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1. Introduction

Proposed sections in the introduction:

1. Exoplanet detection – Dominant detection methods (Transit, RV), basic statistics of exoplanets discovered by Kepler Space Telescope.
2. Planet Formation – MMSN, Core accretion/GI models, Planetesimal Formation, Nice Model.
3. Planet Dynamics – Mean Motion Resonance, (planetesimal and gas) Migration, Stability.
4. Numerical Integration – Hamiltonian Dynamics, integrator types (hybrid, symplectic, high order), coordinate systems (jacobi, heliocentric, whds).

Below I am going to elaborate on Planet Dynamics.

2. Planet Dynamics

2.1. Mean Motion Resonance

Mean motion resonance (MMR) occurs when the orbital period of one planet is an integer ratio of another. Like other types of resonances occuring in nature, MMR results in the amplitude growth of various quantities characterizing the system, like eccentricity, semi-major axis and the longitude of pericentre (Murray & Dermott 1999). As a result, the presence of MMR can strongly affect the formation, evolution and longterm stability of planetary systems in a diversity of ways. For example, Kirkwood gaps are unstable regions in the asteroid belt carved by MMRs with Jupiter, while Pluto and Neptune are protected from going unstable due to a 3:2 MMR.

For every $p:q$ MMR (where p and q are integers) there are two important resonant angles:

$$\begin{aligned}\phi_1 &= p\lambda_1 - q\lambda_2 + \varpi_1\\ \phi_2 &= p\lambda_1 - q\lambda_2 + \varpi_2\end{aligned} \tag{1}$$

where λ is the mean longitude and ϖ is the longitude of periapse. For planets to be in MMR, the time variation of one or both of these resonant arguments must be zero. As a result, MMRs can be modelled in terms of a pendulum oscillating about a stable, fixed point. After some algebra it can be shown that a MMR can be modelled as (Murray & Dermott 1999):

$$\ddot{\phi} = -\omega_0^2 \sin\phi \tag{2}$$

where ω_0 is the amplitude of libration and is dependent upon the orbital parameters of the system (m, e, a, etc.).

The pendulum model facilitates understanding about certain properties of MMR. For energies larger than a critical energy E_{crit}, the pendulum will circulate with ϕ ranging all possible values. For energies smaller than E_{crit} the pendulum will be in MMR and librate about $\phi = 0$. The critical energy, E_{crit}, defines motion on the separatrix, which divides the circulation and libration regimes. In context of a pendulum, this would correspond to the pendulum suspended vertically in the air, with an infinite period of libration.

The strength of a given MMR is related to its width, which in turn is related to the order of the resonance ($= p - q$) and the magnitude of p and q (Murray & Dermott 1999). More fundamentally, the strength of a MMR is related to the mass, eccentricity and mean motions of the planets involved (Murray & Dermott 1999). The lower p, q and the order are the stronger the resonance, making the 2:1 and 3:2 MMRs the most probable resonant locations in nature. Figure 1 shows the distribution of period ratios for planets discovered by *Kepler*, along with the locations of first and second order MMRs. As can be seen, statistical excesses of planets exist near the 2:1 and 3:2 MMR (Lissauer et al. 2011; Fabrycky et al. 2014; Steffen & Hwang 2015), supporting the idea that these resonances are most capable of trapping planets.

The planets from these statistical pileups are typically a few percent away from exact period commensurability, and dissipative mechanisms have been proposed to transport these planets from exact MMR. The most popular of these mechanisms are tidal (Lithwick & Wu 2012; Batygin & Morbidelli 2013; Delisle et al. 2014), protoplanetary (Rein 2012; Baruteau & Papaloizou 2013; Goldreich & Schlichting 2014), and planetesimal (Moore et al. 2013; Chatterjee & Ford 2015). The formation implications for each mechanism are different, and no clear consensus has yet emerged.

2.2. Migration

2.2.1. Planetesimal-Driven Migration

Planetesimals passing through the Hill sphere of a planet will exchange angular momentum via gravity (Ida et al. 2000; Kirsh et al. 2009). If there is an asymmetry to the number of planetesimals interacting with the planet on its near and far sides, a net force will migrate the planet. However, to guarantee migration, planetesimal orbits must decouple from the planet. A massive enough planet (e.g. Jupiter) will directly eject planetesimals from the system, however if the planet is smaller (e.g. Neptune) planetesimals must decouple by interacting with a neighbouring planet. In addition, for sustained migration the planet must constantly encounter fresh, dynamically cold planetesimals (Armitage 2010).

Since protoplanetary growth is proportional to orbital frequency (Rafikov 2003) planetesimal disks are most likely to exist in the outer reaches of planetary systems where fewer orbital cycles have occurred. The primordial Kuiper belt is believed to have caused Neptune to migrate outwards into the Kuiper belt and shepherd planetesimals inwards to Jupiter, which subsequently ejected them from the Solar System (Fernandez & Ip 1984). This idea is well supported by observations of the outer Solar System, which show that Pluto, along with a host of smaller bodies, orbit in stable 3:2 MMRs with Neptune (Malhotra 1993, 1995).

2.2.2. Gas-Driven migration

Since the discovery of the first hot Jupiter (Mayor & Queloz 1995), gas-driven migration is believed to play an important role in shaping exoplanetary systems (Lin et al. 1996). Whenever a fully-formed planet is embedded in a protoplanetary disk, angular momentum can be exchanged via disk-planet torques (Goldreich & Tremaine 1980). The result of this exchange is planetary migration, and this scenario is believed to be common in young planetary systems. Gas-driven migration comes in two main flavours – Type I and Type II.

Type I migration occurs when low-mass planets are fully embedded in a protoplanetary disk and do not significantly perturb the disk structure (Armitage 2010). At particular resonant locations, known as “Linblad resonances”, density waves are excited due to gravitational interactions between the planet and disk (Goldreich & Tremaine 1979). These density waves exchange angular momentum with the planet, and migration occurs when the inner and outer disk interact asymmetrically with the planet (Goldreich & Tremaine 1979). In general, the direction of Type-I migration tends to be inwards towards the central star (Ward 1997).

Type II migration occurs when high-mass Jovian planets significantly modify the structure of the surrounding protoplanetary disk, opening up a gap. This gap locks the planet in place, coupling the migration of the planet to the evolution of the disk (Lin & Papaloizou 1986). The viscous evolution of the disk causes the planet to slowly migrate inwards, at speeds typically one or two orders of magnitude slower than Type-I migration (Ward 1997).

In comparison to planetesimal migration, gas-driven migration is still not well understood. In particular, standard calculations of gas-driven migration are too quick by orders of magnitude (Lin & Papaloizou 1986; Tanaka et al. 2002), causing planets to spiral into their central stars before the protoplanetary disk has dispersed. In contrast to the standard view, recent work (Fung & Chiang 2017) has suggested that planets actually do not migrate that much and tend to be better behaved than originally believed. A consensus on migration has yet to be established, but it is clear that some form of migration must occur in the universe due to the large number of planets in or near MMR (Lissauer et al. 2011; Fabrycky et al. 2014; Steffen & Hwang 2015).

2.3. Stability

The longterm stability of planetary orbits has been studied for hundreds of years by the likes of Newton, Lagrange and Gauss. However, due to the chaotic and non-integrable nature of planetary systems, it has been historically difficult to make progress on N-body problems. The chaos present in planetary systems is caused by overlapping resonances (Chirikov 1979; Lecar et al. 2001), resulting in the divergence of near-identical systems on long timescales. However, with the aid of computers the equations of motion governing planetary systems can be brute-force integrated into the future or past using numerical simulation, allowing scientists to answer fundamental questions that have plagued humans for hundreds of years. For example, it is now known that the Solar

System is marginally stable (Sussman & Wisdom 1988; Laskar 1994; Lecar et al. 2001), with Mercury having a 1% chance of colliding with Venus or the sun within a couple billion years (Laskar & Gastineau 2009). It is also now well established that most known multi-planet systems are packed to capacity, and adding additional planets into these systems would result in dynamical instabilities (Fang & Margot 2013; Pu & Wu 2015).

Although most planetary systems cannot be analytically solved, constraints on these systems can still be derived using analytical means. For example, Wisdom (1980) and Duncan et al. (1989) showed that for small eccentricities in the Restricted 3-Body Problem (R3BP), chaotic orbits (leading to close encounters, collisions and ejections) will occur when the perturber and particle are separated by $\Delta a \leq 1.3\mu_p^{2/7} a_p$ (where subscript p indicates the perturber). Also associated with the R3BP is the Jacobi constant, which can be used to constrain the chaotic motion of the particle. For two massive planets, Gladman (1993) showed that orbits are Hill stable if $\Delta a \geq 3.46 R_H$ (where R_H is the mutual Hill radius), forbidding close encounters for all time.

Since the discovery of numerous exoplanetary systems via *Kepler*, longterm stability has become a popular way to constrain orbital parameters (Lissauer et al. 2011; Steffen et al. 2013; Jontof-Hutter et al. 2014; Tamayo et al. 2015). If one assumes that an observed system is stable over billions of years, grids of N-body integrations can be used to find stable regions of parameter space, further narrowing the range of parameter space constrained by observations. Although this brute-force method is certainly useful it is not without its costs. A 10 billion year integration or the Solar System can take weeks to complete, and due to the chaotic nature of planetary systems hundreds to thousands of realizations must be simulated to acquire statistically rigorous results.

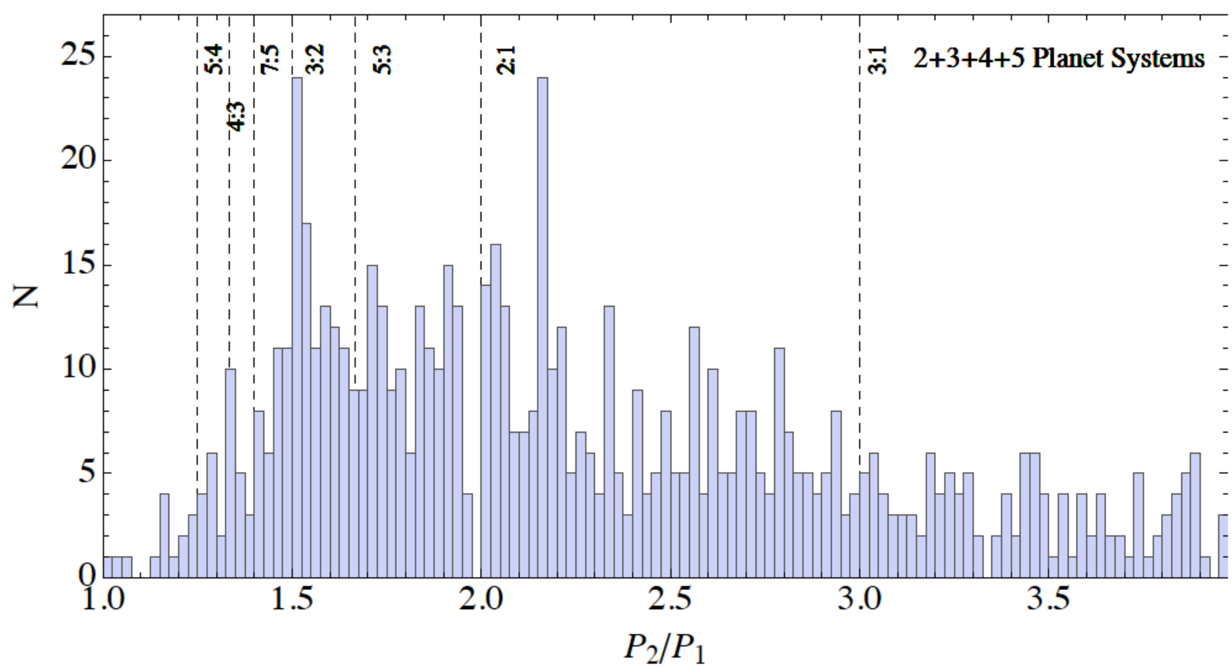

Fig. 1.— Period ratios of Kepler planets, image from Goldreich & Schlichting (2014).

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