

# Synthetic Aperture Radar Point Target Response

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## Abstract

The algorithms for computing the point target response in Synthetic Aperture Radar (SAR) will be presented. The target modeling and simulations will be performed following the procedure developed by McDonough, et. al. (1985) [1] for SEASAT. Simulation results will be provided through the block diagram modeling of the SAR system with Capsim®[2]. The SAR project has been a part of the Capsim® distribution since 1990. The research on SAR was conducted by the author while a Professor at NC State University in 1987. The GitHub repository was created in November 2025.

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# **Chapter 1**

## **Introduction**

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The algorithms for computing the point target response in Synthetic Aperture Radar (SAR) will be presented. The target modeling and simulations will be performed following the procedure developed by McDonough, et. al. (1985) [1] for SEASAT. The algorithms and simulation results for a point target in Synthetic Aperture Radar (SAR) will be presented. The typical SEASAT-SAR geometry is shown in Figure 1 and with parameters given in Table 1. A technique by Fitch [3] will also be used to avoid a Hilbert transform.

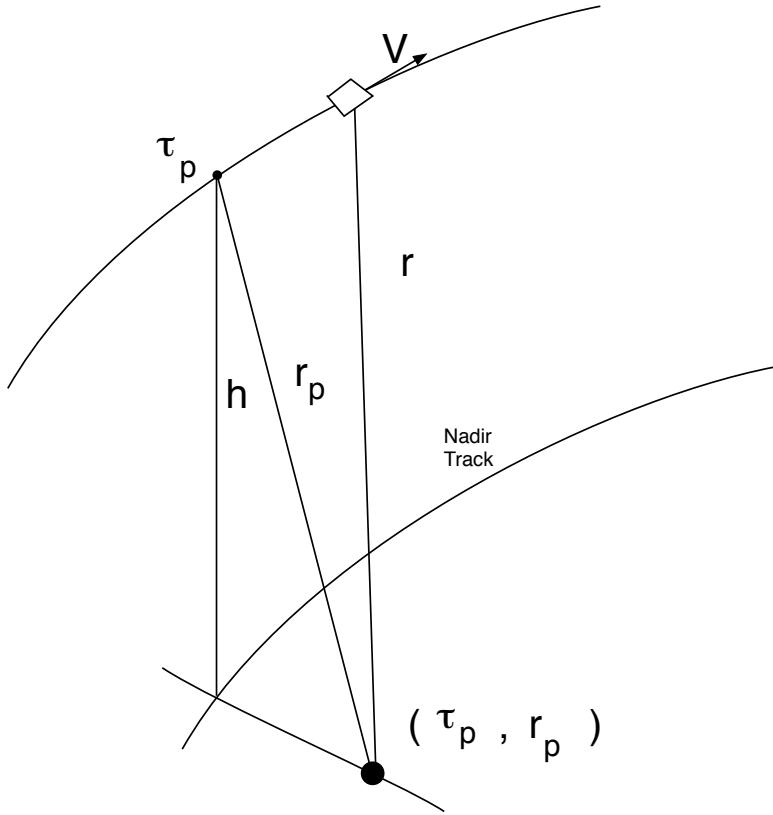


Figure 1: SAR Geometry

## Chapter 2

### Point Target Response

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Consider a point target arbitrarily placed at position  $(r_p, \tau_p)$ . The radar will fly at some height  $h$ , and velocity  $v$ . The slant range to the target changes as a function of viewing position as shown by

$$r(t, \tau) = r_p + \frac{v^2}{2r_p}(\tau_c + \tau_p - \tau)^2 \quad (1)$$

Table 1: SAR Parameters

Item	Parameter	Value	Units
1	Carrier Frequency	1275	MHz
2	Pulse Chirp Rate	0.5621	MHz/uS
3	Pulse Duration	33.8	uS
4	Pulse Bandwidth	19.0	MHz
5	Center Frequency (IF)	11.38	MHz
6	Pulse Repetition Rate(PRF)	1645.0	Hz
7	Sampling Rate	45.03	MHz
8	Doppler Frequency	1150.0	Hz
9	Doppler Rate of Change	501.27	Hz
10	Platform Velocity	7.0	Km/s
11	Integration Time	2.0	s
12	Azimuth Sample Time	0.0	s
13	tc	1.0	s
14	Point Range, rp	840.0	kM
15	Point Azimuth tp in index units	1000	index

where  $\tau$  is the time variable in the azimuth direction,  $\tau_c$  coincides with the mid point of the integration time, and  $\tau_p$  is the azimuthal time location of the point target relative to  $\tau_c$ . To derive 1 , we note that

$$r^2(t, \tau) = r_p^2 + (v \times (\tau_c + \tau_p - \tau))^2 \quad (2)$$

Then using a Taylor series expansion,

$$r(t, \tau) = r_p + \frac{v^2}{2r_p}(\tau_c + \tau_p - \tau)^2 - \frac{v^4}{2r_p^3}(\tau_c + \tau_p - \tau)^4 + \dots \quad (3)$$

Note that  $\frac{v \times (\tau_c + \tau_p - \tau)}{r_p} \ll 1$  so we get Equation 1 .

Equation 1 shows that the range history of the target during the integration time for forming an aperture is described by a parabola. For example, if we consider  $\tau_p$  to coincide with  $\tau_c$ , then the slant range will be a maximum when the target first enters the beam at  $\tau = 0$ . It will then reach a minimum when  $\tau = \tau_c$ , and will again increase to some maximum as the target exits the beam. Now, if the  $\tau_p$  is not identically located at  $\tau_c$ , then the beginning and ending slant ranges will not be equal. The difference in the two range positions is known as range walk, and together with the parabolic curvature introduced into the range, the problem is known as range migration.

To obtain high resolution in the range direction, the radar transmits an FM chirped pulse of the form,

$$s(t) = \cos 2\pi(f_0 t + \pi K_r t^2) \quad |t| \leq \Delta t/2 \quad (4)$$

where  $s(t)$  is the transmitted signal,  $f_0$  is the carrier frequency of the radar,  $K_r$  is the chirp rate and  $\Delta t$  is the pulse width.

Each pulse has a duration of 33.8  $\mu\text{sec}$  and a chirp rate of 0.5621 MHz/ $\mu\text{sec}$  or a pulse bandwidth  $B_r$  of 19 MHz which results in a slant range resolution  $\delta r = \frac{c}{2} B_r \sin \theta$  of 25 meters.

Figure 2 shows plots of the time domain and frequency domain signal for the radar FM chirped pulse for range.

The received pulse  $g(t)$  will be delayed by the round trip time of  $2\frac{r(\tau)}{c}$ , or,

$$g(t) = \cos \left[ (2\pi f_0(t - 2r(\tau)/c) + \pi K_r (t - 2r(\tau)/c)^2) \right] \quad |t - 2r(\tau)/c| \leq \Delta t/2 \quad (5)$$

where  $r(\tau)$  is the slant range from the radar at time  $\tau$ , to the target at position  $(r_p, \tau_p)$ , and  $c$  is the speed of light. To minimize the required sampling rate, the signal is then mixed with a local oscillator at the carrier frequency 1275 MHz and down-converted to some intermediate frequency of  $f_1 = 11.38$  MHz resulting in,

$$g(t) = \cos \left[ (2\pi f_1 t - 2r(\tau)/\lambda) + \pi K_r (t - 2r(\tau)/c)^2 \right] \quad (6)$$

## 2.1 Range Compression

To get any useful information regarding the position of the target, the down-converted received signal can be correlated with a reference function. Thus, when the two signals are coincident at some time and have the same shape the output of the correlation is at a maximum. The correlation is performed as the convolution of  $g'(t)$  and the replica  $s'(-t)$  as [1],

$$s'(-t)*g'(t) = \Delta t/2 \operatorname{Re} \left[ \sin(\pi B_r (t - 2r(\tau)/c)) / (\pi B_r (t - 2r(\tau)/c)) \times e^{-j4\pi r(\tau)/\lambda} e^{j2\pi f_1 t} \right] \quad (7)$$

where

$$|t - \frac{2r(\tau)}{c}| \leq \Delta t \quad (8)$$

and  $B_r$  is the bandwidth.

Hence, the time domain convolution in the above expression results in the desired range compressed signal. Now, the reference function for the pulse compression has  $U = \tau f_s$  or  $33.8 \mu\text{sec} \times 45.03 \text{ MHz} = 1530$  points. Thus the time domain correlation will require on the order of  $U \times U$  or  $2 \times 10^6$  multiplications. The number of operations, however, can be significantly reduced to  $2U \log U$  or  $9.7 \times 10^3$  if the time domain convolution is performed as multiplication in the frequency domain. The range compression in the frequency domain is obtained from:

1. Take the DFFT ( $g'(t)$ ) to get  $G'(f)$  of length  $N$ .
2. Take the DFFT ( $s'(t)$ ) to get  $S'(f)$  of length  $N$ .

3. Truncate the spectrums to half of the original lengths.
4. Range compressed =  $FFT^{-1}(G'(f)S'(f))$  of length  $\frac{N}{2}$ .
5.  $(r_0)$  position of point target =  $\max(|FFT^{-1}(G'(f)S'(f))|)$ .

At the end of operation 4 we are left with the right hand side of Equation 7. Step number 3 is necessary since the azimuth compression requires a complex sequence for correlation. This important approach for real to complex conversion using FFTs was described by Fitch in 1988 [3] and it bypasses the need for a Hilbert transform. Basically, the complex sequence has a real part equal to samples of the real signal taken at half the original rate and a spectrum equal to the first half of the original real sequence spectrum. The real to complex conversion is accomplished by using the first half of the spectral coefficients.

The result of the matched filter for range compression is illustrated in the plot in Figure 3.

The range compression operation begins by fixing an arbitrary reference point within the matrix with a known  $r(t_{ref}, \tau_{ref})$  and computing its impulse response  $g(t)$  using Equation 6. Then, the response  $g(t)$  to the fixed value of  $\tau$ , which is the azimuthal position of  $r(t, \tau)$  of Equation 1. The time extents of both  $s(t)$  and  $g(t)$  are 33.8  $\mu$ sec and the signals are sampled at  $f_s$  or 45.03 MHz. Each pulse is then compressed using a 2048 point FFT according to the procedure described above.

The result of compressing a single pulse is shown in Figure 3. It basically demonstrates the result of the correlation output from Equation 7 showing an envelope of  $\frac{\sin x}{x}$  which is maximized when  $t = 2r_p/c$ , where  $r_p$  is the range to the point target. Hence, the range location,  $r$ , of the point target has been recovered. Figure 4 illustrates the FM Chirped Pulse and the Compression and Resolution Related to Bandwidth. To show that the range compression is indeed a  $\frac{\sin(x)}{x}$  function, Figure 5 shows the range compression of Figure 3 zoomed in.

The range compression process is repeated by moving in the azimuthal direction, fixing the slant range to the point target  $r(\tau)$  and transmitting pulses at the PRF rate. For this particular simulation, using an integration time of 2 seconds and a PRF of 1645 Hz, a total of 3290 pulses were transmitted.

If we consider a number of successive echoes, due to the large change in the range from the satellite to the target during integration time  $T_i$ , the range compressed point response will be a parabolic curve as shown by Equation 1 and :

$$\frac{Z^2}{r_0 c} - \frac{\tau}{2} < Y < \frac{Z^2}{r_0 c} + \frac{\tau}{2} \quad (9)$$

and

$$-\frac{vT_i}{2} < Z < \frac{vT_i}{2} \quad (10)$$

where  $Y$  is the start of each pulse,  $Z = v\tau$ , and  $\tau'$  is the pulse duration.

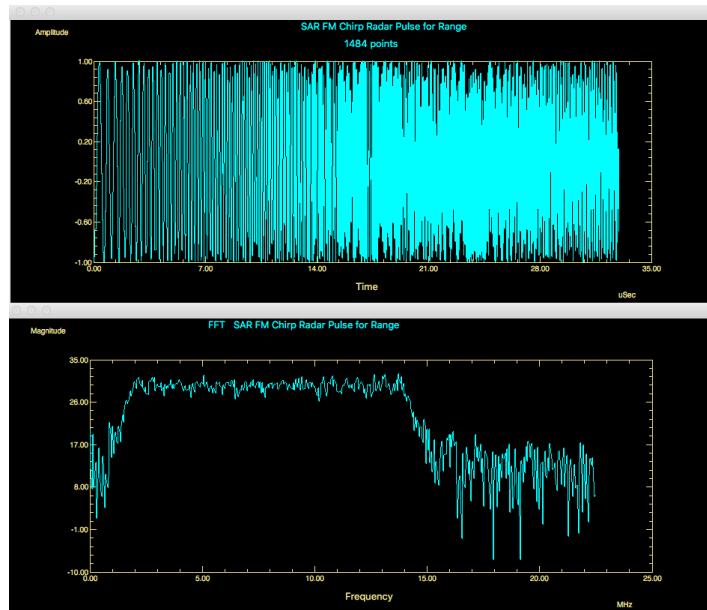


Figure 2: SAR Range FM Chirp Radar Signal

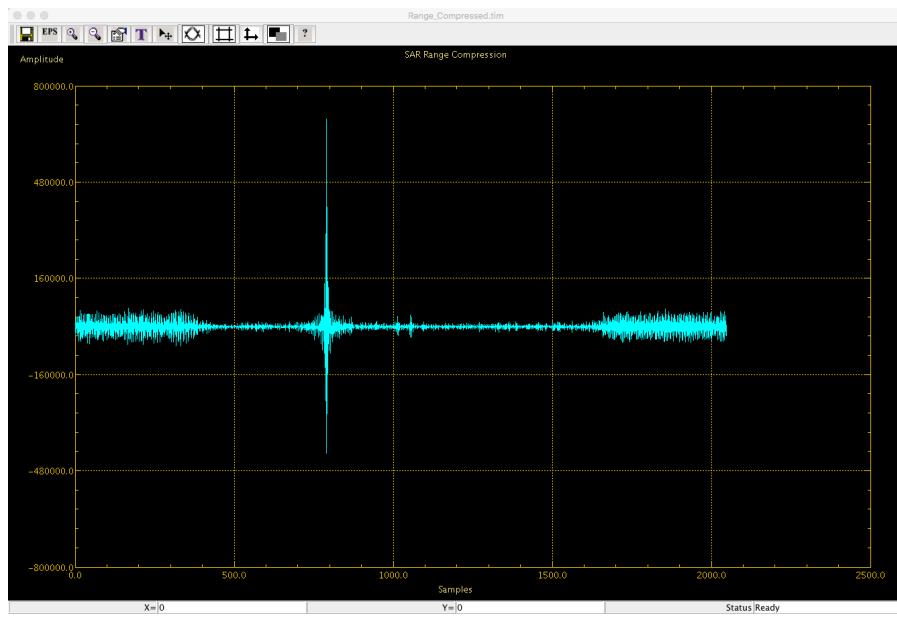


Figure 3: SAR Image Range Compression

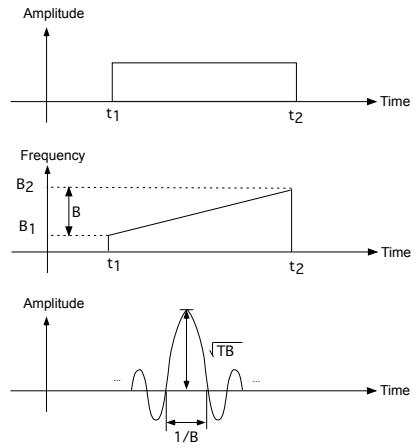


Figure 4: SAR Image Range FM Chirped Radar and Compression

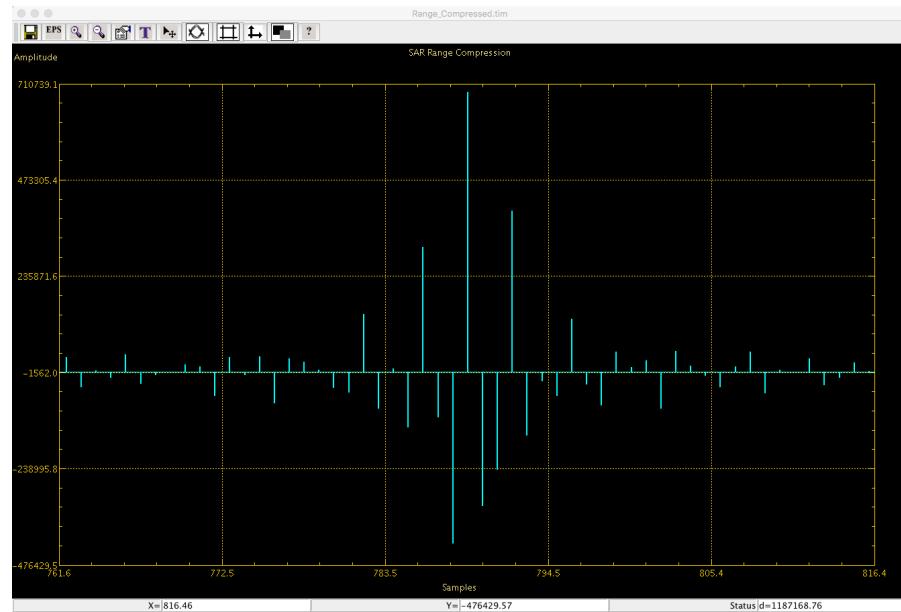


Figure 5: SAR Image Range Compression Zoomed Discrete Showing  $\text{Sin}(x)/x$  Response

As the range returns are received a matrix is formed as shown in Figure 6.

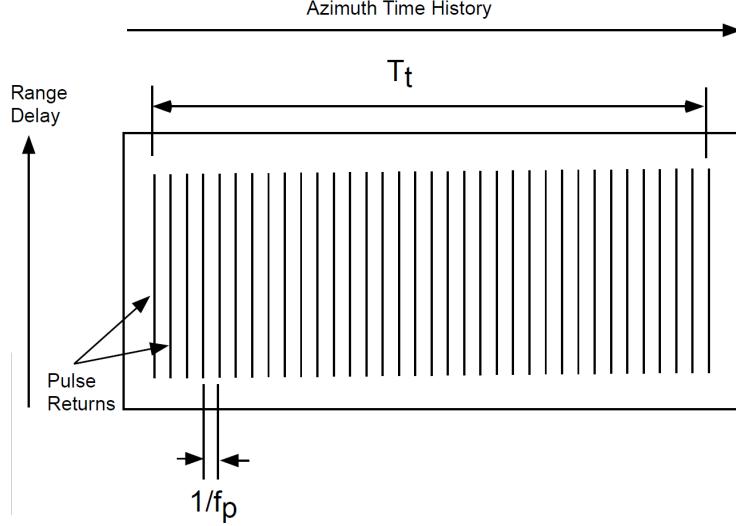


Figure 6: SAR Record Radar Returns

## 2.2 Azimuth Compression

Now, the complex exponential term  $e^{-j4\pi r(\tau)\lambda}$  appearing in Equation 7, represents the azimuth phase history which is encoded in the range compressed signal. If  $r(\tau)$  is expanded about  $\tau_c$ ,

$$r(\tau) = r(\tau_c) + \dot{r}(\tau_c)(\tau - \tau_c) + \frac{1}{2}\ddot{r}(\tau_c)(\tau - \tau_c)^2 + \dots \quad (11)$$

where

$$f_{DC} = -\frac{2\dot{r}}{\lambda} \quad (12)$$

is the Doppler frequency and,

$$K_{az} = -\frac{2\ddot{r}}{\lambda} \quad (13)$$

is the rate of change of Doppler frequency.

$\dot{r}$  and  $\ddot{r}$  are the velocity and acceleration vectors respectively.

Re-writting  $r(\tau)$  in terms of  $f_{DC}$  and  $K_{az}$  we get [1]:

$$g(\tau) = e^{-j4\pi r(\tau)/\lambda} = e^{-j4\pi r_c/\lambda} e^{j2\pi f_{DC}(\tau-\tau_c) + j\pi K_{az}(\tau-\tau_c)^2} \quad (14)$$

$|\tau - \tau_c| \leq \Delta\tau/2 \text{ and } 0 \text{ elsewhere}$

The matched filter for the azimuth compression will have an impulse response of the form:

$$h(\tau) = e^{j2\pi(f_{DC}\tau - 0.5K_{az}\tau^2)} \quad |\tau| \leq \Delta\tau/2 \quad (15)$$

Figure 7 shows a plot of the Azimuth complex matched filter  $h(\tau)$ .

For the azimuth compression an operation similar to the range compression is required. The range compressed lines are read back into memory from external disk storage, and the data are corner turned such that consecutive azimuth cells are aligned for a given range line. In actuality, at this stage a range walk and curvature correction is required since the range data are collected along a parabolic curve in the azimuth direction. Each range compressed line will then have a  $g(t)$  which will then be convolved with  $h(\tau)$  to get[1]:

$$\begin{aligned} b(\tau) = g(\tau) * h(\tau) &= e^{j2\pi f_{DC}\tau} \times e^{-j2\pi(2r_c/\lambda + f_{DC}\tau_c)} \times \\ &\Delta\tau (\pi B_D(\tau - \tau_c)) / (\pi B_D(\tau - \tau_c)) \quad |\tau - \tau_c| \leq \Delta\tau/2 \end{aligned} \quad (16)$$

where  $B_D = 2f_{DC}$ . The matched filter response  $h(\tau)$ , is generated with a  $\tau$  spacing of  $\frac{1}{PRF}$  and  $f_{DC} = 1150$  Hz and  $K_{az} = 501.3$  Hz. Once again because of computational efficiency the above convolution is performed in the frequency domain:

1. Take the  $DFFT(g(\tau))$  to get  $G(f)$ .
2. Take the  $DFFT(h(\tau))$  to get  $H(f)$ .
3. Azimuth compressed  $b(\tau) = FFT^{-1}(G(f)H(f))$
4.  $\tau$  position of point target =  $\max(|b(\tau)|)$

We note that since both  $g(\tau)$  and  $h(\tau)$  are complex, the original length of the sequence for the FFT's were used. The above process is repeated until each of the range compressed lines are compressed in the azimuth. The output of the correlator  $\frac{\sin \pi B_D(\tau - \tau_c)}{\pi B_D(\tau - \tau_c)}$  is a maximum when  $\tau = \tau_c$ , or when the azimuth position of the point target is aligned with the mid point of the beam.

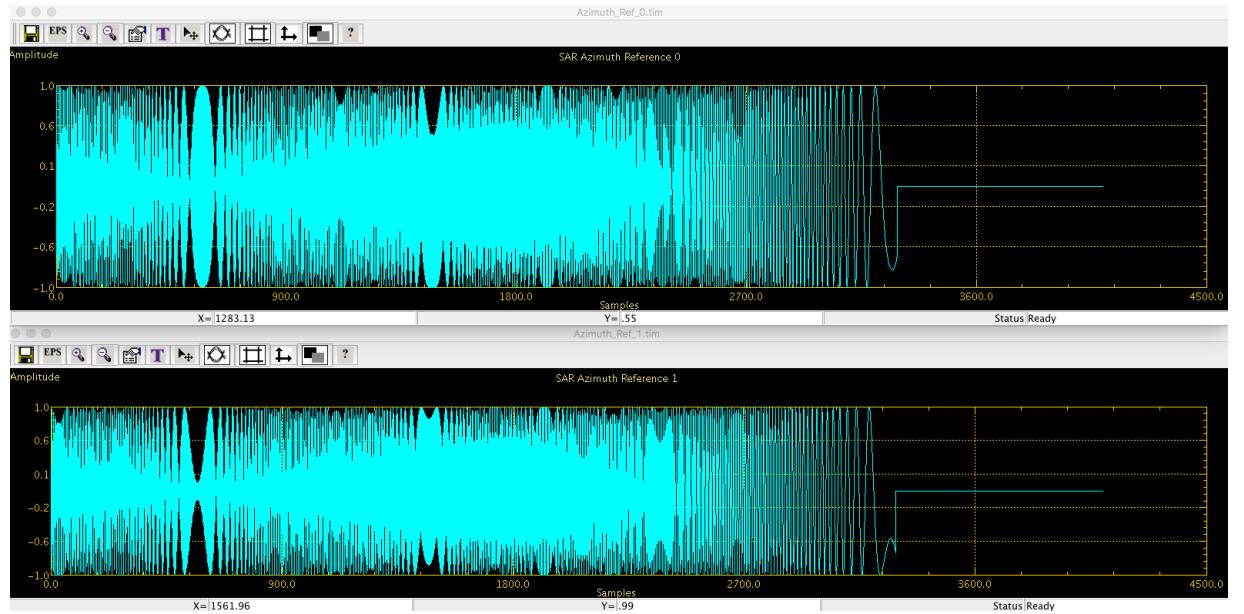


Figure 7: SAR Azimuth Range Reference (Complex)

## Chapter 3

### SAR Image for Point Targets and Block Diagram Modeling and Simulation

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The Capsim® Block Diagram of the SAR Image Formation is shown in Figure 8. The complete C code is provided in the GitHub Repository [2].

An illustration of the SAR image prior to range compression and azimuth compression for a point target created by the block diagram modeling and simulation tool is shown in Figure 9.

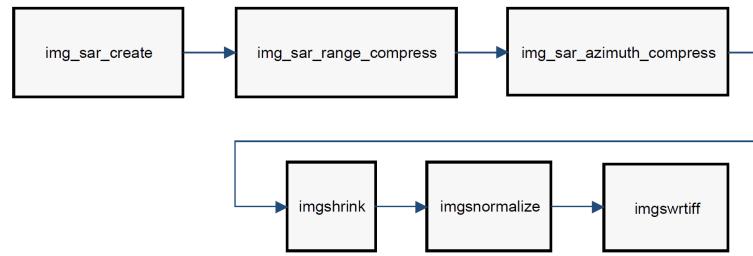


Figure 8: Capsim® SAR Block Diagram

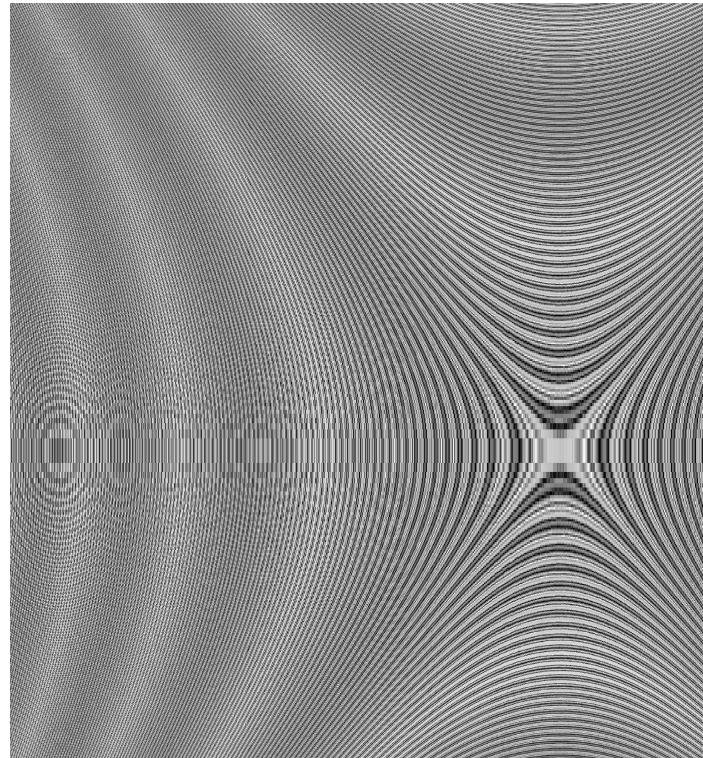


Figure 9: SAR Image of Point Target Range 851 km Azimuth Index 350 Range Time 67.6  $\mu$ s

The SAR image for point targets are shown in Figures 10, and 11.

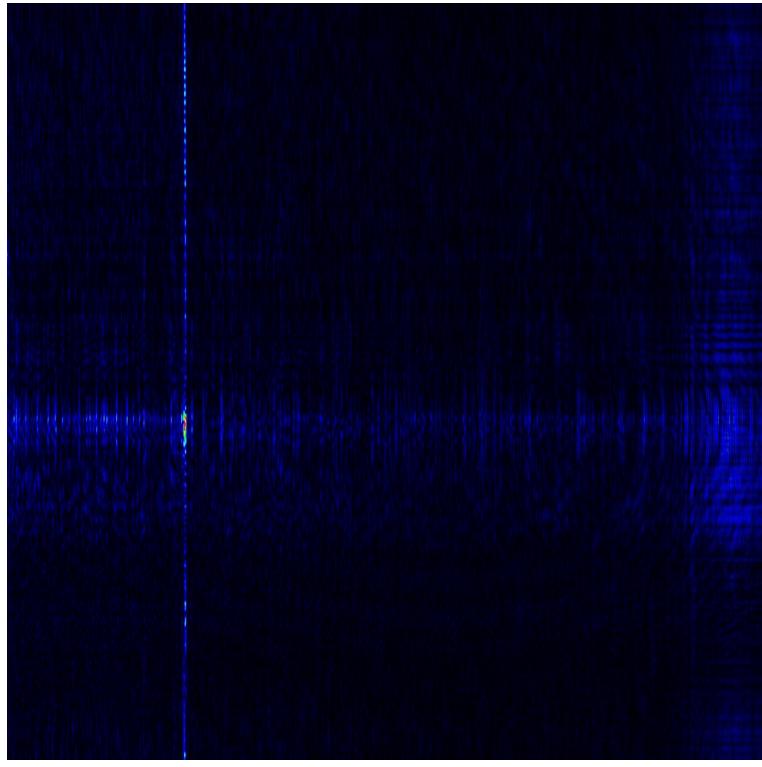


Figure 10: SAR Image Formation Point Target Range 840.8 km Azimuth Index 400 Range Time 33.8  $\mu$ s

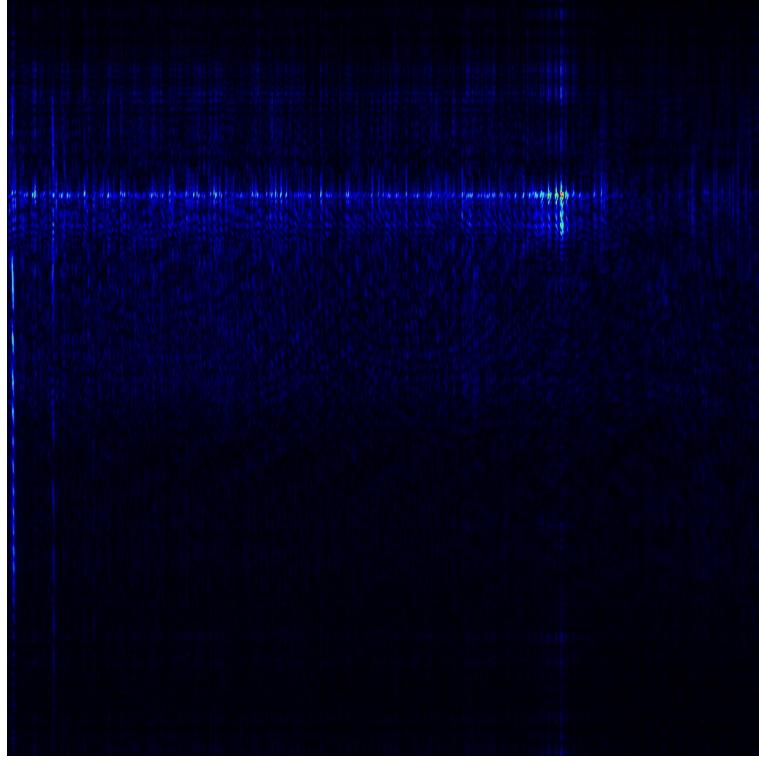


Figure 11: SAR Image Formation Point Target Range 842.5 km Azimuth Index 700 Range Time 33.8  $\mu$ s

## Chapter 4

### Auto-Focusing

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The fundamental parameters in SAR target response are the Doppler frequency and the rate of change of the frequency. These parameters are solely dependent on the rate of change of the relative motion characteristics between the SAR sensor and the position of the target. Therefore, a precise knowledge of the spacecraft state vector is necessary in order to determine these parameters. In situations where a precise satellite tracking and ephemeris are not given and

the Doppler spectrum and frequency sweep rate are not known apriori, these parameters must be derived from the radar data. Estimating the Doppler parameters is the topic of autofocus and clutter-locking. This adds to the system complexity, and a controller must be designed to not only generate the reference functions for the azimuth, but also to provide a feedback loop to refine and readjust the Doppler parameters.

## Chapter 5

### SAR Block Diagram Simulation Repository on GitHub

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Complete C code for SAR signal processing is available for the Capsim® Block Diagram Simulation and Modeling Environment. Visit the GitHub repository at:

<https://github.com/silicondsp/synthetic-aperture-radar>

The repository includes all the C code blocks and documentation. Also visit

<https://www.ccdsp.org>

## **Chapter 6**

### **References**

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- 2- Capsim® Block Diagram Modeling and Simulation Tool, Silicon DSP Corporation, <https://www.silicondsp.com>, 1989-2025. Also see the repository on GitHub: <https://github.com/silicondsp/capsim-tmk>
- 3- Fitch, J.P. , Synthetic Aperture Radar. 1988 Springer-Verlag New York Berlin Heidelberg.

## Chapter 7

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