MAGICAL SHORTCUT METHODS FOR COMPETITIVE EXAM

300. If $\vec{a}$ is vector perpendicular to both $\vec{b}$ or $\vec{c}$, then

(a) $\vec{a}.(\vec{b}\times\vec{c})=0$ (b) $\vec{a}\times(\vec{b}\times\vec{c})=\vec{0}$ (c) $\vec{a}\times(\vec{b}+\vec{c})=\vec{0}$ (d) $\vec{a}+(\vec{b}+\vec{c})=\vec{0}$

301. The value of $\begin{vmatrix} x+y & y+z & z+x \\ x & y & z \\ x-y & y-z & z-x \end{vmatrix}=$

(a) 0 (b) $(x+y+z)^3$ (c) $2(x+y+z)^3$ (d) $2(x+y+z)^2$

302. If x, y, z are all nonzero and $x^2+y^2+z^2=r^2$, then $\tan^{-1}\left(\frac{yz}{xr}\right)+\tan^{-1}\left(\frac{zx}{yr}\right)+\tan^{-1}\left(\frac{xy}{zr}\right)=$

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) none of these

303. If p and q are +ve real number such that $p^2+q^2=1$, then maximum value of $(p+q)$ is

(a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) ½

304. If $\vec{a}$ and $\vec{b}$ are vector Such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then the angle between the vector $\vec{a}$ and $\vec{b}$ is

(a) $90°$ (b) $30°$ (c) $120°$ (d) $60°$

305. $\sqrt{2+\sqrt{2+2\cos 4\theta}}=$

(a) $\cos\theta$ (b) $2\cos\theta$ (c) $\cos 2\theta$ (d) $2\cos 2\theta$

306. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}=$

(a) $1+a+b+c$ (b) $1+ab+bc+ca$ (c) $1+a^2+b^2+c^2$ (d) abc

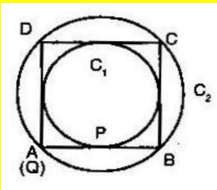
307. If n is non-negative integer and $A=\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ then $A^n=$

(a) $\begin{pmatrix} 1 & 0 \\ n-1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$

308. $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=$

(a) $\left(\frac{x^2+2}{x^2+3}\right)^{1/2}$ (b) $\left(\frac{x^2+3}{x^2+4}\right)^{1/2}$ (c) $\left(\frac{x^2+1}{x^2+2}\right)^{1/2}$ (d) x

309. Let ABCD be a square of side length 2 units, C_2 is a circle through the vertices A, B, C, D and C_1 is the circle touching all the sides of the square ABCD. If P is a point on C_1 and Q is another point on C_2 then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}=$



(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

310. If n is a +ve integer, then n^3+2n is divisible by

(a) 2 (b) 6 (c) 15 (d) 3

311. The value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y z & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

(a) 0 (b) 1 (c) xyz (d) log (xyz)

312. If $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, then a, b, c, are in

(a) G.P (b) H.P (c) equal (d) A.P

313. If $y = \sin^n x \cos nx$ then $\frac{dy}{dx}$ is

(a) n $\sin^{n-1} x \sin(n+1)x$ (b) n $\sin^{n-1} x \sin(n-1)x$

(c) n $\sin^{n-1} x \cos nx$ (d) n $\sin^{n-1} x \cos(n+1)x$

314. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + ... + x^n$, then $f''(1) =$

(a) $n(n-1)2^{n-1}$ (b) $(n-1)\,2^{n-1}$ (c) $(n-1)\,2^{n-2}$ (d) $(n-1)\,2^{n}$

315. $3(\sin x - \cos x)^2 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$

(a) 11 (b) 12 (c) 13 (d) 14

316. If one side of a triangle is double the other and the angles opposite to these sides differ by $60°$, then the triangle is

(a) obtuse angled (b) acute angled (c) isosceles (d) right angled

317. The length of the chord joining the point $(4\cos\theta, 4\sin\theta)$ and $(4\cos(\theta + 60°),\ 4\sin(\theta + 60°))$, is

(a) 4 (b) 8 (c) 16 (d) 2

318. All complex numbers which satisfy the equation $\left|\frac{z-6i}{z+6i}\right| = 1$ lies on

(a) the imaginary axis (b) the real axis (c) neither of the axes (d) none of these

319. The value of $\sin\left[\cot^{-1}\{\cos(\tan^{-1} x)\}\right]$ is

(a) $\sqrt{\frac{1+x^2}{2+x^2}}$ (b) $\sqrt{\frac{2+x^2}{1+x^2}}$ (c) $\sqrt{\frac{x^2-2}{x^2-1}}$ (d) $\sqrt{\frac{x^2-1}{x^2-2}}$

320. $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] =$

(a) $\sin^{-1} x - \sin^{-1}\sqrt{1-x}$ (b) $\sin^{-1} x + \sin^{-1}\sqrt{1-x}$

(b) $\sin^{-1} x - \sin^{-1}\sqrt{x}$ (d) $\sin^{-1} x + \sin^{-1}\sqrt{x}$

321. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is (where $n \in W$)

(a) 0 (b) 2 (c) 7 (d) 8

322. $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) =$

(a) $\sin\theta.\cos\theta$ (b) 1 (c) $\sec\theta + \operatorname{cosec}\theta$ (d) $\sec\theta - \operatorname{cosec}\theta$

323. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, then $A^n =$

(a) $\begin{pmatrix} 1 & 2^{n-2} \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & n^2 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & n^2 \\ 0 & 1 \end{pmatrix}$

324. If rth and (r +1) th terms in expansion of (p + q)^n are equal, then $\frac{(n+1)q}{r(p+q)}$ is.......

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) 0

325. If $\vec{a},\vec{b}$ and $\vec{c}$ are noncoplanar, then the value of

$\vec{a}.\left\{\frac{\vec{b}\times\vec{c}}{3\vec{b}.(\vec{c}\times\vec{a})}\right\}-\vec{b}\left\{\frac{\vec{c}\times\vec{a}}{3\vec{c}.(\vec{a}\times\vec{b})}\right\}$ is ....

(a) $\frac{1}{6}$ (b) $-\frac{1}{6}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{2}$

326. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=$

(a) $2\sin\theta$ (b) $2\cos\frac{\theta}{2}$ (c) $2\sin 2\theta$ (d) $2\cos\theta$

327. $\frac{1}{\sin\theta}-\frac{\sqrt{3}}{\cos\theta}=$

(a) $\frac{4\cos\left(\frac{\pi}{3}-\theta\right)}{\sin 2\theta}$ (b) $\frac{4\sin\left(\frac{\pi}{3}-\theta\right)}{\sin 2\theta}$ (c) $\frac{4\cos\left(\frac{\pi}{3}+\theta\right)}{\sin 2\theta}$ (d) $\frac{4\sin\left(\frac{\pi}{3}+\theta\right)}{\sin 2\theta}$

328. $\tan^{-1}\left(\frac{1}{x+y}\right)+\tan^{-1}\left(\frac{y}{x^2xy+1}\right)=$

(a) $\tan^{-1}x$ (b) $\cot^{-1}x$ (c) $\tan^{-1}y$ (d) $\cot^{-1}y$

329. Area of a triangle formed by tangent and normal to the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at P$\left(\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}\right)$ with the x-axis is.......

(a) $\frac{b(a^2+b^2)}{4a}$ (b) $\frac{ab\sqrt{a^2-b^2}}{4}$ (c) $\frac{ab\sqrt{a^2+b^2}}{4}$ (d) $4ab$

330. If y = sin x . sin 2x . sin 3x...... . sin nx, then y$'$ is ....

(a) $\sum_{k=1}^{n}\cot kx$ (b) y. $\sum_{k=1}^{n}k\tan kn$ (c) y. $\sum_{k=1}^{n}k\cot kx$ (d) $\sum_{k=1}^{n}k\tan kx$

331. $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}=$

(a) $1-(\sin\alpha-\sin\beta)(\sin\beta-\sin\gamma)(\sin\gamma-\sin\alpha)$ (b) $1+\sin\alpha\sin\beta\sin\gamma$

(c) 1 (d) 0

332. If α and β are different complex numbers with $|\beta|=1$, *then* $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ is equal to

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{3}$ (d) 2

333. If $\tan\theta = \dfrac{m}{n}$ then $n\cos 2\theta + m\sin 2\theta$ equal to

(a) n (b) n^2 (c) $\dfrac{n}{m}$ (d) $\dfrac{m^2}{n^2}$

334. If $a = \cos 2\alpha + i\sin 2\alpha$, $b = \cos 2\beta + i\sin 2\beta$, then $\sqrt{\dfrac{a}{b}} + \sqrt{\dfrac{b}{a}} =$

(a) $2i\sin(\alpha-\beta)$ (b) $2i\sin(\alpha-\beta)$ (c) $2\cos(\alpha+\beta)$ (d) $2\cos(\alpha-\beta)$

335. If in ΔABC, $C = \dfrac{\pi}{2}$, then $\tan^{-1}\dfrac{a}{b+c} + \tan^{-1}\dfrac{b}{c+a} =$

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$ (d) none of these

336. A = $(\cos\theta, \sin\theta)$, B = $(\sin\theta, -\cos\theta)$ are two points. The locus of the centroid of ΔOAB, where 'O' is the origin is

(a) $x^2 + y^2 = 3$ (b) $9x^2 + 9y^2 = 2$ (c) $2x^2 + 2y^2 = 9$ (d) $3x^2 + 3y^2 = 2$

337. The value of $\tan^{-1}\left(\dfrac{x}{y}\right) - \tan^{-1}\left(\dfrac{x-y}{x+y}\right)$, $x, y > 0$, is

(a) $\dfrac{\pi}{4}$ (b) $-\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{2}$ (d) $-\dfrac{\pi}{2}$

338. $\displaystyle\int \frac{\cos^{n-1} x}{\sin^{n+1} x}\,dx$, where $n \in N$ is

(a) $\dfrac{\cot^n x}{n}$ (b) $\dfrac{-\cot^{n-1} x}{n-1}$ (c) $\dfrac{-\cot^n x}{n}$ (d) $\dfrac{\cot^{n-1} x}{n-1}$

339. $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ is

(a) parallel to $\vec{a}$ (b) Perpendicular to $\vec{a}$

(d) Parallel to plane containing $\vec{b}$ and $\vec{c}$ (d) none of these

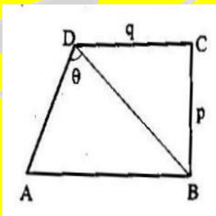
340. If z is a complex number of unit modulus and argument θ, then arg$\left(\dfrac{1+z}{1+\bar{z}}\right)$ equals

(a) $\dfrac{\pi}{2} - \theta$ (b) θ (c) $\pi - \theta$ (d) $-\theta$

341. If x, y, z are in A.P. and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ also in A.P., then which of the following is true ?

(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$ (c) $6x = 4y = 3z$ (d) $x = y = z$

342. ABCD is a trapezium such that AB and CD are parallel and BC is perpendicular to CD. If $\angle ADB = \theta$ BC = p and CD = q, then AB is equal to



(a) $\dfrac{p^2 + q^2 \cdot \cos\theta}{p\cos\theta + q\sin\theta}$ (b) $\dfrac{p^2 + q^2 \cdot \cos\theta}{p^2\cos\theta + q^2\sin\theta}$

(c) $\dfrac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$ (d) $\dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

343. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then

(a) 3 : 2 : 1 (b) 1 : 3 : 2 (c) 3 : 1 : 2 (d) 1 : 2 : 3

344. If $0 \le x \le \pi$ and $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ then $x =$

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(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (d) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

345. The general solution of $\sin x-\cos x=\sqrt{2}$, for any integer n, is

(a) $2n\pi+\frac{3\pi}{4}$ (b) $n\pi$ (c) $(2n+1)\pi$ (d) $2n\pi$

346. A particular solution of $\frac{dy}{dx}=(x+9y)^2$ when $x=0, y=1/127$ is

(a) $3x+27y=\tan 3\left(x+\frac{\pi}{12}\right)$ (b) $3x+27y=\tan^{-1}3\left(x+\frac{\pi}{12}\right)$

(c) $3x+27y=\tan 9\left(x+\frac{\pi}{12}\right)$ (d) $3x+27y=\tan\left(x+\frac{\pi}{12}\right)$

347. If A = $\tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ and B = $\tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$ then A – B =

(a) 0° (b) 45° (c) 60° (d) 30°

348. If $3x\equiv 5$ (mod 7), then

(a) $x\equiv 2$ (mod 7) (b) $x\equiv 3$ (mod 7) (c) $x\equiv 4$ (mod 7) (d) none of these

349. The particular solution of $\frac{y}{x}.\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$, when $x=1, y=2$ is

(a) $5(1+y^2)=2(1+x^2)$ (b) $2(1+y^2)=5(1+x^2)$

(c) $5(1+y^2)=1+x^2$ (d) $1+y^2=2(1+x^2)$

350. If $\frac{3x^2-2x+4}{(x+1)^6}=\frac{A_1}{x+1}+\frac{A_2}{(x+1)^2}+\frac{A_3}{(x+1)^3}+\frac{A_4}{(x+1)^4}+\frac{A_5}{(x+1)^5}+\frac{A_6}{(x+1)^6}$ then $(A_1+A_3+A_{5,}A_2+A_4+A_6)=$

(a) (0, 0) (b) (–8, –12) (c) (8, –12) (d) (–8, 12)

351. If $\log_2(2^{x-1}+6)+\log_2(4^{x-1})=5$, then $x=$

(a) 2 (b) 3 (c) 4 (d) 1

352. If $\frac{1}{2}\le x\le 1$, then $\cos^{-1}x+\cos^{-1}\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right]=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) none of these

353. The general solution of $\tan 5\theta-\cot 2\theta=0$ is

(a) $\theta=\frac{n\pi}{7}+\frac{\pi}{2}$ (b) $\theta=\frac{n\pi}{7}$ (c) $\theta=\frac{n\pi}{7}+\frac{\pi}{14}$ (d) $\theta=n\pi-\frac{\pi}{7}$

354. A value of θ satisfying $\sin 5\theta-\sin 3\theta+\sin\theta=0$ such that $0<\theta<\frac{\pi}{2}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

355. 1.1! + 2.2! + 3.3 ! +……+ n . n ! =

(a) (n + 1)! (b) (n + 1)! + 1 (c) (n + 1)! – 1 (d) n . (n + 1)!

356. If $(1-x+x^2)^n=a_0+a_1x+a_2x^2+.....+a_{2n}x^{2n}$, then $a_0+a_2+......+a_{2n}=$

(a) $3^n - 1/2$ (b) $(3^n - 1)/2$ (c) $(3^n + 1)/2$ (d) $3^n + 1/2$

357. The sum series $\frac{2}{3}+\frac{8}{9}+\frac{16}{27}+$....... upto n terms is

(a) $n-\frac{1}{2}\left(3^n-1\right)$ (b) $n+\frac{1}{2}\left(3^n-1\right)$ (c) $n-\frac{1}{2}\left(1-3^{-n}\right)$ (d) $n-\frac{1}{2}\left(3^{-n}-1\right)$

358. If $S_n = \frac{1}{6.11}+\frac{1}{11.16}+\frac{1}{16.21}+$.........to n terms, then $6\ S_n =$

(a) $\frac{1}{(5n+6)}$ (b) $\frac{(2n-1)}{5n+6}$ (c) $\frac{n}{(5n+6)}$ (d) $\frac{5n-4}{5n+6}$

359. $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+$.............$+\frac{1}{(3n-1)(3n+2)}=$

(a) $\frac{n}{6n+4}$ (b) $\frac{n+1}{6n+4}$ (c) $\frac{n}{6n-4}$ (d) $\frac{n}{6n+3}$

360. $\sum_{k=1}^{n}\frac{{}^nC_k}{k+1}=$

(a) $\frac{2^n-1}{n+1}$ (b) $\frac{2^{n-1}-1}{n+1}$ (c) $\frac{2^{n+1}-1}{n+1}$ (d) $\frac{2^n-1}{n}$

361. The number of terms in $(a+b+c)^n$, where n is a positive integer, is and further putting $n=1$ in choices we get

(a) $\frac{n(n+1)}{n}$ (b) $\frac{(n+1)(n+3)}{2}$ (c) $\frac{(n+1)(n+2)}{2}$ (d) $\frac{n^2}{2}$

362. If C_0, C_1, C_2...........C_n are binomial coefficients of order n, then the value of $\frac{C_1}{2}+\frac{C_2}{4}+\frac{C_3}{6}+$........ =

(a) $\frac{2^n+1}{n+1}$ (b) $\frac{2^n-1}{n+1}$ (c) $\frac{2^n+1}{n-1}$ (d) $\frac{2^n}{n+1}$

363. If $A=\begin{pmatrix}1&2\\0&1\end{pmatrix}$, then $A^n=$

(a) $\begin{pmatrix}1&2^{n-2}\\0&1\end{pmatrix}$ (b) $\begin{pmatrix}1&n^2\\0&1\end{pmatrix}$ (c) $\begin{pmatrix}1&2n\\0&1\end{pmatrix}$ (d) $\begin{pmatrix}1&n^2\\1&1\end{pmatrix}$

364. If the sum of the squares of first n terms an A.P. is cn^2. Then the sum of the squares of the n terms is

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{6}$ (c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{3}$

365. The sum of the n terms of $\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+$.......is.......

(a) $\frac{2n^2-n}{3}$ (b) $\frac{n(n+2)}{3}$ (c) $\frac{2n^2+n}{3}$ (d) $\frac{n^2-2n}{3}$

366. If n is odd positive integer and $\left(1+x+x^2+x^3\right)^n=\sum_{r=0}^{3n}a_r\ x^r$. then $a_0-a_1+a_2-a_3+$........$-a_{3n}$ is

(a) 0 (b) -1 (c) 1 (d) 4

367. If ω is an imaginary cube root of unity, then the value of

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$(1-\omega+\omega^2).(1-\omega^2+\omega^4).(1-\omega^4+\omega^8)$............(2n factor) is........

(a) 0 (b) 1 (c) 2^n (d) 2^{2n}

368. If $N = n!$ where n is a fixed integer ≥ 2, then $\frac{1}{\log_2 N}+\frac{1}{\log_3 N}+............\frac{1}{\log_n N}=$

(a) -1 (b) 0 (c) 1 (d) 4

369. If $(1-x+x^2)^n = a_0+a_1x+.......+a_{2n}x^{2n}$, then $a_0+a_2+a_4+......+a_{2n}=$

(a) $\frac{3^n-1}{2}$ (b) $\frac{3^n+1}{2}$ (c) $\frac{2.3^n-1}{2}$ (d) $\frac{2.3^n+1}{2}$

370 The sum of the series $\frac{1}{2.3}.2+\frac{2}{3.4}.2^2+\frac{3}{4.5}.2^3+.........$to n terms is

(a) $\frac{2^{n+1}}{n+2}+1$ (b) $\frac{2^{n+1}}{n+2}-1$ (c) $\frac{4}{3}+2$ (d) $\frac{4}{3}-2$

371. The value of $\sum_{r=0}^{n}(-1)^r\, {}^nC_r$ is

(a) 2^{-n} (b) 2^n (c) -1 (d) 0

372. If n is an integer greater than 1, than $a-{}^nC_1(a-1)+{}^nC_2(a-2)+....+(-1)^n(a-n)=$

(a) 2^n (b) a^2 (c) 0 (d) a

373. If in the expansion of $(1+x)^n$, a, b, c are three consecutive coefficient, then $n=$

(a) $\frac{2ac+ab+bc}{b^2-ac}$ (b) $\frac{ac+ab+bc}{b^2+ac}$ (c) $\frac{ab+ac}{b^2-ac}$ (d) none of these

374. The value of $C_0+3C_1+5C_2+7C_3+.....+(2n+1)C_n$ is equal to

(a) $2^n(n+1)$ (b) 2^n (c) $2^n+n\cdot 2^{n-1}$ (d) none of these

375. Let n be an odd natural number greater than 1. Then the number of zeroes at the end of the sum 99^n+1 is

(a) 3 (b) 2 (c) 4 (d) 6

376. $\sum_{k=1}^{n} k\left(1+\frac{1}{n}\right)^{k-1}=$

(a) $n(n+1)$ (b) $n(n-1)$ (c) $(n+1)^2$ (d) n^2

377. If $(1+x+x^2)^n = C_0+C_1x+C_2x^2+....$, then the value of $C_0C_1-C_1C_2+C_2C_3....$ is

(a) $(-1)^n$ (b) 2^n (c) 3^n (d) none of these

378. The angle between the pair of straight lines $y^2\sin^2\theta - xy\sin^2\theta + x^2(\cos^2\theta - 1) = 0$, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) None of these

379. If $n \in N$ such that $(7+4\sqrt{3})^n = I + F$, where $I \in N$ and $0 < F < 1$. Then the value of $(I+F)(1-F)$ is

(a) 2^{2n} (b) 7^{2n} (c) 0 (d) 1

380. Co – ordinates of the foot of the perpendicular drawn from $(0,0)$ to the line joining $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$ are

(a) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (b) $\left[\frac{a}{2}(\cos\alpha+\cos\beta), \frac{a}{2}(\sin\alpha+\sin\beta)\right]$

(c) $\left[\cos\frac{\alpha+\beta}{2}, \sin\frac{\alpha+\beta}{2}\right]$ (d) $\left(0, \frac{b}{2}\right)$

381. In the expansion of $(1+x)^n(1+y)^n(1+z)^n$, the sum of the coefficients of the terms of degree r is

(a) ${}^nC_{3r}$ (b) ${}^{3n}C_r$ (c) $3\cdot {}^nC_r$ (d) $\left({}^nC_r\right)^3$

382. The sum of $(n+1)$ terms of the series $\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + ...$ is

(a) $\frac{1}{n(n+1)}$ (b) $\frac{1}{n+1}$ (c) $\frac{1}{n+2}$ (d) none of these

383. The value of $\sum_{r=1}^{n} r\cdot\frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to

(a) $2n$ (b) $n(n+1)$ (c) $\frac{n(n+1)}{2}$ (d) none of these

384. If for $n \in N \sum_{k=0}^{2n}(-1)^k\left({}^{2n}C_k\right)^2 = A$, then value of $\sum_{k=0}^{2n}(-1)^k(k-2n)\left({}^{2n}C_k\right)^2$ is

(a) $-nA$ (b) nA (c) 0 (d) none of these

385. The $(n+1)^{th}$ term from the end in the expansion of $\left(2x - \frac{1}{x}\right)^{3n}$ is

(a) $\frac{3n!}{(n-1)!(2n+1)!}2^{n+1}\cdot x^{n-1}$ (b) $\frac{3n!}{n!2n!}2^n\cdot x^{-n}$

(c) $\frac{3n!}{(n-1)!(2n+1)!}2^{n-1}\cdot x^{-n+1}$ (d) $\frac{3n!}{n!2n!}2^n\cdot x^n$

386. The value of $1^2\cdot C_1 + 3^2\cdot C_3 + 5^2\cdot C_5 + ...$ is

(a) $n(n-1)\cdot 2^{n-3}$ (b) $n(n+1)\cdot 2^{n-3}$ (c) $n(n-1)2^{n-2} + n\cdot 2^{n-2}$ (d) $\frac{3n!}{n!2n!}2^n\cdot x^n$

387. If $S = {}^{2n}C_0\cdot{}^{2n}C_1 + {}^{2n}C_1\cdot{}^{2n-1}C_1 + {}^{2n}C_2\cdot{}^{2n-2}C_1 + ...$ Then S is equal to

(a) 0 (b) 1 (c) 2^{2n} (d) $n\cdot 2^{2n}$

388. Coefficients of $x^r\left[0\le r\le(n-1)\right]$ in the expansion of

$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+....(x+2)^{n-1}$

(a) ${}^nC_r\left(3^r+2^{n-r}\right)$ (b) ${}^nC_r\left(3^{n-r}-2^{n-r}\right)$ (c) ${}^nC_r\left(3^r-2^n\right)$ (d) none of these

389. ${}^nC_0-\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2-....+(-1)^n\frac{{}^nC_n}{n+1}=$

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$ (c) n (d) $\frac{1}{n+1}$

390. If a_1,a_2,a_3,a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$ is equal to

(a) $\frac{1}{2}\frac{a_2}{a_2+a_3}$ (b) $\frac{2a_2}{a_2+a_3}$ (c) $\frac{a_2}{a_2+a_3}$ (d) $\frac{2a_3}{a_2+a_3}$

391. If z_1 and z_2 are any two complex numbers, then $\left|z_1+\sqrt{\left(z_1^2-z_2^2\right)}\right|+\left|z_1-\sqrt{\left(z_1^2-z_2^2\right)}\right|$ is equal to

(a) $|z_1|$ (b) $|z_2|$ (c) $|z_1+z_2|$ (d) None of these

392. If z_1,z_2,z_3 are the vertices of an equilateral triangle in the argand plane, then $\left(z_1^2+z_2^2+z_3^2\right)=k\left(z_1z_2+z_2z_3+z_3z_1\right)$ is true for

(a) $k=1$ (b) $k=2$ (c) $k=3$ (d) $k=4$

393. If z is a complex number and a_1,a_2,a_3,b_1,b_2,b_3, all are real, then $\begin{vmatrix} a_1z+b_1\bar{z} & a_2z+b_2\bar{z} & a_3z+b_3\bar{z} \\ b_1z+a_1\bar{z} & b_2z+a_2\bar{z} & b_3z+a_3\bar{z} \\ b_1z+a_1 & b_2z+a_2 & b_3z+a_3 \end{vmatrix}$

(a) $\left(a_1a_2a_3+b_1b_2b_3\right)^2|z|^2$ (b) $|z|^2$ (c) 3 (d) none of these

394. If $|z|=1$, then $\left(\frac{1+z}{1+\bar{z}}\right)^n+\left(\frac{1+\bar{z}}{1+z}\right)^n$ is equal to

(a) $2\cos n(\arg(z))$ (b) $2\sin(\arg(z))$ (c) $2\cos n\left(\frac{\arg z}{2}\right)$ (d) $2\sin n\left(\frac{\arg z}{2}\right)$

395. If $|a_i|<1$, $\lambda_i\ge 0$ for $i=1,2,3....,n$ and $\lambda_1+\lambda_2+\lambda_3+....+\lambda_n=1$, then the value of $|\lambda_1a_1+\lambda_2a_2+....+\lambda_na_n|$ is

(a) $=1$ (b) <1 (c) >1 (d) none of these

396. If $1,\omega,\omega^2,....,\omega^{n-1}$ are the n, n^{th} roots of unity, then $(2-\omega)(2-\omega^2)....(2-\omega^{n-1})$ equals

(a) 2^n-1 (b) ${}^nC_1+{}^nC_2+....+{}^nC_n$

(c) $\left[{}^{2n}C_0+{}^{2n+1}C_1+{}^{2n+1}C_2+....+{}^{2n+1}C_n\right]^{1/2}-1$ (d) 2^n+1

397. If $(1+x)^n=C_0+C_1+....+C_nx^n$, where n is a positive integer, then

(a) $C_0-C_2+C_4-....=2^{n/2}\cos\left(\frac{n\pi}{4}\right)$ (b) $C_1-C_3+C_5-....=2^{n-2}+2^{(n-2)/2}\cos\left(\frac{n\pi}{4}\right)$

(c) $C_0 + C_4 + C_8 + ... = 2^{n-2} + 2^{(n-2)/2}\cos\left(\frac{n\pi}{4}\right)$ (d) $C_0 + C_3 + C_6 + ... = \frac{1}{3}\left(2^n + 2\cos\left(\frac{n\pi}{3}\right)\right)$

398. If $y = (1+x)(1+x^2)(1+x^4)....(1+x^{2^n})$, then $\frac{dy}{dx}$ at $x = 0$ is :

(a) 0 (b) -1 (c) 1 (d) None of these

399. If $\phi(x)$ be a polynomial function of the second degree. If $\phi(1) = \phi(-1)$ and a_1, a_2, a_3 are in AP, then $\phi'(a_1), \phi'(a_2), \phi'(a_3)$ are in

(a) AP (b) GP (c) HP (d) None of these

400. If $y = \tan^{-1}\left(\frac{1}{1+x+x^2}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right) + ... +$ up to n terms. Then $y'(0)$ is equal to

(a) $-\frac{1}{1+n^2}$ (b) $-\frac{n^2}{1+n^2}$ (c) $\frac{n}{1+n^2}$ (d) None of these

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300. Ans. (b) : Put $\vec{a} = i$, $\vec{b} = j$ and $\vec{c} = k$

(a) $i.(j \times k) = 1$ (b) $i \times (j \times k) = \vec{0}$ (c) $i \times (j + k) = k - j$ (d) $i + j + k = \vec{0}$

$\therefore$ Obviously (b) alone is the correct answer

301. Ans. (a) : Put $x = 1 = y = z$. Then $\Delta = 0$ $\therefore$ (a) is the correct answer

302. Ans. (c) : Put $x = 1 = y = z$. then $x^2 + y^2 + z^3 = 3$ $\therefore$ The correct answer is (c)

303. Ans. (b) : As $\sin^2\theta + \cos^2 = 1$ take p = $\sin\theta$ and q = $\cos\theta$ $\therefore$ p + q = $\cos\theta + \sin\theta$ and its max value is $\sqrt{2}$ $\therefore$ (b) is correct answer.

304. Ans. (a) : Observe that $|\hat{i} + \hat{j}| = \sqrt{2} = |\hat{i} - \hat{j}|$, take $\vec{a} = i$, $\vec{b} = j$. The angle between i and j is $90°$

$\therefore$ (a) is the correct answer

305. Ans. (b) : Put $\frac{\pi}{4}$ then $\cos 4\theta = \cos\pi = -1$

$\therefore$ G.E. $= \sqrt{2 + \sqrt{2 - 2}} - \sqrt{2 + 0} = \sqrt{2}$. Go to the alternatives and put $\theta = \frac{\pi}{4}$

(a) $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ (b) $2\cos\frac{\pi}{4} = 2.\frac{1}{\sqrt{2}} = \sqrt{2}$ (d) $\cos\frac{\pi}{2} = 0$ (d) 0

$\therefore$ (b) is the correct answer

306. Ans. (c) : Put $a = 1$, $b = -1$, $c = 0$

$\Delta = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 4 - 1 = 3$ By expanding along R_3. Now go to the alternatives and put the values of a, b, c we get

(a) : 1 (b) : 0 (c) : $1 + 1 + 1 + 0 = 3$ (d) 0

$\therefore$ (c) is the correct answer.

307. Ans. (c) : Put n = 1 : $A^n = A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. (b) or (c) is the correct answer

Put n = 0. then $A^\circ = I$ $\therefore$ (c) alone gives I $\therefore$ (c) is the correct answer

308. Ans. (c) : Put $x = 0$. Then $\sin(\cot^{-1}x) = \sin(\cot^{-1}0) = \sin\frac{\pi}{2} = 1$

$\therefore$ G.E. $= \cos(\tan^{-1}1) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$. Now go to the alternatives and put $x = 0$, we get

(a) : $\left(\frac{2}{3}\right)^{1/2}$ (b) $\left(\frac{3}{4}\right)^{1/2}$ (c) $\left(\frac{1}{2}\right)^{1/2}$ (d) 0

$\Rightarrow$ (c) is the correct answer.

309. Ans. (d) : Take Q = A and P as shown in the figure then $Pc^2 = 1^2 + 2^2 = PD^2$

G.E. $= \frac{1+1+5+5}{0+4+8+4} = \frac{12}{16} = \frac{3}{4}$ $\therefore$ (d) is the correct answer

310. Ans. (d) Put n = 1: Then $n^3 + 2n = 1 + 2 = 3$, which is divisible only by 3

311. Ans. (a) : Put x = y = z. Then $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$ $\therefore$ Obviously (a) is the correct answer

312. Ans. (a) : If we take a = 1, b = 2, c = 3 $\Rightarrow$ $C_1 \equiv C_3$ $\Rightarrow A = 0$ and 1, 2, 3 are in A.P.

$\therefore$ (a) is the correct answer.

313. Ans. (d) : Put n = 1. Then y = sin$x$ cos$x = \frac{1}{2}\sin 2x \therefore \frac{dy}{dx} = \cos 2x$

Put n = 1: The choices become (a) sin 2x (b) 1 (c) cos x (d) cox 2x

$\therefore$ (d) is the correct answer.

314. Ans. (c) : Put n = 2. Then $f(x) = 1 + 2x + x^2$ $\therefore f'(x) = 2 + 2x, \therefore f''(1) = 2$

Put n = 2: (a) 2.2 = 4 (b) 1.2 = 2 (c) $2.1.2^{\circ} = 2$ (d) $2.1.2^2 = 8$

$\therefore$ (b) and (c) is the correct answer. Now Put n = 3, to get (c) as the correct answer.

315. Ans. (c) : Put x = 0. Then G.E. = $3 + 6 + 4 = 13$

316. Ans. (d) : From choices it is clear that the answer is independent of x hence a = 2b; $A - B = 60°$.

Now $a = 2b \Rightarrow \sin A : \sin B = 2 : 1 = 1 : \frac{1}{2}$ and note that $\sin 90° = 1$ and $\sin 30° = \frac{1}{2}$

$\therefore$ A = 90°, B = 30° and A − B = 60° $\therefore$ Δ is right angled

317. Ans. (a) : Put $\theta = 0$. Then the points are (4, 0) and $\left(4.\frac{1}{2}, 4.\frac{\sqrt{3}}{2}\right)$ i.e. (4, 0) and $\left(2, 2\sqrt{3}\right)$.

Hence required distance == $\sqrt{4+12} = 4$, which is (a)

318. Ans. (b) : Put z = 1. Then $\left|\frac{z-6i}{z+6i}\right| = \left|\frac{1-6i}{1+6i}\right| = \frac{\sqrt{1+36}}{\sqrt{1+36}} = 1$ z = 1 lies on the real axis, which is (b)

319. Ans. (a) : Put x = 0. Then $\tan^{-1} x = 0 \therefore \cos\left(\tan^{-1} x\right) = 1$

Then $\sin\left[\cot^{-1}(1)\right] = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Put $x = 0$: (a) alone gives $\sqrt{\frac{1+0}{2+0}} = \frac{1}{\sqrt{2}}$ and hence (a) is the correct answer.

320. Ans. (c) : Put x = 0: Then G.E. = $\sin^{-1} 0 = 0$

(a) $0 - \frac{\pi}{2}$ (b) $0 + \frac{\pi}{2}$ (c) $0 - 0$ (d) 0 + 0

$\therefore$ (a) and (b) are ruled out. Now put x = 1: G.E. = $\sin^{-1} 0 = 0$

(c) $\frac{\pi}{2} - \frac{\pi}{2} = 0$ (d) $\frac{\pi}{2} + \frac{\pi}{2} \neq 0$ $\therefore$ (c) is the correct answer.

321. Ans. (b) : Put n = 0: then $8^{\circ} - (62)^1 \equiv -61 \equiv 2(\text{mod } 9)$ $\because 63 \equiv 0(\text{mod } 9)$ $\therefore$ (b) is the correct answer.

322. Ans. (c) : Put $\theta = \frac{\pi}{4}$; G.E. = $\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)(1+1) = 2\sqrt{2}$ and (c) alone gives $2\sqrt{2}$

323. Ans. (c) : Put n = 1 : Then G.E. $A^1 = A$. Go to the alternatives :

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \neq A$ (b) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = A$ (c) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = A$ (d) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq A$

$\therefore$ (c) is the correct answer.

324. Ans. (c) : Put n = 1 : Then $(p+q)^n$ becomes $(p+q)^1 = p+q$ Take r = 1. Then $t_1 = t_2 \Rightarrow p = q$

$\therefore \frac{(n+1)q}{r(p+q)} = \frac{(1+1)q}{1(q+q)} = \frac{2}{2} = 1$ $\therefore$(c) is the correct answer.

325. Ans. (b) : Put $\vec{a} = \hat{i}, \vec{b} = \hat{j}, \vec{c} = \hat{k}$. Then $\vec{a}.(\vec{b}\times\vec{c}) = \vec{b}.(\vec{c}\times\vec{a}) = \vec{c}.(\vec{a}\times\vec{b}) = [\text{i j k}] = 1$

$\therefore$ G.E. $= \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$ $\therefore$(b) is the correct answer

326. Ans. (d) : Put $\theta = 2\pi$. Then $\cos 8\theta = 1$ $\therefore \sqrt{2+2\cos 8\theta} = \sqrt{2+2} = 2$

$\therefore$ G.E.$\sqrt{2+\sqrt{2+2}} = \sqrt{2+2} = 2$ Go to alternatives :

(a) $2\sin 2\pi = 0$; (b) $2\cos\pi = -2$ (c) $\sin 4\pi = 0$ (d) $2\cos 2\pi = 2$

$\therefore$ (d) is the correct answer.

327. Ans. (c) : Put $\theta = \frac{\pi}{3}$. Then $\sin\theta = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; and $\cos\theta = \cos\frac{2\pi}{3} = -\frac{1}{2}$

Now, G.E $= \frac{2}{\sqrt{3}} - 2\sqrt{3} = \frac{2-6}{\sqrt{3}} = -\frac{4}{\sqrt{3}}$ Go to the alternatives

(a) $\frac{4(1)(2)}{\sqrt{3}}$ (b) 0 (c) $-$ve (d) $+$ve

Hence (c) is the correct choice.

328. Ans. (a) ; Put $x = -1, y = 0$. Then G.E.$= \tan^{-1}(-1) + \tan^{-1} 0 = -\frac{\pi}{4}$.

Among the given alternatives (a) alone gives $-\frac{\pi}{4}$ when $x = -1$. $\therefore$ (a) is the correct answer.

329. Ans. (a)

(a) Area of the ellipse is $\pi ab < 4ab$ $\therefore$ (d) can't be the correct answer. Look at the alternatives (b) and (c). They are in cubic units and hence they cannot be measure of an area.

$\therefore$ (a) is the correct choice.

330. Ans. (c) : Put n = 1 : Then y = sin x and y' = cos x. Go to alternatives:

(a) $\sum_{k=1}^{n} \cot kx$ i.e. $\cot x$ (b) $\sin x . \tan 1$ (c) $\sin x . \cot x = \cos x$ (d) $\tan x$

$\therefore$ (c) is the correct answer.

331. Ans. (d) : No alternative contains δ. Therefore put $\delta = 0$

Then $\Delta = 0 \because C_1 \equiv C_2$ $\therefore$ (d) is the correct answer

332. Ans. (b) : Put $\alpha = 0$. Then $\bar{\alpha} = 0$. Then $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = \left|\frac{\beta-0}{1-0}\right| = |\beta| = 1$ $\therefore$ (b) is the correct answer.

333. Ans. (a), Put m = n. Then $\tan\theta = 1$ $\therefore \theta = \frac{\pi}{4}$

$\therefore n\cos 2\theta + m\sin 2\theta = n\cos\frac{\pi}{2} = 0 + n = n.$ Go to the alternatives and put m = n :

(a) n (b) 1 (c) n^2 (d) 1

$\therefore$ (a) is the correct answer.

334. Ans. (d) : Put $\alpha = \beta = \frac{\pi}{2}$ Then $a = -1 = b \Rightarrow \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \sqrt{1} + \sqrt{1} = 2$

Go to the alternatives. (a) : 0 (b) : 0 (c) : −2 (d) : 2 $\Rightarrow$ (d) is the correct answer.

335. Ans. (a) : Put A = B = $\frac{\pi}{4}$, If a = b= 1, then c = $\sqrt{2}$, $\frac{a}{b+c} = \frac{b}{c+a} = \frac{1}{\sqrt{2}+1} = \sqrt{2}-1$

$\therefore$G.E.= $\tan^{-1}\left(\sqrt{2}-1\right) + \tan^{-1}\left(\sqrt{2}-1\right) = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$, Hence is (a) correct choice

336. Ans. (b) : When $\theta = 0$, then A = (1, 0) and B = (0, −1) and the centroid of ΔOAB is $\left(\frac{1}{3}, -\frac{1}{3}\right)$ and it lies on $9x^2 + 9y^2 = 2$ and not on any other given circle. $\therefore$ (b) is the correct answer.

337. Ans. (a) : Put $x = 1, y = 1$. Then $G.E. = \tan^{-1} = \frac{\pi}{4}$ $\therefore$ The correct answer is (a)

338. Ans. (c) : Put $n = 1$. Then $I = \int \frac{1}{\sin^2 x} dx = \int \operatorname{cosec}^2 x dx = 1 - \cot x.$

Put n = 1 in the given alternatives to get (c) as correct answer.

339. Ans. (a), Let $\vec{a} = \hat{i}, \vec{b} = \hat{j}, \vec{c} = \hat{k}$, then given = $\left(\hat{k} \times \hat{i}\right) \times \left(\hat{i} \times \hat{j}\right) = \hat{j} \times \hat{k} = \hat{i}$, Hence (a) is the correct choice.

340. Ans. (b) : Put $z = i$. Then $\frac{1+z}{1+\bar{z}} = 1$ $\therefore$ $\arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg(1) = \arg(z)$

Alternative : Put $z = i$. Then $\arg\frac{1+z}{1+\bar{z}} = \arg\frac{1+i}{1+\bar{i}} = \arg\frac{1+i}{1-i} = \arg\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \arg\frac{(1+i)^2}{2} = \arg(i) = \arg(z)$

341. Ans. (d) : Put $x = y = z = 1$. Then obviously the given conditions are satisfied

$\therefore$ the correct answer is (d)

342. Ans. (d) : Use: Units & Dimensions Then (b) and (c) can't be correct answers.

Suppose $\theta \to 0$ than AB $\to 0$ and (d) $\to 0$ but (a) $\to 0$ $\therefore$ the correct answer is (d).

Aliter : Put $\theta = \frac{\pi}{4}$. Then ABCD is a square. Then AB = q etc.

343. Ans. (d) : Suppose $a = 1, b = 2, c = 3$ then both the equations coincide

Obviously they have a common root and a : b : c = 1 : 2 : 3 $\therefore$ The correct answer is (d)

344. Ans. (c) : Take (a) : $\frac{\pi}{4}$ or (b) $\frac{3\pi}{4}$, $\sin^2 x = \cos^2 x = \frac{1}{2}$ *and* $81^{1/2} = 9$ $\therefore 9 + 9 \neq 30$

Take(c) Here $\sin^2 x = \frac{1}{4}$ and $\cos^2 x = \frac{3}{4}$, Hence $81^{\sin^2 x} + 81^{\cos^2 x} = 3 + 3^3 = 30$ $\therefore$(c)is the correct answer.

345. Ans. (a) : Put n = 0, Take (a) : $x = \frac{3\pi}{4}$ (b) : 0 (c) : π (d) : 0

Now itself you can conclude that (a) is the correct answer because neither 0 nor π satisfies the given equation.

346. Ans. (a) : Put $x = 0, y = 1/127$ in the given alternatives

(a) 0 + 1= $\tan\frac{3\pi}{12}$ (b) 0 + 1 = $\tan^{-1}\frac{\pi}{12}$ (c) $1 = \tan\frac{3\pi}{4}$ (d) $1 = \tan\frac{\pi}{12}$

Clearly (a) alone is true and hence (a) is the correct answer

347. Ans. (d) : Put x = 1 = k.

Then A = $\tan^{-1}\frac{\sqrt{3}}{1} = 60°$ and B = $\tan^{-1}\frac{1}{\sqrt{3}} = 30°$ $\therefore A - B = 30°$, which is (d)

348. Ans. (c) : Put $x = 2 : 6 \equiv 5$; Put $x = 2 : 9 \equiv 5$ (mod 7), which are not true

put $x = 4$; $12 \equiv 5$ (mod 7), which is true. $\therefore$ (c) is the correct answer.

349. Ans. (b) : Put $x = 1, y = 2$ in the given alternatives

(a) $5\times5 = 5\times2$ (b) $2(1+4) = 5(1+1)$ (c) $5(1+4) = 1+1$ (d) $1+4 = 2(1+1)$

$\therefore$ (b) alone is true and hence (b) is the correct answer.

350. Ans. (d), Put $x = 0 \Rightarrow 4 = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$

(d) (−8, 12) alone satisfies this condition and the correct answer is (d)

351. Ans. (a) : Go from the alternatives :

Put $x = 2$: LHS $= \log_2 8 + \log_2 4 = 3 + 2 = 5$ $\therefore$ (a) is the correct answer

352. Ans. (c) : $x = 1$: then G.E. $= \cos^{-1}1 + \cos^{-1}\left(\frac{1}{2}+0\right) = 0 + \frac{\pi}{3}$, which is (c)

353. Ans. (c) : Put n = 0 the choice become

Then (a) $\theta = \frac{\pi}{2}$ (b) $\theta = 0$ (c) $\theta = \frac{\pi}{14}$ (d) $\theta = \frac{\pi}{7}$

Obviously $\frac{\pi}{2}$ and $\theta \neq 0$. When $\theta = -\frac{\pi}{7}$, LHS $= +ve \neq 0$ $\therefore$ (c) is the correct answer.

354. Ans. (c) : Go from the alternatives :

Obviously $\theta \neq \frac{\pi}{2}$ $\because 0 < \theta < \frac{\pi}{2}$ .Now when $\theta \neq \frac{\pi}{4}, \sin 5\theta = \sin\frac{5\pi}{4} = -\sin\frac{\pi}{4} = -\sin\ \theta$ but $\sin 3\theta \neq 0$

When $\theta \neq \frac{\pi}{4}, \sin\frac{5\pi}{6} - \sin\frac{\pi}{2} + \sin\frac{\pi}{6} = \frac{1}{2} - 1 + \frac{1}{2}$ $\therefore \theta = \frac{\pi}{6}$

355. Ans. (c) : n = 1: G.E. = 1.1!. Then (c) is the correct answer.

356. Ans. (c) : Put n = 1 : Then $(1 - x + x^2)^1 = a_0 + a_1x + a_2x^2 \Rightarrow a_0 = 1,\ a_1 = -1,\ a_2 = 1$

$\therefore$ $a_0 + a_2 = 1 + 1 = 2$. Now go to the alternatives :

(a) $3 - 1/2 = 5/2$ (b) $(3-1)/2 = 1$ (c) $(3+1)/2 = 2$ (d) $3 + \frac{1}{2}$

$\therefore$ (c) is the correct answer.

357. Ans. (c) : Put n = 1 ; Then the sum = $\frac{2}{3}$. Now put n = 1 in the alternatives :

(a) $1 - \frac{1}{2}(3-1) = 0$ (b) $1 + \frac{1}{2}(3-1) = 2$ (c) $1 - \frac{1}{2}\left(1 - \frac{1}{3}\right) = \frac{2}{3}$ (d) $1 - \frac{1}{2}\left(\frac{1}{3} - 1\right) > 1$

$\therefore$ (c) is the correct answer.

358. Ans. (c) : Put n = 2 in the given alternatives :

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$ (c) $\frac{2}{16} = \frac{1}{8}$ (d) $\frac{6}{16}$

Now $6S_n = 6S_2 = 6\left[\frac{1}{6.11} + \frac{1}{11.16}\right] = 6\left[\frac{16+6}{6.11.16}\right] = \frac{2}{16}$ which is (c)

359. Ans. (a) : Put n = 1 : G.E. $= \frac{1}{2.5} = \frac{1}{10}$. When n = 1, alternative becomes

(a) $\frac{1}{10}$ (b) $\frac{2}{10}$ (c) $\frac{1}{2}$ (d) $\frac{1}{9}$

$\therefore$ (a) is the correct answer.

360. Ans. (c) : Put n = 1. Then G.E. $\sum_{k=0}^{1} \frac{{}^1C_k}{k+1} = \frac{{}^1C_0}{0+1} + \frac{{}^1C_1}{1+1} = 1 + \frac{1}{2} = \frac{3}{2}$ Now put $n = 1$ in choices we get

(a) $\frac{2-1}{1+1} \neq \frac{3}{2}$ (b) $\frac{2^0-1}{1+1} \neq \frac{3}{2}$ (c) $\frac{2^0-1}{1+1} = \frac{3}{2}$ (d) $\frac{2-1}{1} = 1 \neq \frac{3}{2}$

$\therefore$ (c) is the correct answer.

361. Ans. (c) : Put n = 1. Then $(a+b+c)^1$ has 3 terms

(a) 1 (b) 4 (c) 3 (d) $1/2$

$\therefore$ (c) is the correct answer

362. Ans. (b) : Put n = 1: Then G.E. = $\frac{{}^1C_1}{2} = \frac{1}{2}$

(a) $\frac{2+1}{1+1} \neq \frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{0} \neq \frac{1}{2}$ (d) $\frac{2}{2} \neq \frac{1}{2}$

$\therefore$ (b) is the correct answer

363. Ans. (c) : Put n = 1; Then G.E. = A^1=A.Go to the alternatives:

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \neq A$ (b) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \neq A$ (c) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = A$ (d) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq A$

$\therefore$ (c) is the correct answer.

364. Ans. (c) : fix c at 1, $S_n = n^2 \Rightarrow$ the given A.P. is 1, 3, 5, ……

Put n = 1; Then the sum I question = 1^2=1. Go to the alternatives

(a) $\frac{1}{2}$ (b) $\frac{5}{3}$ (c) 1 (d) $\frac{6}{3}$

$\therefore$ (c) is the correct answer.

365. Ans. (b) : Put n = 1: then $S_1 = t_1 = \frac{1^2}{1} = 1$. Observe that (b) and (c) can be a correct answer

Now put n = 2: (b) $\frac{2(2+2)}{3} = \frac{8}{3}$ (c) $\frac{8+2}{3} = \frac{10}{3}$

$S_2 = t_1 + t_2 = 1 + \frac{5}{3} = \frac{8}{3}$ $\therefore$ (b) is the correct answer

366. Ans. (a) : Put n = 1 : Then $1+x+x^2+x^3 = \sum_{r=0}^{3n} a_r x^r$ i.e., $1+x+x^2+x^3 = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$\therefore$ G.E. $= a_0 - a_1 + a_2 - a_3 = 1-1+1-1 = 0$ $\therefore$ (a) is the correct answer

367. Ans. (d), Put n = 1, we get, Given = $(1-\omega+\omega^2)\times(1-\omega^2+\omega^4) = (-2\omega)\times(-2\omega^2) == 4\omega^3 = 4$

(d) $2^{2n} = 2^2$ alone gives the value 4. $\therefore$ (d) is the correct answer.

368. Ans. (c) : Put n = 2. then n = 2! And G.E. $= \frac{1}{\log_2 2} = 1$ $\therefore$ (c) is the correct answer.

369. Ans. (b) Put n = 1. Then LHS = $1 - x + x^2 \Rightarrow a_0 = 1,\ a_1 = -1,\ a_2 = 2,\ a_n = 0,\ n \geq 3$.

$\therefore$ Required $= a_0 + a_2 = 1 + 1 = 2$. Among the given alternatives $\dfrac{3^n + 1}{2}$ alone gives $\dfrac{3^n + 1}{2} = 2$

$\therefore$ (b) is the correct answer.

370 Ans. (b) : Put n = 1. then $S_1 = t_1 = \dfrac{1}{2.3}.2 = \dfrac{1}{3}$. Go to the alternatives and Put n = 1

(a) $\dfrac{4}{3} + 1$ (b) $\dfrac{4}{3} - 1$ (c) $\dfrac{4}{3} + 2$ (d) $\dfrac{4}{3} - 2$

$\therefore$ The correct answer is (b).

371. Ans. (d), Take $n = 1$ we get Given $= \sum_{r=0}^{1} (-1)^r \, {}^1C_r = {}^1C_0 - {}^1C_1 = 1 - 1 = 0$

372. Ans. (c), Let us take $n = 2$ and $a = 3$ then Given $3 - {}^2C_1 \cdot 2 + {}^2C_2 \cdot 1 = 3 - 4 + 1 = 0$

We have not tried $a = 0$ $\therefore$ for $a = 0$ b, c, d are giving same answer

373. Ans. (a), Let $n = 2$ then $a = 1, b = 2, c = 1$.

Now use these values in (a) choice we get $\dfrac{2 + 2 + 2}{4 - 1} = 2$

And (b) choice gives $\dfrac{1 + 2 + 2}{4 + 1} = \dfrac{5}{5} = 1$ and choice (c) gives $\dfrac{2 + 1}{4 - 1} = 1$ Hence (a) is correct choice

374. Ans. (a), Taken = 1 we get Given = 4 and only choice (a) gives 4

375. Ans. (b), Put $n = 1$ we get given $= 99' + 1 = 100$

Hence correct answer is (b)

376. Ans. (d), Put $n = 1$ we get $k = 0$ Given $n = 1$

Now put $n = 1$ in choices we get (d) is correct choice

377. Ans. (d), Put $n = 1$ we get $1 + x + x^2 = C_0 + C_1 x + C_2 x^2 \Rightarrow C_0 = 1, C_1 = 1, C_2 = 1$

Hence Given $C_0, C_1 - C_1 C_2 = 0$ Hence (d) is correct choice

378. Ans.(d) Let us use $\theta = \dfrac{\pi}{2}$ we get $y^2 - xy - x^2 = 0$.

Clearly coefficient of x^2 + coefficient of $y^2 = -1 + 1 = 0$. Hence, lines are $\perp$

379. Ans. (d), Take $n = 1$ we get $I + F = 7 + 4\sqrt{3} = 7 + 4\sqrt{3} - 6 + 6 = 13 + 4\sqrt{3} = 6$

Clearly I = 17, F $= 4\sqrt{3} - 6$ Hence $1 - F = 7 - 4\sqrt{3}$

$\therefore$ Given $= 7 + 4\sqrt{3}$

$\left(7 - 4\sqrt{3}\right) = 1$

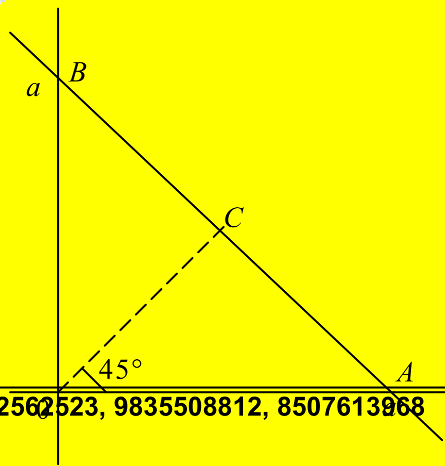
380. Ans.(b) Let $\alpha = 0, \beta = \dfrac{\pi}{2}$ then point become $(a, 0), (0, a)$

$\Rightarrow$ foot of perpendicular C midpoint

Of AB i.e., $\left(\dfrac{a}{2}, \dfrac{a}{2}\right)$

clearly putting these values in (b) we get $\left(\dfrac{a}{2}, \dfrac{a}{2}\right)$

Hence, (a) or (b) may be correct but (c) and (d) are surely wrong.



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But it is clear that as α or β varies that foot of $\perp$ will vary .

Hence, (a) cannot be correct . Hence, (b) is correct choice.

381. Ans. (b), Take $n=1$ then Given $=(1+x)(1+y)(1+z)$

382. Ans. (d), Take $n=1$ we get given series $=\frac{{}^1C_0}{2}-\frac{{}^1C_1}{2}=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

But putting $n=1$ in the choices no any choice given $\frac{1}{6}$ Hence correct choice is (d)

383. Ans. (a), Putting $n=1$ we get given $=1\cdot\frac{{}^1C_1}{{}^1C_0}=1$

Clearly choice (c) also gives 1 for $n=1$ Hence (c) is correct choice

384. Ans. (a), Taken $n=1$ we get $\sum_{k=0}^{2}(-1)^k\cdot\left({}^2C_k\right)^2=A$

$\Rightarrow\left({}^2C_0\right)^2-\left({}^2C_1\right)^2+\left({}^2C_2\right)^2=A\Rightarrow 1-4+1=A\ \Rightarrow A=-2$

Now $\sum_{k=0}^{2n}(-1)^k(k-2n)\left({}^{2n}C_k\right)^2$ becomes

$\sum_{k=0}^{2}(-1)^k\cdot(k-2)\left({}^2c_k\right)^2=(0-2)\left({}^2C_0\right)^2-(-1)\left({}^2c_1\right)^2+0\left({}^2c_2\right)^2=-2+2^2=2$

Taking $n=1$ we get choice (a) gives $-1\times(-2)=2$ Hence choice (a) is correct

385. Ans. (b), Take $n=1$

We have to find 2nd from end for $\left(2x-\frac{1}{x}\right)^3$

$\left(2x-\frac{1}{x}\right)^3=(2x)^3-3\cdot(2x)^2+3\cdot 2x\cdot\frac{1}{x^2}-\left(\frac{1}{x}\right)^3\ =8x^3-12x+\frac{6}{x}-\frac{1}{x^3}$

Hence second term from end $=\frac{6}{x}$

Now putting $n=1$ in choice we see that only choice (d) gives $\frac{3!}{2!}\cdot 2^1x^{-1}=\frac{6}{x}$

Hence choice (b) is correct choice

386. Ans. (b), Put $n=1$ in the choices we get

Given $=1^2\cdot c_1=1$

And choice is gives $1\times 2\times 2^{1-2}=2\times\frac{1}{2}=1$ Hence choice (b) is correct

387. Ans. (d), Take $n=1$ we get $S={}^2C_0\cdot{}^2C_1+{}^2C_1\cdot{}^1C_1=2+2=4$

Putting $n=1$ in all choices we get (c) and (d) both gives 4

Hence let us put $n=2$ we get $S={}^4C_0\cdot{}^4C_1+{}^4C_1\cdot{}^3C_1+{}^4C_2\cdot{}^2C_1+{}^4C_3\cdot{}^1C_1\ =4+12+12+4=32$

Now putting $n=2$ we get only (d) choice gives 32

Hence (d) choice gives correct answer

388. Ans. (b), put $n=2$ and $r=1$ we get

Given $(x+3)+x+2=2x+5$ Now putting $n=2$ in all choices we get

Choice (b) and choice (c) both give 2 Hence let us take $n=3$ and $r=1$

Given becomes $(x+3)^2+(x+3)(x+2)+(x+2)^2 = 3x^2+15x+19$

Now we have to find the coefficient of x which is 15 here

Now put $n=3$ and $r=1$ is choices we get ${}^3C_1(3^2-2^2)=3\times(9-4)=15$

Hence (b) is correct answer

389. Ans. (d), Take $n=1$ we get

Given $={}^1C_0-\frac{1}{2}\times{}^1C_1=1-\frac{1}{2}=\frac{1}{2}$

Now putting $n=1$ we see that only (d) choice gives $\frac{1}{2}$ Hence (d) is correct choice

390. Ans. (b), Let us take $n=3$

Hence a_1,a_2,a_3,a_4 by the expansion of $(1+x)^3=1+3x+3x^2+x^3$ we get $a_1=1, a_2=3, a_3=3, a_4=1$

Hence $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{1}{1+3}+\frac{3}{3+1}=\frac{1}{4}+\frac{3}{4}=1$

And using then values we get (b) choice gives $\frac{2\times3}{3+3}=1$ and (d) choice gives $\frac{2a_3}{a_2+a_1}=\frac{2\times3}{3+3}=1$

Hence both give same answer

Hence use $n=4$ We get $(1+x)^4=1+4x+6x^2+4x^3+x^4$

Let $a_1=1, a_2=4, a_3=6, a_4=4$

Now (b) choice gives $\frac{2\times4}{4+6}=\frac{4}{5}$ and $\frac{a_1}{a_1+a_2}+\frac{a_b}{a_3+a_4}=\frac{1}{1+4}+\frac{6}{6+4}=\frac{1}{5}+\frac{6}{10}=\frac{8}{10}=\frac{4}{5}$

Hence (b) choice is correct

391. Ans. (c), Taking $z_1=1$ and $z_2=1$. We get given $=2\times1=2$. Hence, correct choice is (c).

392. Ans. (a), For z_1,z_2,z_3 to form equilateral triangle. Let $z_1=1$, $z_2=-1$ and $z_3=\sqrt{3}\,i$

and $1+1-3=k\left(-1+\sqrt{3}\,i-\sqrt{3}\,i\right)\Rightarrow k=1$. Hence, correct choice is (a).

393. Ans. (d), Taking $z=1$ we get the value of determinant$=0$. Hence, correct choice is (d)

394. Ans. (a), Let $z=i$ and $n=1$ we get given $\frac{1+i}{1-i}+\frac{1-i}{1+i}=\frac{(1+i)^2+(1-i)^2}{2}=0$. Hence, we take $n=2$

and $z=i$. We get $\left(\frac{1+i}{1-i}\right)^2+\left(\frac{1-i}{1+i}\right)^2=-1+(-1)=-2$. Using it in choices we get

For choice (a) gives $2\cos\left(2\times\frac{\pi}{2}\right)=-2$, For choice (b) gives $2\sin\frac{\pi}{2}=2$

For choice (c) gives $2\cos\left(2\times\frac{\pi}{4}\right)=0$, For choice (d) gives $2\sin\left(2\times\frac{\pi}{4}\right)=2$

395. Ans. (b), Take $n=1$ we get $\lambda_1=1$ and given $|\lambda_1 a_1|=|a_1|<1$. Hence, correct choice is (b)

396. Ans. (a,b,c), Taking $n=2$ we get $w=-1$. Hence, given $=2-w=3$. Hence, (a), (b), (c) give correct result.

397. Ans. (a,b,c,d), Take $n=2$ we get $(1+x)^2 = 1+2x+x^2$. $\therefore$ $C_0 = 1, C_1 = 2, C_2 = 1$

$\therefore$ (a) gives L.H.S $= C_0 - C_2 = 0$ and R.H.S $2^{\frac{2}{2}}\cos\dfrac{\pi}{2} = 0$

(b) gives L.H.S $= C_1 = 2$ and R.H.S $= 1+0 = 1$ (c) gives L.H.S $= C_0 = 1$ and R.H.S $= 1$

(d) gives L.H.S $= C_0 = 1$ and R.H.S $= \dfrac{1}{3}(4-1) = 1$ hence, all (a), (b), (c), (d) are correct.

398. Ans. (c), Let us take $n=1$. Hence, given equation becomes

$$y = (1+x)(1+x^2) \Rightarrow \frac{dy}{dx} = (1+x^2) + 2x(1+x) = 1+2x+3x^2 \Rightarrow \left.\frac{dy}{dx}\right]_{x=0} = 1$$

399. Ans. (a), Let $\phi(x) = x^2 + 1$ (As $\phi(x)$ is given as any quadratic polynomial) $\Rightarrow \phi'(x) = 2x$.

Hence, $\phi'(a_1) = 2a_1$, $\phi'(a_2) = 2a_2$, $\phi'(a_3) = 2a_3 \Rightarrow \phi'(a_1), \phi'(a_2), \phi'(a_3)$ are in A.P.

400. Ans. (b), Let $n=1$. Hence, given becomes

$$y = \tan^{-1}\left(\frac{1}{1+x+x^2}\right) \Rightarrow \frac{dy}{dx} = \frac{1}{1+\left(\frac{1}{1+x+x^2}\right)^2} \times \frac{-1}{(1+x+x^2)^2} \times (1+2x) \Rightarrow \left.\frac{dy}{dx}\right]_{x=0} = \frac{1}{2} \times \left(\frac{-1}{12}\right) \times 1 = -\frac{1}{2}$$

Hence, both (a) and (b) can be correct. $\therefore$ for surety take $n=2$.

We get $y = \tan^{-1}\dfrac{1}{1+x+x^2} + \tan^{-1}\dfrac{1}{x^2+3x+3} \Rightarrow y = \cot^{-1}(1+x+x^2) + \cot^{-1}(x^2+3x+3)$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+1)}{1+(1+x+x^2)^2} + \frac{-(2x+3)}{1+(x^2+3x+3)^2} \Rightarrow \left.\frac{dy}{dx}\right]_{x=0} = \frac{-1}{1+1^2} + \frac{-3}{1+3^2} = -\frac{1}{2} - \frac{3}{10} = -\frac{4}{5}.$$

Hence, correct choice is (b).

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