Survival Analysis with Applications in Medicine: Take-home examination

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A. Weibull regression models

Q1. For a proportional hazards model with $S_0(t)$ that is a Weibull distribution, show that survival $S(t)$ is also from a Weibull distribution.

Given a proportional hazards model with survival function $S(t|x) = S_0(t)^{\exp(\beta x)}$ for time t and a given covariate x, $S_0(t)$ is the baseline survival function, and β is the log hazard ratio.

We have the Weibull survival function $S_0(t) = \exp(-\lambda t^k)$ for a scale parameter λ and a shape parameter k.

Substituting $S_0(t)$ in the proportional hazards model, we get the survival function as

$S(t|x) = \left[\exp(-\lambda t^k)\right]^{\exp(\beta x)}$

$S(t|x) = \exp(-\lambda t^k \exp(\beta x))$

which is in form $S(t|x) = \exp(-\tilde{\lambda}_a t^k)$ where $\tilde{\lambda}_a = \lambda \exp(\beta x)$ is the new scale parameter for the same shape parameter k.

This means that the survival function $S(t|x)$ for a proportional hazards model is also from a Weibull distribution.

Here, $S(t) = E_X[S(t \mid X)]$, which will also be from a Weibull distribution.

Q2. For an accelerated failure time model with $S_0(t)$ that is a Weibull distribution, show that survival $S(t)$ is also from a Weibull distribution.

Given an accelerated failure time model with survival function $S(t|x) = S_0(t \exp(-\tilde{\beta}x))$ for time t and a given covariate x, $S_0(t)$ is the baseline survival function, and $\tilde{\beta}$ is the log time ratio.

Substituting $S_0(t)$ in the accelerated failure time model, we get the survival function as

$S(t|x) = \exp(-\lambda\big(t \exp(-\tilde{\beta}x)\big)^k)$

$S(t|x) = \exp(-\lambda t^k \exp(-k\tilde{\beta}x))$

which is in form $S(t|x) = \exp(-\tilde{\lambda}_b t^k)$ where $\tilde{\lambda}_b = \lambda \exp(-k\tilde{\beta}x)$ is the new scale parameter for the same shape parameter k.

This means that the survival function $S(t|x)$ for an accelerated failure time model is also from a Weibull distribution.

Similarly, $S(t) = E_X[S(t \mid X)]$, which will also be from a Weibull distribution.

Q3. What is the relationship between β and $\tilde{\beta}$ if both models have a Weibull baseline survival function?

For the proportional hazards model, we have $S(t|x) = \exp(-\lambda t^k \exp(\beta x))$.

For the accelerated failure time model, we have $S(t|x) = \exp(-\lambda t^k \exp(-k\tilde{\beta}x))$.

Comparing the two models, we get $\exp(\beta x) = \exp(-k\tilde{\beta}x)$.

Taking the natural logarithm of both sides, we get $\beta x = -k\tilde{\beta}x$.

Therefore, the relationship between β and $\tilde{\beta}$ is $\beta = -k\tilde{\beta}$ if both models have a Weibull baseline survival function.

B: Interval-censored likelihood

C: Truncated distributions

D: Cox's partial likelihood with a time-varying effects

E: Data analysis of a randomised controlled trial for hormonal treatment of breast cancer patients in Germany

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F: Analysis plan for a randomised controlled trial
