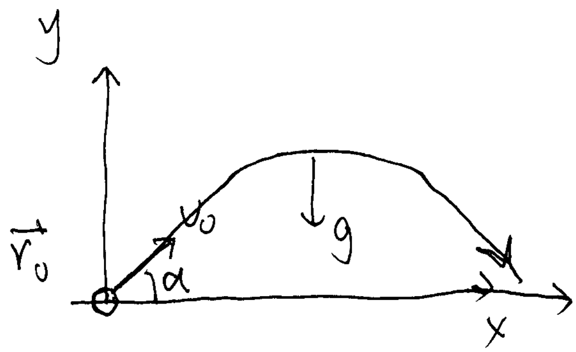
8/25 Sec. 13.2.

example: the Ideal Projectile Motion.



$$\vec{V_0} = (|\vec{V_0}|\cos\alpha,\ |\vec{V_0}|\sin\alpha)$$

$$ma = F$$

$$m \cdot \frac{d^2\vec{r}}{dt^2} = -mg\vec{j}$$

$$\begin{cases} \dfrac{d^2\vec{r}}{dt^2} = -g\vec{j} & ① \\ \vec{r}(0) = \vec{r_0} \\ \dfrac{d\vec{r}}{dt}(0) = \vec{V_0} \end{cases}$$

Integral ① w.r.t time t.

$$\int_0^t \frac{d^2\vec{r}}{ds^2}\,ds = \int_0^t -g\vec{j}\,ds.$$

$$\frac{d\vec{r}}{ds}(t) - \underline{\frac{d\vec{r}}{ds}(0)} = -(gt)\vec{j}$$

Eqn for velocity $\leftarrow$ $\dfrac{d\vec{r}}{ds}(t) = \vec{V_0} - (gt)\vec{j}$

$$\underline{\int_0^t \frac{d\vec{r}}{ds}\,ds} = \int_0^t (\vec{V_0} - (gt)\vec{j})\,ds.$$

$\downarrow$

Eqn for position: $\vec{r}(t) - \vec{r}(0) = \vec{V_0}t - (\frac{1}{2}gt^2)\vec{j}$

$$\vec{r}(t) = \vec{r}(0) + \vec{V_0}t - (\tfrac{1}{2}gt^2)\vec{j}$$

$$\vec{r}(0) = (x_0, y_0)$$

$$\vec{v}(0) = (|\vec{v_0}|\cos\alpha, |\vec{v_0}|\sin\alpha)$$

$$\vec{r}(t) = \left(x_0 + |\vec{v_0}|\cos\alpha t,\ y_0 + |\vec{v_0}|\sin\alpha t - \tfrac{1}{2}gt^2\right)$$

In Summary: (If $(x_0, y_0) = (0,0)$)

Ideal Projectile Motion:

$$\boxed{\begin{aligned}\vec{r}(t) &= \cancel{(V_0\cos\alpha)t\,\vec{i}} \\ &\underbrace{(V_0\cos\alpha t)}_{\neq\ \cos(\alpha t)}\vec{i} + \left(\underline{V_0\sin\alpha t - \tfrac{1}{2}gt^2}\right)\vec{j}\end{aligned}}$$

$$x(t) = V_0\cos\alpha\, t \qquad t = \frac{x(t)}{V_0\cos\alpha} = \frac{x}{V_0\cos\alpha}$$

$$y(t) = V_0\sin\alpha\, \underline{t} - \tfrac{1}{2}g\underline{t^2}$$

plug into

Trajectory is a parabola:

$$y = V_0\sin\alpha\cdot\frac{x}{V_0\cos\alpha} - \frac{1}{2}g\frac{x^2}{V_0^2\cos^2\alpha}$$

$$y = \underline{A}x^2 + Bx + C.$$

$$A = \frac{-g}{2V_0^2\cos^2\alpha} \qquad B = \tan\alpha.$$

① Max Height:

② Flight time:

③ Range:

$(V_0\cos\alpha\, t,\ V_0\sin\alpha\, t - \frac{1}{2}gt^2)$

① $y_{max} = \max(\underline{-\frac{1}{2}gt^2 + V_0\sin\alpha\, t})$

$y = Ax^2 + Bx + C.$ $\qquad A = -\frac{1}{2}g$

$= A(\underline{x^2 + \frac{B}{A}x} + \frac{C}{A})$ $\qquad B = V_0\sin\alpha$

$= A[(x + \frac{B}{2A})^2 - (\frac{B}{2A})^2 + \frac{C}{A}]$ $\qquad C = 0$

$= A(x + \frac{B}{2A})^2 + \underline{A(\frac{C}{A} - \frac{B^2}{4A^2})}$

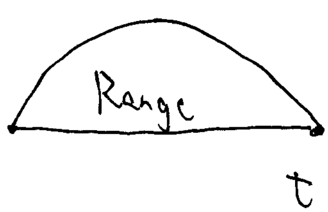
$= A(x + \frac{B}{2A})^2 + \boxed{\frac{4AC - B^2}{4A}}$

$y = -\frac{1}{2}g(x + \frac{V_0\sin\alpha}{-g})^2 + \cancel{\frac{V_0^2\sin^2\alpha}{4g^2}} \quad \frac{-V_0^2\sin^2\alpha}{-2g}$

$= \underline{-\frac{1}{2}g(x - \frac{V_0\sin\alpha}{g})^2} + \frac{V_0^2\sin^2\alpha}{2g}$

$y_{max} = \frac{V_0^2\sin^2\alpha}{2g} \quad \leq 0 \longrightarrow$ Max Height.

②



$y(t) = \underline{0 = -\frac{1}{2}gt^2 + V_0\sin\alpha\, t}$

$\frac{1}{2}gt = V_0\sin\alpha$

$t = \frac{2V_0\sin\alpha}{g} \rightarrow$ Flight time.

③ $X(\frac{2V_0\sin\alpha}{g}) = V_0\cos\alpha \cdot \frac{2V_0\sin\alpha}{g}$

$= \frac{V_0^2}{g}\underline{2\sin\alpha\cos\alpha} = \underbrace{\frac{V_0^2}{g}\sin 2\alpha}_{\text{Range.}}$