A MATHEMATICAL MODEL OF AMERICAN FOULBROOD TRANSMISSION THROUGH DRIFTING BEHAVIOUR OF EUROPEAN HONEYBEES (APIS MELLIFERA L.)

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**A MATHEMATICAL MODEL OF INTER-COLONY SPREAD OF AMERICAN FOULBROOD IN EUROPEAN HONEYBEES (APIS MELLIFERA L.)**

by

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# BIOGRAPHICAL SKETCH

Born on August 30, 2001, in Makati City, Kieffer P. Santos is the eldest child of Antonio P. Santos and Ruth P. Santos, and sibling to Seymour P. Santos, Calahan P. Santos, and Austein P. Santos. Growing up, he received his primary education at Golden Lampstand Grade School in San Pedro, Laguna, followed by his secondary education at Immaculate Heart of Mary School, also in San Pedro. In September 2020, Kieffer embarked on his academic journey at the University of the Philippines Los Baños (UPLB), where he pursued a degree in BS Applied Mathematics. With a keen interest in mathematical applications, Kieffer is committed to leveraging his education to contribute meaningfully to his field and beyond.

KIEFFER P. SANTOS

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# ABSTRACT

KIEFFER P. SANTOS, University of the Philippines Los Baños, June 2023

**A MATHEMATICAL MODEL OF INTER-COLONY SPREAD OF AMERICAN FOULBROOD IN EUROPEAN HONEYBEES (APIS MELLIFERA L.)**

**Major Professor: Eduardo O. Jatulan**

American Foulbrood (AFB) remains a critical concern for European honeybee colonies globally, exerting profound impacts on bee health and apiary productivity. This study endeavors to deepen our comprehension of the inter-colony transmission dynamics of AFB among European honeybee colonies by developing a comprehensive mathematical model. The model incorporates influential factors such as bee drifting, providing a nuanced exploration of disease ecology in honeybee communities. This study contributes to the advancement of our understanding of disease ecology in honeybee populations and represents a significant step forward in the quest for sustainable beekeeping practices and the preservation of honeybee populations.

**Mathematics Subject Classification (2020):** 92-10, 92B05, 92D25, 92D30, 92D50

**Keywords:** American Foulbrood (AFB), European honeybees (Apis mellifera L.), epidemiology, population dynamics

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# INTRODUCTION

American Foulbrood (AFB), caused by Paenibacillus larvae, poses a significant threat to honeybee colonies primarily during the pre-pupal and pupal stages, exhibiting characteristic symptoms like sunken cappings and a foul smell, and spreading through various means including robbing, drifting bees, and human activity [1, 5, 15]. The lifecycle of European honeybees progresses through distinct stages from egg to adult, vital for colony survival dependent on factors like foraging behavior and resource availability, with peak seasons comprising substantial foraging activity requiring approximately 50 pounds of pollen and 100 pounds of honey annually for colony sustenance [6, 14, 17]. Influential factors impacting colony survival and disease dynamics include transmission rates and disease-induced death rates, with disease spread through robbing emphasizing the importance of reducing drift for disease mitigation and understanding disease dynamics elucidating mechanisms underlying colony survival or collapse [2, 3, 10]. Inter-colony behavior, including drifting and robbing, plays a pivotal role in disease transmission within apiaries, with mite immigration contributing to varroa mite infestations, highlighting the role of worker bee behavior and guard bees in disease transmission [13, 18, 20]. Control and treatment methods for AFB, such as sealing and burning infected colonies, require rigorous colony management practices for prevention and effective disease extinction, with an understanding of transmission routes and proper hive management crucial for controlling AFB spread and maintaining healthy bee colonies [1, 4, 5, 15]. Comparisons between environmental hazards and infectious diseases underscore the complexity of honeybee colony dynamics, emphasizing the severity of colony-level infections and the interplay between environmental factors and evolutionary adaptations [2, 3].

The primary objective of this study is to develop a mathematical model (SI model) to investigate the dynamics of inter-colony spread of American Foulbrood (AFB) in European honeybee (Apis mellifera L.) populations, aiming to incorporate key factors such as drifting rate, effectiveness of drifting bees in integrating into other colonies, and return rate of unsuccessful drifting bees into the model to comprehensively capture AFB transmission dynamics between honeybee colonies. Through analysis and numerical simulations, the study seeks to identify factors influencing the spread of AFB and assess the accuracy and effectiveness of the model. The significance of the study lies in its comprehensive analysis of AFB transmission dynamics, providing insights crucial for devising effective disease management strategies and safeguarding the economic viability of beekeeping operations. Additionally, the study's implications extend to biodiversity conservation, public health, and scientific research, highlighting its importance in addressing a pressing threat to honeybee populations and the ecosystems they support. While the study focuses on understanding the influence of factors like colony density, bee movement, behavior, and health on AFB transmission dynamics and offers insights into long-term AFB exposure effects and critical control points for mitigating epidemics, it acknowledges limitations inherent in mathematical modeling, including simplifications and data availability constraints, which may affect the accuracy and generalizability of findings to real-world scenarios. Therefore, it's important to exercise prudence when applying the findings of this study to practical beekeeping and disease management practices.

Recent studies have delved into understanding disease dynamics in honeybee populations. Betti, Wahl, and Zamir [2] (2014) developed a mathematical model showing that high drifting levels among colonies could rapidly spread diseases, emphasizing the need for effective guarding behavior. Similarly, Betti and Shaw [3] (2021) highlighted the significant threat of infectious diseases to honeybee colonies compared to other hazards. Datta, Bull, Budge, and Keeling [4] (2013) analyzed an American Foulbrood outbreak in 2010, offering insights into epidemic reconstruction. Gavina, Rabajante, and Cervancia [7] (2014) presented models to optimize hive locations for bee populations and pollination. Meanwhile, Jatulan, Rabajante, Banaay, Fajardo, and Jose [8] (2015) emphasized the importance of controlling infection transmission for AFB management. Khoury, Myerscough, and Barron [10] (2011) focused on forager death rates' impact on colony failure, and Prado, Requier, Crauser, Le Conte, Bretagnolle, and Alaux [12] (2020) explored honeybee lifespan dynamics. Additionally, Šekulja, Pechhacker, and Licek [20] (2014) studied drifting's role in AFB transmission, especially during young bee orientation flights. These studies collectively enhance our understanding of disease management in honeybee populations.

# METHODOLOGY

## Main Assumptions

In this model, Same max number of cells in apiary

## Mathematical Model

This mathematical model builds upon the model presented in Jatulan et al.’s paper [8] by incorporating an inter-colony interaction. Specifically, it focuses on modeling two colonies, designated as Colony 1 and Colony 2, to illustrate how different behaviors impact disease transmission between colonies. The mathematical model for Colony 1 is given by the following system:

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |
|  | (4) |
|  | (5) |
|  | (6) |

The dynamics of Colony 2 exhibit similarities to Colony 1, albeit with distinct subscripts of the state variables and parameters , , , , , and . The dynamics of Colony 2 are governed by the following system:

|  |  |
| --- | --- |
|  | (7) |
|  | (8) |
|  | (9) |
|  | (10) |
|  | (11) |
|  | (12) |

The drifting component's dynamics are governed through the following system:

|  |  |
| --- | --- |
|  | (13) |
|  | (14) |
|  | (15) |
|  | (16) |

From the equations, the following terms , , , and . The parameters introduced, not originally in Jatulan et al.’s paper [8], include and , representing the drifting rate of honey bees times the percentage of their foragers; and , indicating the rejection rate of drifting honey bees not accepted by the other colony; and finally and , signifying the return rate of drifting adult bees to their original colony upon failure to integrate into the other colony.

The other key parameters that are in Jatulan et al.’s paper [8], together with the new added ones are presented in the table below:

Table 1. Additional significant parameters utilized in the model.

|  |  |  |  |
| --- | --- | --- | --- |
| **Parameters** | **Description** | **Value** | **Reference** |
|  | The queen’s maximum laying rate per day | , | [8, 9, 14] |
|  | A coefficient that influences the rate at which the term approaches as the number of adult bees in a colony gets large [10]. |  | [8] |
|  | The rate at which broods are infected by spore-carrier adult bees |  | [8] |
|  | The rate at which adult bees become infected by the infected cell |  | [8] |
|  | The rate at which adult bees become infected upon contact with spore-carrier adult bees |  | [8] |
|  | The rate at which broods become immune with the spores |  | [8], calculated from [11] |
|  | The rate at which immune broods become adult bees |  | calculated from [11] |
|  | The death rate of infected broods | variable, | [8, 11] |
|  | The rate at which a diseased dead brood is cleaned by adult bees | variable, | [8, 11] |
|  | The maximum number of cells in the colony |  | [8] |
|  | The percentage of clean cell combs that will be contaminated by AFB spores |  | [8] |
|  | The death rate of adult bees (both spore-carrier and spore-free) |  | [8] |
|  | The rate at which honeybees drift multiplied by the percentage of foraging honeybees | \* | calculated from [17] and [20] |
|  | The rejection rate of drifting honeybees that are not accepted by the receiving colony. |  | [16] |
|  | The rate at which drifting adult bees return to their original colony after failing to integrate into the other colony. | , | [11, 12] |

\*the subscript representing the two colonies.

\*~1% drift rate [20] x ~30% are foragers [17]

In the model, we examined 16 state variables, with parameter details provided in the previous table, **Table 1**. We applied the Susceptible-Infectious (SI) framework to depict the transmission of American foulbrood (AFB) among broods and adult bees between colonies.

Table 2. Description of the state variables.

|  |  |
| --- | --- |
| **Variables** | **Description** |
|  | The number of infected broods in colony 1 |
|  | The number of healthy broods in colony 1 before hours old ( depends on the susceptibility of bees) |
|  | The number of healthy broods in colony 1 beyond hours old |
|  | The number of infected cell combs in colony 1 (due to diseased dead brood) |
|  | The number of adult bees with AFB spores in colony 1 |
|  | The number of adult bees without AFB spores in colony 1 |
|  | The number of infected broods in colony 1 |
|  | The number of healthy broods in colony 2 before hours old ( depends on the susceptibility of bees) |
|  | The number of healthy broods in colony 2 beyond hours old |
|  | The number of infected cell combs in colony 2 (due to diseased dead brood) |
|  | The number of adult bees with AFB spores in colony 2 |
|  | The number of adult bees without AFB spores in colony 2 |
|  | The number of infected adult bees from colony 2 that drift into colony 1. |
|  | The number of adult bees from colony 2 that drift into colony 1. |
|  | The number of infected adult bees from colony 1 that drift into colony 2. |
|  | The number of adult bees from colony 1 that drift into colony 2. |

To be able to come up with equations, it is important to identify the relationships among the established variables and parameters. **Figure 1** depicts the intra-colony transmission flow diagram of AFB. The intra-colony behavior mirrors that described in Jatulan et al.’s paper [8], with supplementary parameters outlining inter-colony behavior that **Figure 2** illustrates.

|  |
| --- |
| **(a)** |
| **(b)** |

Figure 1. Compartmental diagram illustrating the interaction of honeybees and the spread of AFB within Colony 1 (a) and Colony 2 (b). Red arrows represent infection, while blue arrows denote transitions between states (refer to Table 2 for the definitions of state variables).

|  |
| --- |
| **(a)** |
| **(b)** |

Figure 2. Diagram illustrating inter-colony honeybee behavior between the two colonies. Subfigure (a) depicts the drifting behavior dynamics, while (b) illustrates infection dynamics. In (a), dark blue arrows represent honeybees leaving their colony to drift to the other colony, while orange arrows indicate the return movement of foreign honeybees either back to their original colony or outside the two colonies.

Upon merging all the diagrams, the resulting comprehensive diagram is displayed in **Figure 3**. **Figure 3** provides an overview of the complete mathematical model depicting the spread of AFB through drifting behavior, constituting the primary focus of this study.

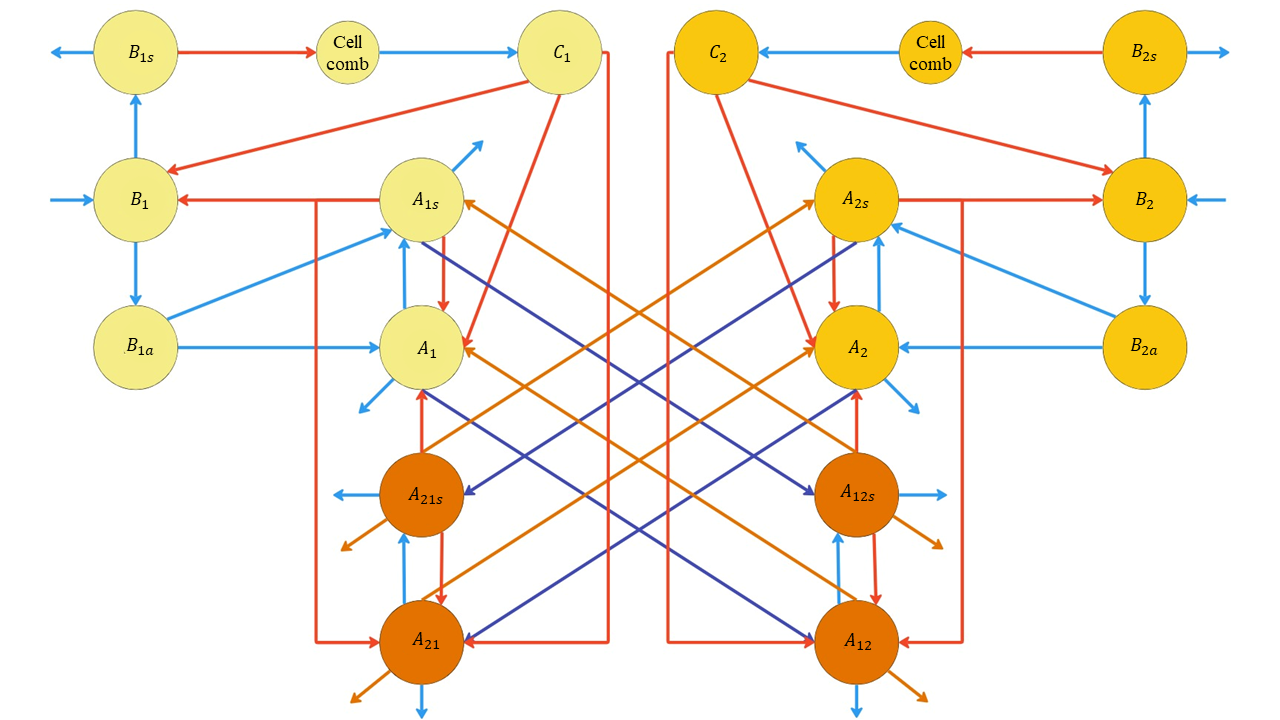


Figure 3. The resulting comprehensive model diagram depicting the AFB transmission dynamics.

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