	Undergraduate Course
ELEMENTS OF COMPUTATION THEORY	College of Computer Science
Chapter 4	ZHEJIANG UNIVERSITY
	Fall-Winter, 2014

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4.1.2 Let $M = (K, \sum, \delta, s, \{h\})$, where $K = \{q_0, q_1, q_2, h\}$, $\sum = \{a, b, \sqcup, \triangleright\}$, $s = q_0$, and δ is given by the following table (the transitions on $\triangleright$ are $\delta(q, \triangleright) = (q, \triangleright)$, and are omitted).

q	σ	$\delta(q, \sigma)$
q_0	a	$(q_1, \leftarrow)$
q_0	b	$(q_0, \rightarrow)$
q_0	$\sqcup$	$(q_0, \rightarrow)$
q_1	a	$(q_1, \leftarrow)$
q_1	b	$(q_2, \rightarrow)$
q_1	$\sqcup$	$(q_1, \leftarrow)$
q_2	a	$(q_2, \rightarrow)$
q_2	b	$(q_2, \rightarrow)$
q_2	$\sqcup$	$(h, \sqcup)$

(a) Trace the computation of M starting from the configuration $(q_0, \triangleright a\underline{b}b \sqcup bb \sqcup \sqcup \sqcup aba)$.
(b) Describe informally what M does when started in q_0 on any square of a tape.

Solution:

(a)

$$\begin{aligned}
(q_0, \triangleright a\underline{b}b \sqcup bb \sqcup \sqcup \sqcup aba) &\vdash_M (q_0, \triangleright ab\underline{b} \sqcup bb \sqcup \sqcup \sqcup aba) \\
&\vdash_M (q_0, \triangleright abb\underline{\sqcup}bb \sqcup \sqcup \sqcup aba) \\
&\vdash_M (q_0, \triangleright abb \sqcup \underline{b}b \sqcup \sqcup \sqcup aba) \\
&\vdash_M (q_0, \triangleright abb \sqcup b\underline{b} \sqcup \sqcup \sqcup aba) \\
&\vdash_M (q_0, \triangleright abb \sqcup bb\underline{\sqcup} \sqcup \sqcup aba) \\
&\vdash_M (q_0, \triangleright abb \sqcup bb \sqcup \underline{\sqcup} \sqcup aba) \\
&\vdash_M (q_0, \triangleright abb \sqcup bb \sqcup \sqcup\underline{\sqcup} aba) \\
&\vdash_M (q_0, \triangleright abb \sqcup bb \sqcup \sqcup \sqcup \underline{a}ba) \\
&\vdash_M (q_1, \triangleright abb \sqcup bb \sqcup \sqcup\underline{\sqcup} aba) \\
&\vdash_M (q_1, \triangleright abb \sqcup bb \sqcup \underline{\sqcup} \sqcup aba) \\
&\vdash_M (q_1, \triangleright abb \sqcup bb\underline{\sqcup} \sqcup \sqcup aba) \\
&\vdash_M (q_1, \triangleright abb \sqcup b\underline{b} \sqcup \sqcup \sqcup aba) \\
&\vdash_M (q_2, \triangleright abb \sqcup bb\underline{\sqcup} \sqcup \sqcup aba) \\
&\vdash_M (h, \triangleright abb \sqcup bb\underline{\sqcup} \sqcup \sqcup aba)
\end{aligned}$$

(b) M scans right until it finds an a, then left until it finds a b, then right again until it finds a $\sqcup$, and then halts.

4.1.7 Design and write out in full a Turing machine that scans to the right until it finds two consecutive a's and then halts. The alphabet of the Turing Machine should be $\{a, b, \sqcup, \triangleright\}$.

Solution:
$K = \{q_0, q_1, h\}$, $\sum = \{a, b, \sqcup, \triangleright\}$, $s = q_0$, $H = \{h\}$, δ is given by the following table:.

q	σ	$\delta(q, \sigma)$
q_0	a	$(q_1, \rightarrow)$
q_0	b	$(q_0, \rightarrow)$
q_0	$\sqcup$	$(q_0, \rightarrow)$
q_0	$\triangleright$	$(q_0, \rightarrow)$
q_1	a	(h, a)
q_1	b	$(q_0, \rightarrow)$
q_1	$\sqcup$	$(q_0, \rightarrow)$
q_1	$\triangleright$	$(q_0, \rightarrow)$

4.1.10 Explain what this machine does.

$$> R \xrightarrow[a \neq \sqcup]{} R \xrightarrow[b \neq \sqcup]{} R_\sqcup a R_\sqcup b$$

Solution:
This machine scans to right, remembering the first and the second nonblank symbols (respectively a and b) it encounters. It then continues to the right, writing a in the first blank it encounters, and b in the second.

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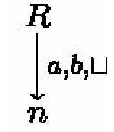
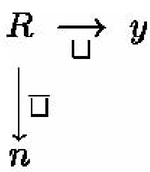
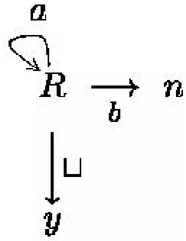
4.2.2 Present Turing machines that decide the following languages over $\{a, b\}$.
(a) $\emptyset$
(b) $\{e\}$
(d) $\{a\}^*$
Solution:



(a)



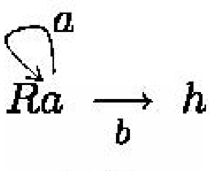
(b)



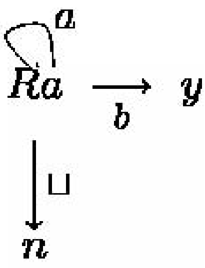
(d)

4.2.4 (a) **Given an example of a TM with one halting state that does not compute a function from strings to strings.**
(b) **Given an example of a TM with two halting states, y and n, that does not decide a language.**
(c) **Can you given an example of a TM with one halting state that does not semidecide a language**

Solution:



(a)



(b)

(c) **On each input, a TM either halts or does not. The language semidecided by TM M is simply the set of input strings on which M halts. No Turing can fail to semidecide some language.**

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4.6.2 Find grammars that generate the following languages:
(a) $\{ww : w \in \{a,b\}^*\}$ (b) $\{a^{2^n} : n \geq 0\}$ (c) $\{a^{n^2} : n \geq 0\}$

Solution:

(a)

$$
\begin{aligned}
&V = \{a, b, A, B, S, T, U, [,], \$, x\}\\
&\Sigma = \{a, b\}\\
&R = \{S \to [T]\\
&\qquad T \to xTx\\
&\qquad T \to \$U\\
&\qquad Ux \to AaU\\
&\qquad Ux \to BbU\\
&\qquad xA \to Ax\\
&\qquad xB \to Bx\\
&\qquad [A \to a[\\
&\qquad [B \to b[\\
&\qquad [\$ \to e\\
&\qquad U] \to e\}
\end{aligned}
$$

$$
\begin{array}{ll}
(b) & V = \{a, S, M, \$\} \\
& \Sigma = \{a\} \\
& R = \{S \to Ta\$ \\
& \quad T \to TM \\
& \quad T \to e \\
& \quad Ma \to aaM \\
& \quad M\$ \to \$ \\
& \quad \$ \to e\} \\
(c) & V = \{a, S, T, B, C, \$\} \\
& \Sigma = \{a\} \\
& R = \{S \to \$T\$ \\
& \quad T \to BTC \\
& \quad T \to e \\
& \quad BC \to CaB \\
& \quad \$C\$ \to \$ \\
& \quad B\$ \to \$ \\
& \quad \$ \to e\}
\end{array}
$$

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4.7.2 Show the following functions are primitive recursive:

(a) $factoria(n) = n!$

(b) $gcd(m, n)$**, the greast comon divisor of** m **and** n

Solution:

(a) $factoria(n)$ **is the function defined recursively by** $g(0) = 1$ **and** $h(m, r) = (m + 1) \cdot r$

(b) $gcd(m, n) = \begin{cases} n, & rem(m, n) = 0, \\ gcd(n, rem(m, n)), & \textbf{otherwise} \end{cases}$