14.1.

5. we require $y - x - 2 \geq 0$. $\Rightarrow$ $y \geq x + 2$.

6. $x^2 + y^2 - 4 > 0$. $\Rightarrow$ $x^2 + y^2 > 4$

7. we require $y \neq x$ $y = x^3$.

~~ex~~ except the two curves.

18. a). $y > x$.

b). $f \geq 0$.

c) The level curve are obilique lines $y - x = b \geq 0$.

d). Boundary $y = x$. e). closed. f) unbounded $y \to \infty$ is possible

26. a). $\mathbb{R}^2$

b). $f > 0$.

c). level curves are circles centered at origin. $x^2+y^2 = b \geq 0$.

d). No boundary.

e). Both open and closed.

f). unbounded.

42. a). b)

14.2.

1. plug in $(0,0)$ $\lim = \frac{5}{2}$

9. $\lim\limits_{\substack{x\to 0 \\ y\to 0}} \frac{e^y \cdot \sin x}{x} = 1 \cdot 1 = 1.$

12. plug in $x = \frac{\pi}{2}$ $y=0$.

$\frac{\cos y + 1}{y - \sin x} = \frac{1+1}{0-1} = -2.$

13. $\lim \frac{(x-y)^2}{x-y} = \lim\limits_{x,y\to 1} x-y = 0$

17. $= \lim\limits_{x,y\to 0,0} (\sqrt{x}+\sqrt{y})+2.$

$= 2.$

20. $= \lim\limits_{\substack{x\to 4\\ y\to 3}} \dfrac{1}{\sqrt{x}+\sqrt{y+1}}$

$= \dfrac{1}{2+2} = \dfrac{1}{4}.$

33. a) $\mathbb{R}^2 \setminus \{(0.0)\}.$

b). $\mathbb{R}^2$

38 a) At $\mathbb{R}^3 \setminus \mathbb{R}\times\{0\}\times\{0\}.$

41. suppose $y = kx$

$\lim\limits_{\substack{x\to 0\\ y\to 0}} f = \lim - \dfrac{x}{\sqrt{(k^2+1)x^2}}$

$= \dfrac{1}{k^2+1}$ depends on k

The limit doesn't exist.

48. $y = kx^2.$

$\lim f = \lim\limits_{x\to 0} \dfrac{k\cdot x^4}{x^4+k^2x^4}$

$= \dfrac{k}{1+k^2}$ depends on $k.$

49 let $x = 1+k.$ $k\to 0.$

$y = 1+qk.$

$\Rightarrow \lim \dfrac{(1+k)(1+qk)^2-1}{qk.}$

$= \lim\limits_{k\to 0} \dfrac{(k+2qk)}{qk}$

$= \dfrac{1+2q}{q}$ depends on $q.$

52. a). 9

b) -8

c). No exist.

56. $2 - \dfrac{|xy|}{6} < \dfrac{4-4\cos\sqrt{xy}}{|xy|} < \dfrac{2|xy|}{|xy|} = 2.$

Apply limit at each side:

$\Rightarrow \quad 2 \leq \lim \dfrac{4-4\cos\sqrt{xy}}{|xy|} \leq 2.$

$\Rightarrow \lim = 2.$

58. Yes. it tells that

$\lim\limits_{\substack{x\to 0\\ y\to 0}} x\cdot\cos\dfrac{1}{y} = 0.$

62. $\left|\dfrac{x^3-y^3}{x^2+y^2}\right| = \left|\dfrac{(x-y)(x^2+y^2+xy)}{x^2+y^2}\right|$

$\leq \dfrac{|x-y|\cdot\frac{3}{2}\cdot(x^2+y^2)}{\ldots} = |x-y|\cdot\frac{3}{\ldots}$

This means

$$\lim_{\substack{x\to 0\\ y\to 0}} \left|\frac{x^3-y^3}{x^2+y^2}\right| \to 0.$$

$\Rightarrow$ ~~lim~~

$$\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3-y^3}{x^2+y^2}\right) = 0.$$

70. $\left|\frac{y}{x^2+1}\right| < 0.05$

we can see if $|y| < 0.05$

Then $\left|\frac{y}{x^2+1}\right| < |y| < 0.05$

$\Rightarrow \quad x^2+y^2 < (0.05)^2$

$\Rightarrow \delta$ can be 0.05.

14.3.

1). $\frac{\partial f}{\partial x} = 4x \qquad \frac{\partial f}{\partial y} = -3.$

5). $\frac{\partial f}{\partial x} = 2(xy-1)\cdot y \qquad \frac{\partial f}{\partial y} = \cancel{2(x-1)}\ 2(xy-1)\cdot x.$

17). $\frac{\partial f}{\partial x} = 2\cdot\sin(x-3y)\cdot\cos(x-3y)$

$\frac{\partial f}{\partial y} = 2\cdot\sin(x-3y)\cdot\cos(x-3y)\cdot(-3)$

43. $\frac{\partial^2 g}{\partial x^2} = 2y - y\cdot\sin x. \qquad \frac{\partial^2 g}{\partial x\partial y} = 2x + \cos x.$

$\frac{\partial^2 g}{\partial y^2} = -\cos y$

51. $\frac{\partial w}{\partial x\partial y} = \frac{\partial}{\partial y}(\frac{2}{2x+3y}) = -\frac{6}{(2x+3y)^2}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \|$

$\frac{\partial w}{\partial y\partial x} = \frac{\partial}{\partial x}(\frac{3}{2x+3y}) = -\frac{6}{(2x+3y)^2}$

57. $\frac{\partial f}{\partial x} = -1-6xy \Big|_{(1,2)} = -1-12 = -13.$

$\frac{\partial f}{\partial y} = 1-3x^2\Big|_{(1,2)} = -2.$

70. $\frac{\partial f}{\partial x} = \lim_{\substack{x\to 0\\ y\to 0}} \frac{\frac{\sin(x^3+y^4)}{x^2+y^2} - 0}{x-0}\Bigg|_{(0,0)}$

$= \lim_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x^3+y^4)}{x^3+xy^2}$

Let $x=ky$

$= \frac{k^2}{k^2+1}$ depends on k

The $\frac{\partial f}{\partial x}$ doesn't exist.

$\frac{\partial f}{\partial y} = \lim_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x^3+y^4)}{x^2y+y^3}$

Let $x=ky$

$= \frac{k^3}{k^2+1}$ depends on k

doesn't exist.

61, the line is $x=2$.

$f(x,y) = 3y$.

$\Rightarrow$ the slope is 3. at the plane $x=2$.

65 $\frac{\partial}{\partial x}\left((xy)+z^3x-2yz\right) = 0$

$\Rightarrow y+z^3+3z^2x\cdot\frac{\partial z}{\partial x} - 2y\cdot\frac{\partial z}{\partial x} = 0$

$\Rightarrow \frac{\partial z}{\partial x} = \frac{-y-z^3}{-2y+3z^2x}$

$= \frac{-2}{1} = -2.$

72. $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x}}$ when $x\geq 0$.

$= 2x$ when $x<0$.

$\frac{\partial f}{\partial y} = 0$ always.

$f_{xy} = 0$ always

$f_{yx} = 0$ always.

88. Yes. Since the 1st derivative exists,

the limit $\frac{f(x,y)-f(x+\Delta x,y)}{\Delta x}$ makes sense

$\Rightarrow$ $f(x,y)$ is continuous in x inside the domain

92. $f_x(0.0)=0$

$f_y(0.0)=0$

It's not diffrentiable,

Sine f is not continous at (0.0)

14.4 1. a). $\frac{dw}{dt}=2x\frac{dx}{dt}+2y\cdot\frac{dy}{dt}$

$=2\cdot\cos t\cdot(-\sin t)+2\cdot\sin t\cdot\cos t$

$=0$

b). $\frac{dw}{dt}=0$ at $t=\pi$.

25. $\frac{d}{dx}(x^3-2y^2+xy)$

$=3x^2-4y\cdot\frac{dy}{dx}+y+x\cdot\frac{dy}{dx}=0.$

$\Rightarrow 3-4\frac{dy}{dx}+1+\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{4}{3}.$

31. $\cancel{\frac{d}{dx}}\ \frac{\partial}{\partial x}\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-1\right)=0$

$\Rightarrow -\frac{1}{x^2}+0+-\frac{1}{z^2}\cdot\frac{\partial z}{\partial x}=0$

$\Rightarrow -\frac{1}{4}+\left(-\frac{1}{36}\right)\cdot\frac{\partial z}{\partial x}=0$

$\Rightarrow \frac{\partial z}{\partial x}=-9.$

33. $\frac{\partial w}{\partial r}=2(x+y+z)\cdot(1+\sin(r+s)+\cos(r+s))$

plug in $r=1$ $s=-1$

$x=2$

$y=1$

$z=0$

$\Rightarrow \frac{\partial w}{\partial r}=3\cdot 2\cdot(1-0+1)$

$=12.$

39. $\frac{\partial w}{\partial t}=\cancel{\frac{d}{dx}}\ f'(x)\cdot 2t$

$=e^{(s^3+t^2)}\cdot 2t$

$\frac{\partial w}{\partial s}=e^{(s^3+t^2)}\cdot 3\cdot s^2.$

43. $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=\frac{\partial f}{\partial u}-\frac{\partial f}{\partial w}+\frac{\partial f}{\partial v}-\frac{\partial f}{\partial u}$

$+\frac{\partial f}{\partial w}-\frac{\partial f}{\partial v}=0.$

51. $F'(x)=\int_0^{x^2}\frac{3x^2}{2\sqrt{t^4+x^3}}\,dt+2x\sqrt{x^8+x^3}$

14.5. 1. $\nabla f = (1, -1)$

3. $\nabla g = (y^2, 2xy)\Big|_{(2,-1)}$

$= (1, -4)$

7. $\nabla f = \left(2x + \frac{z}{x},\ 2y,\ -4z + \ln x\right)\Big|_{(1,1,1)}$

$= (3, 2, -4)$

11. $\nabla f = (2y, 2x - 6y)\Big|_{(5,5)} = (10, -50)$

$\left((10, 50) \cdot \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right) = 8 + 30 = 38.$

23. $\nabla f = \left(\frac{2}{x}, \frac{2}{y}, \frac{2}{z}\right)$

Increase most rapidly. $\frac{1}{\sqrt{3}} \cdot (1, 1, 1)$

Decrease most rapidly $-\frac{1}{\sqrt{3}}(1, 1, 1)$

26. $\nabla f = (2x, -1)$

Tangent line: $(2\sqrt{2}, -1)\begin{pmatrix} (x - \sqrt{2}) \\ (y - 1) \end{pmatrix} = 0.$

29. a). $\nabla f = (2x - y,\ 2y - x - 1)$

plug in $(1, -1)$

$= (3, -4)$

max $(D f(1, -1))$ ...

31. $\nabla f\Big|_{3,2} = (y, x+2y)\Big|_{(3,2)} = (2, 7)$

$\vec{u} = (7, -2)\cdot\frac{1}{\sqrt{53}}$ or $(7, -2)\frac{-1}{\sqrt{53}}$

36. a) $(1, 1, -1)\cdot\frac{1}{2\sqrt{3}}$ is the ~~deri~~ Gradient.

b) $(1, 1, 0)\cdot\frac{1}{\sqrt{2}} \cdot (1, 1, -1)\cdot\frac{1}{2\sqrt{3}} = \frac{1}{\sqrt{6}}$.

14.6.

1. $f(x, y, z) = x^2+y^2+z^2-3$,

$\nabla f = (2x, 2y, 2z)$

$\Rightarrow$ Tangent plane: $2(x-1)+2(y-1)+2(z-1)=0$.

Normal line is: $\begin{cases} x = 2t+1 \\ y = 2t+1 \\ z = 2t+1 \end{cases}$

3. a). $\nabla f = (-2x, 0, 2)\Big|_{(2,0,2)} = (-4, 0, 2)$

$-4(x-2)+2(z-2)=0$. Tangent plane.

b) Normal line is $x = -4t+2$

$y = 0$

$z = 2t+2$.

9. $\nabla f = \left(\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}, -1\right)\Big|_{(1,0,0)}$

$= (2. \ 0. \ -1)$

Tangent surface.

$2(x-1) + 0 + (-1)\cdot z = 0$

$\Rightarrow \quad z = (2x-1)$

13. $y^2+2z=3$. in $x=1$ plane.

$\nabla(y^2+2z) = (2y \ 2)\Big|_{(1.1)} = (2. \ 2)$

$$\begin{cases} x = 1 \\ y = 2t+1 \\ z = 2t+1 \end{cases}$$ is the parametic line.

19. $\nabla f = \frac{1}{2}\left(\frac{2x}{x^2+y^2+z^2}, \frac{2y}{x^2+y^2+z^2}, \frac{2z}{x^2+y^2+z^2}\right)\Big|_{(3.4.12)}$

$= \left(\frac{2}{169}. \ \frac{2}{169}. \ \frac{2}{169}\right)$

$\Delta f = 0.1 \cdot (3. \ 6. \ -2) \cdot \frac{1}{\sqrt{3^2+6^2+2^2}} \cdot \frac{2}{169}(1. \ 1. \ 1)$

$= 0.1 \cdot \frac{2}{169}.$

27. $\nabla f = (3. -4)$

a). $f \approx f(0.0) + \nabla f\big|_{(0.0)} \cdot \begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$

$= 5 + 3x - 4y$

b). $f \approx f(1.1) + \nabla f\big|_{(1.1)} \cdot \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$

$= 4 + 3(x-1) - 4(y-1)$

33. $\nabla f = (2x-3y, -3x)\big|_{(2,1)} = (1,-6)$

$f \approx 3+(x-2)-6(y-1)$

The magnitude is $\frac{1}{2}M\cdot(0.1+0.1)^2 = 0.06$

53. $f(a,b,c,d) = ad-bc$. $|a|$ large

$\Rightarrow$ f is sensitive to d.

57. $f(x,y,z) = x^2+y^2-z-3$

$\nabla f = (2x, 2y, -1)$

$t=1$

$\nabla f\big|_{t=1} = (2x, 2y, -1)\big|_{(1,1,-1)} = (2,2,-1)$

$r'(t)\big|_{t=1} = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{4}\right)$ proportional to

$(2,2,-1)$

58. $r(1) = (1,1,1)$

$\nabla f\big|_{(1,1,1)} = \cancel{2x\ldots}\ (2x, 2y, -1) = (2,2,-1)$

$r'(1) = \left(\frac{1}{2}, \frac{1}{2}, 2\right)$

$\left(\frac{1}{2}, \frac{1}{2}, 2\right)\cdot(2,2,-1) = 0$

$\Rightarrow$ $r'(1)$ is orthogonal to ∇f.

14.7

1. $\nabla f = (2x+y+3,\ x+2y-3)$

$\Rightarrow$ critical point $(-3,3)$

$\nabla^2 f = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \geq 0$

$\Rightarrow (-3,3)$ is local minimum.

22. $\nabla f = 0 \Rightarrow (k\pi, 0)$

$\nabla^2 f = \begin{pmatrix} 0 & (-1)^k \\ (-1)^k & 0 \end{pmatrix}$ at $(k\pi, 0)$

It's saddle pt.

31 $\nabla f = 0 \Rightarrow x = 1$
$y = 2$.

$f(1,2) = -5$

on $x=0$. $f(0,y) = y^2 - 4y + 1$ $y \in [0,2]$.

~~f(0,2) = … max~~

~~f~~

$f(0,0) = 1$ max

$f(0,2) = -3$ min.

on $y=2$. $x \in (0,1)$

$f(x,2) = 2x^2 - 4x - 3$.

$f(0,2) = -3$ max

on $y = 2x$

$f(x,2x) = 2x^2 - 4x + 4x^2 - 8x + 1$

$= 6x^2 - 12x + 1$.

$x \in (0,1)$.

$\Rightarrow$ max $= 1$ min $= -5$

$\Rightarrow$ maxima $= 1$ at $(0,0)$
minima $= -5$ at $(1,2)$

33. minima $= 0$ when $(x,y) = (0,0)$
maxima $= 4$ when $(x,y) = (0,2)$

49 ~~tat~~ Notice when $x \in (-3,2)$

$6 - x - x^2 > 0$.

$\Rightarrow a = -3$
$b = 2$.

$\int_{-3}^{2} (6 - x - x^2)\,dx = \frac{49}{3}$

43. a) $(f_x, f_y) = 0 \Rightarrow (x,y) = (0,0)$

$\nabla^2 f = \begin{pmatrix} 2 & -4 \\ -4 & 2 \end{pmatrix}$ det < 0.

$\Rightarrow (0,0)$ is saddle point.

44 a). $\cancel{f(x,y)}$ $f = x^2y^2$

$(0,0)$ is minima.

Since $f(x,y) \geq 0$ always.

45. $k=0$: $f = x^2+y^2$

$\nabla f = (2x, 2y)$

$\nabla f = 0 \Leftrightarrow (x,y) = 0$.

$k \neq 0$: $f = x^2+y^2+kxy$

$\nabla f = (2x+ky,\ 2y+kx)$

$(2x+ky,\ 2y+kx)\big|_{(0,0)} = (0,0)$

$\Rightarrow$ critical point.

50. $f(x,y,z) = x^2+y^2+10-z$.

$\nabla f = (2x,\ 2y,\ -1)$

The normal vector of $x+2y-z=0$

is $(1, 2, -1)$

$\Rightarrow$. $x = \frac{1}{2}$ $y = 1$

$\Rightarrow$. $(\frac{1}{2}, 1, \frac{45}{4})$

54. $x+y+z=3$.

max xyz ?

$xyz \leq (\frac{x+y+z}{3})^3 = 1$ when $x=y=z$

60. $\nabla f = (2x+2y-1,\ 2y+2x-1)$

$\Rightarrow$ $2x+2y=1$ makes (x,y) critical points

$\Rightarrow f(x,y)\big|_{2x+2y=1} = \frac{3}{4}$

Notice $f(x,y) = (x+y)^2-(x+y)+1$

$0 \leq x+y \leq 2$.

$f(0,0) = 1$ $f(1,1) = 3$.

$\Rightarrow$ $\min f = \frac{3}{4}$ when $2x+2y=1$ $\max f = 3$ when $(x,y)=(1,1)$

61. a). $f(t) = 2\cos t + 2\sin t$

$= 2\sqrt{2}\sin(t+\frac{\pi}{4})$

$t \in [0, \pi]$.

$\Rightarrow \max f(t) = 2\sqrt{2}$. when $t = \frac{\pi}{4}$.

$\min f(t) = 2\sqrt{2}\cdot(-\frac{\sqrt{2}}{2})$

$= -2$ when $t = \pi$.

$f(t) = 2\cdot\sqrt{2}\cdot\sin(t+\frac{\pi}{4})$

$t \in [0, \frac{\pi}{2}]$.

$\max f(t) = 2\sqrt{2}$ $t = \frac{\pi}{4}$

$\min f(t) = 2$. $t = 0$. or $\frac{\pi}{2}$.

b). $g(t) = 4 \cdot \cos t \cdot \sin t$

$= 2 \cdot \sin 2t$

$t \in (0, \pi)$

$\Rightarrow \max g(t) = 2$. when $t = \frac{\pi}{4}$

$\min g(t) = -2$ when $t = \frac{3}{4}\pi$

c) $h(t) = 4 + x^2 = 4 + 4 \cdot \cos^2 t$

$t \in (0, \pi)$

$\max h(t) = 8$ when $t = 0, \pi$

$\min h(t) = 4$ when $t = \frac{\pi}{2}$.