**4.2.1 Model**

This mathematical model builds upon the model presented in [] by incorporating an inter-colony interaction. Specifically, it focuses on modeling two colonies, designated as Colony 1 and Colony 2, to illustrate how different behaviors impact disease transmission between colonies. The mathematical model for Colony 1 is given by the following system:

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |
|  | (4) |
|  | (5) |
|  | (6) |

The dynamics of Colony 2 exhibit similarities to Colony 1, albeit with distinct subscripts of the state variables and parameters , , , , , and . The dynamics of Colony 2 are governed by the following system:

|  |  |
| --- | --- |
|  | (7) |
|  | (8) |
|  | (9) |
|  | (10) |
|  | (11) |
|  | (12) |

The drifting component's dynamics are governed through the following system:

|  |  |
| --- | --- |
|  | (13) |
|  | (14) |
|  | (15) |
|  | (16) |

From the equations, the following terms , , , and . The additional parameters that were added are and for the drifting rate, and for the success rate representing the likelihood of adult bees being integrated into the colony they've drifted to, and lastly and for the return rate of the drifting adult bee to their original colony if they fail to integrate into the other colony.