Synthetic Aperture Radar Simplified

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Abstract

The size of the SAR image matrix in terms of range cells and azimuth cells is derived from parameters such as frequency, chirp bandwidth, range, and antenna sizes. The equations for the number of range and azimuth cells clearly show the tradeoffs in the design of SAR sytems as well as predict the complexity of image formation.

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1 Introduction

Define the number of azimuth cells as N_x and the number of range cells as N_y for the area that is being imaged. The final processed SAR image will be an $N_x \times N_y$ image for a one look case. The image formation process will consist of operating on a matrix of this size. The complexity of SAR image formation is thus directly related to the number of azimuth and range cells. Below, we will present a simple derivation of N_x and N_y in terms of SAR parameters.

2 Derivation of Number of Range Cells N_y

The geometry of SAR is illustrated in Figure 1. In Figure 2 the cross section along the range direction is shown (the range plane). The "look angle" is θ and ψ is the beam width along the range direction. The beam width is,

$$\psi = a_y \frac{\lambda}{D_y} \tag{1}$$

where a_y is the aperture factor of the antenna and D_y is the antenna length along the range direction. Now from Figure 2,

$$R_0 \psi = L_y \cos\theta \tag{2}$$

from which

$$L_y = \frac{R_0 \psi}{\cos\theta} \tag{3}$$

where L_y is the illuminated area along the range plane. Thus,

$$L_y = a_y \frac{\lambda R_0}{D_y \cos\theta} \tag{4}$$

Now the range resolution is related to the time interval between two received echoes from two targets and the ability of the receiver to distinguish between the two received pulses. This time interval is related inversely to the bandwidth. We assume that the chirp pulse compression technique is used. The ambiguity diagram for a single frequency-modulated pulse is shown in Figure 3 in which the time resolution is seen to be inversely proportional to the chirp bandwidth. The transmitted pulse and the compressed pulse are illustrated in Figure 4. Thus, the range resolution along the beam is,

$$\delta r = \frac{c}{2}\delta T = \frac{c}{B} \tag{5}$$

By projecting the range ground resolution (xy plane) on the along-beam range direction we can obtain the expression for the ground range resolution.

$$\delta y = \frac{\delta r}{\sin\theta} = \frac{c}{2B\sin\theta} \tag{6}$$

The total number of range cells is, therefore,

$$N_y = \frac{L_y}{\delta y} \tag{7}$$

$$N_y = 2a_y \frac{R_0 B \lambda}{D_y c} \tan\theta \tag{8}$$

3 Derivation of number of Azimuth Cells N_x

The viewing geometry in the azimuth plane is shown in Figure 5. From the figure we have the length of the illuminated area along the azimuth direction at range R_0 is,

$$L_x = R_0 \psi \tag{9}$$

where ψ is the beam width,

$$\psi = a_x \frac{\lambda}{D_x} \tag{10}$$

Let the pulse repetition rate be f_p. The transmitter pulses at

$$T_p = \frac{1}{f_p} \tag{11}$$

intervals apart. Now, a target in the azimuth direction at a constant range R_0 remains in the antenna beam for a distance L_x. Thus if the vehicle velocity is v, the total time in which the target is in view is,

$$T_t = \frac{L_x}{v} \tag{12}$$

Thus the total number of pulse echoes that the radar receives during the interval T_t is (see Figure 6)

$$N_x = \frac{T_t}{T_p} \tag{13}$$

Now, the pulse repetition rate must be twice the doppler bandwidth along the azimuth direction. The doppler bandwidth is related to the "synthesizing" of the aperture as the vehicle approaches the target and then recedes. The doppler shift at each extreme is,

$$f_D = \frac{f_0 v_r}{c} = \frac{v_r}{\lambda} \tag{14}$$

Projecting vehicle velocity along the azimuth path onto the path along the beam,

$$f_D = \frac{v \sin\frac{\psi}{2}}{\lambda} \tag{15}$$

The doppler bandwidth B_D is twice f_d. Thus, assuming that ψ is small,

$$B_D = \frac{v\psi}{\lambda} \tag{16}$$

Or substituting for ψ,

$$B_D = \frac{a_x}{\lambda}\frac{v\lambda}{D_x} = a_x\frac{v}{D_x} \tag{17}$$

Since $f_p = 2B_D$ to satisfy the Nyquist criterion,

$$N_x = 2a_x\frac{v}{D_x}\frac{L_x}{v} \tag{18}$$

Or,

$$N_x = 2a_x^2\frac{R_0\lambda}{D_x^2} \tag{19}$$

We are done!

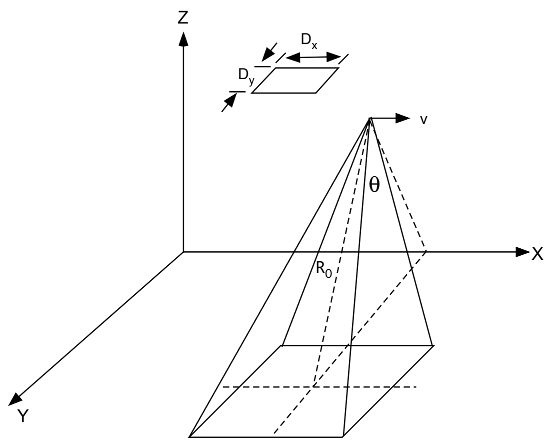

Figure 1: SAR Geometry

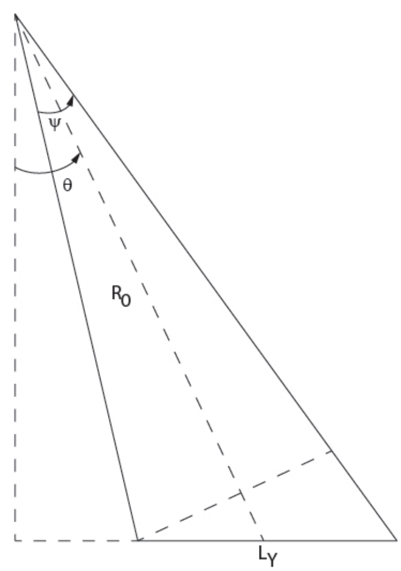

Figure 2: Geometry in Range Plane

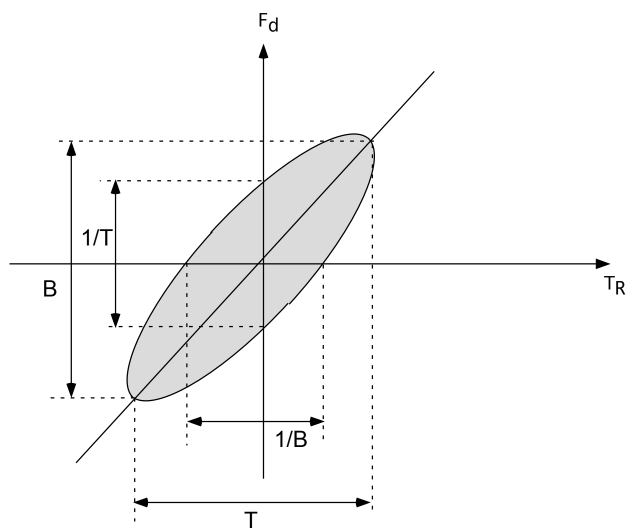

Figure 3: Ambiguity Diagram Chirp Pulse Compression

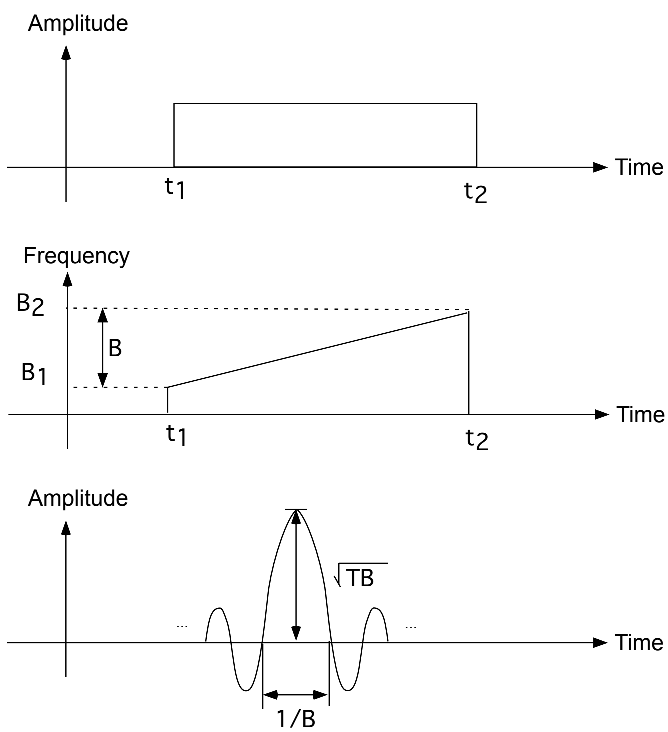

Figure 4: Chirp Pulse Compression

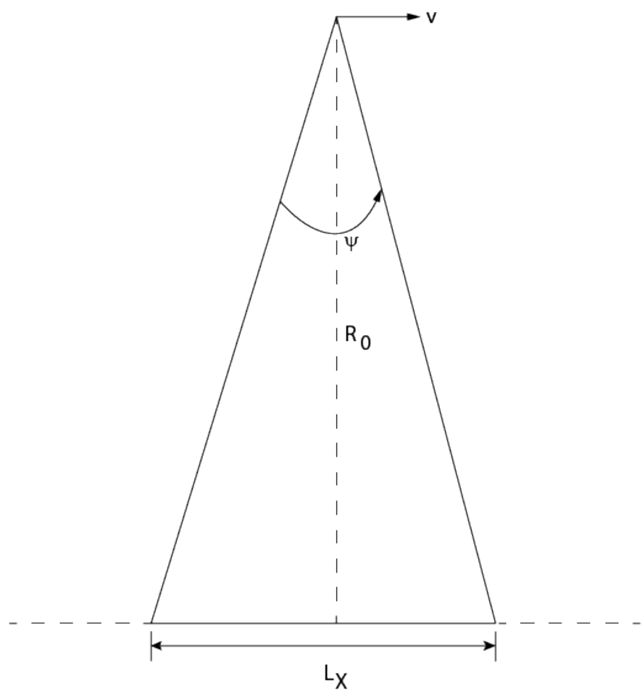

Figure 5: Viewing Geometry Azimuth Plane

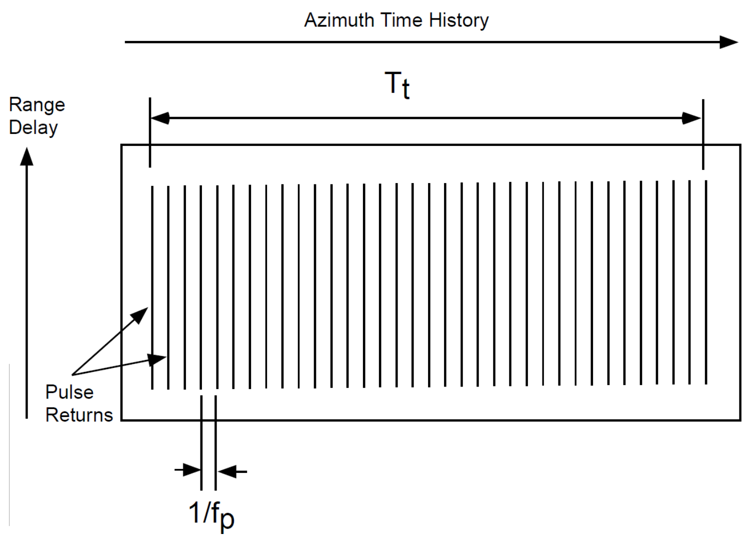

Figure 6: SAR Record Radar Returns