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3.1.3 (c)

$\{w \in \{a,b\}^* : w = w^R\}$

$G = (V, \Sigma, R, S)$ 其中

$V = \{a, b, S\}$

$\Sigma = \{a, b\}$

$R = \{S \to aSa,$

$S \to bSb,$

$S \to a,$

$S \to b$

$S \to e\}.$

3.1.9 (a) 令 $V = \{a, b, S\}$，$\Sigma = \{a, b\}$. $R = \{S \to aSb, S \to aS, S \to e\}$.

(b) 设 $m+n = p+q = k$ 则 $n = k-m$, $q = k-p$.

$a^m b^n c^p d^q = a^m b^{k-m} c^{k-q} d^q$

不失一般性 设 $m > q$ 则 $a^m b^n c^p d^q = a^q a^{m-q} b^{k-m} c^{k-m} c^{m-q} d^q$.

然后可以定义 $V = \{a, c, b, d, S, A, B\}$. $\Sigma = \{a, b, c, d\}$.

$R = \{S \to aSd, S \to A, A \to aAc, A \to B, B \to bBc, B \to e$

3.3.2 (c) $M = (K, \Sigma, \Gamma, \Delta, s, F)$. 其中. $K = \{q, r\}$, $\Sigma = \{a, b\}$. $\Gamma = \{a, b\}$.

$\Delta = \{((q,a,e),(q,a)), ((q,b,e),(q,b)), ((q,e,e),(r,e)),$

$((q,a,e),(r,e)), ((q,b,e),(r,e)),$

$((q,a,a),(r,e)), ((q,b,b),(r,e))\}.$

(d) $M = (K, \Sigma, T, \Delta, s, F)$ $K = \{q\}$, $\Sigma = \{a, b\}$, $T = \{A, a, b\}$, $F = \{q$

$\Delta = \{((q, a, e), (q, A)), ((q, b, e), (q, b)),$

$((q, a, b), (q, a)), ((q, b, A), (q, a)),$

$((q, b, a), (q, e))\}.$

3.4.1. $M = (\{p, q\}, \{(,)\}, \{(,), S\}, \Delta, p, \{q\})$.

$\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, SS)), ((q, e, S), (q, (S))),$

$((q, e, S), (q, e)), ((q, (, (), (q, e)), ((q,),)), (q, e))\}.$

会是 $(p, (())(), e) \vdash_M (p, (())(), S)$

$\vdash_M (p, (())(), (S))$

$\vdash_M (p, ())(), S))$

$\vdash_M (p, ()(), S)S))$

$\vdash_M (p,)(), S))$

$\vdash_M (p, (), (S)))$

$\vdash_M (p,)), S)))$

$\vdash_M (p,)),)))$

$\vdash_M (p,))$

$\vdash_M (p, e, e))$

3.5.1 (b) $\{a,b\}^* - \{a^n b^n : n \geq 0\}$.

即 $\{a^m b^n | m \neq n\} \cup \Sigma^* a \Sigma^* b \Sigma^* a \cup \Sigma^* b \Sigma^* a \Sigma^* b$

各国均为上下文无关，故原语言也上下文无关

(c) $\{a^m b^n c^p d^q : n = q, or\ m \leq p\ or\ m+n = p+q\}$.

即 $\{a^m b^n c^p d^q : n = q\} \cup \{a^m b^n c^p d^q : m \leq p\} \cup \{a^m b^n c^p d^q : m+n = p+q\}$

设为 L_1, L_2, L_3

其中 $L_1 = \{a^* b^n c^* d^n : n \geq 0\}$ 为 CF

$L_2 = \{a^m b^* c^p d^* : m \leq p\}$ 也为 CF

$L_3 = \{a^m b^n c^p d^q : m+n = p+q\}$ CF 故 $L_1 \cup L_2 \cup L_3$ 为 CF.

(d) $\{a,b\}^* - L$, $L = \{babaabaaab \ldots ba^{n-1}ba^n b : n \geq 1\}$.

即 $\Sigma^* bb \Sigma^* \cup \Sigma^* \{ba^m ba^n : m - n \neq 1\} \Sigma^* \cup a\Sigma^* \cup \Sigma^* a$.

每一项均为 CF 故原语言为 CF.

3.5.2 (a) $\{www : w \in \{a,b\}^*\}$.

设若 L 为 CF. 由泵引理 存在 $k > 0$ 使得 $\forall w \in L$, $|w| \geq k$.

存在 $u, v, x, y, z \in \Sigma^*$. $w = uvxyz$ 其中 $|vxy| \leq k$, $vy \neq e$ 且

~~$uv^n xy^n$~~ $uv^n xy^n z \in L$ $(n \geq 0)$

对于 $w = a^k b a^k b a^k b \in L$ 且 $|w| \geq k$. 由上述假设 v 或 y 均不能含 b

~~否~~ 否则 uv^2xy^2z 将含多于 3 个 b 而不属于 L. 故 $v, y \in$ 且 $\{a^*\}$. 今 $v = a^m$, $y = a^n$

则 $uv^2xy^2z = a^{k+m} b a^{k+n} b a^k b$. 由 $vy \neq e$ 故 uv^2xy^2z 不再属于 L.

综上，L 不满足泵引理 即 L 不是上下文无关的

3.5.14 (a) Yes.

$\{a^m b^n c^p \mid m=n \text{ or } n=p \text{ or } m=p\} = \{a^m b^m c^p \mid m,p\in N\} \cup \{a^m b^n c^n \mid m,n\in N\} \cup$

$\{a^m b^n c^m \mid m,n\in N\}$. 均为CF 故并也为CF.

(b) Yes

因为$\{a^m b^n \mid m\neq n\}$是上下文无关的

进而$\{a^m b^n c^p \mid m\neq n\}$、$\{a^m b^n c^p \mid m\neq p\}$、$\{a^m b^n c^p \mid n\neq p\}$均为CF.

故$\{a^m b^n c^p \mid m\neq n\} \cup \{a^m b^n c^p \mid m\neq p\} \cup \{a^m b^n c^p \mid n\neq p\}$即原L为CF.

(c) No

$\{a^m b^m c^m \mid m\in N\}$不是CF. 可由泵引理推知

(d) Yes.

$L = \{w\in\{a,b,c\}^* : w$ 不包含同样数目a, b, c$\}$.

即$L = \{w\in\{a,b,c\}^* : w$包含不同数目a和b$\} \cup \{w\in\{a,b,c\}^* : w$含不同b, c$\} \cup$

$\{w\in\{a,b,c\}^* : w$含不同a, c$\}$ 3个L均为CF 故L也为CF.

3.5.15

$L-R = L\cap\bar{R}$, R为正则 那么$\bar{R}$也为正则, $\bar{R}$为正则 必然也CF

于是$L\cap\bar{R}$ 即$L-R$也是CF.

$R-L$ 不一定为CF. 令$R=\Sigma^*$ 则$R-L=\bar{L}$, $\bar{L}$不一定为CF.