

# A unified framework for decision making and motor control

Alaa A. Ahmed



Erik Summerside  
CU Boulder



Reza Shadmehr  
Johns Hopkins University



Megan O'Brien  
CU Boulder



Helen Huang  
University of Central Florida



University of Colorado  
Boulder

Integrative  
Physiology  
University of Colorado at Boulder



Neuromechanics  
Lab

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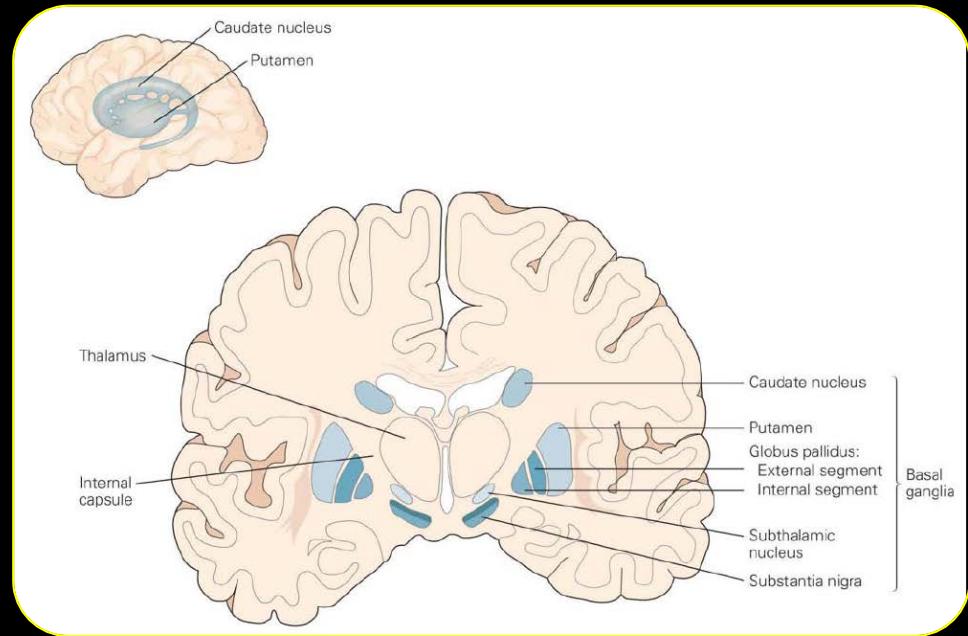
# To move fast or to move slow ...





# Parkinson Disease (PD)

- Bradykinesia
- PD patients “choose” to move more slowly
- Disease of the basal ganglia
- Loss of dopaminergic neurons in the substantia nigra



# Dopamine

Source:

Substantia nigra  
VTA

Projections to:

Striatum  
Frontal Cortex

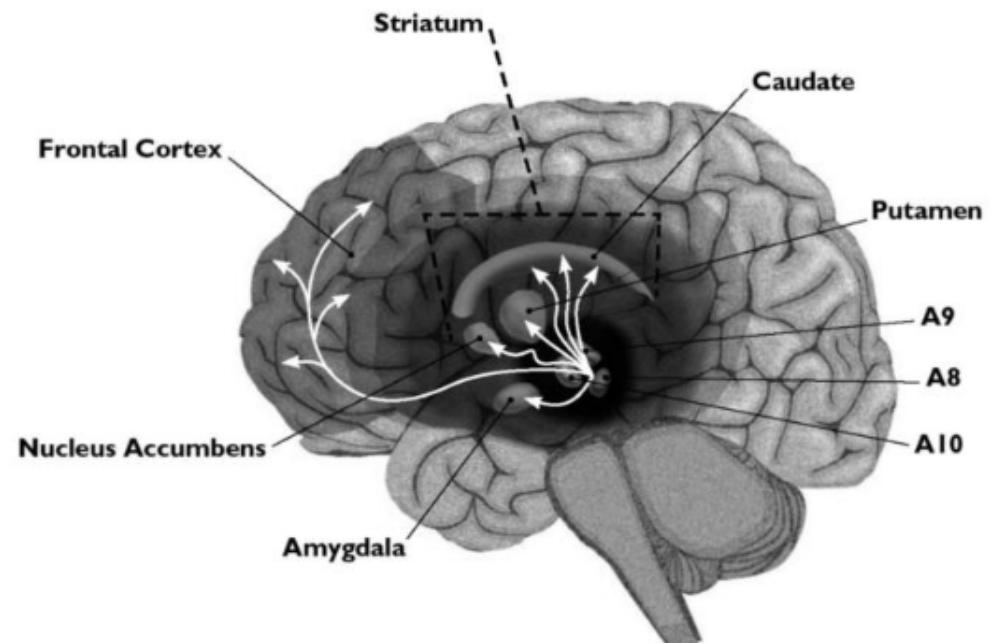
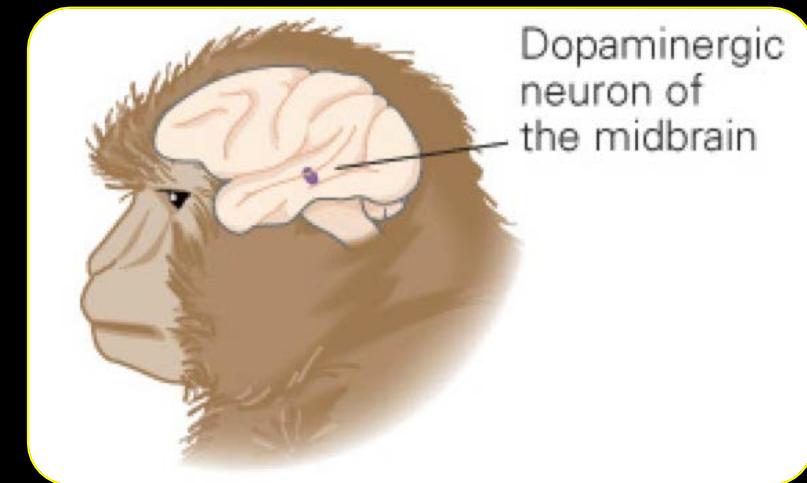
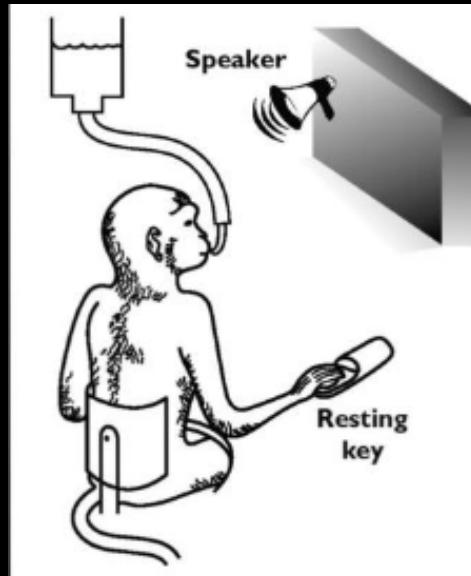


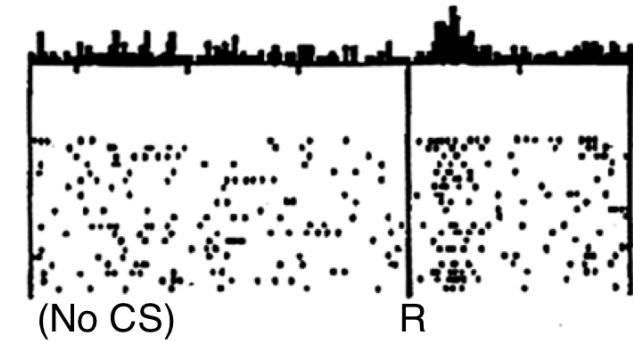
FIGURE 13.2 The A8, A9 and A10 cell groups.

# Dopamine

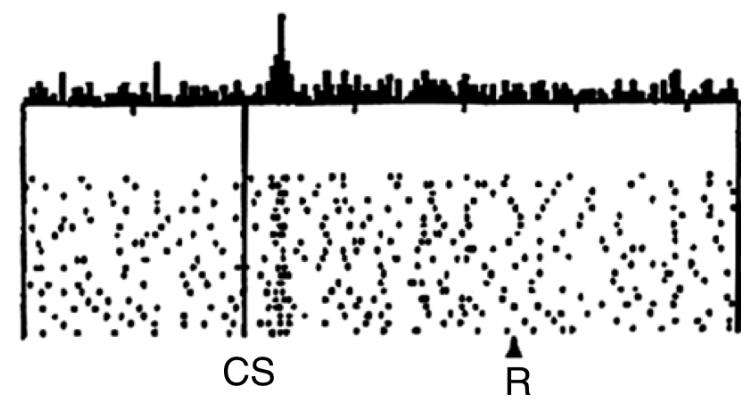


Do dopamine neurons report an error in the prediction of reward?

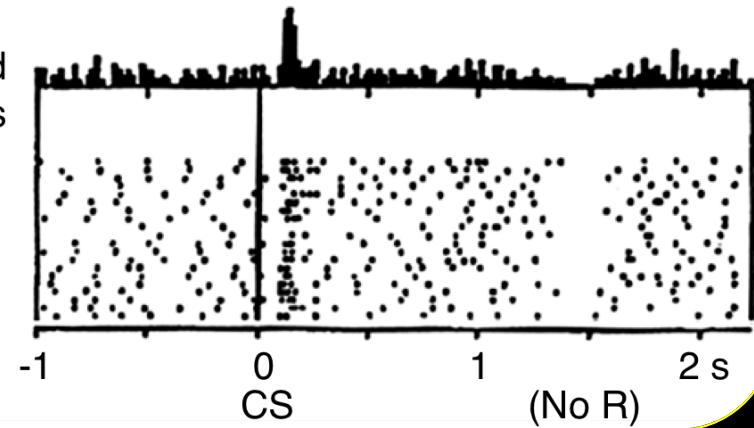
No prediction  
Reward occurs



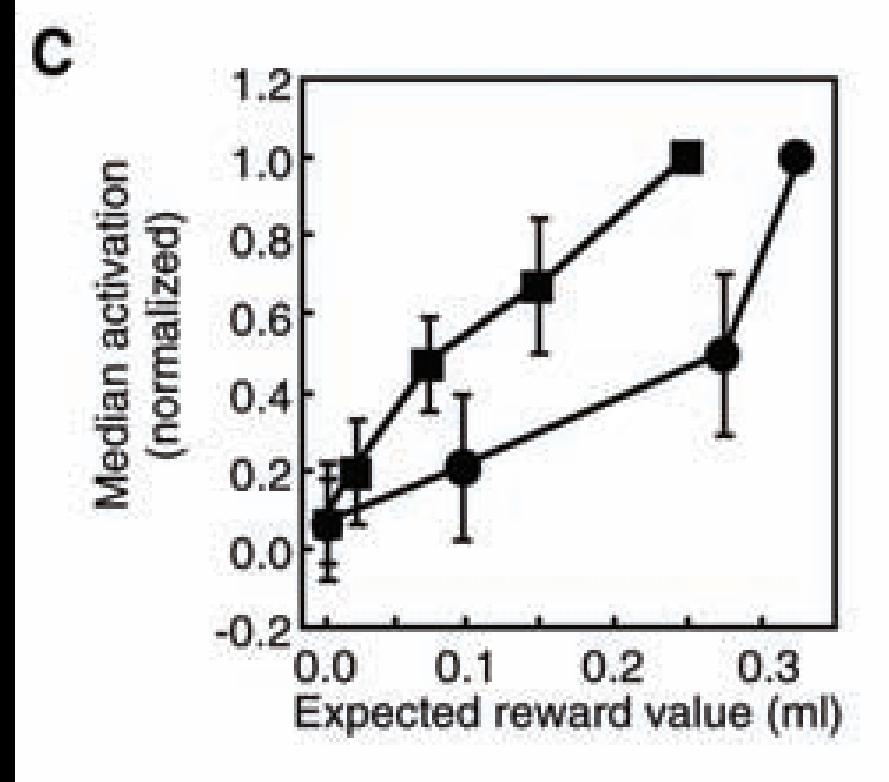
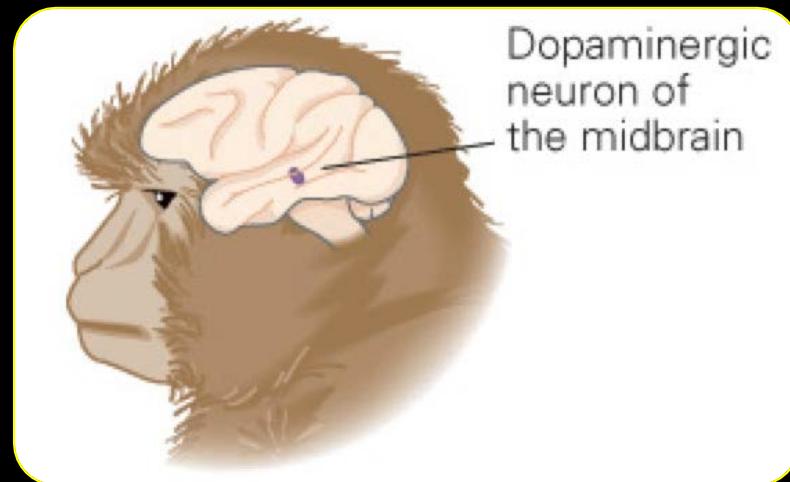
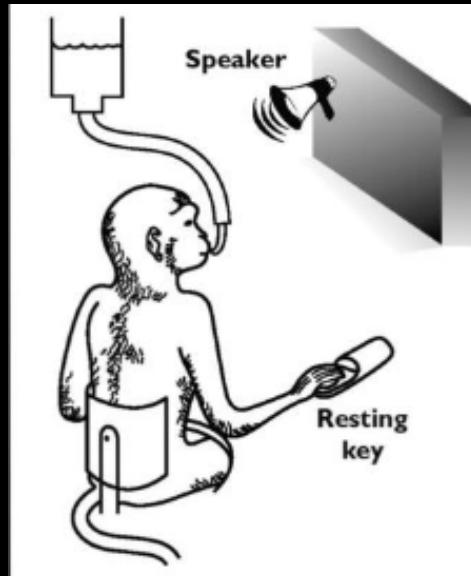
Reward predicted  
Reward occurs



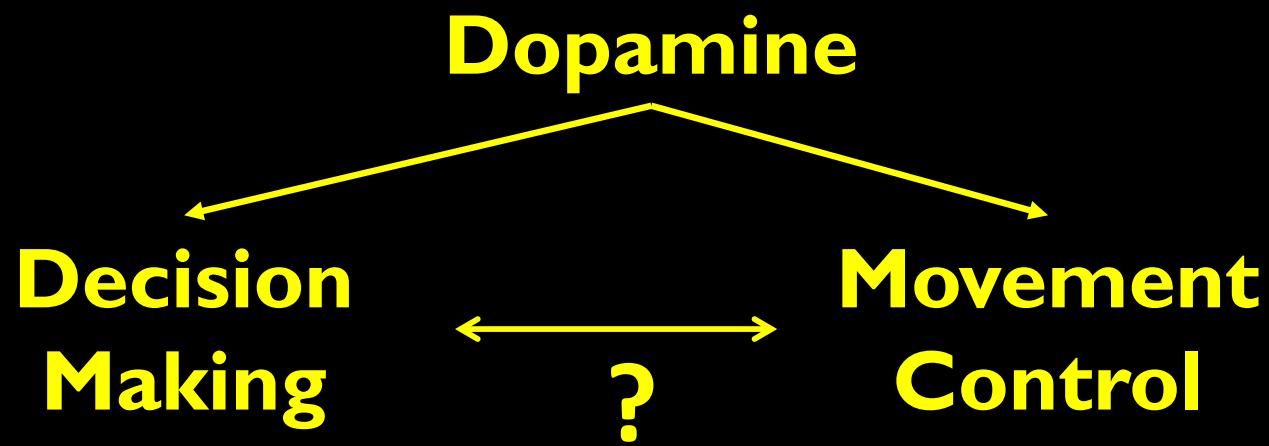
Reward predicted  
No reward occurs



# Dopamine



Firing rate increases in proportion to the expected value of the stimulus



# Decision making and movement

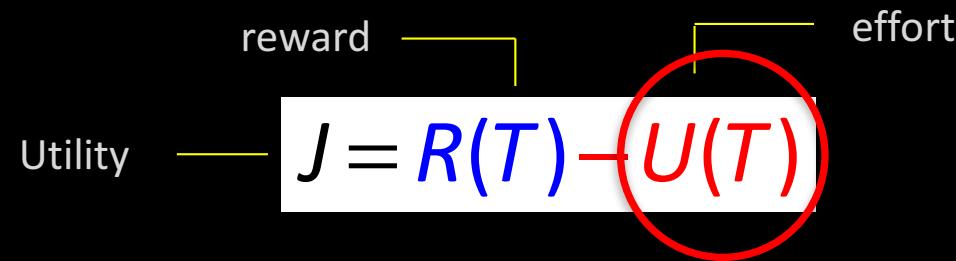
What: cookie or apple?

How: fast or slow?



People reach faster for candy associated with greater preference.<sup>1</sup>

# Movement utility as sum of reward and effort



reward

effort

Utility

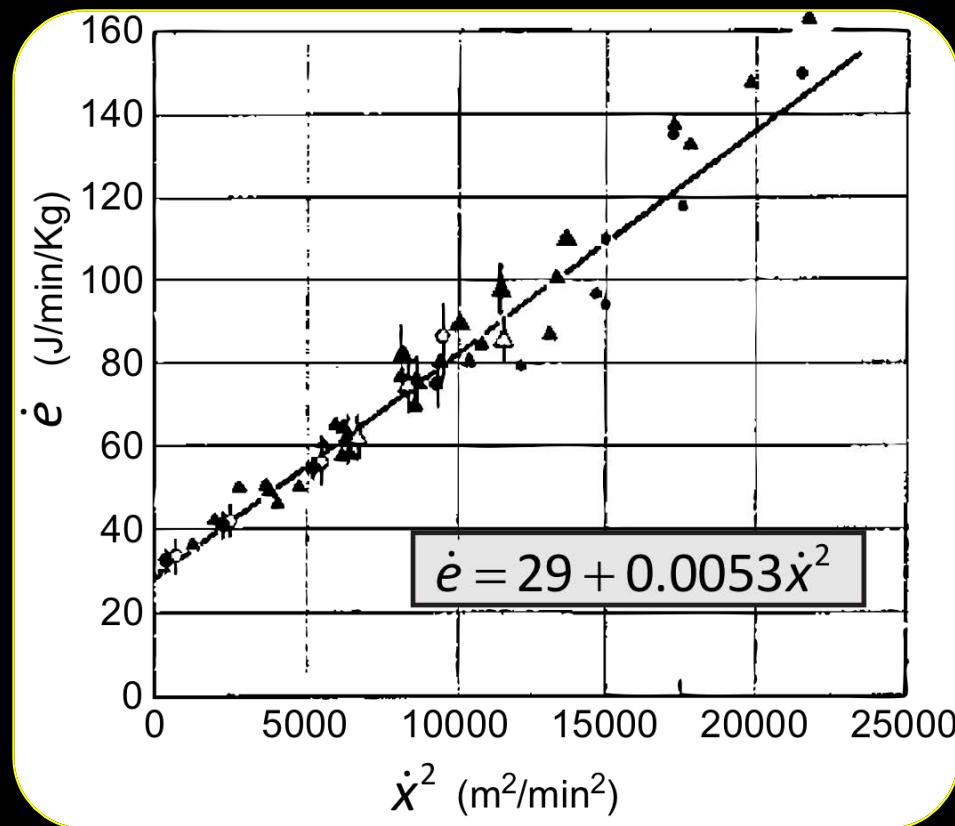
$$J = R(T) - U(T)$$

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

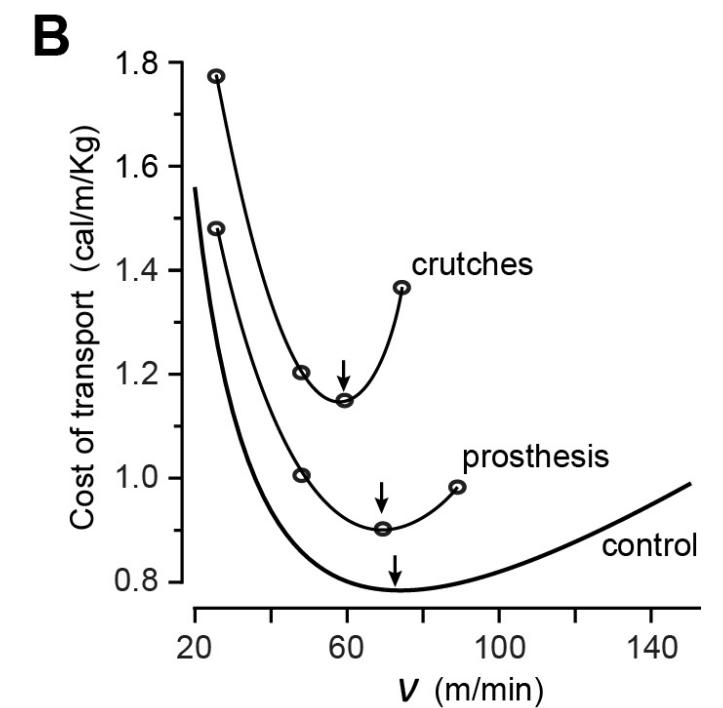
H: The brain represents effort as the metabolic cost of the action.

# Locomotion and metabolic rate

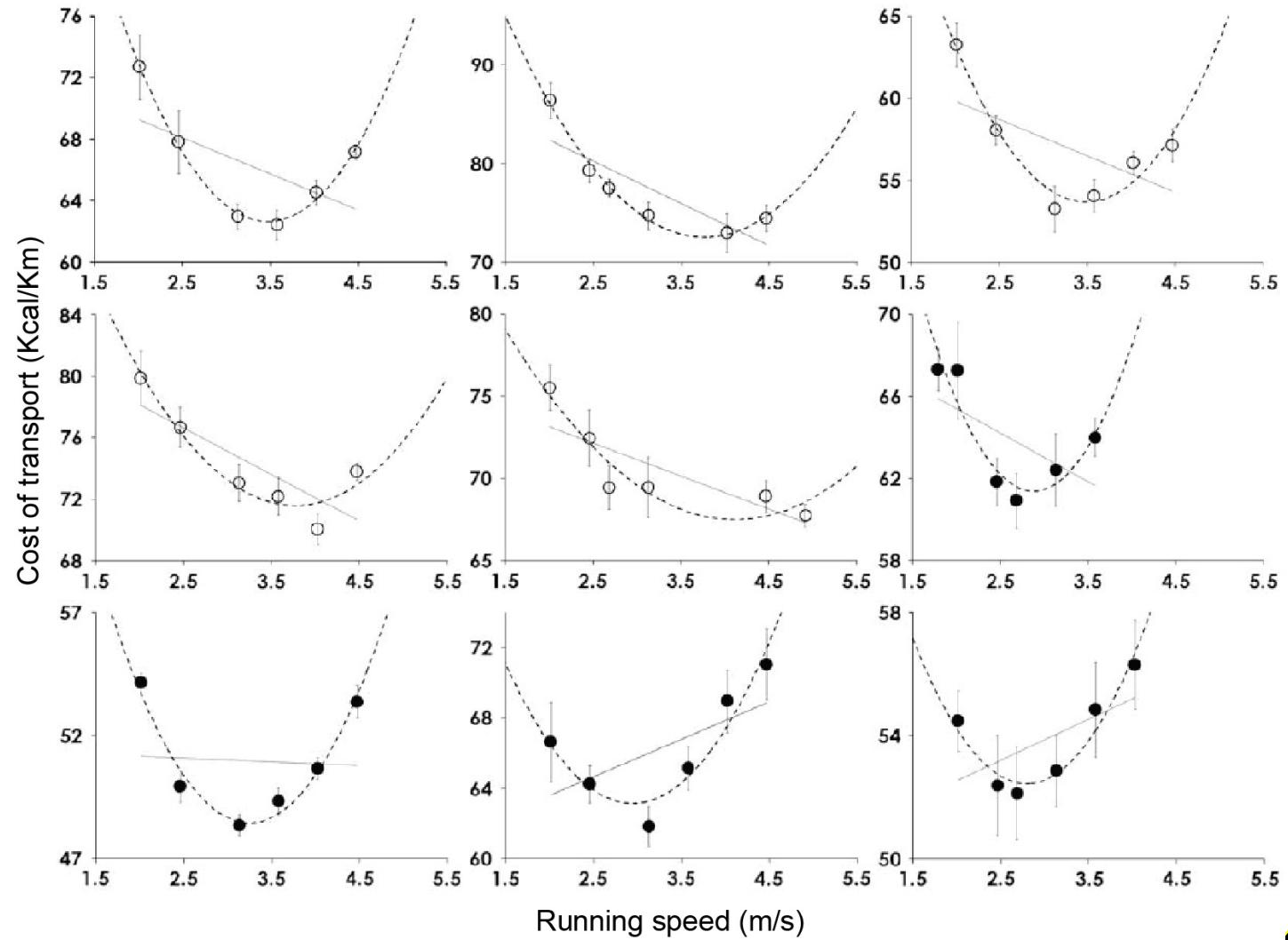
$$COT = \frac{\dot{e}}{v} = e = \frac{29}{v} + 0.0053v$$



Ralston (1958)

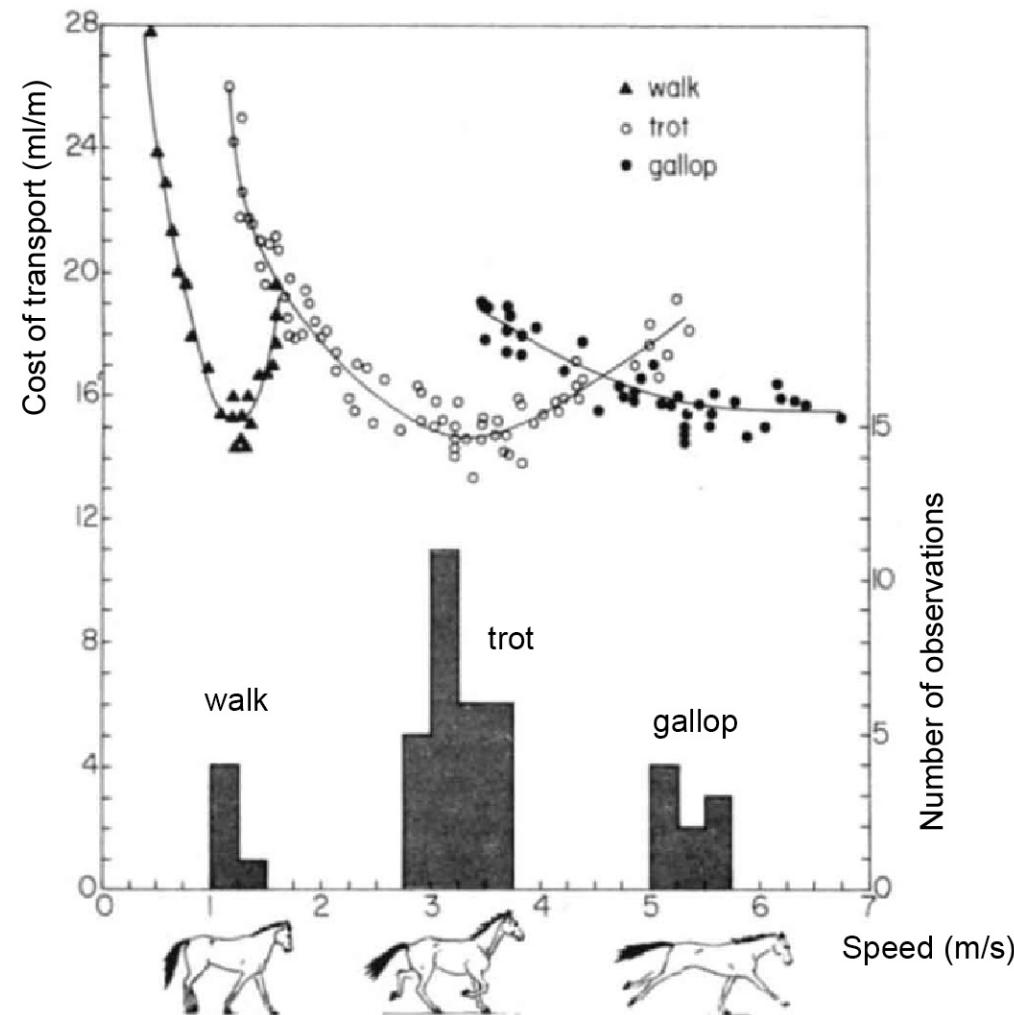


# An energetically optimal running speed



Steudel-Numbers and Wall-Scheffler (2009)

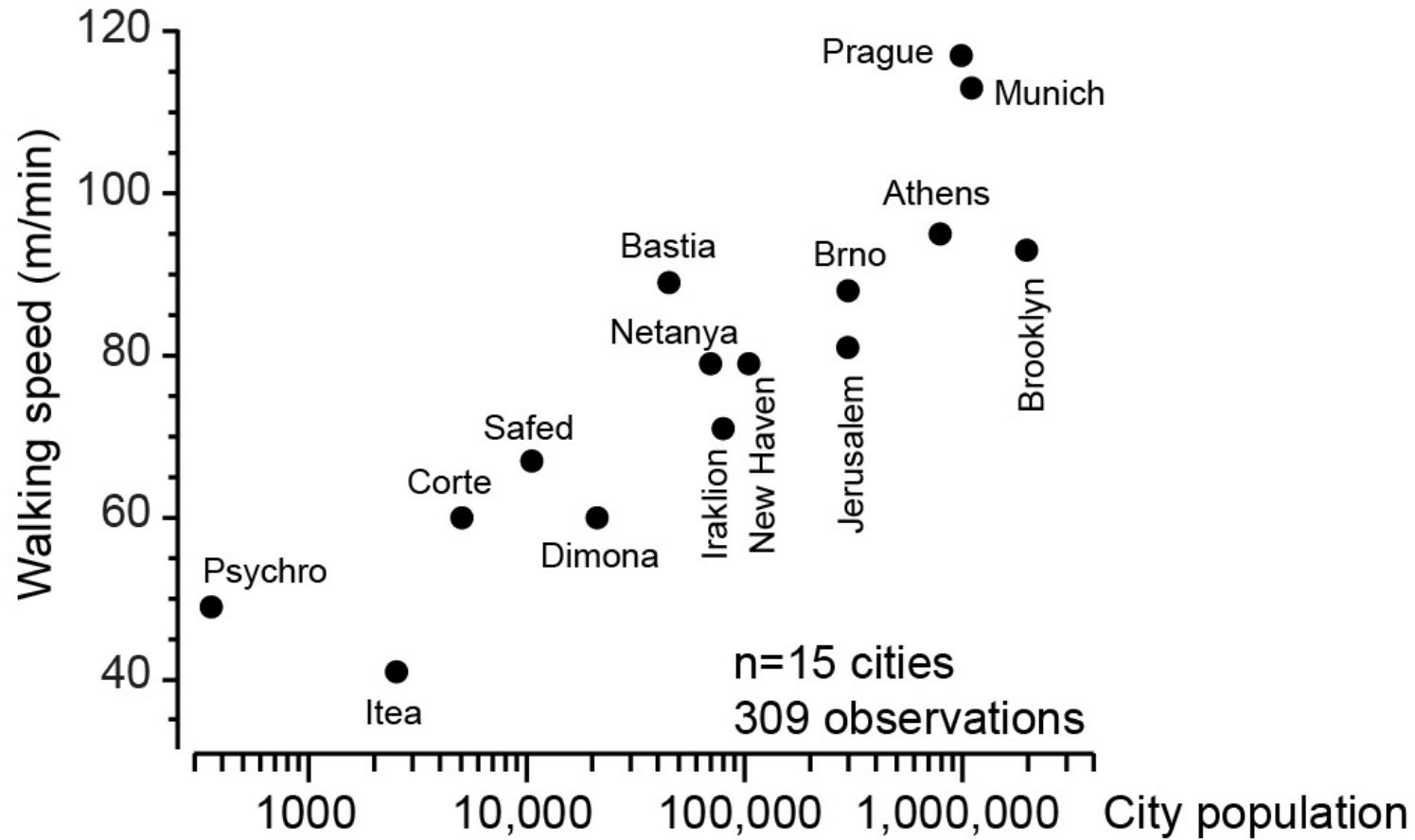
# Horses appear to minimize COT



Hoyt and Taylor (1981)



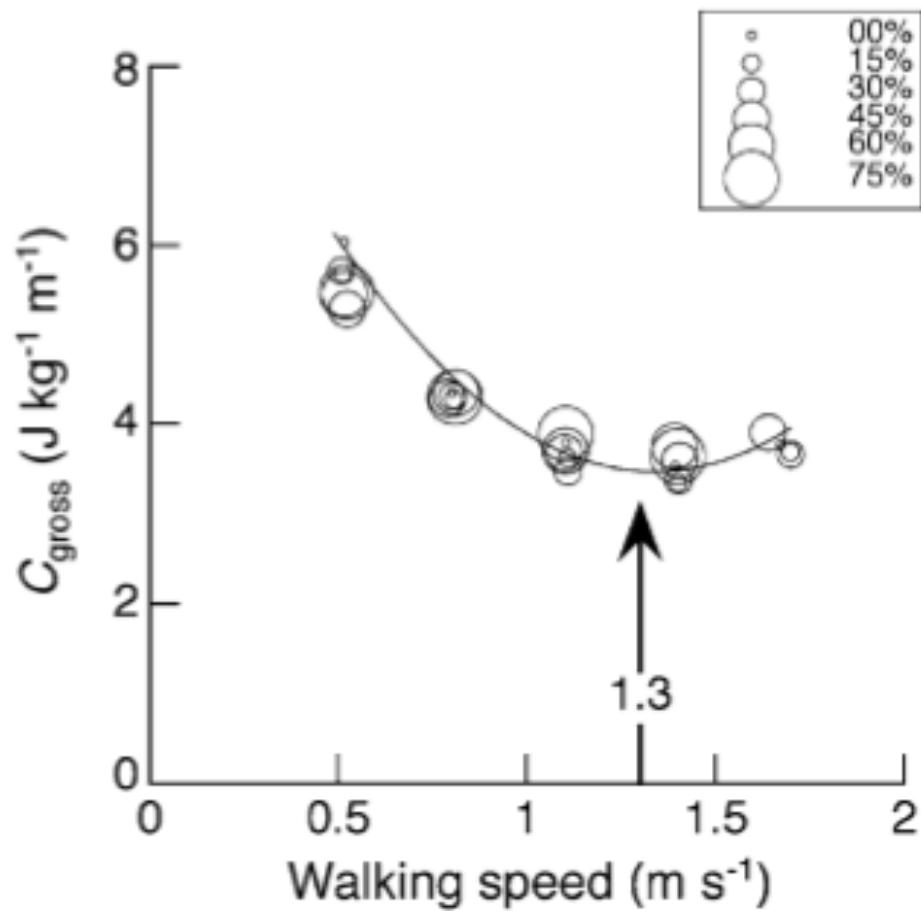
# Walking speed varies across cities



Bornstein and Bornstein (1976)



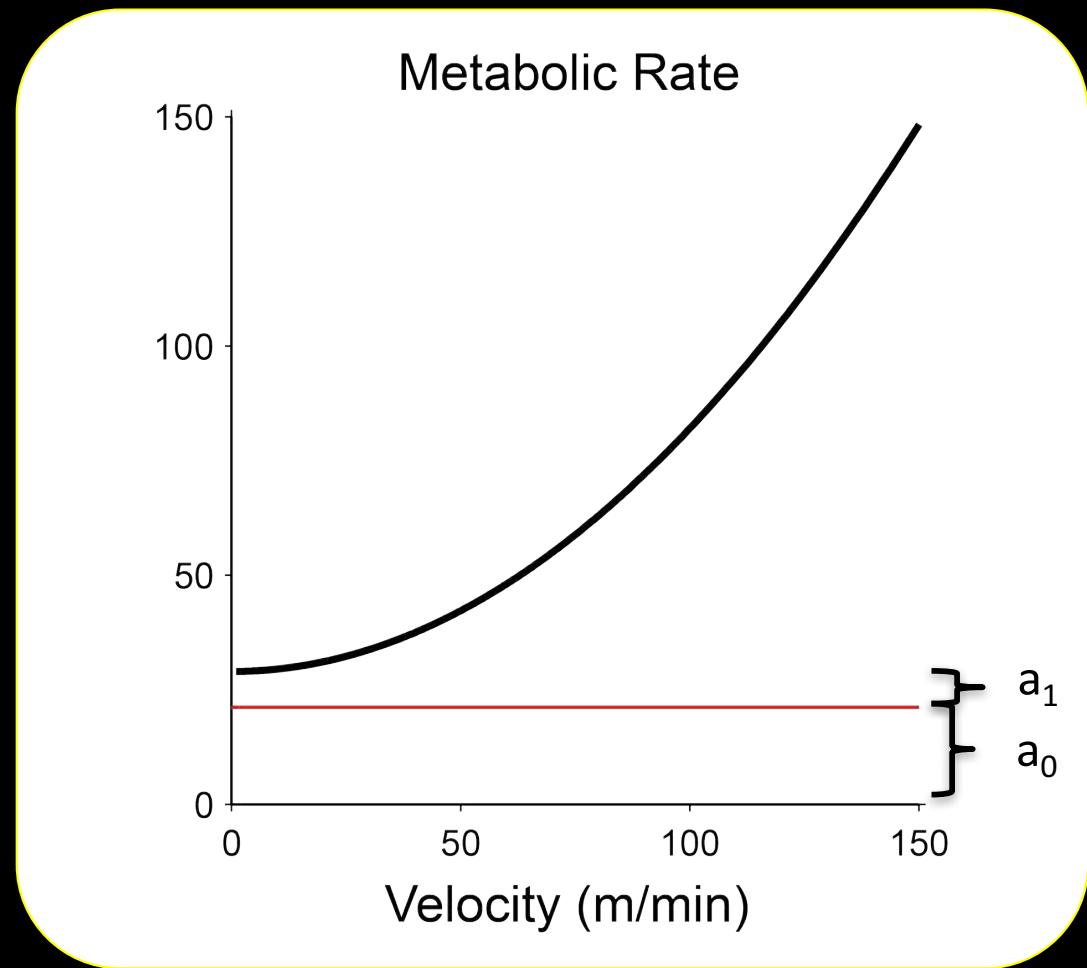
## Mass effects



Increased mass does not change metabolically optimal speed.

People generally slow down when carrying loads.

# Gross vs. Net cost of transport?



Gross Metabolic Rate

$$\dot{e}_{gross} = a + cv^2$$

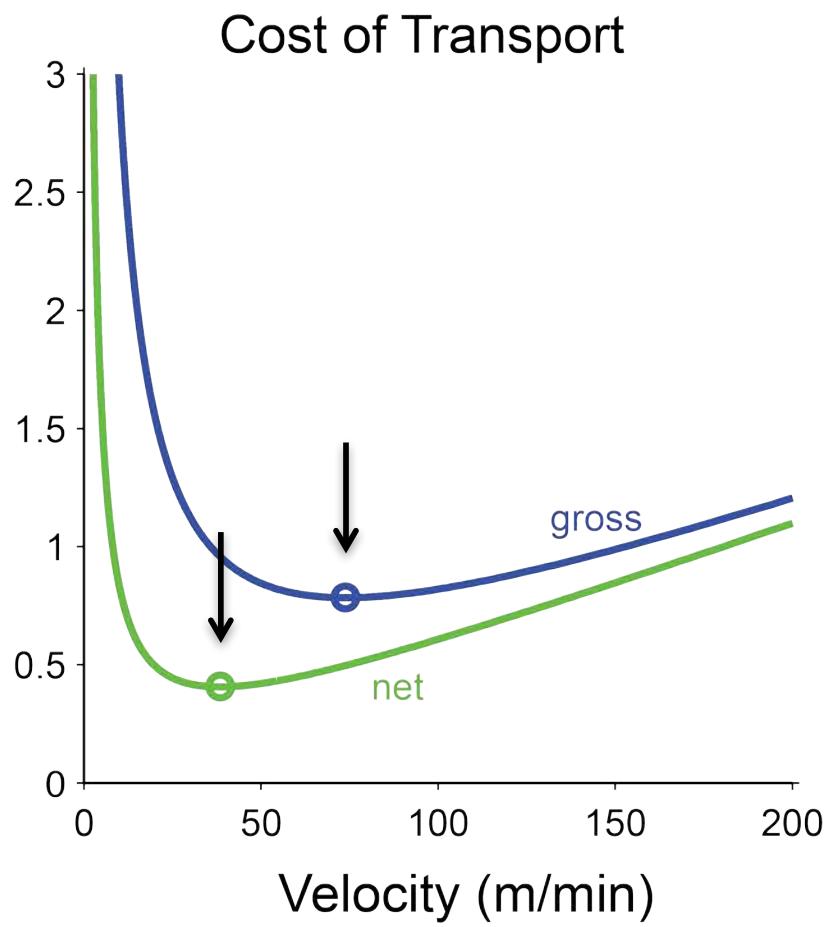
$$\dot{e}_{gross} = a_0 + a_1 + cv^2$$

Cost of being alive

Net Metabolic Rate

$$\dot{e}_{net} = a_1 + cv^2$$

# Gross vs. Net cost of transport?



Gross Metabolic Rate

$$\dot{e}_{gross} = a + cv^2$$

$$\dot{e}_{gross} = a_0 + a_1 + cv^2$$

Cost of being alive

$$e_{gross} = a_0/v + a_1/v + cv$$

Net Metabolic Rate

$$\dot{e}_{net} = a_1 + cv^2$$

$$e_{net} = a_1/v + cv$$

# Movement utility as sum of reward and effort

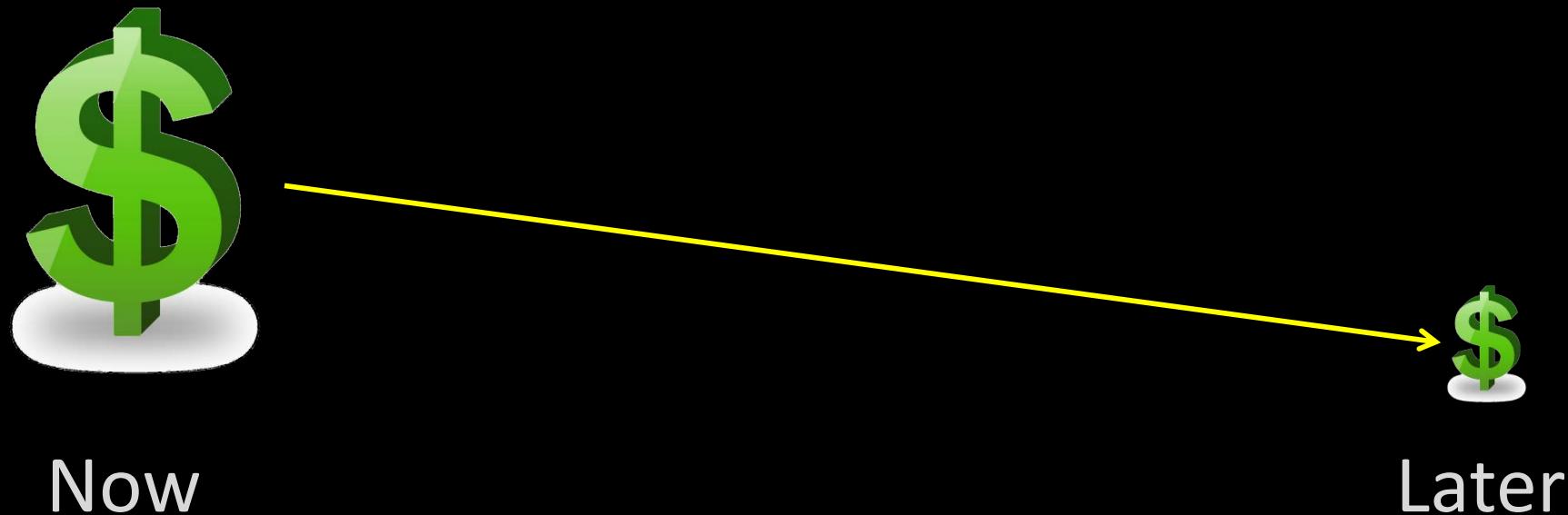
The diagram illustrates the utility equation  $J = R(T) - U(T)$  on a black background. A horizontal line with arrows at both ends represents the equation. The term  $R(T)$  is in blue and is circled in blue. The term  $- U(T)$  is in red. Above the equation, a bracket labeled "reward" spans the width of  $R(T)$ . Another bracket labeled "effort" spans the width of  $U(T)$ . To the left of the equation, a vertical line labeled "Utility" points to the center of the equation.

$$J = R(T) - U(T)$$

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

H: The brain represents effort as the metabolic cost of the action.

# Temporal discounting of reward



# Movement utility as sum of reward and effort

$$J = \frac{\alpha}{1 + \gamma T} - U(T)$$

reward      effort

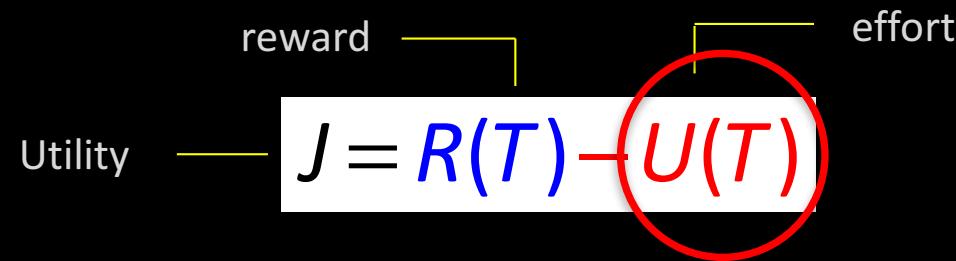
Utility

Temporal discounting factor      Duration of the movement

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

H: The brain represents effort as the metabolic cost of the action.

# Movement utility as sum of reward and effort



reward

effort

Utility

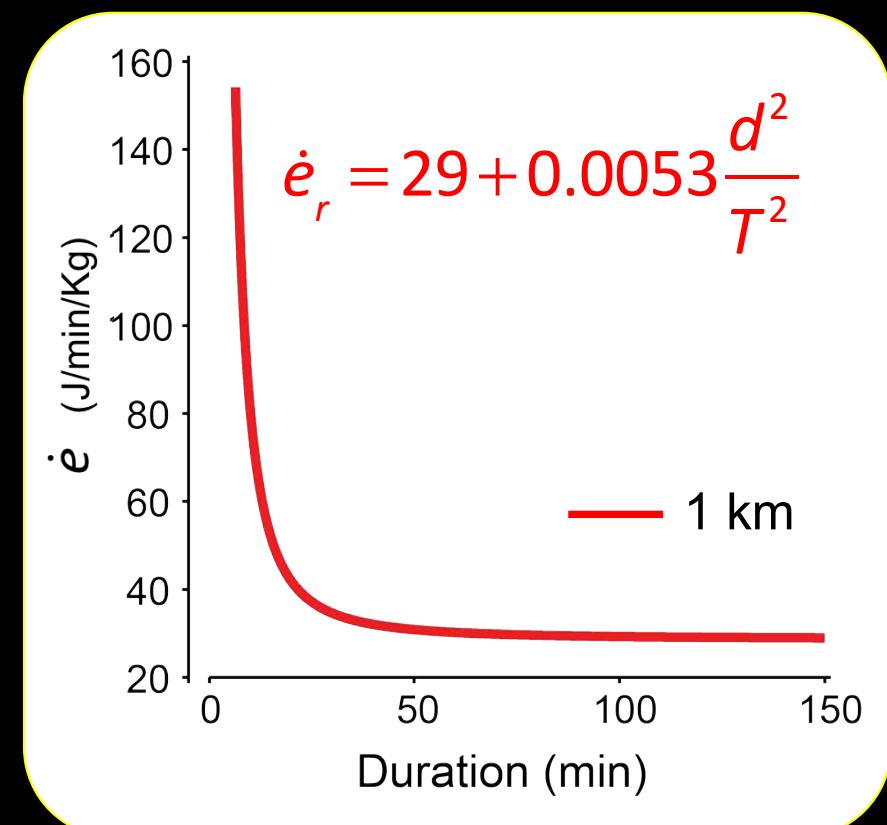
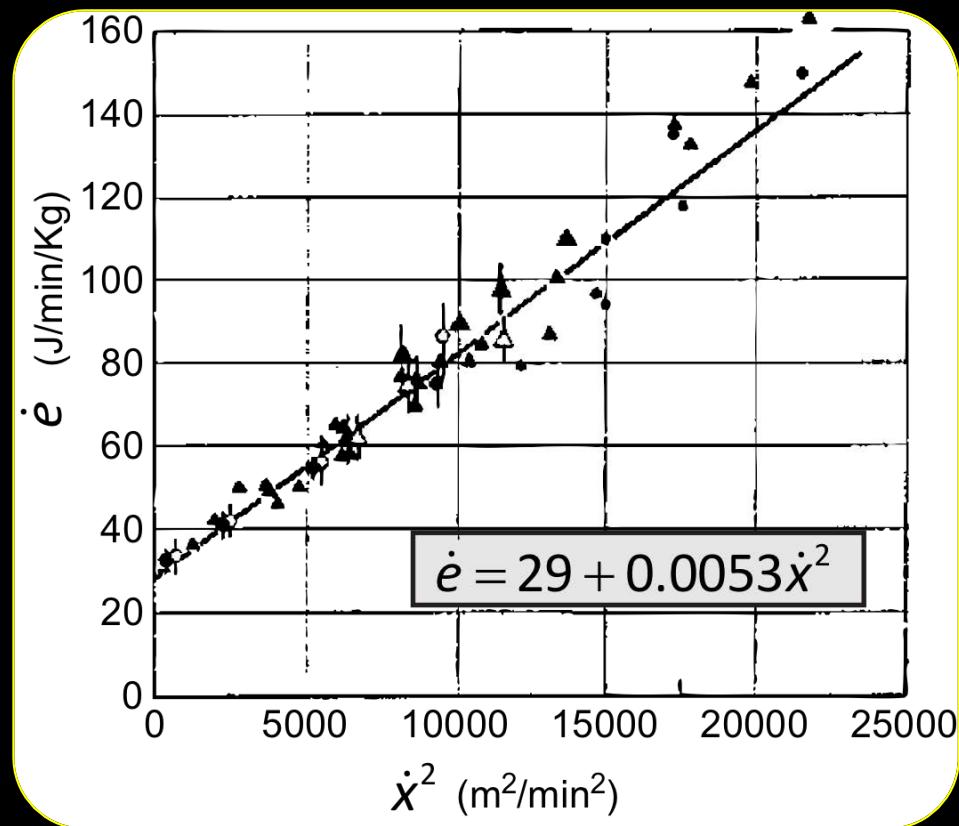
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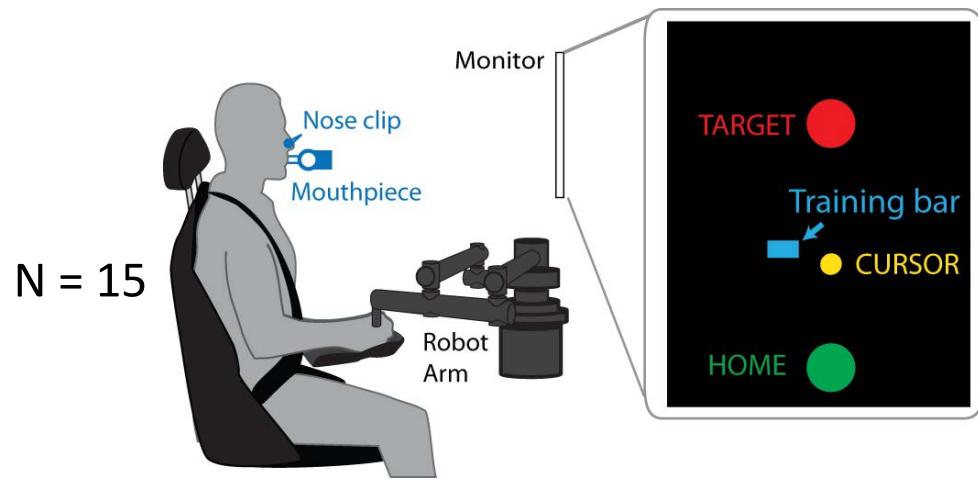


# Locomotion and metabolic rate

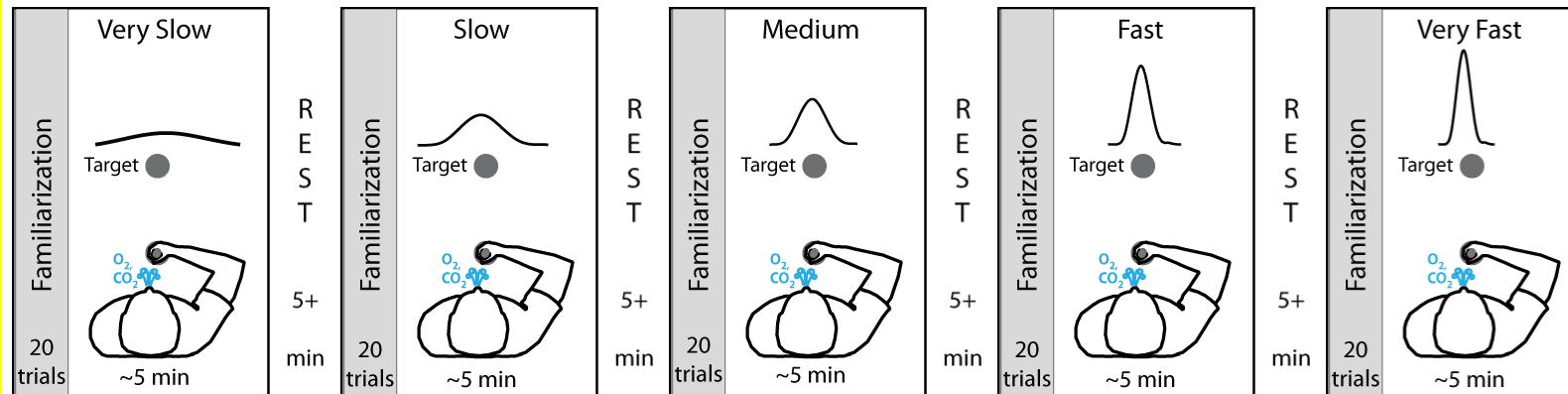


Ralston (1958)

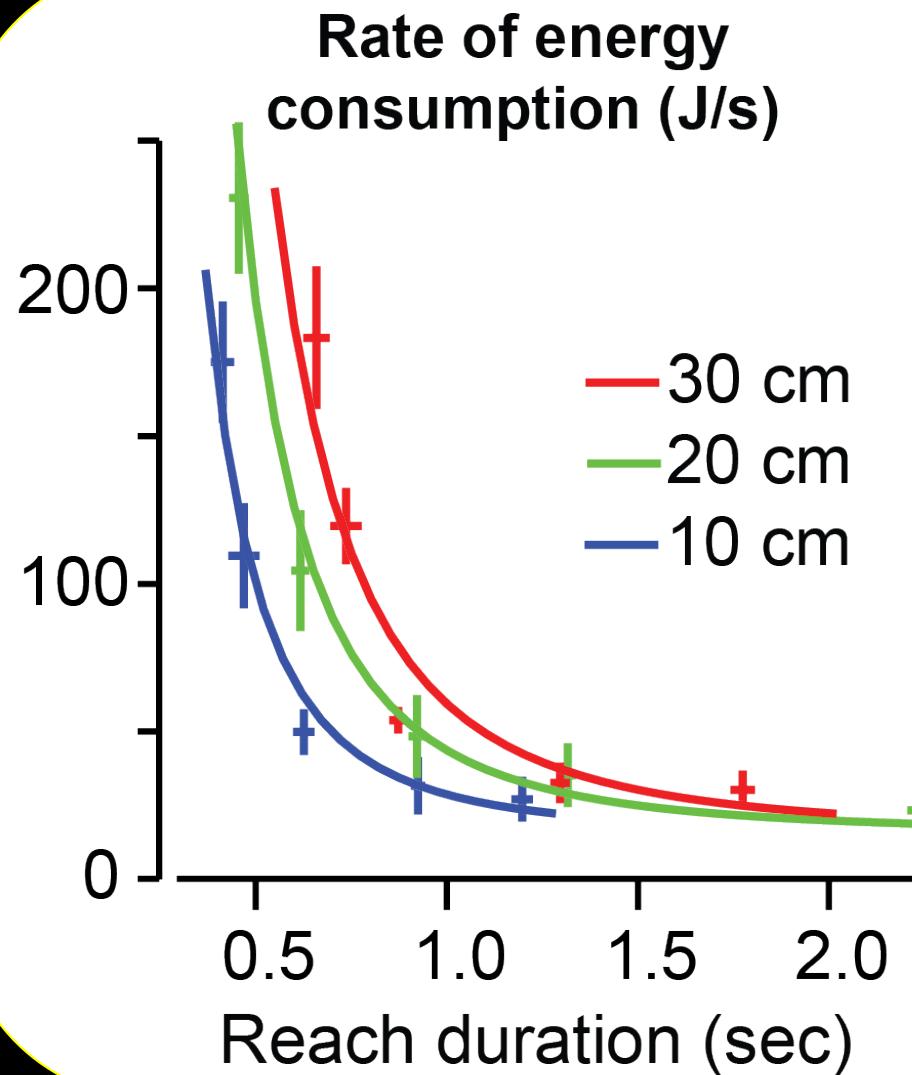
# Reaching and metabolic rate



## 5-minute reaching blocks at different speeds



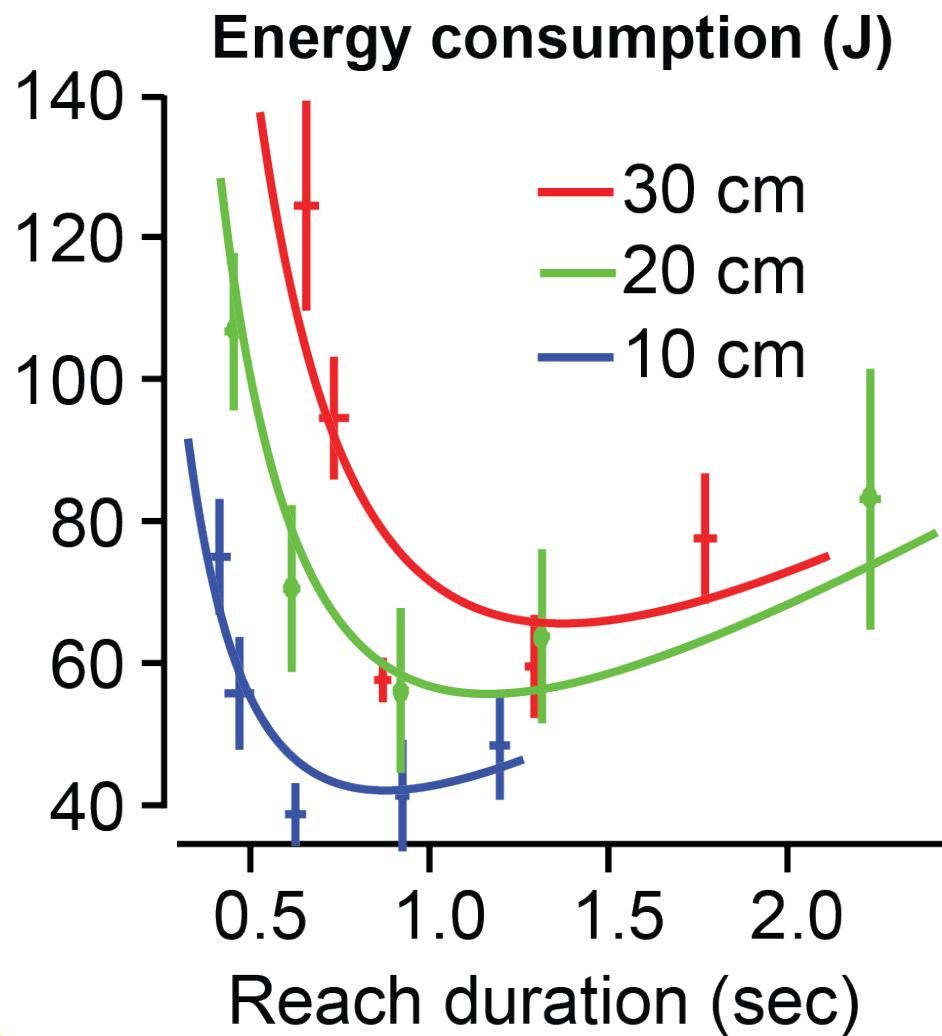
# Reaching and metabolic rate



Metabolic Rate:

$$\dot{e}_r = a + c \frac{md^{1.1}}{\pi^{j3}}$$

# Reaching and metabolic energy



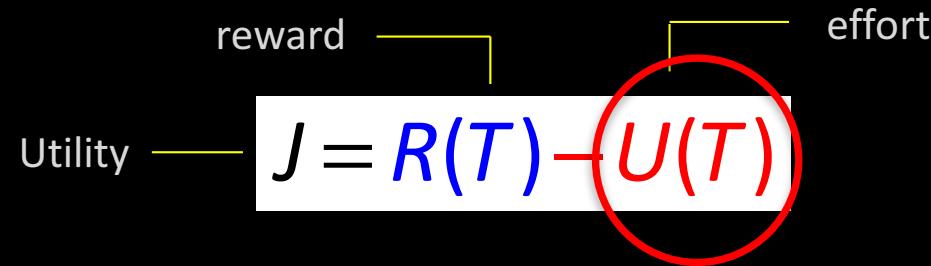
Metabolic Rate:

$$\dot{e}_r = a + c \frac{md^{1.1}}{T^3}$$

Movement Energy:

$$e_r = aT + c \frac{md^{1.1}}{T^2}$$

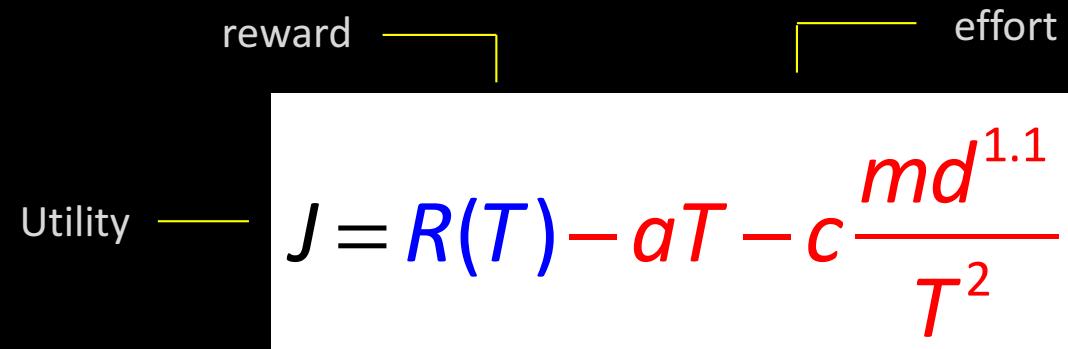
# Movement utility as sum of reward and effort


$$\text{Utility} = \text{reward} - \text{effort}$$
$$J = R(T) - U(T)$$

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

H: The brain represents effort as the metabolic cost of the action.

# Movement utility as sum of reward and effort



reward

effort

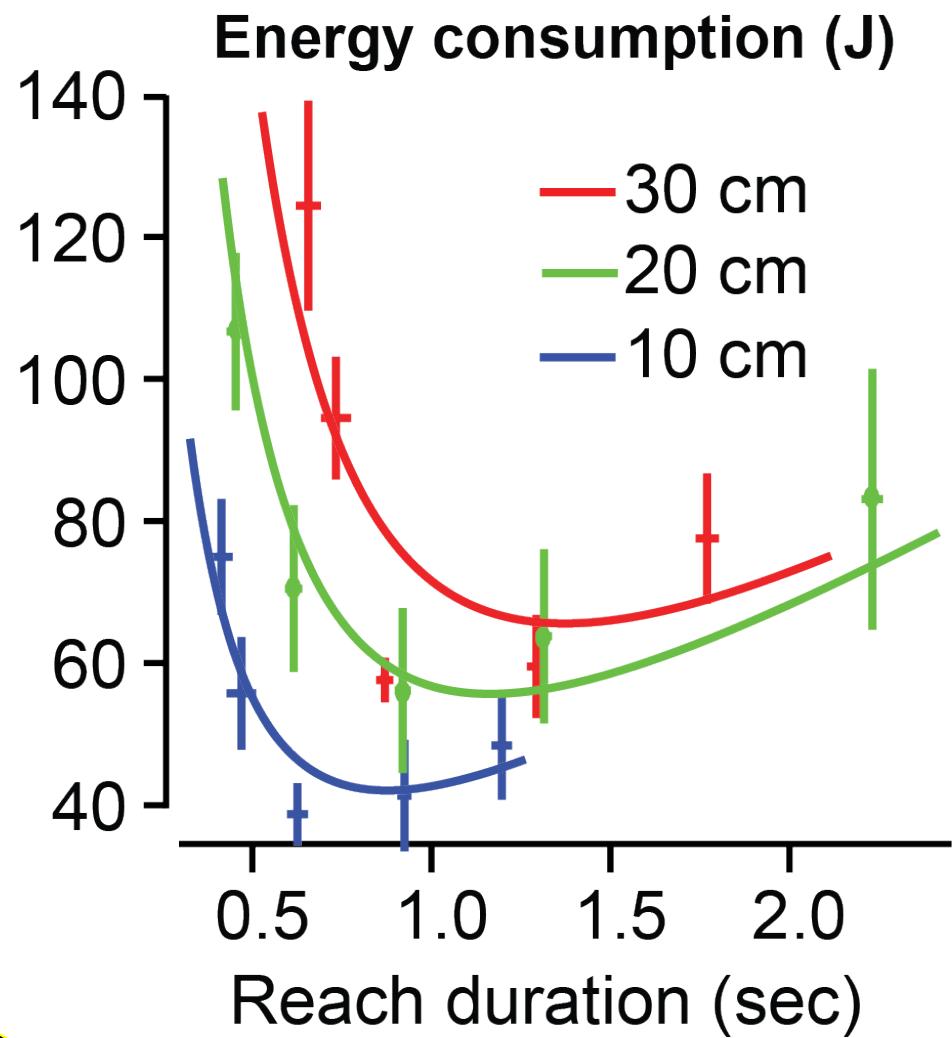
Utility

$$J = R(T) - aT - c \frac{md^{1.1}}{T^2}$$

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

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# An energetically optimal movement duration



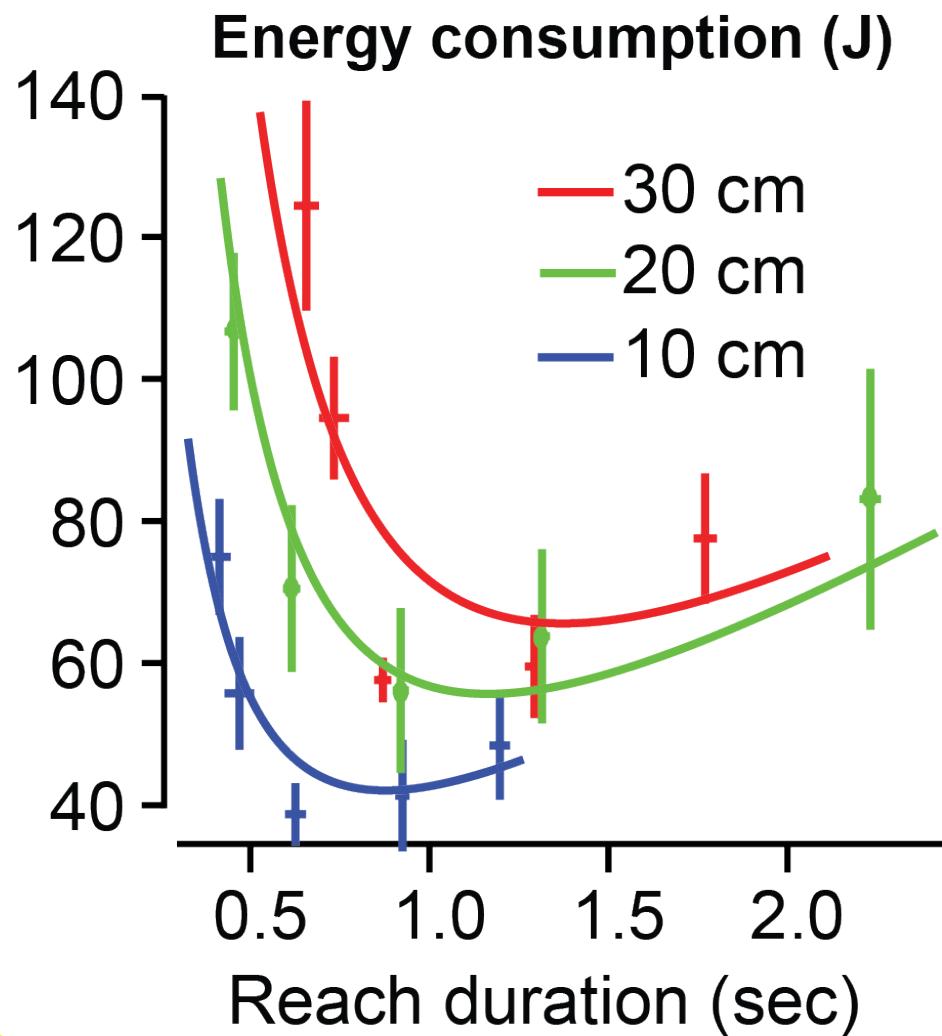
Metabolic Rate (J/s):

$$\dot{e}_r = a + c \frac{md^{1.1}}{T^3}$$

Movement Energy (J):

$$e_r = aT + c \frac{md^{1.1}}{T^2}$$

# An energetically optimal movement duration



Metabolic Rate (J/s):

$$\dot{e}_r = a + c \frac{md^{1.1}}{T^3}$$

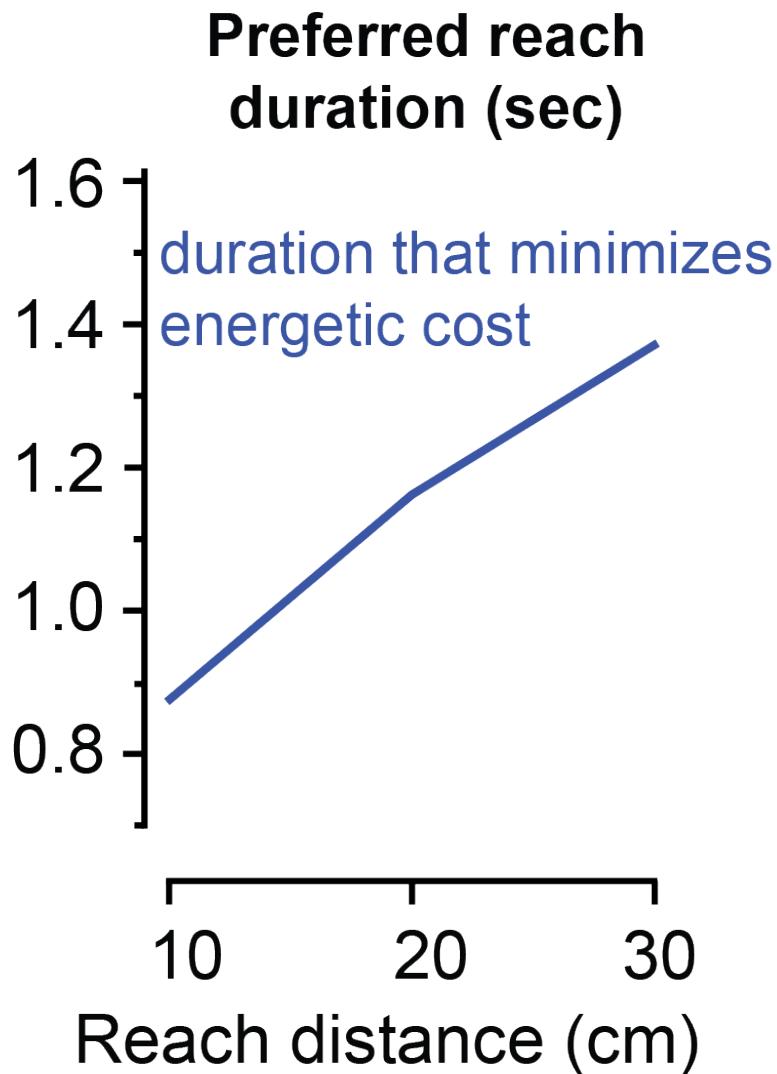
Movement Energy (J):

$$e_r = aT + c \frac{md^{1.1}}{T^2}$$

Duration that minimizes energy:

$$T^* = \left( \frac{bmd^i(j-1)}{a} \right)^{\frac{1}{j}}$$

# Preferred reach duration

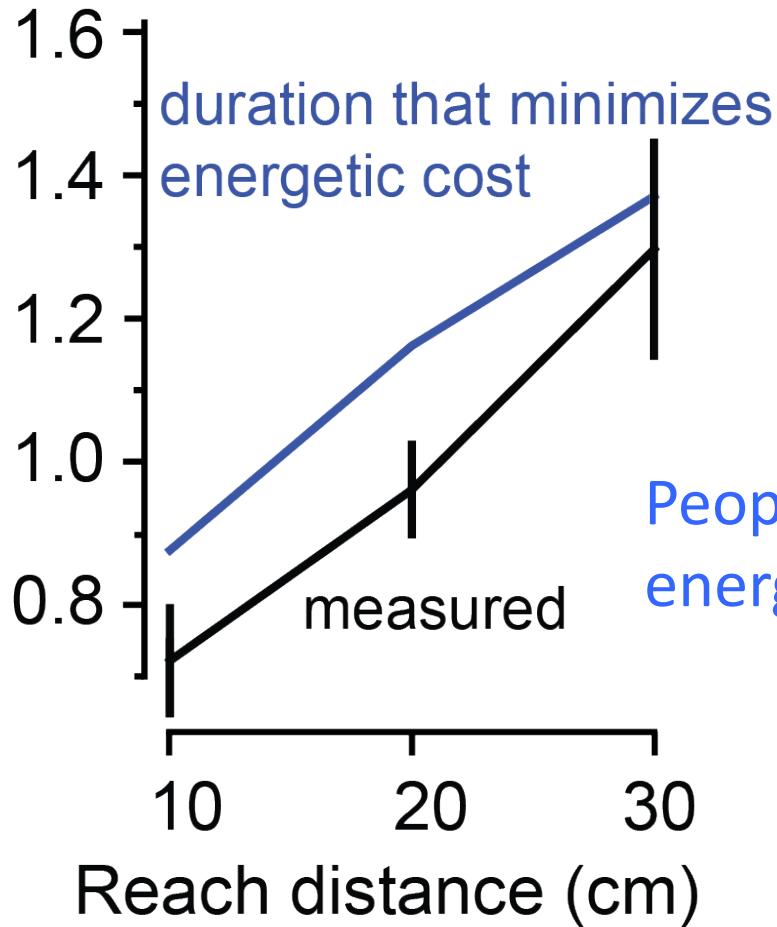


Duration that minimizes energy:

$$T^* = \left( \frac{bmd^i(j-1)}{a} \right)^{\frac{1}{j}}$$

# Preferred reach duration

Preferred reach duration (sec)

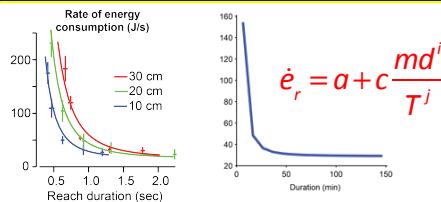


Duration that minimizes energy:

$$T^* = \left( \frac{bmd^i(j-1)}{a} \right)^{\frac{1}{j}}$$

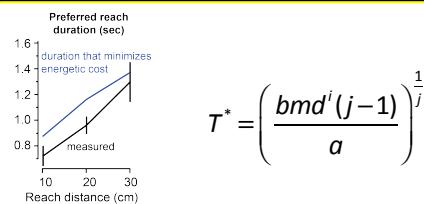
People reach faster than energetically optimal

# Summary



$$\dot{e}_r = a + c \frac{md^i}{T^j}$$

In both walking and reaching, metabolic energy increases with movement distance and decreases with duration.



$$T^* = \left( \frac{bmd^i(j-1)}{a} \right)^{\frac{1}{j}}$$

There is an energetically optimal reach speed.  
People prefer to reach faster than energetically optimal.

# Movement utility as sum of reward and effort

The diagram illustrates the utility equation  $J = R(T) - U(T)$  on a black background. A horizontal line with arrows at both ends represents the equation. The term  $R(T)$  is in blue and is circled in blue. The term  $U(T)$  is in red. Above the equation, a bracket labeled "reward" spans the width of  $R(T)$ . Another bracket labeled "effort" spans the width of  $U(T)$ . To the left of the equation, a vertical line labeled "Utility" points to the center of the equation.

$$J = R(T) - U(T)$$

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

H: The brain represents effort as the metabolic cost of the action.

# Movement utility as sum of reward and effort

$$J = \frac{\alpha}{1 + \gamma T} - U(T)$$

reward      effort

Utility

Temporal discounting factor      Duration of the movement

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

H: The brain represents effort as the metabolic cost of the action.

# Movement utility as sum of reward and effort

$$J = \frac{\alpha}{1 + \gamma T} (aT + cmd^{1.1}/T^2)$$

reward

effort

Utility

Temporal discounting factor

Duration of the movement

**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

H: The brain represents effort as the metabolic cost of the action.

# Movement utility as sum of reward and effort

$$J = \frac{\alpha}{1+\gamma T} \frac{(aT + cmd^{1.1}/T^2)}{1+\gamma T}$$

Utility

reward

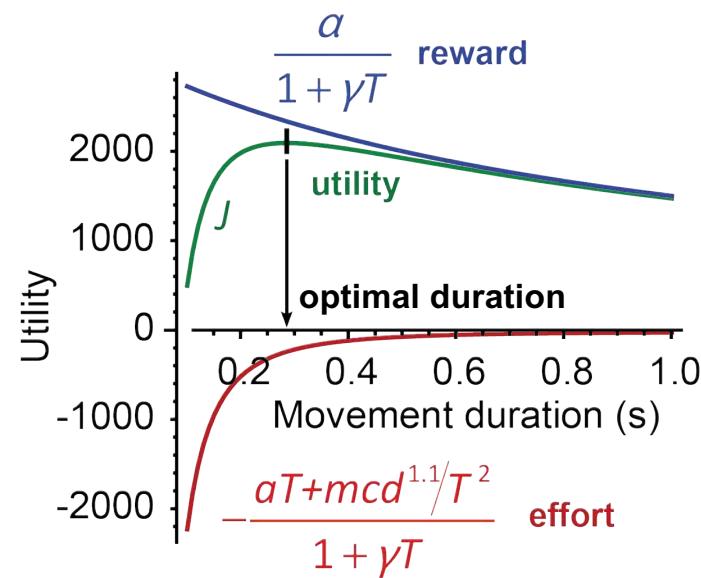
effort

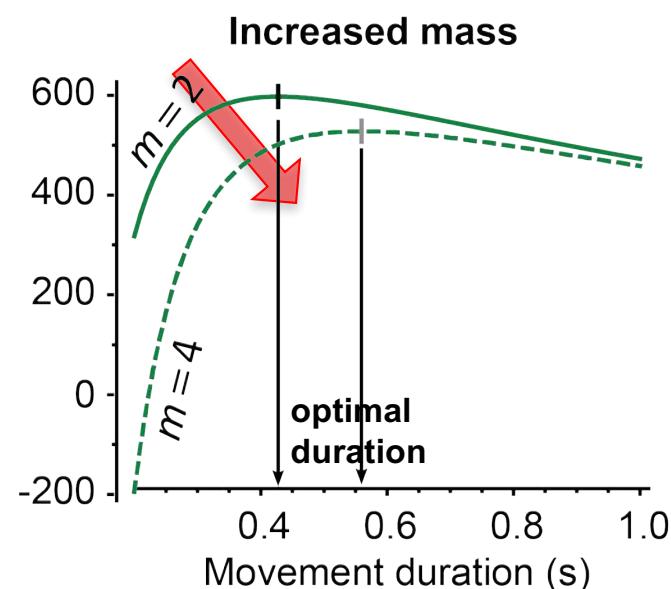
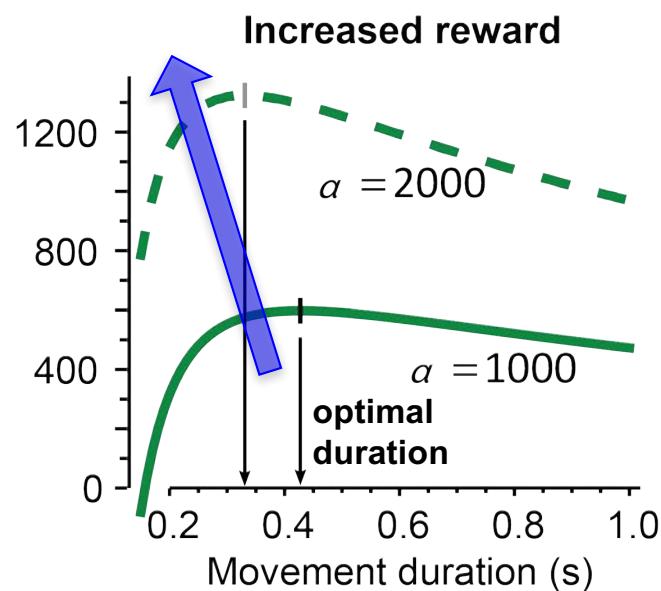
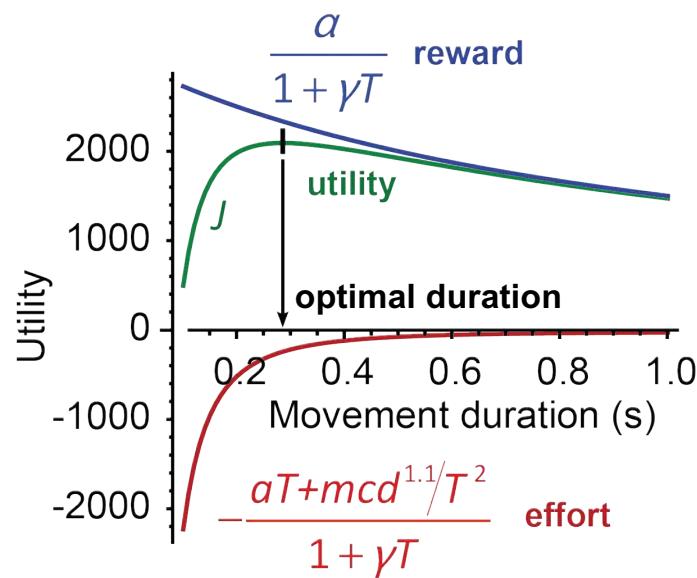
Temporal discounting factor

Duration of the movement

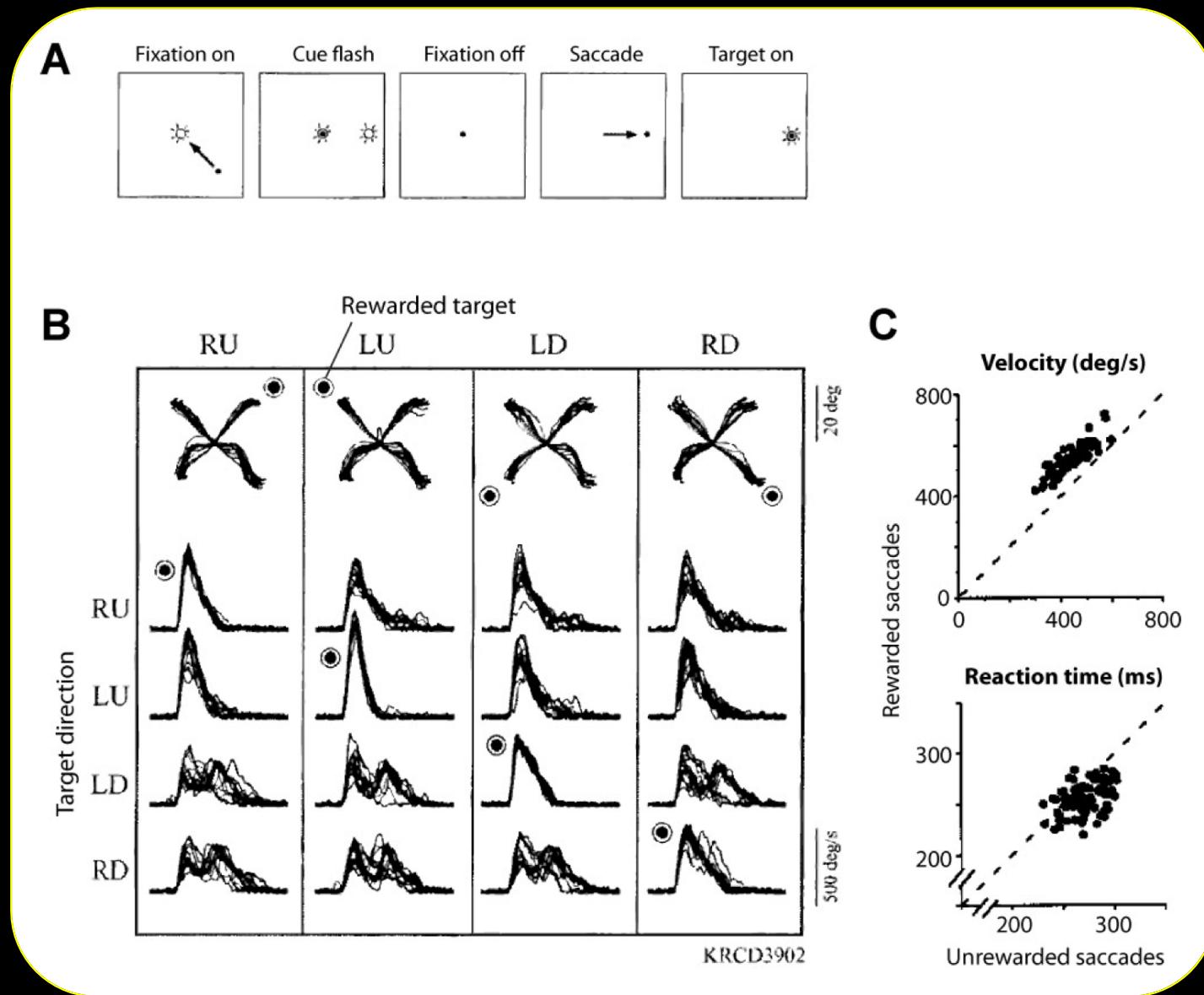
**Basic idea:** our choice of action is a reflection of the interplay between reward and effort.

H: The brain represents effort as the metabolic cost of the action.





# Saccade kinematics are affected by reward

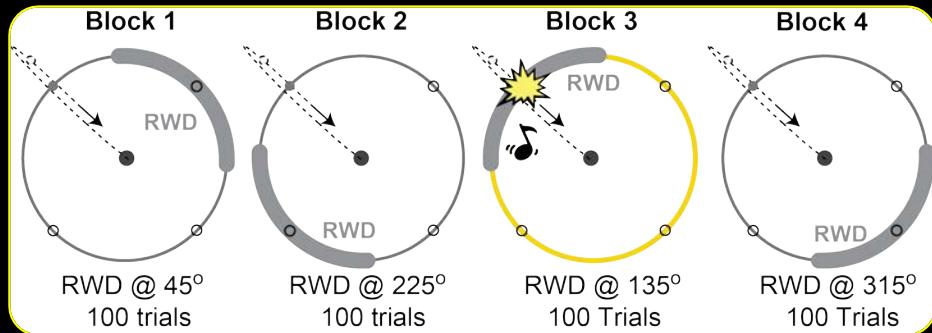


Takikawa et al. 2002, Figure: Shadmehr & Mussa-Ivaldi 2012

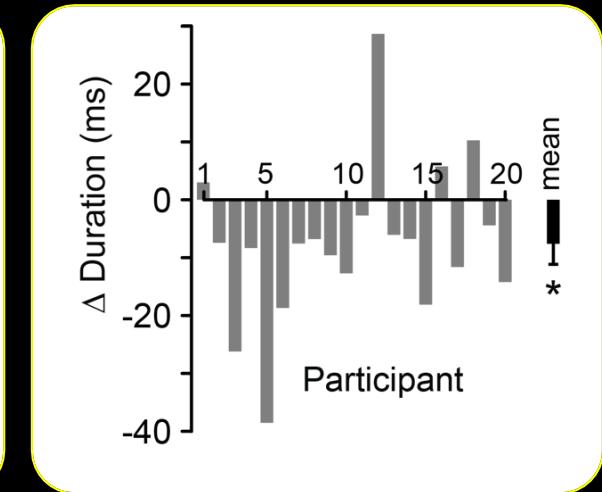
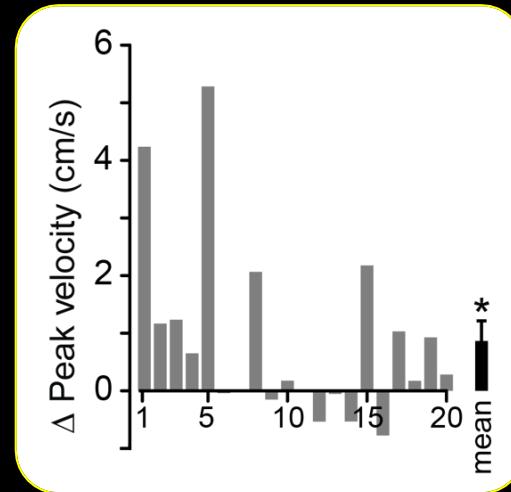
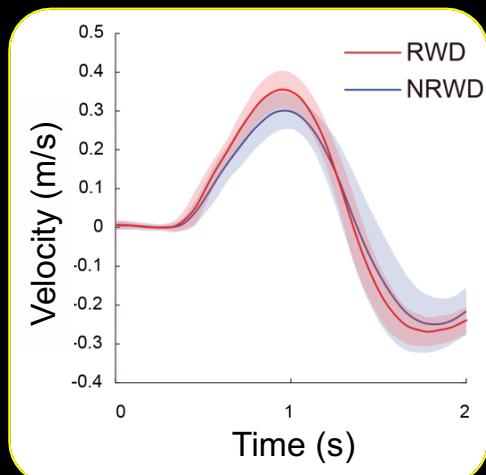
# Reward increases movement vigor

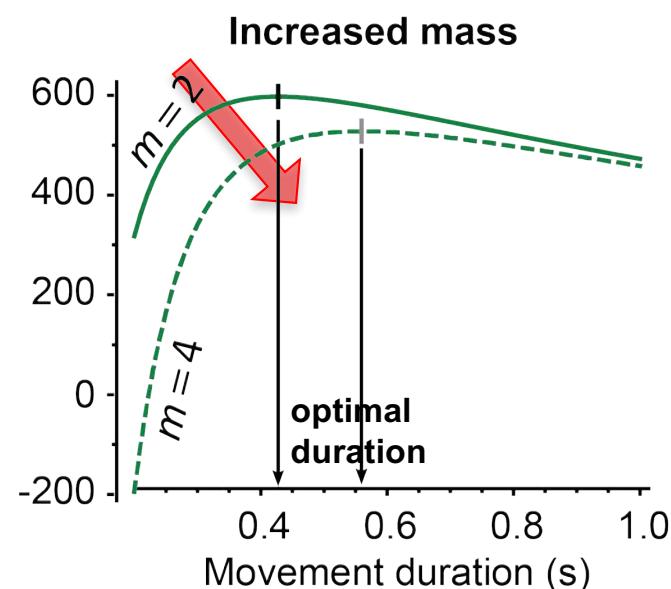
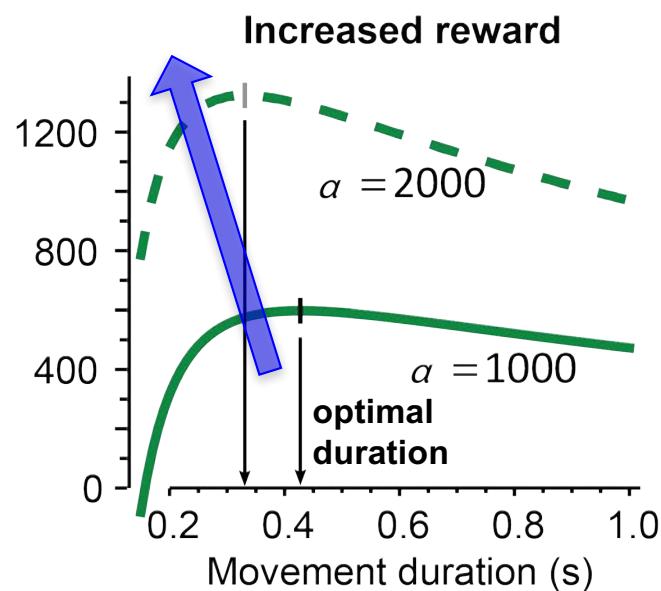
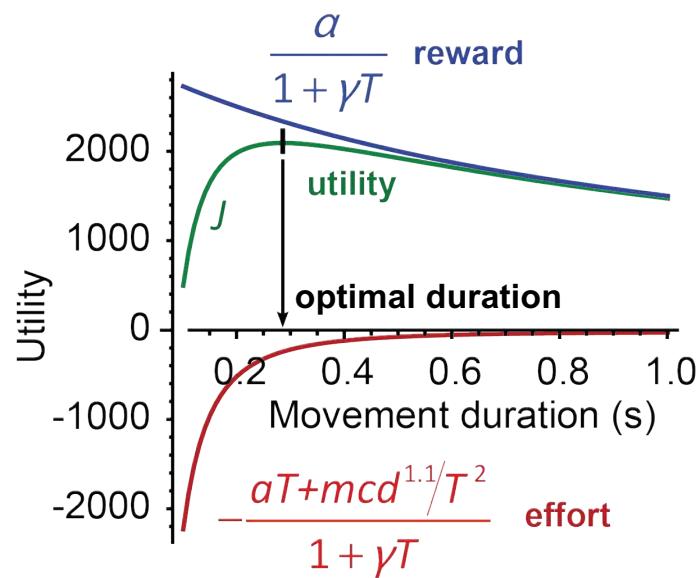


Erik Summerside



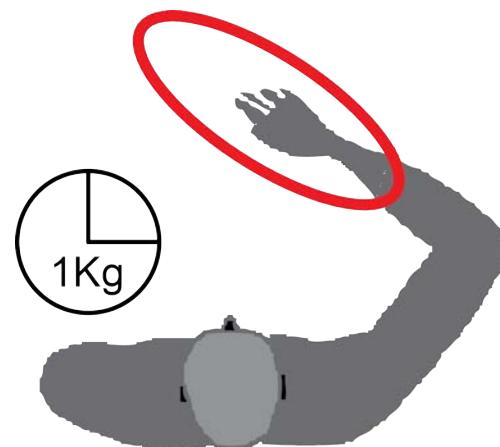
Subjects (n=20) made 10 cm reaching movements. In each block of 100 trials, one target direction was consistently rewarded.





# Mass of the arm affects movement speed

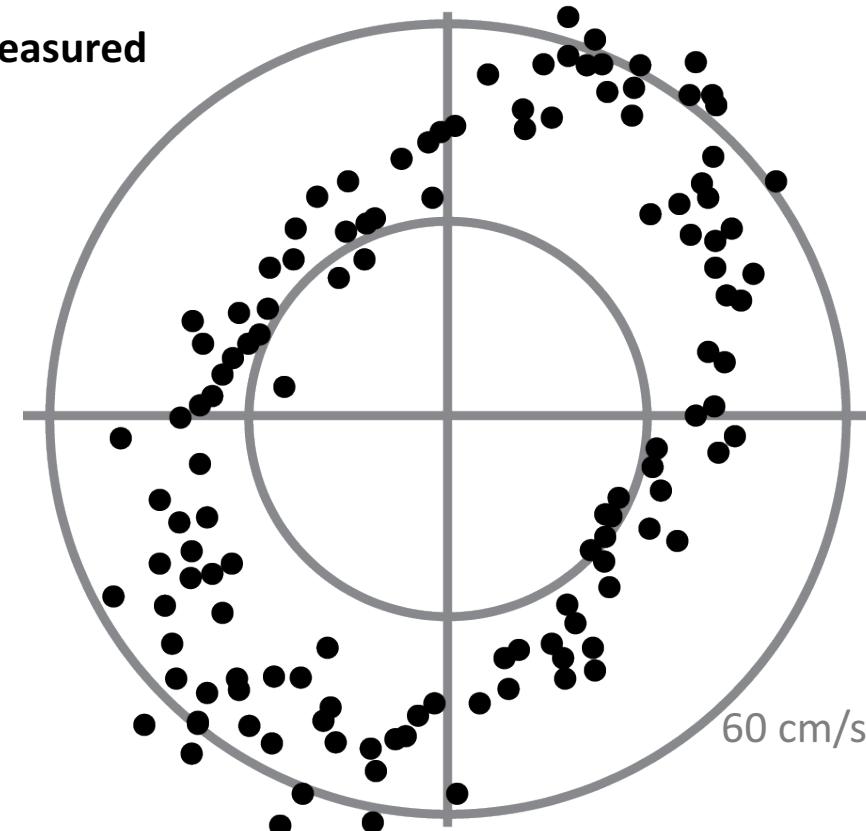
## arm effective mass



$$J = \frac{\alpha}{1+\gamma T} - \frac{aT + cmd^{1.1}/T^2}{1+\gamma T}$$

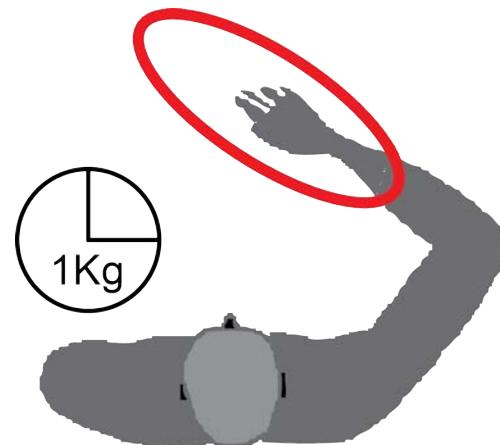
## peak velocity

measured



# Mass of the arm affects movement speed

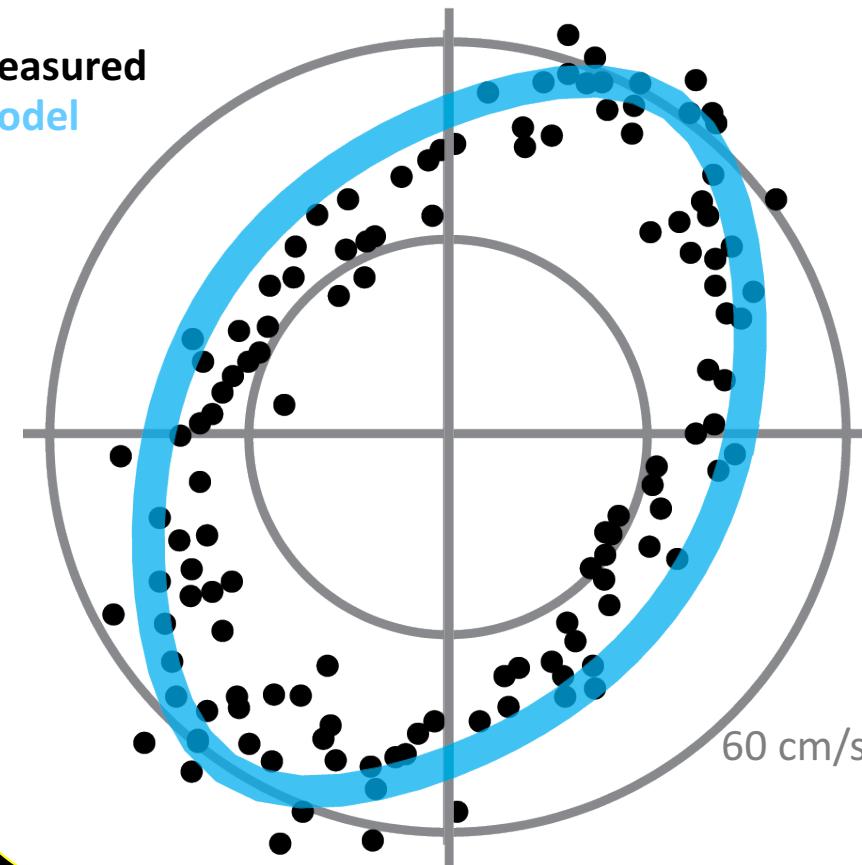
## arm effective mass



$$J = \frac{\alpha}{1+\gamma T} - \frac{aT + cmd^{1.1}/T^2}{1+\gamma T}$$

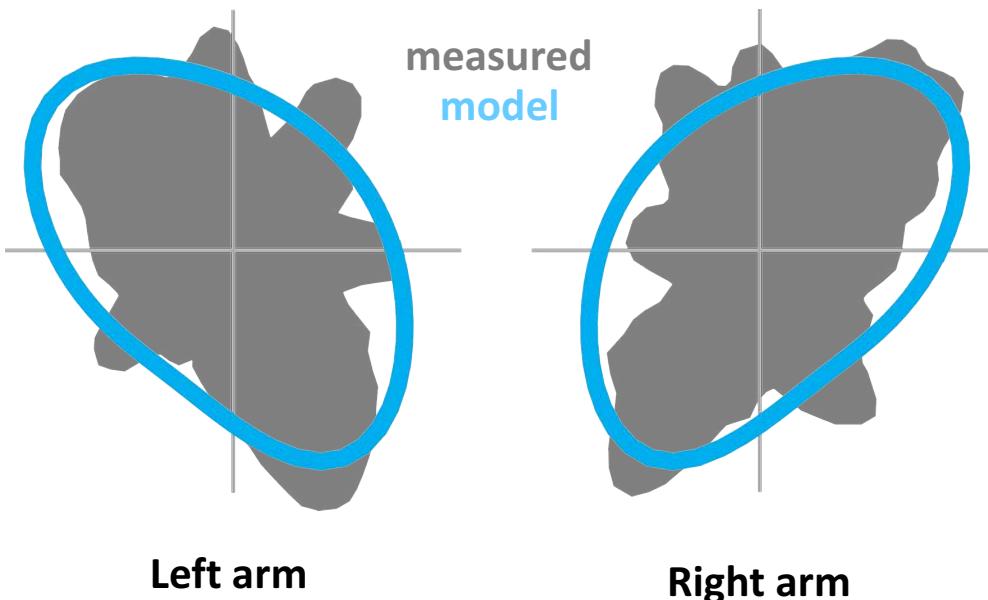
## peak velocity

measured  
model



# Mass of the arm affects decision making

## choice probability



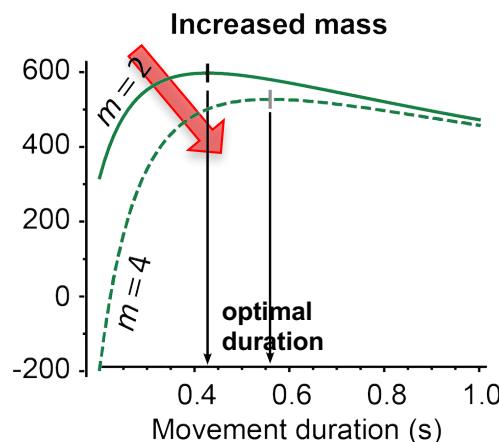
$$\rightarrow J = \frac{\alpha}{1+\gamma T} - \frac{aT + cmd^{1.1}/T^2}{1+\gamma T}$$

$$\Pr(\theta_j) = \left( \frac{J(\theta_j)}{\sum_j J(\theta_j)} \right)$$

The same utility that described the velocity of movements as a function of movement direction, also described the movement choices that people made when free to reach in any direction.

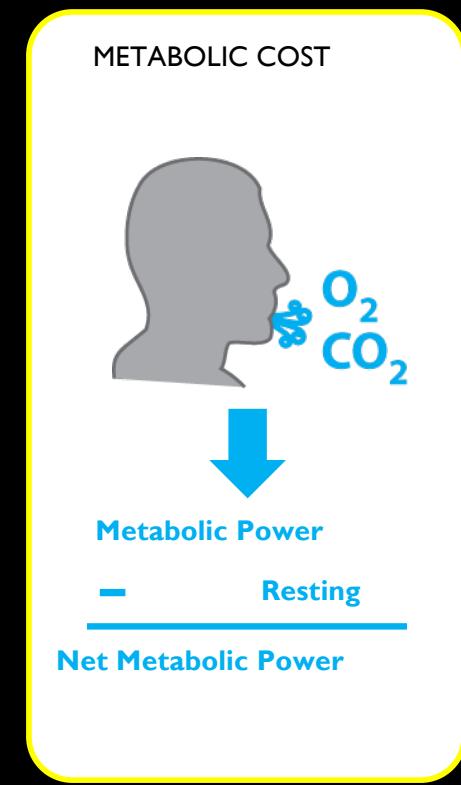
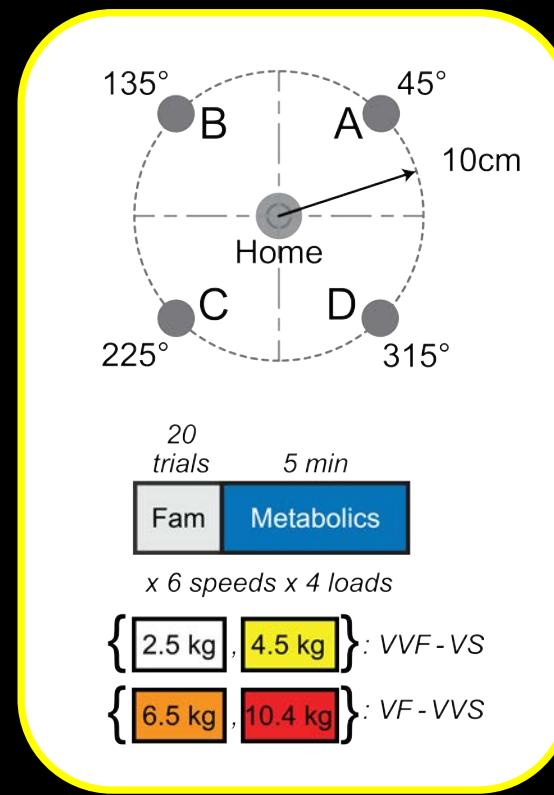
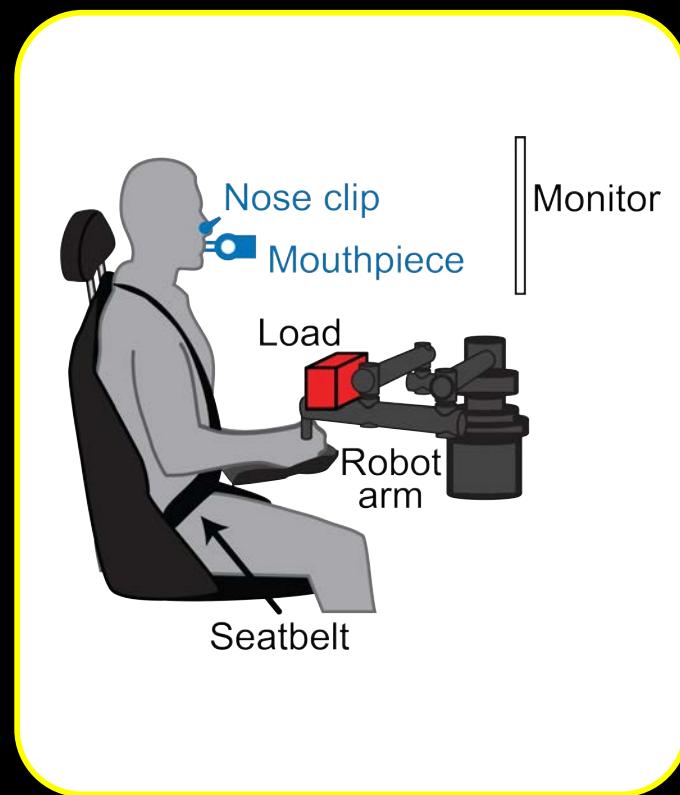
Goble et al., 2007; Wang and Dounskoia, 2012

# Can metabolic cost explain mass-based changes in preferred reaching speed?

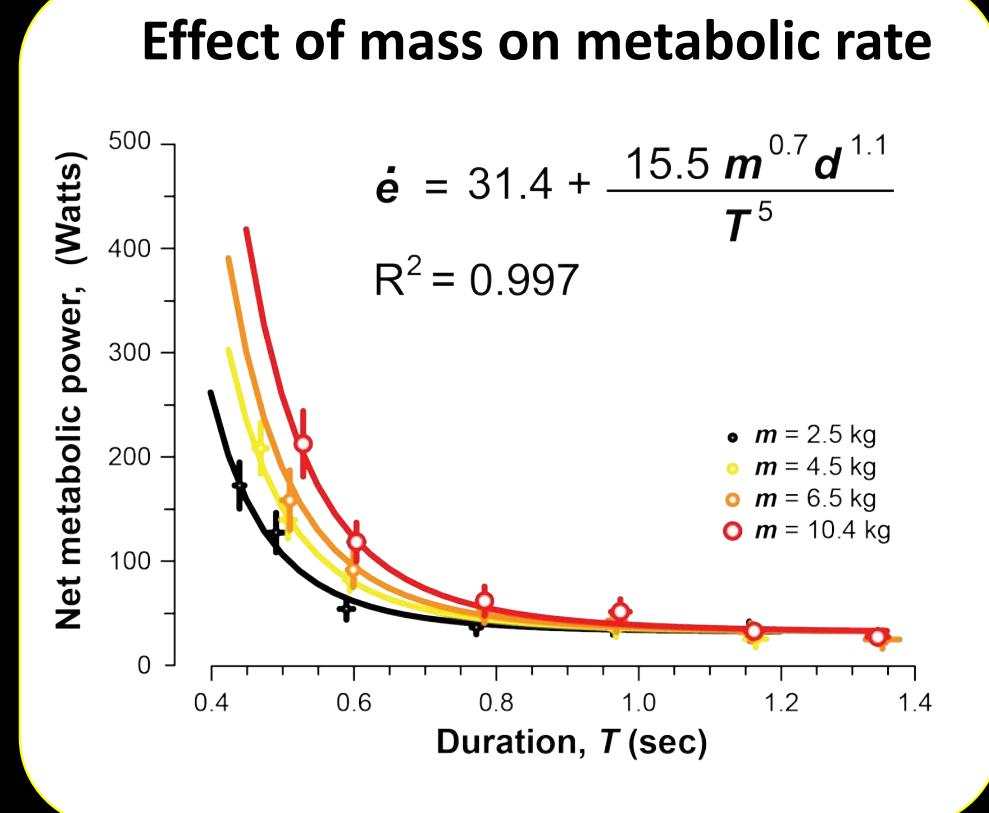
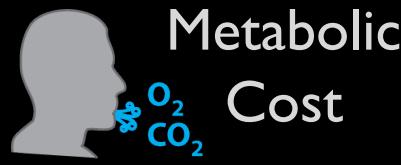


- What is the effect of mass on the metabolic cost of reaching?
- What is the effect of mass on preferred reaching speed?

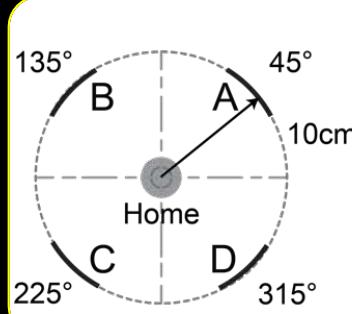
# Effect of mass on metabolics of reaching



# Added mass increases metabolic rate



# Effect of mass on preferred reach kinematics



Subjects (N=12) perform reaching movements with added mass.

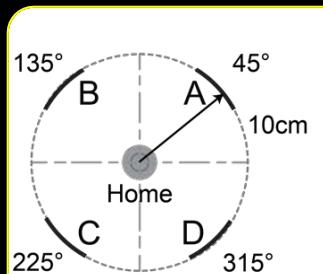


Gary Bruening

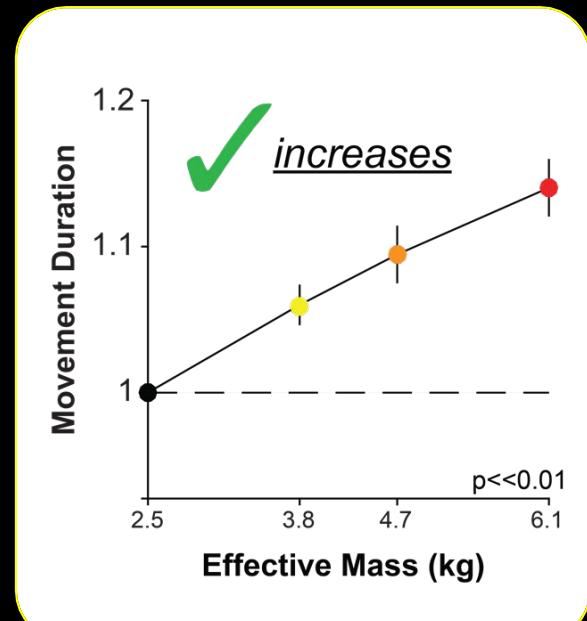
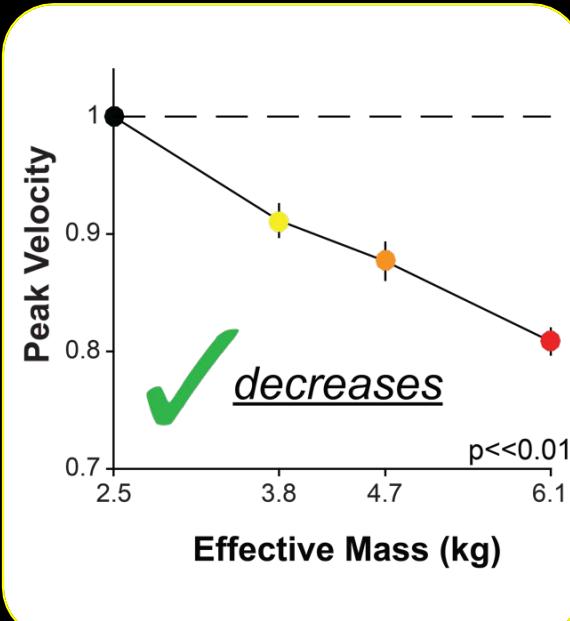
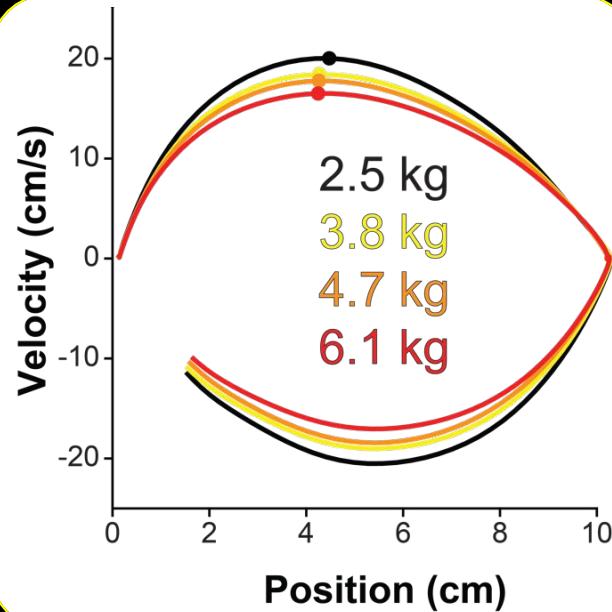


Megan O'Brien

# Mass of the arm reduces movement vigor



Subjects (N=12) perform reaching movements with added mass.

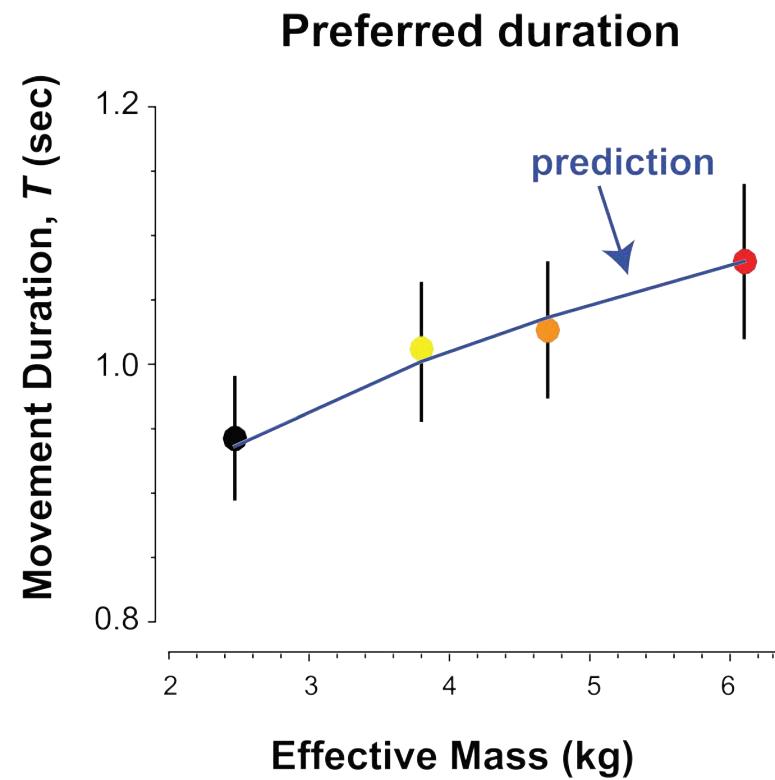


# Does metabolic cost explain preferred movements?

$$J = \frac{a-e}{1 + \gamma T} \longrightarrow e = \frac{15.5 m^{0.7} d^{1.1}}{T^4} + 31.4T$$

**Metabolic cost**

**Movement preferences can be explained by a representation of effort as metabolic cost**



O'Brien, et al., in preparation

# Alternative models of utility

$$J = \frac{a - e}{1 + \gamma T} \longrightarrow e = \frac{15.5 m^{0.7} d^{1.1}}{T^4} + 31.4 T$$

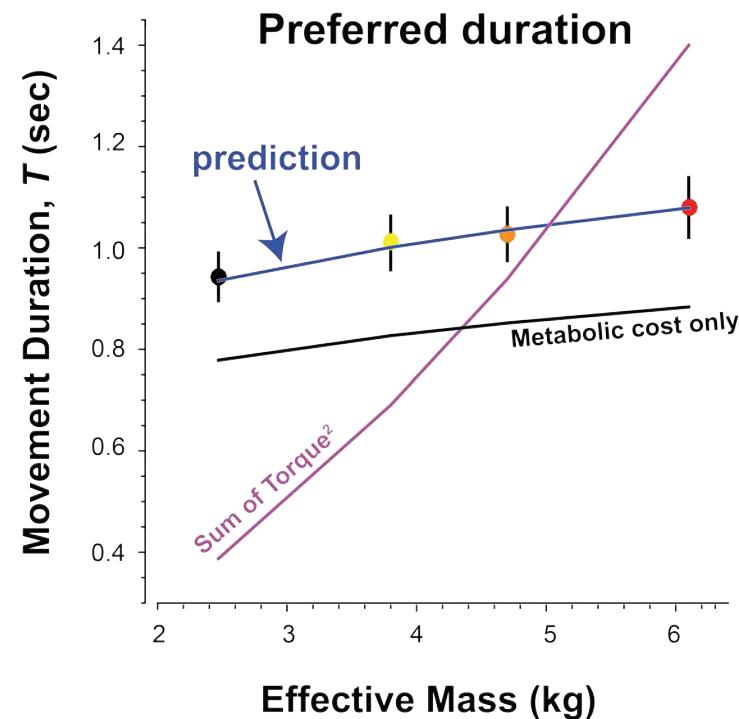
**Metabolic cost**

Sum of Torque<sup>2</sup>:

$$J = \frac{a - e}{1 + \gamma T} \longrightarrow e = \frac{0.8 m^2 d^{1.6}}{T^3}$$

Metabolic cost only:

$$J = \frac{15.5 m^{0.7} d^{1.1}}{T^4} + 31.4 T$$



# Movement utility as sum of reward and effort

$$J = \frac{\alpha - e(T)}{1 + \gamma T}$$

Utility

reward

effort

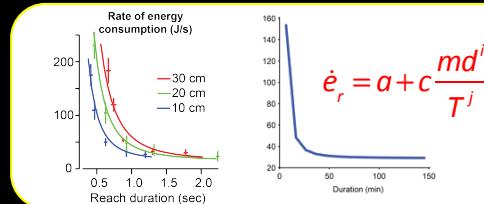
Temporal discounting factor

Duration of the movement

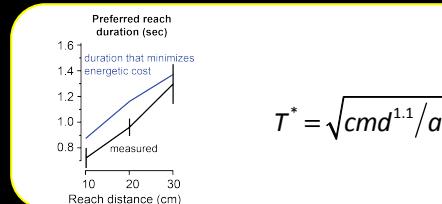
## Summary

- 1) Reward leads to faster movements.
- 2) People choose to reach slower with added mass.
- 3) Preferred movement speed can be explained with a utility that represents effort as the metabolic cost of the movement.

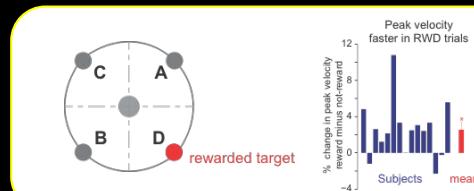
# Summary



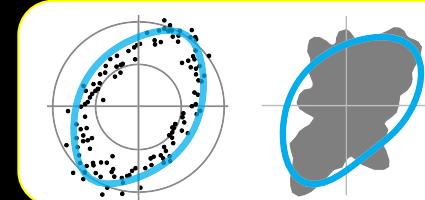
In both walking and reaching, metabolic rate increases with movement distance and decreases with duration.



There is an energetically optimal reaching speed. People prefer to reach faster than is energetically optimal.



Subjects move faster toward a rewarded stimulus compared with a non-rewarded stimulus. (Xu-Wilson et al. 2009, Nikooyan et al. 2015)

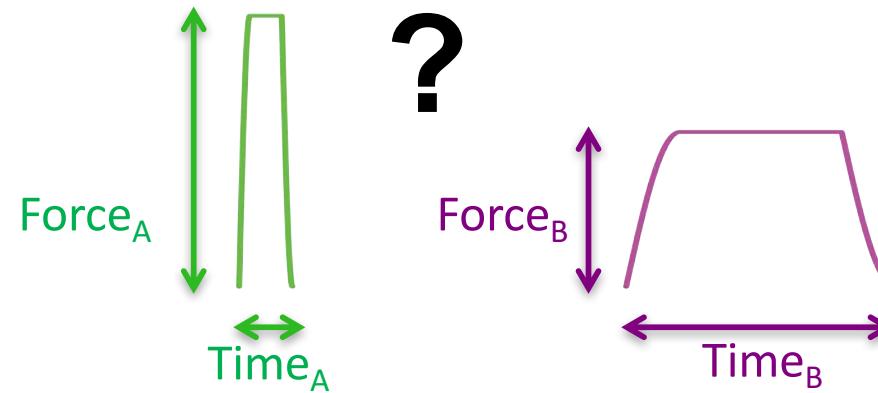
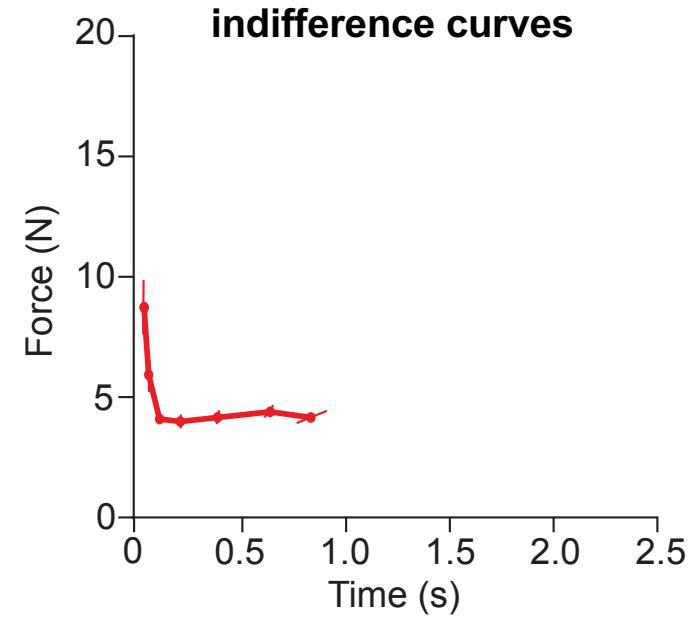


Subjects preferred to reach to the stimuli that required transport of a smaller mass (Wang and Dounskaia 2012), and did so with a higher velocity (Gordon et al., 1994).

# Temporal discounting of effort?

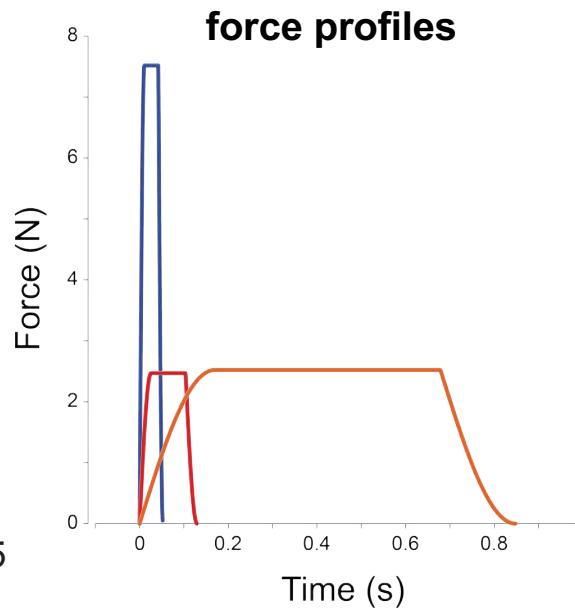
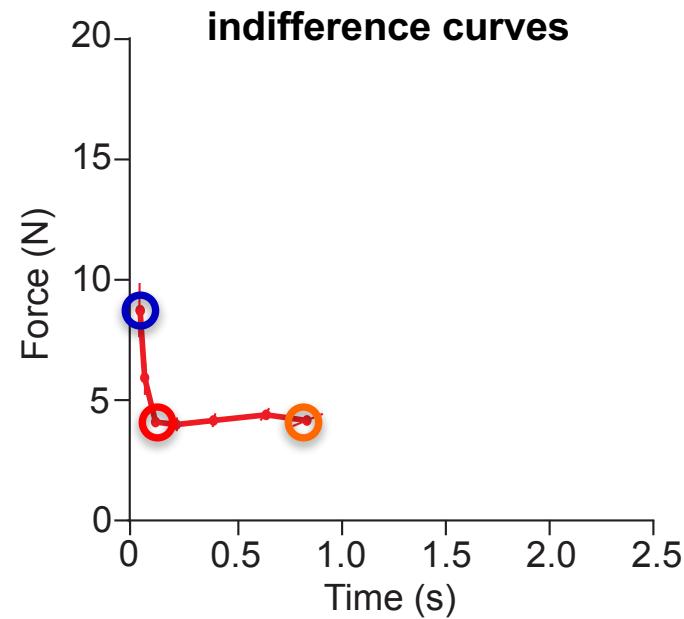
$$J = \frac{\alpha}{1 + \gamma T} - \frac{(aT + cmd^{1.1}/T^2)}{1 + \gamma T}$$

# Temporal discounting of effort?



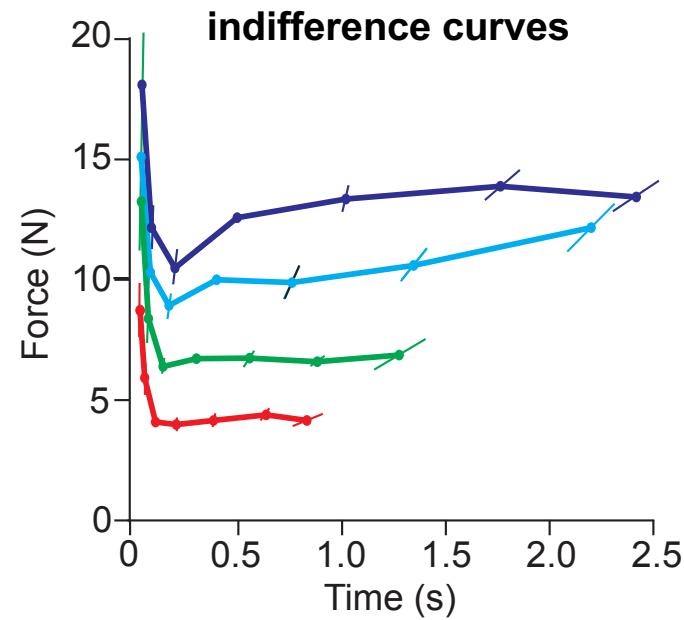
Kording et al. 2004

# Temporal discounting of effort



Kording et al. 2004

# Temporal discounting of effort

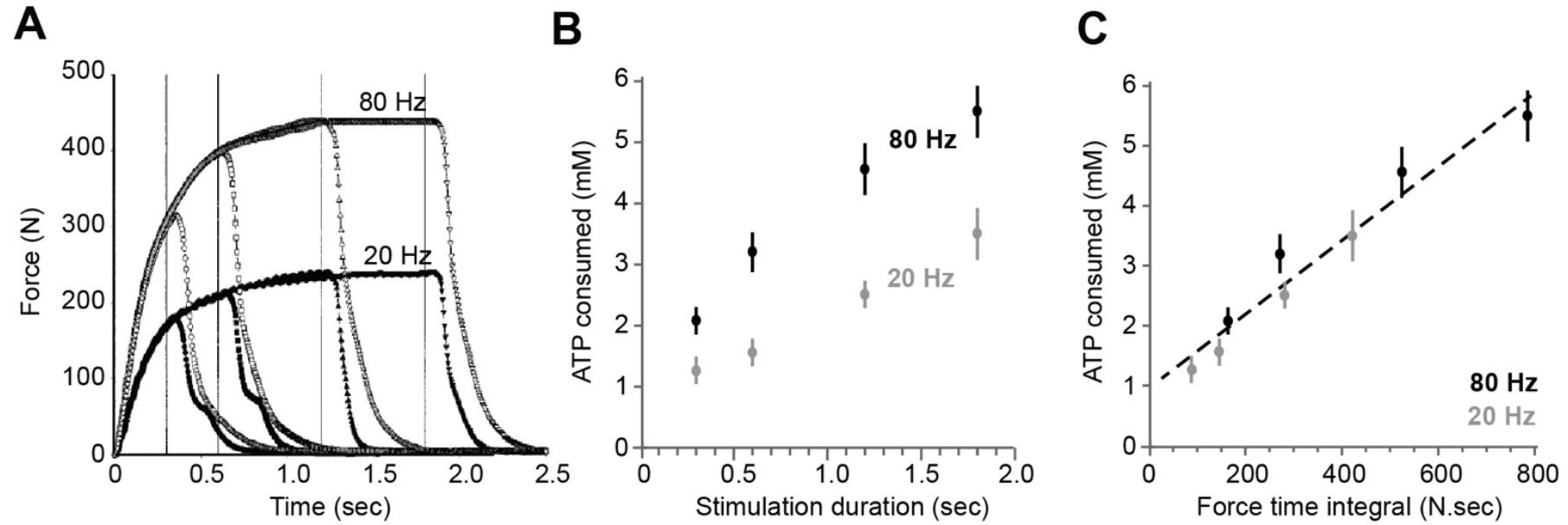


Kording et al. 2004

# Metabolic cost and force

- Thus far relied on an inverse relation between metabolic cost and movement duration.
- How do we relate metabolic cost to isometric force?

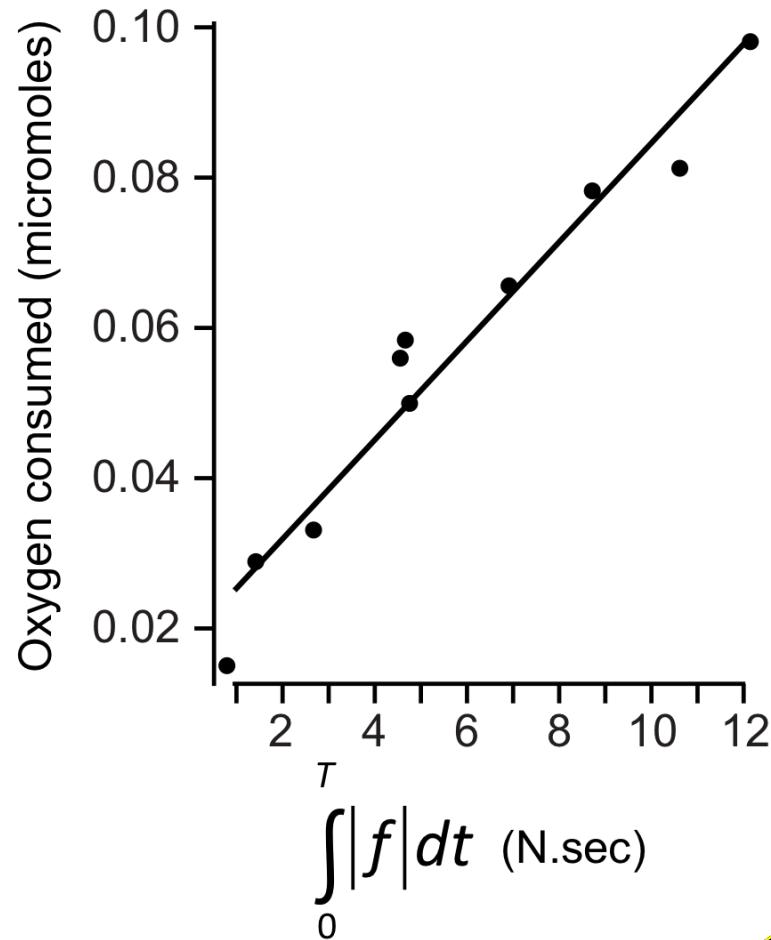
# Metabolic cost and Force



Human subjects were engaged in an isometric task while the experimenters measured the metabolic cost of force production. Spectroscopy was used to estimate concentration of ATP per gram of muscle in the human gastrocnemius. They electrically stimulated the muscle with trains of 20Hz or 80Hz pulses and measured the resulting forces and energy consumption.

# Metabolic cost and Force

C



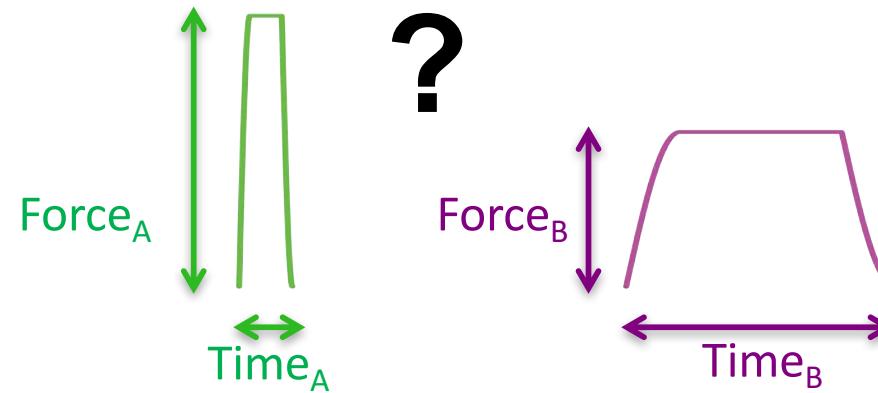
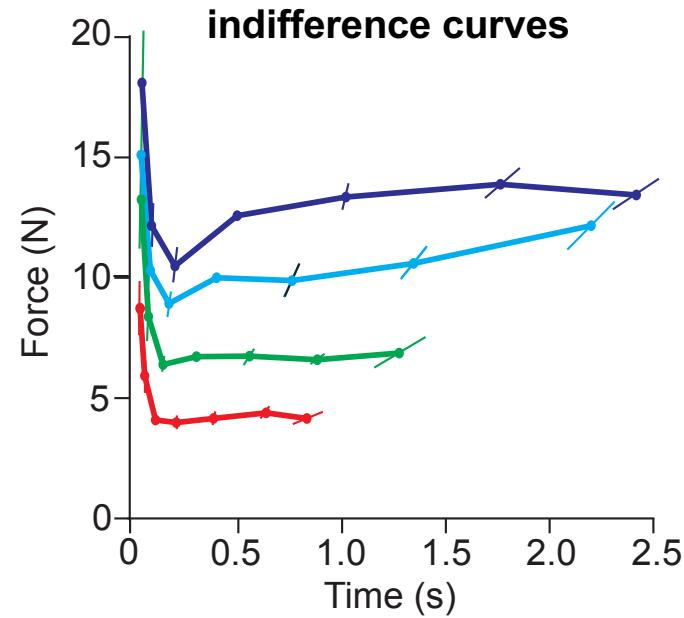
Kushmerick and Paul (1976) electrically stimulated a frog muscle for various durations and measured the resulting oxygen consumption. An analysis of their data suggests that oxygen consumption grows linearly with the force-time integral.

# Metabolic cost and force

- Thus far relied on an inverse relation between metabolic cost and movement duration.
- How do we relate metabolic cost to force?
  - Metabolic cost can be approximated with the Force-Time integral

$$e(f(t)) = a_1 \int_0^T |f(t)| dt + a_2$$

# Temporal discounting of effort



Kording et al. 2004

# Temporal discounting of effort

Energy is linearly related to the force-time integral

$$e(f(t)) = a_1 \int_0^T |f(t)| dt + a_2$$

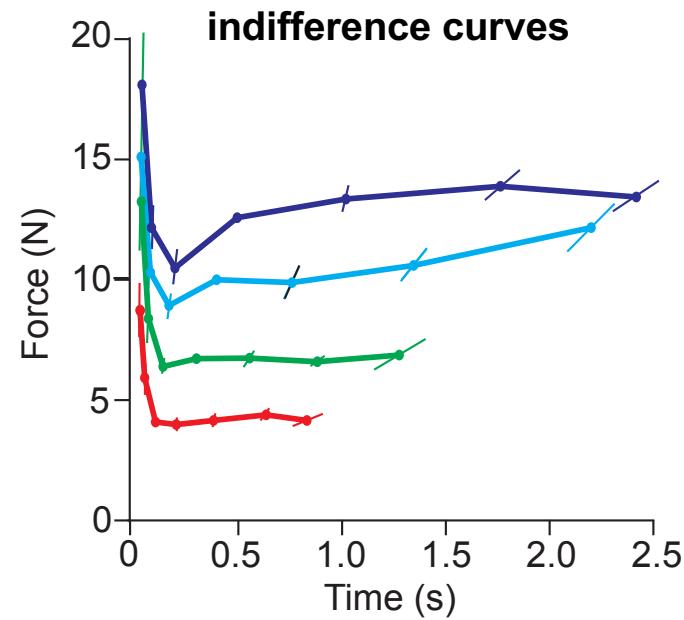
Utility of Effort

$$U(f(t)) = -\frac{c}{1 + \gamma T} \left( a_1 \int_0^T |f(t)| dt + a_2 \right)$$

Effort utility for a constant force

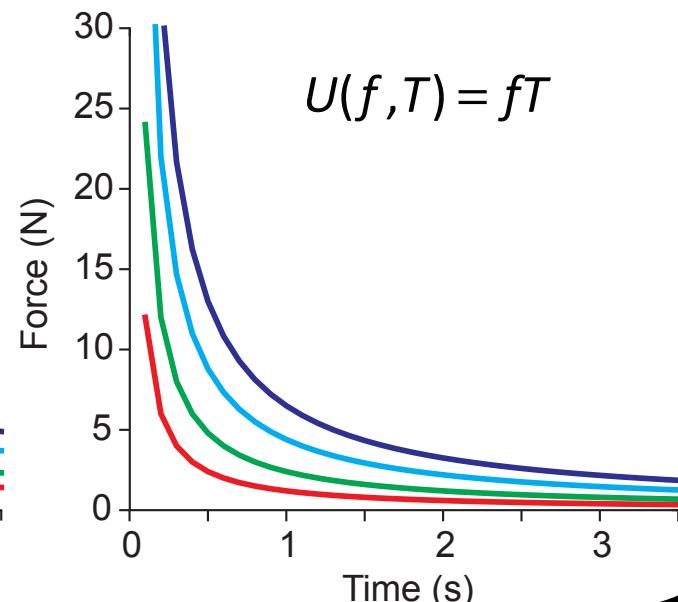
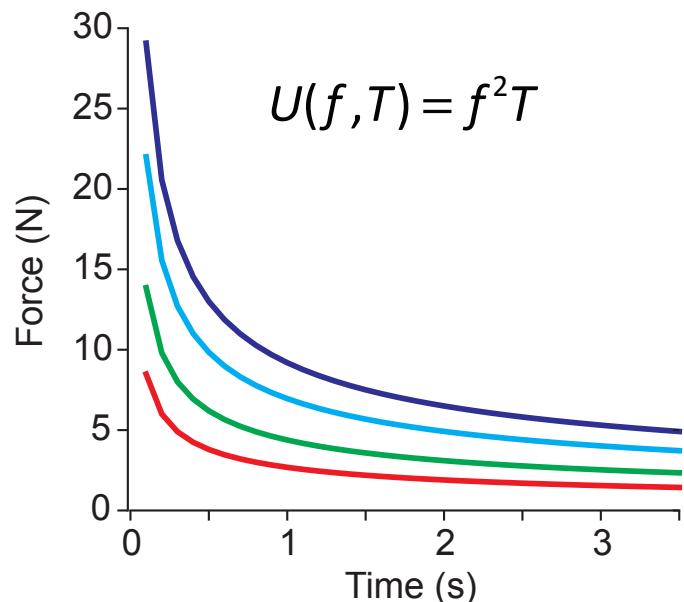
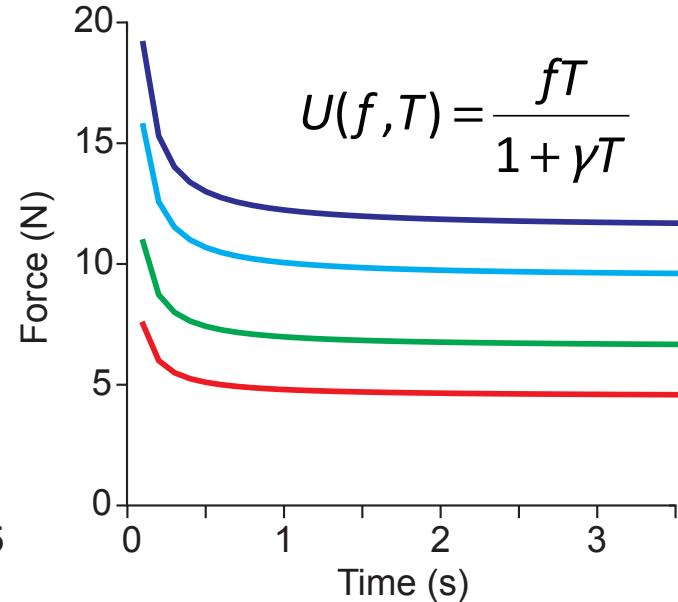
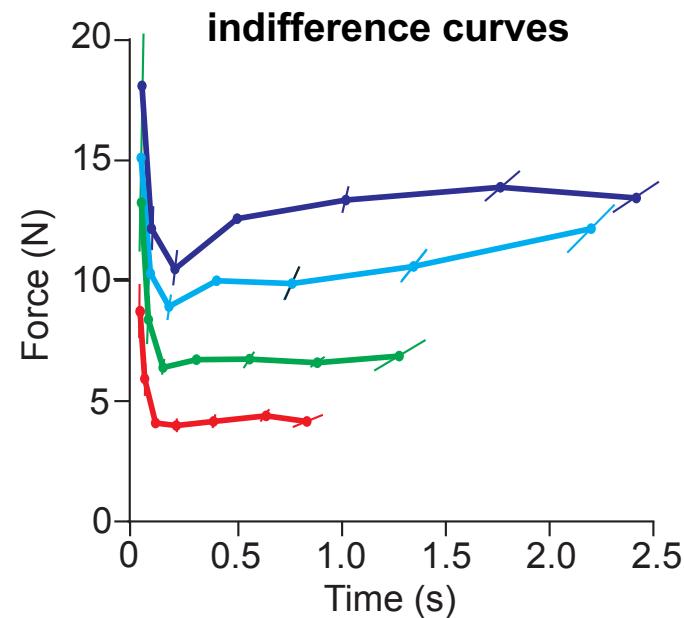
$$U = -\frac{c}{1 + \gamma T} (a_1 FT + a_2)$$

# Temporal discounting of effort

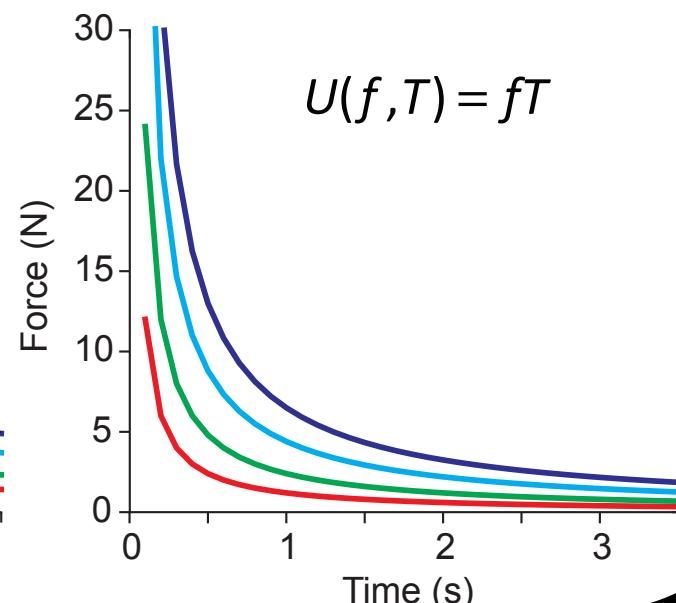
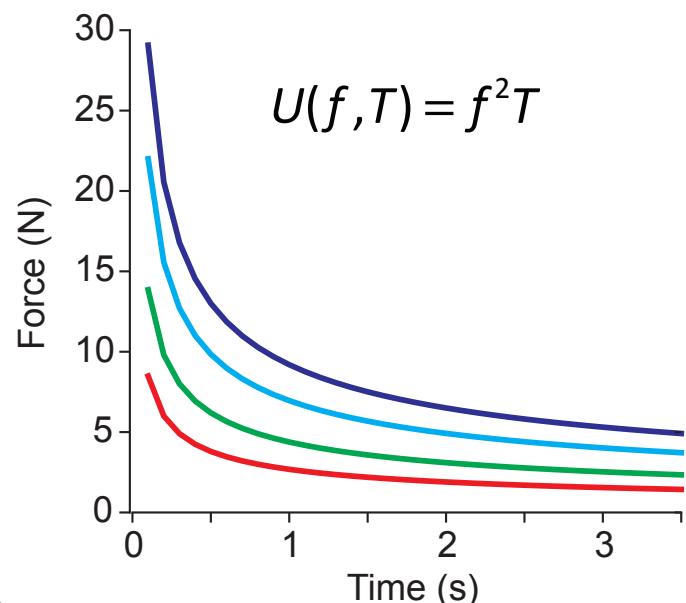
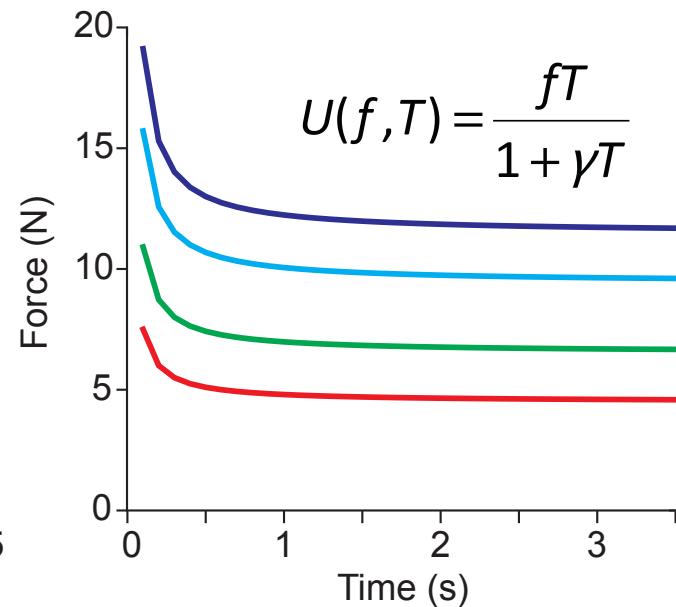
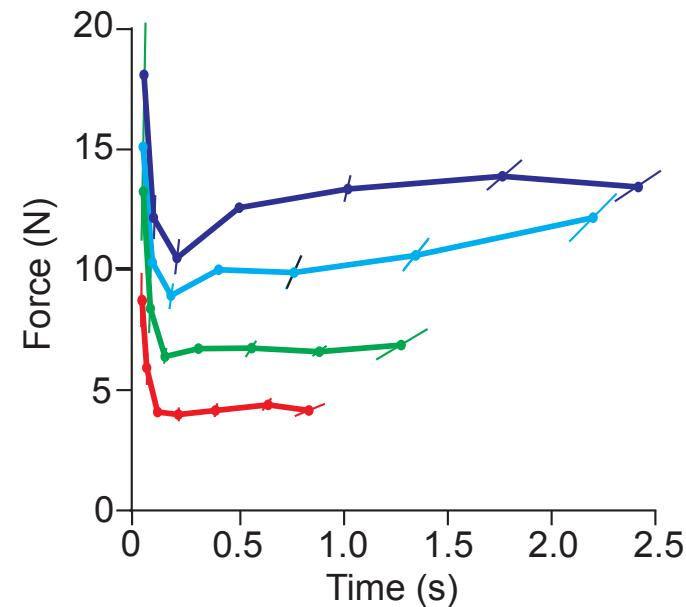


Kording et al. 2004

# Temporal discounting of effort



As the duration of generating an isometric force increased, the utility did not continue to increase, but rather reached a plateau, consistent with a utility in which effort is discounted by time.



# Reward value?

$$J = \frac{\alpha - mcd^2/T}{1 + \gamma T}$$

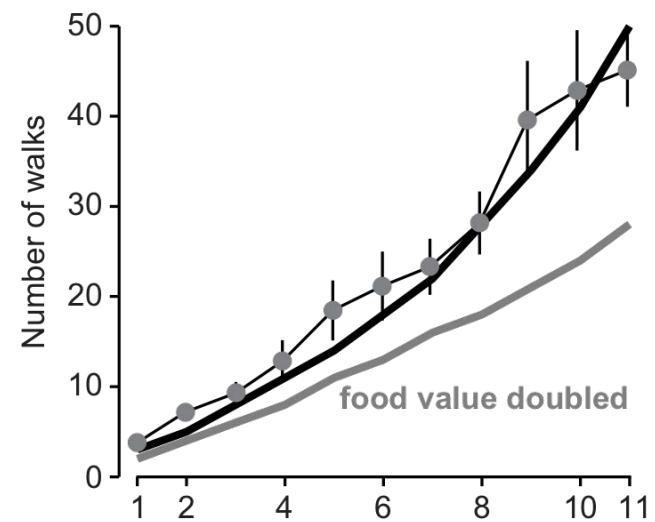


Bautista et al. 2001

$$J = \frac{\alpha - e_m}{1 + \gamma T}$$

**A**

Temporal discounting of reward and effort



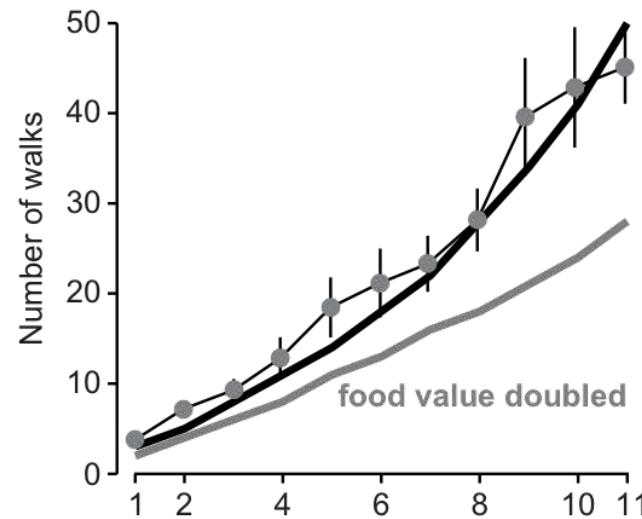
$$J = \frac{\alpha - e_m}{1 + \gamma T}$$

$$J = \alpha - e_m$$

$$J = \frac{\alpha}{1 + \gamma T} - e_m$$

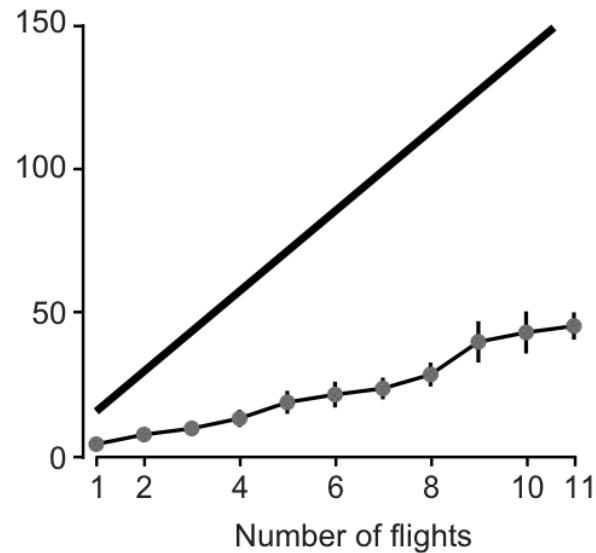
**A**

Temporal discounting of reward and effort



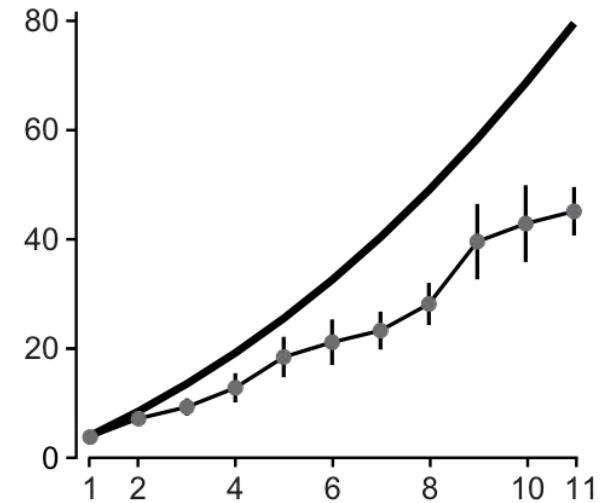
**B**

No temporal discounting

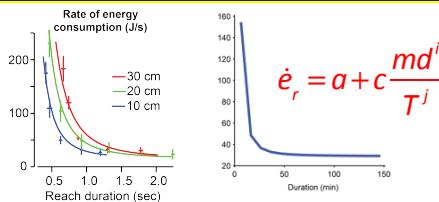


**C**

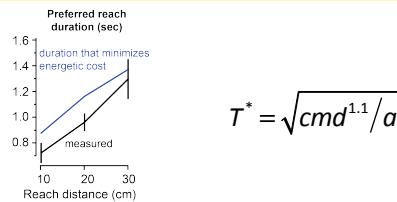
Temporal discounting of reward only



# Summary



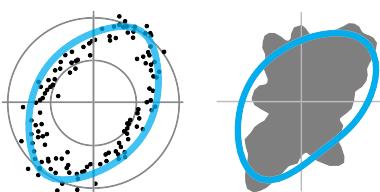
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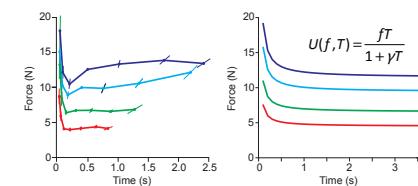
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