13.1

5. At $t=\frac{\pi}{4}$: position: $\frac{\sqrt{2}}{2}i+\frac{\sqrt{2}}{2}j$

velocity: $\frac{\sqrt{2}}{2}i+(-\frac{\sqrt{2}}{2})j$

acc : $-\frac{\sqrt{2}}{2}i+(-\frac{\sqrt{2}}{2})j$.

At $t=\frac{\pi}{2}$: position: $i+0j$

velocity: $0i-j$

acc : $-i+0j$.

7. At $t=\pi$: position: $(\pi, 2)$

velocity: $2i+0j$

acc : $0i-j$.

At $t=\frac{3\pi}{2}$ position: $(\frac{3\pi}{2}+1)i+j$

velocity: $i-j$

acc : $-i$

9. velocity: $r'(t)=i+2tj+2k$

$r''(t)=2j$.

speed at $t=1$: $i+2j+2k$

speed $=\sqrt{1^2+2^2+2^2}=3$

direction: $\frac{1}{3}i+\frac{2}{3}j+\frac{2}{3}k$.

15. $r'(t) = 3i + \sqrt{3}\,j + 2tk$

$r''(t) = 2k.$

$$\cos\theta = \frac{r'(t) \cdot r''(t)\big|_{t=0}}{|r'(t)| \cdot |r''(t)|\,\big|_{t=0}}$$

$$= 0$$

$\Rightarrow \theta = 90^\circ.$

19. $r'(t) = \cos t\, i + (2t + \sin t)\, j + e^t k.$

$r(0) = 0\, i + (-1)\, j + k.$

$r'(0) = i + 0 j + k.$

$$\therefore \begin{cases} x(t) = t \\ y(t) - (-1) = 0 \cdot (t - 0) \\ z(t) - 1 = (t - 0) \end{cases}$$

$$\Rightarrow \begin{cases} x = t \\ y + 1 = 0 \\ z = t + 1 \end{cases}$$

21. $r(1) = 0 \cdot i + 0 j + 0 k.$

$r'(1) = \frac{1}{t} \cdot i + \left(1 + \frac{3}{(t+2)^2}\right) j + (\ln t + 1)\, k. \Big|_{t=1}$

$= i + \frac{4}{3} j + k.$

~~$x = t$~~ $\quad x = t - 1$

~~$y = \frac{4}{3}(t$~~ $\quad y = \frac{4}{3}(t - 1)$

2). i). Yes the speed is 1.

ii). Yes $r''(t)\cdot r'(t) = 0$

iii). counter-clockwise.

iv). Yes. it begins at. (1.0)

25. $y^2 = 2x$

At (2.2). $y' = \frac{\sqrt{2}}{2}\cdot\frac{1}{\sqrt{x}}\Big|_{x=2} = \frac{1}{2}$.

we can assume the velocity is $(\bar{x}.\ \frac{1}{2}\bar{x})$

$\bar{x}^2 + (\frac{1}{2}\bar{x})^2 = 25 \quad \Rightarrow \quad \bar{x} = \sqrt{20}$

$\Rightarrow$ velocity is $(\sqrt{20}, \sqrt{5})$

27. $\frac{d(r\cdot r)}{dt} = 2\cdot\frac{dr}{dt}\cdot r = 0$.

$\Rightarrow \quad |r|^2 = \text{Const}.$

28 we know. $\frac{d}{dt}(u\cdot v) = \frac{du}{dt}\cdot v + u\cdot\frac{dv}{dt}$

$\frac{d}{dt}(u\times v) = \frac{du}{dt}\times v + u\times\frac{dv}{dt}$.

$\Rightarrow \frac{d}{dt}(u\cdot v\times w) = \frac{du}{dt}\cdot(v\times w) + u\cdot\frac{d}{dt}(v\times w)$

$= \frac{du}{dt}\cdot(v\times w) + u\cdot(\frac{dv}{dt}\times w) + u\cdot(v\times\frac{dw}{dt})$

ii) $\frac{d}{dt}\cdot\left(r\left(\frac{dr}{dt}\times\frac{d^2r}{dt^2}\right)\right) = \overset{0}{\overset{\parallel}{\cancel{\frac{dr}{dt}\cdot\left(\frac{dr}{dt}\times\frac{d^2r}{dt^2}\right)}}} + \overset{0}{\overset{\parallel}{\cancel{r\cdot\left(\frac{d^2r}{dt^2}\times\frac{d^2r}{dt^2}\right)}}}$

$+\ r\cdot\left(\frac{dr}{dt}\times\frac{d^3r}{dt^3}\right)$

32.

$$\lim_{t\to t_0} r_1 \times r_2 = \lim_{t\to t_0} \begin{vmatrix} i & j & k \\ f_1(t) & f_2(t) & f_3(t) \\ g_1(t) & g_2(t) & g_3(t) \end{vmatrix}$$

$$= \lim_{t\to t_0} \Big((f_2 \cdot g_3 - g_2 f_3) \cdot i - (f_1 \cdot g_3 - g_1 f_3)\, j + (f_1 \cdot g_2 - f_2 \cdot g_1) k \Big)$$

Since $\lim_{t\to t_0} r_1(t) = A = (A_1, A_2, A_3)$

$\lim_{t\to t_0} r_2(t) = B = (B_1, B_2, B_3)$

$$= (A_2 \cdot B_3 - A_3 \cdot B_2)\, i - (A_1 \cdot B_3 - A_3 \cdot B_1)\, j + (A_1 B_2 - A_2 \cdot B_1) k$$

$$= A \times B.$$

34. $\vec{u} = \text{const.} \Rightarrow \vec{u} = (u_1\ u_2\ u_3) = \text{const}$

$$\frac{d\vec{u}}{dt} = \left(\frac{du_1}{dt}, \frac{du_2}{dt}, \frac{du_3}{dt}\right) = \left(\frac{d\,\text{const}}{dt}, \frac{d\,\text{const}}{dt}, \frac{d\,\text{const}}{dt}\right) = (0.\ 0.\ 0) = \vec{0}.$$

13.2.

1 $\int_0^1 t^3 \cdot i + 7j + (t+1)k \cdot dt$

$= \frac{t^4}{4} i + 7t j + (\frac{t^2}{2} + t)k \Big|_0^1$

$= \frac{1}{4} j + 7j + \frac{3}{2} k$

6. $\int_0^1 \left(\frac{2}{\sqrt{1-t^2}}\, i + \frac{\sqrt{3}}{1+t^2} \cdot j\right) dt$

$= 2 \cdot \arcsin t\, i + \sqrt{3} \tan t \Big|_0^1$

$= 2 \cdot \frac{\pi}{2} i + \sqrt{3} \cdot \frac{\pi}{4} j$

13. $\frac{dr_1}{dt} = \frac{3}{2}(t+1)^{\frac{1}{2}}$ $\quad r_1(0)=0 \quad \Rightarrow \quad r_1(t) = (t+1)^{\frac{3}{2}} - 1$

$\frac{dr_2}{dt} = e^{-t}$ $\quad r_2(0)=0 \quad \Rightarrow \quad r_2(t) = 1 - e^{-t}$

$\frac{dr_3}{dt} = \frac{1}{t+1}$ $\quad r_3(0)=1. \quad \Rightarrow \quad r_3(t) = \ln(t+1)+1$

20. a) Suppose the initial speed is v, angel is α.

The range is $\frac{2 \cdot v \cdot \sin\alpha}{g} \cdot v \cdot \cos\alpha = \frac{v^2 \sin 2\alpha}{g}$

when replace v by $2v$, it will generate a coefficient 4

b) If we want to double the range we should let v be $\sqrt{2}v$.

If we want to double the height:

height $= \frac{\frac{1}{2}(v \cdot \sin\alpha)^2}{g}$, we should also let v be $\sqrt{2}v$

23. a). $10 \text{ miles} = \frac{v^2 \cdot \sin 2\alpha}{g} = \frac{v^2}{g}$.

$\Rightarrow v = \sqrt{10 \text{ miles} \times g} \approx 12.5 \, m/s$.

b). $\frac{v^2 \cdot \sin 2\alpha}{g} = 6 \text{ miles} \quad \Rightarrow \quad \sin 2\alpha = \frac{3}{5}$.

$\alpha = \frac{1}{2} \cdot \arcsin\frac{3}{5}$

or $\frac{1}{2}(\pi - \arcsin\frac{3}{5})$

29. In x - axis : $\frac{d^2x}{dt^2} = 0$, $x(0) = x_0$ $x'(0) = V_0 \cos\alpha$

$\Rightarrow$ $x(t) = x_0 + V_0 \cos\alpha . t$.

In y - axis $\frac{d^2y}{dt^2} = -g$ $y(0) = y_0$ $y'(0) = V_0 \cdot \sin\alpha$

$\Rightarrow$ $y(t) = y_0 + V_0 \cdot \sin\alpha t - \frac{1}{2} g t^2$.

32. in x-axis: $\frac{d^2x}{dt} = -k \cdot \frac{dx}{dt}$ $x(0) = 0$ $x'(0) = V_0 \cdot \cos\alpha$

$\Rightarrow$ $e^{kt} \cdot \frac{dx}{dt} = \text{Const} = V_0 \cdot \cos\alpha$

$\Rightarrow$ $\frac{dx}{dt} = V_0 \cdot \cos\alpha (e^{-kt})$

$\Rightarrow$ $x(t) = \frac{V_0}{k} (1 - e^{-kt}) \cdot \cos\alpha$

in - y - axis. $\frac{d^2y}{dt^2} = -g - k \cdot \frac{dy}{dt}$ $y(0) = 0$ $y'(0) = V_0 \cdot \sin\alpha$.

$\Rightarrow$ $e^{kt} \frac{dy}{dt} = -gt + V_0 \cdot \sin\alpha$.

$\Rightarrow$ $\frac{dy}{dt} = (-gt + V_0 \sin\alpha) e^{-kt}$

$\Rightarrow$ $y(t) = (1 - e^{-kt}) \cdot \frac{V_0}{k} \cdot \sin\alpha + \frac{g}{k^2} (1 - kt - e^{-kt})$

40. a) $\vec{r}(t) = (r_1(t), r_2(t), r_3(t))$

r_i are continuous. $i = 1, 2, 3$.

By continuous function property (scalar form).

$u \cdot r_1, u r_2, u r_3$ are continous.

$\Rightarrow$ $u \cdot r$. continuous.

b). $\frac{d u\cdot\vec{r}}{dt} = \left(\frac{du\cdot r_1}{dt}, \frac{du\, r_2}{dt}, \frac{du\cdot r_3}{dt}\right)$

$$= \left(u\cdot\frac{dr_1}{dt} + \frac{du}{dt}\cdot r_1,\ \frac{du}{dt} r_2 + u\cdot\frac{dr_2}{dt},\ \frac{du}{dt} r_3 + \frac{dr_3}{dt} u\right)$$

$$= u\frac{dr}{dt} + \vec{r}\cdot\frac{du}{dt}.$$

42. a) $\vec{r}(t) = (r_1(t).\ r_2(t).\ r_3(t))$. we can apply fundemental Thm to $r_1\ r_2\ r_3$.

we know $\frac{d}{dt}\int_a^t r_i(\tau)\cdot dt = r_i(t)$. for $\forall t \in [a, b]$

$\Rightarrow \frac{d}{dt}\int_a^t \vec{r}(\tau)\cdot dt = \vec{r}(t)$ holds.

b). Suppose. $R(t)+C$ is ~~the~~ one antiderivative. of $r(t)$.

$\frac{d}{dt}\int_a^t r(\tau)\, dt = r(t) \quad \Rightarrow \quad \int_a^t r(\tau)\cdot d\tau = R(t)+C.$

$\int_a^a r(\tau)\cdot d\tau = R(a)+C = 0.$

$\int_a^b r(t)\cdot d\tau = R(b)+C = R(b) - R(a).$