答案

No.

Date

一. D B D C A C B A

二. 1. $\frac{2}{5}$ 2. $3^x \ln 3$ 3. 0 4. $-\cot x - \tan x + C$

5. $f(x)dx$ 6. $\frac{3\pi}{2}$ 7. $3x^2 - \frac{1}{2}$ 8. $\pi \int_0^1 (x^2 - x^4) dx$

三. 1. 解 原式 $= \lim_{x \to 0} \frac{4x^2 - 2x + 1}{3x + 2} = \frac{1}{2}$

2. 解: 左式 $\xlongequal{\text{洛}} \lim_{x \to a} \frac{f(x)}{1} = f(a)$

3. 解: $\frac{dy}{dx} = \frac{9t^2}{2t} = \frac{9}{2}t$

$\frac{d^2y}{dx^2} = \frac{\frac{9}{2}}{2t} = \frac{9}{4t}$

4 解. $\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \frac{1}{2} \ln^2 x + C$

5. 解: $\int x \arctan x dx = \int \arctan x d \frac{x^2}{2}$

$= \frac{x^2}{2} \arctan x - \int \frac{x^2}{2(1+x^2)} dx$

$= \frac{x^2}{2} \arctan x - \frac{1}{2} \int (1 - \frac{1}{1+x^2}) dx$

$= \frac{x^2}{2} \arctan x - \frac{1}{2} x + \frac{1}{2} \arctan x + C$

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6. 解: $\int_3^{+\infty} \frac{dx}{\sqrt{x}+x\sqrt{x}} = \int_3^{+\infty} \frac{dx}{\sqrt{x}(1+x)}$

令 $\sqrt{x}=t$, $x=t^2$ $\int_3^{+\infty} \frac{2t\,dt}{t(1+t^2)} = 2\int_3^{+\infty} \frac{dt}{1+t^2}$

$= 2\arctan t\Big|_3^{+\infty} = 2\left(\frac{\pi}{2} - \arctan 3\right)$

7. 解: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x}\,dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{\cos x}\cdot|\sin x|\,dx = -\int_{-\frac{\pi}{4}}^{0} \sqrt{\cos x}\sin x\,dx$

$+\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\sin x\,dx = \frac{2}{3}\left[\cos^{\frac{3}{2}} x\right]_{-\frac{\pi}{4}}^{0} - \frac{2}{3}\left[\cos^{\frac{3}{2}} x\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{3}(4-\sqrt{2})$

8. 解:

$y=ex$

交点 $\begin{cases} y=e^x \\ y=ex \end{cases} \Rightarrow (1,e)$

$\therefore A = \int_0^1 (e^x - ex)\,dx = \left[e^x - \frac{1}{2}e\cdot x^2\right]_0^1 = \frac{e}{2} - 1$

9. 证: 设 $f(x) = \arctan x + \operatorname{arccot} x$

则 $f'(x) = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0$ $\therefore f(x) = C$

取 $x=0$ 时 $\frac{\pi}{2} = C$ $\therefore f(x) = \frac{\pi}{2}$