MAT 201B Homework 4
Winter 2020
Professor Qinglan Xia
Due Date: Wednesday, February 5th at 9:00am

1. For each $N \in \mathbb{N}$, let
$$f_N(x) = \frac{1}{2\pi} \sum_{n=-N}^{N} e^{inx}.$$
Show that for any $g \in L^2(\mathbb{T})$, the sequence $\{f_N * g\}$ converges to g in $||\cdot||_{L^2}$ norm.

2. Let
$$f(x) = a\left(2\chi_{[-\frac{\pi}{2},\frac{\pi}{2}]}(x) - 1\right) \in L^2(\mathbb{T})$$
for some real number a, and let
$$f_n = f * f * \cdots * f$$
be the n-fold convolution of f. If the sequence $\{f_n\}$ converges to a non-zero function g in the $||\cdot||_{L^2}$ norm, find the function g.

3. Suppose $f \in L^2(\mathbb{T})$ and $\frac{\tau}{\pi}$ is an irrational number. If the rotation $g(x) = f(x - \tau)$ satisfies $f = g$, then show that f must be a constant function.

4. Exercise 7.2 in the textbook "Applied Analysis", page 183.

5. Exercise 7.3 in the textbook "Applied Analysis", page 183.