17~18

$\varphi_k = N \times N \times N \times \cdots N \mapsto N$, and $k \in N, k \geq 2$

$\varphi_k(n_1, n_2, n_3, n_4 \cdots n_k) = \max_k\{n_1, n_2 \cdots n_k\}$ 是原始递归的.

$$\varphi_k(n_1, n_2 \cdots n_k) = \begin{cases} \max_2\{n_1, n_2\} & , k=2 \\ \max_2\{\max_{k-1}\{n_1, n_2 \cdots n_{k-1}\}, n_k\} & , k \geq 3 \end{cases}$$

$\max_2\{n_1, n_2\} = n_1 \cdot (n_1 \geq n_2) + n_2 \cdot (1 \sim (n_1 \geq n_2))$ and $n_1 \geq n_2$ 是原始递归函数，并且原函数被递归定义，则证毕.

11~12

$$\varphi(x, y) = \begin{cases} (x+1)^y & , x, y \text{ are composite number} \\ 0 & , \text{otherwise} \end{cases}$$

$\varphi(x, y) = (1 - prime(x))(1 - prime(y))exp(x+1, y)$

$prime(x) = (1 - equals(x, 2))\prod_{i=2}^{x-1}(rem(x, i) \neq 0) + equals(x, 2)$

$exp(x, y) = (1 \sim equals(y, 0))(multi(exp(x, y-1), x)) + equals(y, 0)$

故原函数是原始递归的.

08~09

$g(x, y)$ 是原始递归函数

$$e(x, y) = \begin{cases} 1, & \text{if } \exists t_{(0 \leq t < y)}(g(x, t) = 0) \\ 0, & \text{otherwise} \end{cases}$$

$e(x, y) = \sum_{t=0}^{y-1}(g(x, t) = 0)$

$g(x, y)$ 原始递归, is zero 判定也是，析取也是，故证毕

06~07

$$f(x) = \begin{cases} x+1 & , x \text{ is odd} \\ 4x & , x \text{ is even} \end{cases}$$

$f(x) = (rem(x, 2))(x+1) + (1 \sim rem(x, 2)) \cdot 4x$

04~05

$P(x,y)$ 原始递归，证明 $\forall y \le u \, P(x,y)$

$\forall y \le u \, P(x,y) \Longleftrightarrow \prod_{y=0}^{u} P(x,y) \neq 0$

类比：$\exists_{t (0 \le t < y)} (g(x,y) \neq 0) \Longleftrightarrow \sum_{t=0}^{y-1} (g(x,y) \neq 0)$

$fac(n) = n!$:

$(1 - equals(n,0))\, fac(n-1) \cdot n + equals(n,0)$

$$gcd(m,n) = \begin{cases} n, & \text{if } m\%n = 0 \\ gcd(n, m\%n), & \text{if } m\%n \neq 0 \end{cases}$$

$gcd = (1 \sim rem(m,n))\, n + rem(m,n)\, gcd(n, m\%n)$

$prime(n) = (1 - equals(n,2)) \prod_{i=2}^{n-1} (rem(n,i) \neq 0) + equals(n,2)$

总结：类似于 mua 编程，需要细心。

$$\varphi(x) = \begin{cases} |x-y|, & (x \equiv y) \bmod 3 \\ x+y, & \text{otherwise} \end{cases}$$

$mod3(x,y) = ((x-y) \ge 0)(1 \sim rem(x-y, 3)) + ((x-y) < 0)(1 \sim rem(y-x, 3))$

$\varphi(x) = mod3(x,y)\big(((x-y) \ge 0)(x-y) + ((x-y) < 0)(y-x)\big) + (1 \sim mod3(x,y))(x+y))$