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ASSIGNMENT – 2

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Gödel's incompleteness theorems are two theorems of mathematical logic that are concerned with the limitations of provability in formal axiomatic theories. These conclusions, announced by Kurt Gödel in 1931, are fundamental both in mathematical logic and in the philosophy of mathematics. The theorems are commonly, but not universally, understood as proving that Hilbert's quest to establish a full and consistent set of axioms for all mathematics is impossible.

First incompleteness theorem asserts that no consistent system of axioms whose theorems can be stated by an effective process (i.e., an algorithm) is capable of demonstrating all facts about the arithmetic of natural numbers. For any such consistent formal system, there will always be claims about natural numbers that are true, but that are unprovable inside the system. The second incompleteness theorem, an extension of the first, indicates that the system cannot verify its own coherence.

The incompleteness theorems apply to formal structures that are of sufficient complexity to describe the basic arithmetic of the natural numbers and which are consistent, and properly axiomatized, these notions being discussed below. Discussed in the case of first-order logic, formal systems are sometimes termed formal theories. In generally, a formal system is a deductive apparatus that consists of a particular set of axioms combined with rules of symbolic manipulation (or rules of inference) that allow for the derivation of new theorems from the axioms. One example of such a system is first-order Peano arithmetic, a system in which all variables are meant to signify natural numbers. In other systems, such as set theory, only some words of the formal system convey claims about the natural numbers. The incompleteness theorems are about formal provability inside these systems, rather than about "provability" in an informal sense.

The incompleteness theorem is strongly connected to numerous theorems regarding undecidable sets in recursion theory.

Stephen Cole Kleene (1943) gave a demonstration of Gödel's incompleteness theorem utilising basic findings of computability theory. One such conclusion indicates that the halting issue is undecidable: there is no computer programme that can properly identify, given any programme P as input, whether P finally halts when executed with a certain supplied input. Kleene proved that the presence of a complete effective system of arithmetic with particular consistency qualities would compel the halting issue to be decidable, a contradiction. This form of evidence has already been provided by Shoenfield (1967, p 132); Charlesworth (1980); and Hopcroft and Ullman (1979).