

2004

(a) 给出 $L_3 = \{xy \mid x, y \in \{a, b\}^*, |x| = |y| \text{ 且 } x \text{ 与 } y^R \text{ 只有 1 位不同}\}$ 的上下文无关文法.

$V = \{a, b, S, A, B\}, \Sigma = \{a, b\}$

$R = \{S \rightarrow aSa, S \rightarrow bSb, S \rightarrow aAb, S \rightarrow bBa,$

$A \rightarrow aAa, A \rightarrow bAb, A \rightarrow \epsilon,$

$B \rightarrow aBa, B \rightarrow bBb, B \rightarrow \epsilon\}$

(b) 为 L_3 设计 1T PDA.

$M = \{K, \Sigma, \Gamma, \Delta, s, F\}$

$K = \{p, q\},$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, S, A, B\}$

$s = p$

$F = \{q\}$

(q, δ, β)	(p, γ)
(p, ϵ, ϵ)	(q, S)
(q, ϵ, S)	(q, aSa)
(q, ϵ, S)	(q, bSb)
(q, ϵ, S)	(q, aAb)
(q, ϵ, S)	(q, bBa)
(q, ϵ, A)	(q, aAa)
(q, ϵ, A)	(q, bAb)
(q, ϵ, A)	(q, ϵ)
(q, ϵ, B)	(q, bBb)
(q, ϵ, B)	(q, aBb)
(q, ϵ, B)	(q, ϵ)
(q, a, a)	(q, ϵ)
(q, b, b)	(q, ϵ)

2006

给出下推自动机 $M = \{K, \Sigma, \Gamma, \Delta, s, F\}, K = \{s, f\}, \Sigma = \{a, b\}, \Gamma = \{b\}, F = \{f\}$

$\Delta: (s, a, \epsilon), (f, \epsilon)$

$(s, b, \epsilon), (s, b)$

$(s, a, b), (s, b)$

$(s, \epsilon, \epsilon), (f, \epsilon)$

$(f, a, \epsilon), (f, \epsilon)$

$(f, b, \epsilon), (s, b)$

(a) M 可以接受 $aaaaababab$ 吗?

(b) 描述 M 接受的语言.

(c) 给出描述此语言的 TM.

(a) 不接受 $(s, aaaaababa, e) \vdash (f, baba, e) \vdash (s, aba, b) \vdash (s, ba, b) \vdash (s, a, bb) \vdash (s, e, bb) \vdash (t, e, bb)$

(b) M 接受 $\{a\}^*$

(c) $\rightarrow R \xrightarrow{a} n$
 $\downarrow U$
 y

2007

(a) 写文法: $L_3 = \{a^m b^m c a^{2n} b^{2n} \mid m, n \in \mathbb{N}\}$

(b) 设计 PDA

(a) $S \rightarrow UcV \quad V \rightarrow aaVbb$

$U \rightarrow aUb \quad V \rightarrow e$

$U \rightarrow e$

(b) $K = \{p, q\} \quad \Gamma = \{a, b, c, s, u, v\} \quad \Sigma = \{a, b, c\} \quad F = \{q\}$

$\delta: (p, e, e), (q, UcV)$

$(q, e, u), (q, aUb)$

$(q, e, u), (q, e)$

$(q, e, v), (q, aaVbb)$

$(q, e, v), (q, e)$

$(q, b, b), (q, e)$

$(q, a, a), (q, e)$

$(q, c, c), (q, e)$

2008

$L = \{a^m b^n c w w^k \mid m, n \in \mathbb{N}, n \leq m \leq 2n, w \in \{a, b\}^*\}$

(a) $V = \{a, b, c, s_1, s_2\} \quad \Sigma = \{a, b, c\}$

$R = \{s \rightarrow s_1 c s_2, s_1 \rightarrow a s_1 b, s_1 \rightarrow a a s_1 b, s_1 \rightarrow e,$

$s_2 \rightarrow a s_2 a, s_2 \rightarrow b s_2 b, s_2 \rightarrow e\}$

(b) 同理

2009

$$L = \{a^m b^n c^k \mid m, n, k \in \mathbb{N}, m+n \leq k\}$$

$$(a) S \rightarrow aSc, S \rightarrow T,$$

$$T \rightarrow bTc, T \rightarrow Tc, T \rightarrow e$$

(b) 同构

2010

$$L = \{a^m b^n \mid m, n \in \mathbb{N}, m \neq n\}$$

$$S \rightarrow aSb, S \rightarrow aS_1, S \rightarrow S_2b,$$

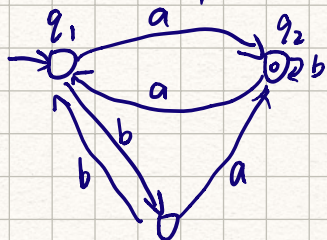
$$S_1 \rightarrow aS_1, S_1 \rightarrow e, S_2 \rightarrow S_2b, S_2 \rightarrow e$$

2011

$$L_3 = \{xy^k, x, y \in \{a, b\}^*, |x|=|y| \text{ 且 } x, y \text{ 只存在第 } i \text{ 个字符不同}\}$$

$$S \rightarrow aTb, S \rightarrow bTa, T \rightarrow aTa, T \rightarrow bTb, T \rightarrow e.$$

2006~2007



(a) 此FA接受有奇数个T的字符串 $(aa \cup bb)^*(a \cup ba)b^*((aa)^* \cup (ab^*a)^*)$

(b) $b^*a(b^*ab^*ab^*)^*b^*$

$$S \rightarrow bS, S \rightarrow Sb, S \rightarrow aT$$

$$T \rightarrow bT, T \rightarrow Tb, T \rightarrow aTa, T \rightarrow e$$

2010~2011

$$L = \{xcycz \mid x, y, z \in \{a, b\}^*, |x|=|z|\}$$

$S \rightarrow aSa, S \rightarrow aSb, S \rightarrow bSa, S \rightarrow bSb, S \rightarrow cTc,$

$T \rightarrow aT, T \rightarrow bT, T \rightarrow e$

2012~2013

$L = \{w \mid w \in \{a,b,c\}^*, \#_b(w) = \#_c(w)\}$

$S \rightarrow aS, S \rightarrow Sa, S \rightarrow bSc, S \rightarrow cSb, S \rightarrow e \quad R = \{S \rightarrow bSc \mid cSb \mid SS \mid AS \mid e, A \rightarrow aA \mid e\}$

2013~2014

$L = \{ab^m c^n a^{m+2n} c \mid m, n \in \mathbb{N}\}$

$S \rightarrow aS_1c, S_1 \rightarrow bS_1a, S_1 \rightarrow S_2, S_2 \rightarrow cS_2aa, S_2 \rightarrow e$

2014~2015

$L = \{xcy \mid x, y \in \{a,b\}^*, |x| = |y|, x \neq y^R\}$

$S \rightarrow aSa \mid bSb \mid aSb \mid bSa \mid S_1$

$S_1 \rightarrow aS_2b \mid bS_2a$

$S_2 \rightarrow aS_2a \mid bS_2b \mid e.$

总结:

① CFG: $G = \{V, \Sigma, S, R\}$

V 是字母表; Σ 是终结符集合; $S \in V - \Sigma$ 是起始符; R 是规则集合.

注意可以冗余但是不能少写

② PDA: $M = (K, \Sigma, \Gamma, \Delta, s, F)$

K 是有穷的状态集合; Σ 是输入字符的字母表; Γ 是所有栈符号的字母表;

Δ 是转移函数; $s \in K$ 是初始状态; $F \subseteq K$ 是终结状态集合.

Δ 直接全由CFG的 R 改写即可, 注意第一个关系是 $((p, e, e), (q, s))$