Research Summary

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My research lies in the field of mathematical physics and I am generally interested in algebraic and geometric aspects of quantum field theory and string theory.

1 Factorization Algebra and Index Theory

Supersymmetric field theory leads to exact computations in ∞-dimensional path integrals. This often gives surprising mathematical predictions that are not visible via traditional finite dimensional geometry. The goal of this program is to build up the connection between factorization algebra and index type theories that arise from topological/holomorphic twists of supersymmetric field theories.

The notion of factorization algebra was first introduced by Beilinson and Drinfeld to reformulate chiral algebras arising from two-dim conformal field theories. In quantum field theory, Costello-Gwilliam established the factorization algebra structure of quantum observables via the Batalin–Vilkovisky formalism. This constitutes the basic motivation for this program.

1.1 Topological Quantum Mechanics and Algebraic Index

In [1][2]

We studied topological quantum mechanical model via the free loop space of the phase space and established an exact corresondence between

Batalin–Vilkovisky quantization $\Longleftrightarrow$ Fedesov's deformation quantization

As a result, we found an explicit model of S^1 factorization complex for quantum observables that relates

Renormalization Group Flow (via integrating out modes) = Hochschild-Kostant-Rosenberg Theorem

In this framework, we established an S^1-equivariant BV localization formalism that leads to a geometric proof of the algebraic analogue of Aityah-Singer index theorem by Fedosov and Nest-Tsygan.

In [3], the above results are generalized to orbifold phase space and orbifold index theorem. In [4], we find a stochastic interpretation of this topological model in terms of large variance limit.

In [5]

We study the 2d analogue of the above construction in topological B-model and find Calabi-Yau condition as the quantum consistency condition for solving quantum master equations. This established the low energy effective theory of topological B-twisted σ-model (without coupling to 2d gravity).

1.2 Chiral Conformal Field Theory and Chiral Algebraic Index

In [6]

We studied the two dimensional chiral conformal field theories that arise from chiral deformation of conformal field theories and established the corresponding effective Batalin–Vilkovisky quantization theory. We proved an exact correspondence between

$$\text{quantum master equation} \Longleftrightarrow \text{MC equation for chiral modes Lie algebra.}$$

As an application, we solved the full **higher genus mirror symmetry conjecture on elliptic curves**.

In [7]

Based on the Batalin–Vilkovisky quantization theory established in [6], we studied the corresponding chiral algebra and factorization homology, and established an **elliptic trace map** that intertwines a quasi-isomorphism between Beilinson-Drinfeld's elliptic chiral complex and an explicit BV algebra. This can be viewed as a chiral analogue of quantum Hochschild-Kostant-Rosenberg map. This construction leads to a version of **elliptic chiral algebraic index**, which can be viewed as the chiral analogue of algebraic index formalism as in [2]. The Witten genus arises as the elliptic trace in the presence of the universal background.

In [8][9]

In establishing the chiral trace map in [7], we need to regularize and evaluate (singular) integrals of local correlation functions on the configuration space of Riemann surfaces. Such regularized integral technique is fully established in [8], which gives a precise relation between the elliptic curve integrals (partition function) and the traditional A-cycle integrals (q-trace) of chiral correlation functions on elliptic curves. A version of holomorphic anomaly equation is also established in [9].

2 B-twisted Topological String and Twisted Gauge/Gravity Duality

Mirror symmetry, as a duality of topological field theories, has been a great influence in mathematics. It predicts a duality between symplectic geometry (A-model) and complex geometry (B-model). The A-model has been well-developed in mathematics, known as Gromov-Witten theory, and has been extended into various different contexts. The B-model is related to Calabi-Yau geometry and Hodge theory. The classical geometry of B-model encodes variation of Hodge structures and period maps. The higher genus B-model, which can be viewed as a quantization of variation of Hodge structures, has been a mysterious object and big challenge in mathematics and less known comparing to its mirror theory. Nevertheless, B-model has a beautiful physics description in terms of string field theory on Calabi-Yau manifolds. The open-string field of B-model is proposed by Witten via **holomorphic Chern-Simons** theory, and the closed-string field of B-model is developed by Bershadsky-Cecotti-Ooguri-Vafa, known as **Kodaira-Spencer gravity**. This part of my research concerns mathematical studies and geometric applications of B-model string field theories, as well as its physical applications in string theory and supergravity.

2.1 B-twisted Topological String Field Theory

In [10]

In [10], we discovered an extension of BCOV's Kodaira-Spencer gravity on Calabi-Yau 3-folds to incorporate with gravitational descendants. This extension not only allows us to put Kodaira-Spencer gravity on Calabi-Yau manifolds of arbitrary dimension, but also puts B-model closed string field into the framework that allows quantization in the Batalin-Vilkovisky formalism. This also fits into Zwiebach's general form of closed string field theory. What we find is, however, a *degenerate BV* theory, which is only defined though Feynman diagrams. The B-model closed string field theory is not renormalizable in the usual physics sense, but can be quantized in terms of Costello's effective renormalization theory. This is systematically developed in [10] and reviewed in [11] [12]. The natural appearance of integrable hierarchy in such Kodaira-Spencer gravity was explored in [13].

In [14] [15]

In [14] [15], we developed the B-model open-closed string field theory by coupling Kodaira-Spencer gravity with holomorphic Chern-Simons theory. The coupling is highly nontrivial. At the disk level, the coupling is fully encoded by Kontsevich's graph formula in deformation quantization. At the quantum level, we found a remarkable new anomaly-cancellation mechanism which allows us to quantize Kodaira-Spenser theory coupled to holomorphic Chern-Simons theory directly on target space-time. Our mechanism cancels all anomalies occurring beyond one loop and shows that all counter-terms are uniquely determined by the absence of anomalies. At one loop the anomaly cancellation is analogous to the Green-Schwarz mechanism. In the work [15], we also generalize such Green-Schwarz mechanism to a type I Kodaira-Spencer gravity.

2.2 Solution of Higher Genus Mirror Symmetry Conjecture on Elliptic Curves

In [6]

As a nontrivial test, we consider the quantum B-model on elliptic curves. The quantum Kodaira-Spencer gravity is fully carried out in [6], where all higher loops corrections (as required by BV quantum master equation) are explicitly found in terms of $W_{1+\infty}$-algebras. These data correspond exactly to all Gromov-Witten invariants on the mirror elliptic curve, as a consequence of boson-fermion correspondence. This fully establishes in mathematics the first *compact* Calabi-Yau manifold example of higher genus mirror symmetry.

2.3 Twisted Holography and Koszul Duality

In [16]

Based on the open-closed string field theory as in [14] [15], we described in [16] a conjectured relationship between twisted IIB supergravity in nontrivial **superghost background** and Kodaira-Spencer gravity on $\mathbb{C}^5$. This nontrivial superghost background in supergravity solution is what we called **twisted supergravity**. Thus our conjecture can be formulated as

$$\text{BCOV= Twisted Type II-B Supergravity}$$

In [14] [15], the open and closed string fields are coupled by solving a version of Maurer-Cartan equation

$$\mathcal{MC}(\text{open} \otimes \text{closed})$$

This is in a manifest fashion of Koszul duality. Thus it is natural to expect Koszul duality to play an important role in understanding gauge/gravity duality in twisted background, or **twisted holography**.

In [17]

Koszul duality also arises naturally for boundary operators in quantum field theories with transversal boundary conditions. This suggests a possible extension of the notion of Koszul duality to a large class of algebras living in higher dimensions (the usual notion of algebra can be viewed as living in dimension 1, as in quantum mechanics). In [17], we have indeed pushed one step further and find a notion of quadratic duality for chiral algebras (which live in dimension 2).

2.4 Relative Periods and Open Mirror Symmetry

In [18]

We studied the relative period map on Calabi-Yau manifold in classical topological B-model. This is proposed by Walcher is his study of open string mirror symmetry on Calabi-Yau 3-fold. We generalized GKZ system to relative period maps on toric Calabi-Yau hypersurfaces, which gives an effective way to compute inhomogeneous Picard-Fuchs equations encoding open string instanton counting.

3 Singularity Theory and Landau-Ginzburg Mirror Symmetry

Frobenius manifolds were introduced by Dubrovin as the fundamental algebraic structure of 2d topological field theories. The first series of examples arise from K. Saito's study of primitive period maps associated to the germ of a holomorphic map $f : X \to \mathbb{C}$ with an isolated critical point. The pair (X, f) is termed as the Landau-Ginzburg model, and f is called the superpotential. K. Saito's theory is nowadays recognized as describing LG B-models. Givental extended Saito's primitive period map in Gromov-Witten theory in terms of J-functions, and Barannikov and Kontsevich vastly generalized Frobenius manifold constructions to Calabi-Yau geometries. This part of my research mainly concerns the study of quantum invariants of Landau-Ginzburg (LG) models and their mirror symmetries.

3.1 Frobenius Manifolds of Landau-Ginzburg B-model

Correlation functions of LG B-models were little known beyond ADE and simple elliptic singularities. This was due to the fact that K.Saito's primitive forms are rarely known besides their abstract existence. Explicit formula are only known for the above two simple cases. This problem has been a main obstacle is understanding mirror symmetry between LG models, where LG A-models are mathematically well established as Fan-Jarvis-Ruan-Witten theory.

In [19][20]

In [19], we developed a differential geometric model of K.Saito's primitive form in terms of polyvector fields. This is the Barannikov-Kontsevich construction twisted by the superpotential. In [19], we find a recursive algorithm to compute genus zero invariants of LG B-models in general. This algorithm is fully developed in our later work [20], where it is applied to unimodular exceptional singularities and we proved that they reproduce FJRW-invariants in the mirror. This gives the first examples of mirror theorem between two LG models with central charge > 1. These results are reviewed in works [21] [22]

3.2 Solution of Landau-Ginzburg Mirror Symmetry Conjecture

In [23]

Based on the method of [20], we established a strong reconstruction result for quantum invariants of LG models in [23]. This allows us to determine the full data of LG invariants in terms of simple 3-point and 4-point functions. As an application, we give a complete proof of the general mirror symmetry conjecture between LG models in [23].

3.3 Categorical Primitive Form Theory

In [24]

We developed a categorical analogue of Saito's theory fitting into Kontsevich's framework of homological mirror symmetry. This is based on a study of variation of $\frac{\infty}{2}$-Hodge structures arising from the category of matrix factorizations. As a nontrivial test, the categorical primitive form of A-type singularities are computed and shown to give the same answer as in Saito's theory.

3.4 Deformation Theory- L^2 approach

In [25]

We developed a L^2-Hodge theoretical theory of LG B-models for superpotential with compact critical locus. We introduced the notion of f-twisted Sobolev spaces for the pair (X, f) (which is also compatible with the product structure) and prove the corresponding Hodge-to-de Rham degeneration property via L^2-Hodge theoretical methods when f satisfies an asymptotic condition of strongly ellipticity. This leads to a Frobenius manifold via the Barannikov-Kontsevich construction, unifying the Landau-Ginzburg and Calabi-Yau geometry. As an application, we established Frobenius manifolds for orbifold Landau-Ginzburg B-models which admit crepant resolutions.

3.5 Deformation Theory- Algebraic Approach

In [26] [27]

In [26], we studied orbifold LG B-models in terms of Hochschild theory of G-twisted curved algebras. It is the first time in the literature that we presented explicit 3-point functions for orbifold LG B-models for a large class of invertible polynomials. This computation is based on a construction of G-twisted brace structures and explicit homotopy between Koszul resolution and bar resolution in the G-twisted case. In [27], we carried out some explicit orbifold ADE examples of such noncommutative deformations and showed that the associated quantum invariants match the mirror.

3.6 Primitive Form and Seiberg-Witten Geometry

In [28]

We revealed the connection between the Seiberg–Witten differential of four dimensional N = 2 SCFT arising from isolated rational singularity and the Gelfand–Leray form of K. Saito's primitive form. This result extends the Seiberg–Witten solutions to include irrelevant deformations.

4 Noncommutative Quantum Field Theory

My research in this area mainly concerns geometric and representation theoretic aspects of quantum field theories on noncommutative spaces.

4.1 Noncommutative Chern-Simons Theory

In [29]

We study the noncommutative Chern-Simons theory and the algebra of quantum observables of the Chern-Simons matrix model which was originally proposed by Susskind and Polychronakos to describe electrons in fractional quantum Hall effects. We establish the exact commutation relations for its generators and study the large N limit of its representation. This leads to a complete proof (and extension) of the large N emergence of the u(p) current algebra as proposed by Dorey, Tong and Turner.

In [30]

We study the Hilbert space of the Chern-Simons matrix model from a geometric quantization point of view. We show that the Hilbert space of the Chern-Simons matrix model can be identified with the space of sections of a line bundle on the quiver variety associated to a framed Jordan quiver. We compute the character of the Hilbert space using localization technique. Using a natural isomorphism between vortex moduli space and a Beilinson-Drinfeld Schubert variety, we prove that the ground states wave functions are flat sections of a bundle of conformal blocks associated to a WZW model. In particular they solve a Knizhnik-Zamolodchikov equation. We also define and study the conformal limit of the Chern-Simons matrix model and construct the Yangian action.

4.2 Higher Spin Theory

In [31]

We explore various algebraic aspects of higher spin theory and Vasiliev equation in terms of homotopy algebras. We present a systematic study of unfolded formulation developed for the higher spin equation in terms of the Maurer-Cartan equation associated to differential forms valued in L_∞-algebras. This leads to a closed combinatorial graph formula for all the vertices of higher spin equations in the unfolded formulation.

5 Phenomenology

In [32] [33]

These are some earlier works in phenomenological aspects of quantum field theory and gravity.

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