

MAT201A Homework 9
Fall 2019

Professor Qinglan Xia

Due Date: Wednesday, November 27th at 9:00am

1. Exercise 6.1 in the textbook, page 144.
2. Exercise 6.2 in the textbook, page 144.
3. Write a detailed proof for Theorem 6.9 in page 128.
4. Prove that there does not exist an inner product on $C([0, 1])$ such that the supremum norm is derived from this inner product.
5. Let M be a subset of a Hilbert space \mathcal{H} . Show that

$$(M^\perp)^\perp = \overline{\text{span}(M)}.$$

6. Let \mathcal{H} be a Hilbert space and $x, y \in \mathcal{H}$. Define the linear transformation R by

$$Rz = \langle y, z \rangle x, \quad \forall z \in \mathcal{H}.$$

Prove that R is bounded and compute its norm.