

MAT201A Homework 6

Fall 2019

Professor Qinglan Xia

Due Date: Wednesday, November 6th at 9:00am

1. Exercise 5.3 of the textbook, page 121.
2. Prove that the sequence (f_n) defined in Example 5.11 in the textbook is a Schauder basis of $(C([0, 1]), \|\cdot\|_\infty)$.
3. Exercise 5.4 of the textbook, page 121.
4. Exercise 5.5 of the textbook, page 121. Is it possible that $\|K\| = 1$ but $\|K^2\| = 0$? Justify your answers.
5. Let $\|\cdot\|_1$ and $\|\cdot\|_2$ be two norms on a linear space X . We say $\|\cdot\|_2$ is stronger than $\|\cdot\|_1$ if for any sequence (x_n) in X , $\|x_n\|_2 \rightarrow 0$ always implies $\|x_n\|_1 \rightarrow 0$. Show that $\|\cdot\|_2$ is stronger than $\|\cdot\|_1$ if and only if there exists a constant $C > 0$ such that

$$\|x\|_1 \leq C\|x\|_2, \quad \forall x \in X.$$