MAT 201B Homework 3
Winter 2020
Professor Qinglan Xia
Due Date: Wednesday, January 29th at 9:00am

1. Suppose that $f : X \to [-\infty, \infty]$ is $\mu-$integrable. Show that $\forall \epsilon > 0$, there exists $\delta > 0$ such that
$$0 \le \int_A |f| d\mu < \epsilon$$
whenever A is a measurable subset of X with $\mu(A) < \infty$.

2. Show that there are Lebesgue integrable functions $f \in L^1(\mathbb{R})$ and a sequence $\{f_n\}$ with $f_n \in Ł^1(R)$ such that
$$||f - f_n||_{L^1} \to 0$$
but $f_n(x) \to f(x)$ for no x.

3. Consider the convolution
$$(f * g)(x) := \int_{\mathbb{R}} f(x-y)g(y)dy.$$
   - Show that $f * g$ is uniformly continuous when f is integrable and g is bounded.
   - If in addition g is integrable, prove that $(f * g)(x) \to 0$ as $|x| \to \infty$.

4. Exercise 12.8 in the textbook “Applied Analysis”, page 375.

5. Show that the set of simple functions is dense in $L^p(X, \mu)$ for $1 \le p < \infty$.