Laplace's method

Suppose $f(x)$ is a twice continuously differentiable function on $[a, b]$, and there exists a unique point $x_0 \in (a, b)$ such that:

$$f(x_0) = \max_{x \in [a,b]} f(x) \quad \text{and} \quad f''(x_0) < 0.$$

Then:

$$\lim_{n \to \infty} \frac{\int_a^b e^{nf(x)}\, dx}{e^{nf(x_0)} \sqrt{\frac{2\pi}{n(-f''(x_0))}}} = 1. \tag{1}$$

Euler Product Formula

Let's take $s \in \mathbb{C}$. The Euler Product Formula, when $\Re(s) > 1$, is given by:

$$\prod_{p \in \mathbb{P}} \left(\frac{1}{1 - p^{-s}} \right) = \prod_{p \in \mathbb{P}} \left(\sum_{k=0}^{\infty} \frac{1}{p^{ks}} \right) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \zeta(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} \frac{x^{s-1}}{e^x - 1}\, \mathrm{d}x \tag{2}$$

Where:

$$\Gamma(s) = \int_0^{\infty} x^{s-1}\, e^{-x}\, \mathrm{d}x$$

Stirling's formula

It is also called Stirling's approximation for factorials:

$$\lim_{n \to +\infty} \frac{n\,!}{\sqrt{2\pi n}\, (n\, /\, e)^n} = 1 \tag{3}$$

Also frequently written as:

$$n\,! \sim \sqrt{2\pi n} \left(\frac{n}{e} \right)^n$$

One can easily derive the following limit from Stirling's formula:

$$\lim_{n \to \infty} \frac{(n!)^{1/n}}{n} = \frac{1}{e}$$