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1. Introduction

Proposed sections in the introduction:

1. Exoplanet detection – Dominant detection methods = Transit, RV. Basic statistics of exoplanets discovered by Kepler Space Telescope.
2. Planet Formation – MMSN, Core accretion/GI models, Planetesimal Formation, Nice Model.
3. **Planet Dynamics – Mean Motion Resonance, (planetesimal and gas) Migration, Stability.**
4. Numerical Integration – Hamiltonian Dynamics, Close Encounters, etc.

Below I am going to elaborate on planet dynamics.

2. Planet Dynamics

2.1. Mean Motion Resonance (MMR)

MMR occurs when the orbital period of one planet is an integer ratio of another. Like other types of resonances occuring in nature, MMR results in the amplitude growth of various quantities that characterize the system like eccentricity, semi-major axis and the longitude of pericentre (Murray & Dermott 1999). As a result, the presence of MMR can strongly affect the formation, evolution and longterm stability of planetary systems in diverse ways. For example, Kirkwood gaps are unstable regions in the asteroid belt carved by MMRs with Jupiter, while Pluto and Neptune are on crossing orbits but are protected from colliding due to a 3:2 MMR.

For every $p : q$ MMR (where p and q are integers) there are two important resonant angles:

$$\phi_1 = p\lambda_1 - q\lambda_2 + \varpi_1$$

$$\phi_2 = p\lambda_1 - q\lambda_2 + \varpi_2$$

where λ is the mean longitude and ϖ is the longitude of periapse. For planets to be in MMR the time variation of one or both of these resonant arguments, $\dot{\phi}$, must be zero.

The strength of a given MMR is related to its width, which in turn is related to the order ($= p - q$) of the resonance and the magnitude of p and q (Murray & Dermott 1999). In general the lower the order, p and q are the stronger the resonance (Murray & Dermott 1999), making the 2:1 and 3:2 MMRs the most probable resonant locations in nature. Figure 1 shows the distribution of period ratios for planets discovered by *Kepler*, along with the locations of first and second order MMRs. As can be seen, statistical excesses of planets exist near the 2:1 and 3:2 MMR (Lissauer et al. 2011; Fabrycky et al. 2014; Steffen & Hwang 2015), supporting the idea that these resonant locations trap planets.

However, most planets belonging to these statistical pileups are a few percent away from exact period commensurability, and dissipative mechanisms have been proposed to transport these planets from exact MMR. The most popular of these mechanisms are tidal (Lithwick & Wu 2012; Batygin & Morbidelli 2013; Delisle et al. 2014), protoplanetary (Rein 2012; Baruteau & Papaloizou 2013; Goldreich & Schlichting 2014), and planetesimal (Moore et al. 2013; Chatterjee & Ford 2015). The formation implications for each mechanism are different, and no clear consensus has yet emerged.

2.2. Migration

2.2.1. Planetesimal-Driven Migration

Planets can migrate in the presence of a massive planetesimal disk. Planetesimals that pass through the Hill sphere of a planet will strongly interact gravitationally, exchanging angular momentum with the planet (Ida et al. 2000; Kirsh et al. 2009). If there is an asymmetry to the number of planetesimals interacting with the planet on its near and far sides, migration will result. However, to guarantee migration, planetesimal orbits must decouple from the planet, which is achieved by either ejecting the planetesimals from the planetary system or by passing the planetesimals to another, neighbouring planet. In addition, for continuous migration the planet must constantly encounter fresh, dynamically cold planetesimals.

It is believed that such migration occurred for Neptune, which migrated outwards into the Kuiper belt, shepherding planetesimals inwards to Jupiter which subsequently ejected them from the Solar System (Fernandez & Ip 1984). This idea is well supported by the observations of the outer Solar System, which show that Pluto along with a host of smaller bodies orbit in stable 3:2 MMRs with Neptune (Malhotra 1993, 1995), a natural outcome from outward, planetesimal-driven migration.

2.2.2. Gas-Driven migration

A planet embedded in a protoplanetary disk can migrate via an exchange of angular momentum from disk-planet torques (Goldreich & Tremaine 1980). Since the discovery of the first hot Jupiter (Mayor & Queloz 1995), gas-driven migration is believed to play an important role in shaping exoplanetary systems (Lin et al. 1996). Planet migration comes in three main flavours, Type I, Type II and Type III.

Type I migration occurs when low-mass planets are fully embedded in a protoplanetary disk and do not significantly perturb the disk structure. At particular resonant locations, known as “Linblad resonances”, density waves are excited due to gravitational interactions between the planet and disk (Goldreich & Tremaine 1979). These density waves exchange angular momentum with the planet, and migration occurs when the inner and outer disk interact asymmetrically with the planet (Goldreich & Tremaine 1979).

Type II migration occurs when high-mass Jovian planets significantly modify the structure of the surrounding protoplanetary disk. In particular, a gap in the disk is opened up by the planet, and is locked in place by the boundaries of the gap. The planet then migrates inwards the same rate as the local disk due to ordinary viscous evolution (Armitage 2010).

Type III.

In comparison to planetesimal migration, gas-driven migration is still not well understood. In particular, standard calculations of gas-driven migration are too quick by an order of magnitude (Lin & Papaloizou 1986; Tanaka et al. 2002), causing planets to spiral into their central stars before the protoplanetary disk has dispersed. In contrast, recent work (Fung & Chiang 2017) suggests that planets actually do not migrate that much and tend to be better behaved than originally believed. A consensus on migration has yet to be established, but it is clear that some form of migration occurs in the universe due to the large number of planets in MMR.

2.3. Stability

Although the longterm stability of planetary orbits has been studied for hundreds of years by the likes of Newton, Lagrange and Gauss, it has been historically difficult to make progress in the field due to the chaotic and non-integrable nature of planetary systems. This chaos is caused by overlapping resonances (Chirikov 1979; Lecar et al. 2001), resulting in the divergence of near-identical systems on long timescales. However, with the aid of computers the equations of motion governing planetary systems can be brute-force integrated into the future or past with N-body simulation, allowing scientists to answer fundamental questions that have plagued humans for hundreds of years. For example, it is now known that the Solar System is marginally stable (Sussman & Wisdom 1988; Laskar 1994; Lecar et al. 2001) with Mercury having a 1% chance of colliding with Venus or the sun within a couple billion years (Laskar & Gastineau 2009).

Since the discovery of numerous exoplanetary systems via *Kepler*, longterm stability has become a popular way to constrain the orbital parameters of the planets in that system (Lissauer et al. 2011; Steffen et al. 2013; Jontof-Hutter et al. 2014; Tamayo et al. 2015). Assuming that the observed system is stable over billion year timescales, grids of N-body integrations are run using initial conditions that are consistent with the observational constrains. Although this brute-force method is useful for ruling out additional regions of parameter space, it is not without its costs. A typical 10 billion year integration or the Solar System can take weeks or even months to perform, and due to the chaotic nature of planetary systems, hundreds to thousands of realizations must typically be simulated to acquire statistically rigorous answers.

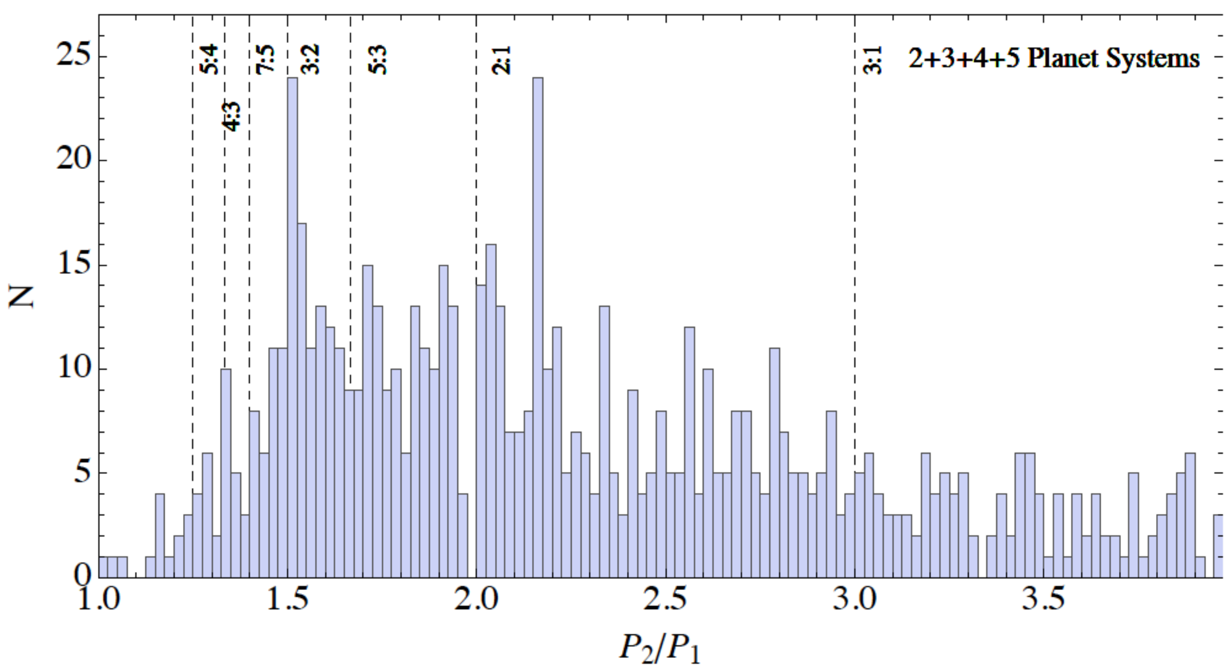

Fig. 1.— Period ratios of Kepler planets, image from Goldreich & Schlichting (2014).

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