浙江大学 2010-2011 学年 秋冬 学期

《计算理论》课程期末考试试卷

课程号：21120520　开课学院：计算机学院

考试试卷：A卷

考试形式：闭卷，允许带 ________ 入场

考试日期: 2011 年 1 月 13 日，考试时间：120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名________________ 学号________________ 所属院系________________

题序	1	2	3	4	5	6	总分
得分							
评卷人							

Zhejiang University
Theory of Computation, Fall-Winter 2010
Final Exam

1. **(24%) Determine whether the following statements are true or false. If it is true fill a T otherwise a F in the bracket before the statement.**

   (a) (　) Let L be a language, then $(L^+)^+ = L^+ \circ L^+$, where $L^+ = L \circ L^*$.

   (b) (　) Let L be a regular language, so is $\{w | w \in L$ and w with even length$\}$.

   (c) (　) For languages L_1, L_2 and L_3, if $L_1 \subseteq L_2 \subseteq L_3$ and both L_1 and L_3 are regular, then L_2 is also regular.

   (d) (　) Just as Turing Machine's encoding, DFAs M can also be encoded as strings "M", then the language $\{$"M"$|$ DFA M rejects "M"$\}$ is not regular but recursive.

   (e) (　) Language $\{a^m b^n c a^{2n} b^{2m} | m, n \in \mathbb{N}\}$ is context-free.

   (f) (　) Let L be a context-free language, so is $\{w | w \in L$ and $|w| = 3k$ for some $k \in \mathbb{N}\}$.

   (g) (　) Every recursive function is primitive recursive.

   (h) (　) Language $\{$"M" : Turing machine M accepts at least 2011 distinct strings$\}$ is recursively enumerable.

   (i) (　) A language is recursive if and only if it is Turing-enumerable.

   (j) (　) Let L be a language, if there is a Turing machine M halts on x for every $x \in L$, then L is recursive.

   (k) (　) To simulate a computation of n steps for the nondeterministic Turing machine, it requires exponentially many steps in n for a deterministic Turing machine.

   (l) (　) If there is reduction τ from language A to $\{$"M"$|$ Turing machine M halts on empty string$\}$, where τ is a recursive function, then A is undecidable.

2. **(16%)** Decide whether the following languages are regular or not and provide a formal proof for your answer.

   (a) $L_1 = \{xcycz | x, y, z \in \{a, b\}^* \text{ and } y = y^R\}$.

   (b) $L_2 = \{xyz | x, y, z \in \{a, b\}^* \text{ and } y = y^R\}$.

3. **(20%) On Context-free Languages**

   (a) Construct a context-free grammar that generates the language

   $$L_3 = \{xcycz | x, y, z \in \{a, b\}^* \text{ and } |x| = |z|\}.$$

   (b) Construct a pushdown automata that accepts L_3.

   Solution:

   (a)

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

	(q, σ, β)	(p, γ)
$K = \{\underline{\qquad\qquad}\}$		
$\Sigma = \{a, b, c\}$		
$\Gamma = \{\underline{\qquad\qquad}\}$		
$s = \underline{\qquad\qquad}$		
$F = \{\underline{\qquad\qquad}\}$		

4. **(14%) On Turing machines**

Design a single tape Turing machine M that decides the language L_4 on $\{a, b, c\}$:

$$L_4 = \{xcycz | x, y, z \in \{a, b\}^* \text{ and } z = z^R\}.$$

When describing the Turing machines above, you can use the elementary Turing machines described in textbook. Always assume that your Turing machine starts from the configuration $\rhd\underline{\sqcup}w$.

5. **(12%) On Primitive Recursive Functions**

   Show that the following function:

$$\varphi(x, y) = \begin{cases} |x - y|, & \text{if } x \equiv y \pmod{3} \\ x + y, & \text{otherwise.} \end{cases}$$

   is primitive recursive.

6. **(14%) On Undecidability**

   Let $ODD_{TM} = \{\text{“}M\text{”} | M \text{ is a TM that does not accept any string of odd length}\}$.

   (a) Show that $\overline{ODD_{TM}}$ is recursively enumerable.

   (b) Show that ODD_{TM} is undecidable using a reduction from Halting Problem :

$$H = \{\text{“}M\text{” “}w\text{”} | \text{ Turing machine } M \text{ halts on input string } w\}.$$

Enjoy your Spring Festival!