, \Phi_j^{-1}(\beta)\} \right\}$ to relax the value of $\Phi_j^{-1}(\beta)$, especially when standard deviation is large.

We can represent γ_{ij} by $\gamma_{ij} = \overline{\mu}_j / \sum_{l \in C_i} \overline{\mu}_l$ or $\gamma_{ij} = \widehat{\psi}_j / \sum_{l \in C_i} \widehat{\psi}_l$. However, in the following illustration, we use the latter to take into account the effect of the standard deviation.

Illustrative example 2:

Consider the network shown in Figure 1, the strengths of available colonies as presented in Table 1, and the carrying capacities of the plant clusters in Table 2 (using $\Phi_j^{-1}(0.1)$). Suppose the fuzzy priority weights given to the plant clusters are the values shown in Table 4.

The corresponding mathematical program is as follows:

$Maximize$

$2(1000g_1 - P_1) + 2.67(1000g_2 - P_2) + 3(1000g_3 - P_3) + 0.33(1000g_5 - P_5) + 4.67(1000g_6 - P_6)$

$subject\ to$

	Fuzzy weight
PC 1	$(1, 2, 3)$
PC 2	$(1, 2, 5)$
PC 3	$(2, 3, 4)$
PC 4	$(-3, -2, -1)$
PC 5	$(-2, 1, 2)$
PC 6	$(3, 5, 6)$

Table 4: Priority weights assigned to the plant clusters.

$$
\begin{aligned}
&0.21(0.5Z_{11} + 1.5Z_{21} + 0.01Z_{31} + 2.5Z_{41} + 0.01Z_{51} + 3.5Z_{61}) + P_1 - 0.47g_1 \geq 0\\
&0.29(0.5Z_{11} + 1.5Z_{21} + 0.01Z_{31} + 2.5Z_{41} + 0.01Z_{51} + 3.5Z_{61})\\
&\quad +0.12(0.5Z_{13} + 1.5Z_{23} + 0.01Z_{33} + 2.5Z_{43} + 0.01Z_{53} + 3.5Z_{63}) + P_2 - 0.65g_2 \geq 0\\
&0.5(0.5Z_{11} + 1.5Z_{21} + 0.01Z_{31} + 2.5Z_{41} + 0.01Z_{51} + 3.5Z_{61})\\
&\quad +0.2(0.5Z_{13} + 1.5Z_{23} + 0.01Z_{33} + 2.5Z_{43} + 0.01Z_{53} + 3.5Z_{63}) + P_3 - 1.14g_3 \geq 0\\
&0.54(0.5Z_{13} + 1.5Z_{23} + 0.01Z_{33} + 2.5Z_{43} + 0.01Z_{53} + 3.5Z_{63}) + P_5 - 3.045g_5 \geq 0\\
&0.14(0.5Z_{13} + 1.5Z_{23} + 0.01Z_{33} + 2.5Z_{43} + 0.01Z_{53} + 3.5Z_{63}) + P_6 - 0.80g_6 \geq 0\\
&Z_{11} + Z_{13} = 1\\
&Z_{21} + Z_{23} = 1\\
&Z_{31} + Z_{33} = 1\\
&Z_{41} + Z_{43} = 1\\
&Z_{51} + Z_{53} = 1\\
&Z_{61} + Z_{63} = 1.
\end{aligned}
$$

The solution suggests that colonies 1, 2, 3 and 5 be located at site 1; and colonies 4 and 6 be located at site 3. All g_i ($i = 1, 2, 3, 5, 6$) are equal to 1, which implies that there is at least one colony assigned to forage on each plant cluster (except the plant cluster located in the hazardous area). However, there is still a portion of plant cluster 1 that is expected not to be pollinated. Refer to Table 5 for the list of optimal values.

A relatively minimal number of colonies necessary to sustain $P_j = 0\ \forall j$ can be identified

Decision variable	Optimal value
g_1,g_2,g_3,g_5,g_6	1
P_1	0.05
P_2,P_3,P_5,P_6	0
Z_{11},Z_{21},Z_{31},Z_{51},Z_{43},Z_{63}	1
Z_{41},Z_{61},Z_{13},Z_{23},Z_{33},Z_{53}	0

Table 5: Optimal solution to the mathematical program. P_i $\times 10,000$.

by increasing or decreasing the number of colonies in the model. We do this to determine the reasonable number of hives to support optimal pollination. In this example, we have $P_1 = 0.05$. Suppose the beekeeper can avail an additional colony (Colony 7) with strength $\hat{b}_7 - \Delta\hat{b}_7 = 1$. The solution to the corresponding mathematical program suggests that colonies 1, 3, 4 and 5 be placed at site 1; and colonies 2, 6 and 7 be placed at site 3. The additional colony results in $P_j = 0\ \forall j$.

3.3. An alternative model

This model is specific for cases where we assume that food from plant cluster j is allocated to nearby location sites. For simplicity, we assume that the beekeeper prefers to evenly allocate the carrying capacity of a plant cluster to all connected location sites (if the number of bee colonies is enough, this will force each location site to have at least one bee colony). The major difference between this alternative model and the previous model (with IFD) is the construction of Constraint 1.

Let $\widehat{P}_i$ represent the approximate carrying capacity of nearby plant clusters inadequately utilized by colonies located in site i, and $\widehat{\gamma}_j$ be the number of location sites connected to plant cluster j less the number of location sites connected to hazardous places.

$$Objective\ function:\ Maximize \sum_{j\in S_+} w_j M g_j - \sum_{i\in D_j,\ j\in S_+} \widehat{P}_i \tag{3.7}$$

$subject\ to$

$$Constraint\ 1: \sum_{k=1}^{n} b_k Z_{ki} + \widehat{P}_i \geq \sum_{j \in C_i} \frac{y_j}{\widehat{\gamma}_j} g_j,\ \forall i \in D_j,\ j \in S_+ \tag{3.8}$$

$$Constraint\ 2: \sum_{i \in D_j,\ j \in S_+} Z_{ki} = 1,\ \forall k = 1, 2, ..., n. \tag{3.9}$$

Take note that P_j and $\widehat{P}_i$ are not equivalent since we use different assumptions in their calculations. Figures 2 and 3 illustrate the assumptions used in the model with IFD and in this alternative model, respectively. In the model with IFD, total strength of colonies located in a certain site is distributed among the connected plant clusters; while in this alternative model, carrying capacity (food) of a plant cluster is allocated to the connected possible location sites. Moreover, the auxiliary deterministic program that represents the stochastic version of this alternative model may lead to a nonlinear program [42] (because of the term $\sum_{j \in C_i} y_j/\widehat{\gamma}_j$ in Constraint 3.8).

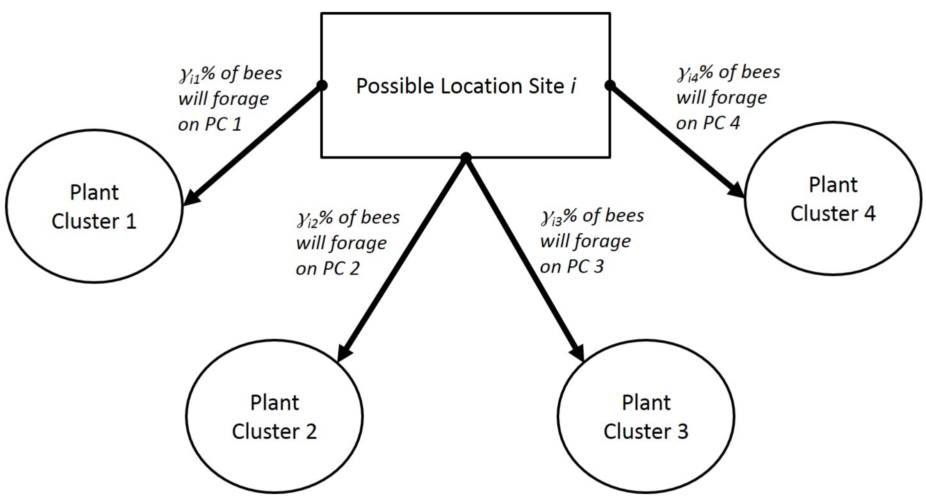

Figure 2: A sample network showing the distribution of the total strength of colonies to the connected plant clusters following the Ideal Free Distribution theory.

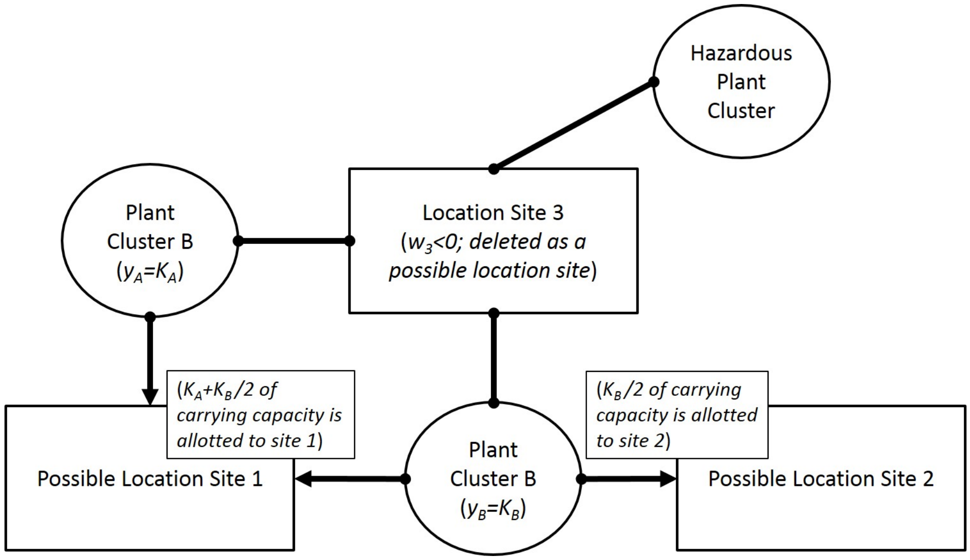

Figure 3: A sample network showing the allocation of the carrying capacity of a plant cluster to the connected possible location sites. Site 3 is not a possible location site since it is connected to a plant cluster situated in a hazardous area (e.g., with insecticides).

4. Concluding Remarks

In this paper, we formulated two auxiliary mixed-integer linear programs for (1) minimizing "over-population" of bee species and for (2) maximizing pollination of flowers, where some parameters are either fuzzy or stochastic. Keen observation of the spatial orientation of the apiary is necessary in writing the appropriate mathematical program, and a close collaboration between beekeepers and mathematical modelers is indispensable. The derived mathematical program can be solved using any Operations Research (OR) software capable of dealing with mixed-integer programming problems. After obtaining the results, it is advised to do sensitivity analysis by identifying the changes in the optimal solution when parameter values are perturbed. Our model can be extended by considering a weighted network (spatial orientation of the apiary), where a weight is a function of distance between nodes.

The optimal solution derived from the model can help beekeepers in selecting the best beehive location sites. The optimal distribution of bee colonies in an apiary can help minimize the foraging stress of the bees, which may result in increased production of honey, pollen and wax. Optimal distribution of beehives can also aid in augmenting the yield of high-value crops. However, the computed optimal solution is not permanent. When ecological season changes, a new spatial distribution of beehives may be necessary, bearing in mind the cost of transferring beehives.

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