Assessing Group Dominance: A Novel Method for Ranking and Analysis

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Abstract

This paper introduces a novel method for calculating group-level dominance scores, incorporating rank sums, weight bias adjustments for unequal group sizes, and the concept of the Upper Dominant Half. This method offers a fair and scalable approach for comparing group performance, addressing limitations in existing dominance ranking methods. Empirical validation and theoretical implications are discussed.

Keywords: Normalized Rank Comparison, Dominance Analysis, Weighted Bias Adjustment, Group Performance Metrics, Statistical Methods

Assessing Group Dominance: A Novel Method for Ranking and Analysis

- Overview of dominance ranking methods in psychology and related fields.
- Limitations of existing methods (e.g., David's Score, Elo-Rating).
- Purpose of this research: Introducing a group-level dominance calculation method.

Method

Key Components

- **Ranks**: Sequential ranks assigned to items within and across groups. Tied ranks are averaged out.
- **Items**: There are

$$k = \text{number of groups}, \quad n_i = \text{number of items in each group}$$

$$\mathrm{N} = \sum_{i}^{k} n_i, \quad \text{where N = total number of items} \tag{1}$$

- **Upper Dominant Half**:

$$S_{UDH} = \frac{N(N+1)}{2} - \frac{a(a+1)}{2}, \quad a = \lfloor N/2 \rfloor \tag{2}$$

- **Weight Bias** (w_i):

$$w_i = \frac{N}{kn_i} \tag{3}$$

Dominance Score Formula

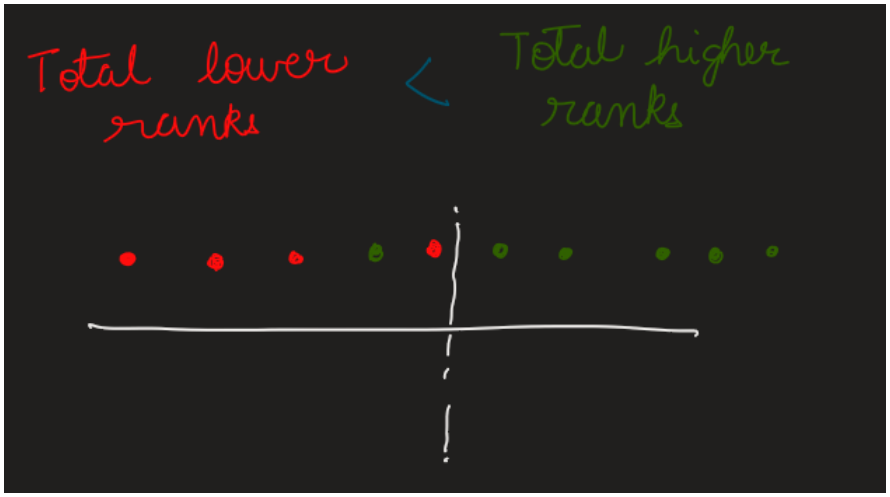
$$\boxed{\mathbf{U_i = w_i \cdot \frac{\sum R_i}{S_{UDH}}}} \tag{4}$$

- $\sum R_i$: Sum of ranks for group i.
- S_{UDH}: Sum of the upper half acting as a benchmark
- w_i: Adjustment for group size bias.
- U_i: Normalized Dominance Measured for that particular group.

Visual Understanding

Figure 1

Upper Dominance Intuition



Derivation

Derivation of UDH (Upper Dominant Half)

1. Purpose of UDH

The **Upper Dominant Half (UDH)** represents the **rank sum of the dominant half** of a dataset, providing a benchmark for dominance potential in rank-based comparisons.

2. Total Rank Sum (Full Dataset)

For a dataset with **N items**, the **total rank sum** is the sum of integers from **1 to N**:

$$S = 1 + 2 + 3 + \ldots + N \tag{5}$$

Using the formula for the sum of the first N integers:

$$S = \frac{N(N+1)}{2} \tag{6}$$

3. Split the Dataset into Halves

- $a = \lfloor \frac{N}{2} \rfloor$ represents the size of the **nonn-dominant half** (lower half).

- The **dominant half** includes the **highest-ranked elements**.

4. Rank Sum of Non-Dominant Half (Lower Half)

The **non-dominant half** (lower half) consists of the **smallest** a **ranks**. Its rank sum is:

$$S_{lower} = 1 + 2 + \ldots + a \tag{7}$$

Using the formula for sum of integers:

$$S_{lower} = \frac{a(a+1)}{2} \tag{8}$$

5. Rank Sum of Dominant Half (Upper Half)

The **Upper Dominant Half** is calculated by subtracting the rank sum of the **lower half** from the **total rank sum**:

$$S_{UDH} = S - S_{lower} \tag{9}$$

Substituting the formulas:

$$S_{UDH} = \frac{N(N+1)}{2} - \frac{a(a+1)}{2} \tag{10}$$

6. Final Formula

$$S_{UDH} = \frac{N(N+1)}{2} - \frac{a(a+1)}{2} \tag{11}$$

Terms

- N: Total number of items.
- a: Size of the lower **dominant half**, calculated as the floor of net total number of items across all groups:

$$a = \lfloor \frac{N}{2} \rfloor \tag{12}$$

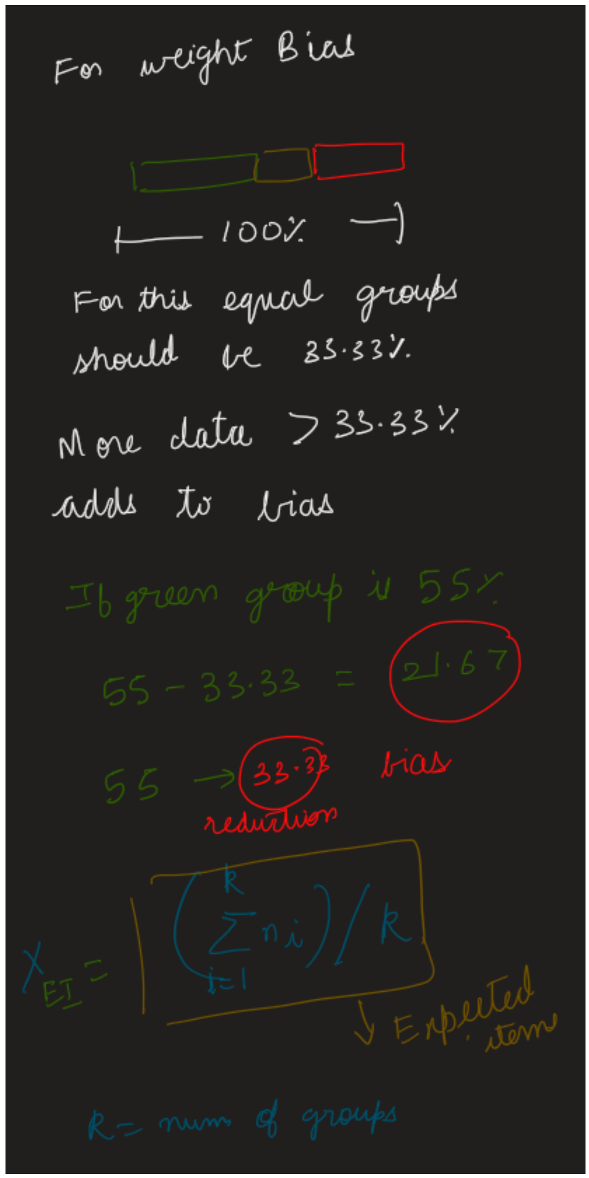
Key Note. This formula ensures a **normalized benchmark** for dominance evaluation, making it useful for **rank-based comparisons** in group analysis.

Weight Bias Derivation

Weight Bias Intuition. Add a figure to intuitively describe all the terms magnified into an image to represent what each means. The original weight bias formula is

Figure 2

Weight Bias Intuition



given as:

$$w_i = 1 - \frac{n_i - X_{EI}}{n_i}, \quad X_{EI} = \frac{N}{k} \tag{13}$$

where:

- w_i: Weight bias for group i.
- n_i: Number of items in group i.
- N: Total number of items.
- k: Number of groups.
- X_{EI}: Expected item count per group.

Simplification

Step 1: Substitute X_{EI} into the formula

$$w_i = 1 - \frac{n_i - \frac{N}{k}}{n_i} \tag{14}$$

Step 2: Simplify the terms inside the fraction

$$w_i = 1 - \left(1 - \frac{N}{kn_i}\right) \tag{15}$$

Step 3: Combine terms

$$w_i = \frac{N}{kn_i} \tag{16}$$

Final Formula

The simplified weight bias formula is:

$$w_i = \frac{N}{kn_i} \tag{17}$$

Interpretation

- N: Total number of items.
- k: Number of groups.
- n_i: Number of items in group i.

This formula directly adjusts for group size differences, ensuring fairness when comparing dominance scores.

Detailed Example of Dominance Method

To explicitly demonstrate the utility and robustness of the normalized dominance scoring method, we consider a complex example meeting the following conditions:

- Multiple groups (in example four groups)
- Unequal number of items per group (minimum three items per group)
- Tied ranks
- Odd total number of participants ($N = 15$)

Data and Initial Rankings

Participants from four intervention groups (A, B, C, D) were ranked based on their effectiveness scores. Ties were assigned average ranks explicitly in Table 1:

Calculations

Step 1: Rank Sums

- $R_A = 13.5 + 9 + 6 + 2.5 = 31$
- $R_B = 12 + 10 + 15 = 37$
- $R_C = 11 + 5 + 8 = 24$
- $R_D = 13.5 + 2.5 + 4 + 7 + 1 = 28$

Step 2: Define Parameters

Total participants: $N = 15$, Number of groups: $k = 4$.

Group sizes:

- $n_A = 4$, $n_B = 3$, $n_C = 3$, $n_D = 5$

Compute explicitly $a = \lfloor N/2 \rfloor = \lfloor 15/2 \rfloor = 7$.

Step 3: Calculate Upper Dominant Half

Table 1

Effectiveness Scores and Verified Assigned Ranks

Participant	Group	Score	Rank
1	A	95	13.5 (tie)
2	A	48	9
3	A	25	6
4	A	15	2.5 (tie)
5	B	83	12
6	B	57	10
7	B	100	15
8	C	70	11
9	C	20	5
10	C	40	8
11	D	95	13.5 (tie)
12	D	15	2.5 (tie)
13	D	18	4
14	D	30	7
15	D	10	1

Lower Dominant Half (LDH):

$$S_{lower} = \frac{a(a+1)}{2} = \frac{7 \times 8}{2} = 28 \tag{18}$$

Total sum of ranks:

$$S = \frac{N(N+1)}{2} = \frac{15 \times 16}{2} = 120 \tag{19}$$

Upper Dominant Half (UDH):

$$S_{UDH} = S - S_{lower} = 120 - 28 = 92 \tag{20}$$

Step 4: Weight Bias Calculation

$$w_i = \frac{N}{k \cdot n_i} \tag{21}$$

Calculate explicitly:

- $w_A = \frac{15}{4\times4} = 0.9375$
- $w_B = \frac{15}{4\times3} = 1.25$
- $w_C = \frac{15}{4\times3} = 1.25$
- $w_D = \frac{15}{4\times5} = 0.75$

Step 5: Dominance Scores

Dominance scores explicitly computed as:

$$U_i = w_i \cdot \frac{R_i}{S_{UDH}} \tag{22}$$

Calculate:

- $U_A = 0.9375 \times \frac{31}{92} = 0.316$
- $U_B = 1.25 \times \frac{37}{92} = 0.503$
- $U_C = 1.25 \times \frac{24}{92} = 0.326$
- $U_D = 0.75 \times \frac{28}{92} = 0.228$

Summary of Dominance Results

Results have been summarized in Table 2

Interpretation of Results

Group B achieves the highest dominance score, demonstrating superior effectiveness. Despite the variations in group size and tied ranks, the method accurately balances these complexities. Group D has the lowest dominance score, emphasizing the method's robustness and fairness in clearly assessing relative performance.

Table 2

Dominance Calculation Results

Group	Rank Sum (R_i)	Weight Bias (w_i)	Dominance Score (U_i)
A	31	0.9375	0.316
B	37	1.2500	0.503 (Most Dominant)
C	24	1.2500	0.326
D	28	0.7500	0.228 (Least Dominant)