A debt-financed real estate boom with an endogenous credit crunch*

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October 15, 2021

JEL codes: E30, E44

Abstract

I present a continuous-time macro model of the real estate market, where exogenous relaxation of a financing constraint produces a housing boom as well as a subsequent endogenous credit crunch without an additional shock reversal. I consider two types of agents: (i) productive, but credit constrained, *entrepreneurs* and (ii) patient *savers*, with differing productivity and saving propensities. An exogenous credit liberalization allows the entrepreneurs to increase their ownership of business capital. The savers selling the capital readjust by increasing their portfolio share on housing, creating a housing boom. Different saving propensities between the agents, however, lead to an eventual endogenous reversal with decreasing output and decreasing interest rate during the bust. I also propose a novel house price decomposition, dynamics of which are characterized in a closed form phase diagram. Quantitative analysis suggests that the model is capable of explaining the house price boom preceding The Great Recession.

*I greatly appreciate useful feedback from Veronica Guerrieri, Lars Hansen, Greg Kaplan, Paymon Khorrami, Ralph Koijen, Alan Moreira, Nancy Stokey, Harald Uhlig, Joseph Vavra, Eric Zwick, and the participants in the Capital Theory, and Economic Dynamics and Financial Markets working groups at the University of Chicago, and the Macroeconomics working group at Helsinki Graduate School of Economics. Any errors that remain are my sole responsibility.

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1 Introduction

Real estate bubbles and subsequent market crashes are often preceded by financial liberalization; for example, the housing booms and banking crises in Finland, Norway and Sweden in late 1980s and early 1990s, the "Asian financial crisis" in mid-1990s, the "Japanese asset price bubble" of 1986 – 1991, and the housing boom of early 2000s in the U.S.A. and much of the Europe. Discussing the sources of asset price bubbles, Kindleberger and Aliber (2011) write that while "most expansions of money and credit do not lead to mania . . . every mania has been associated with the expansion of credit".

The theoretical literature is, however, still divided on the drivers behind such boom-bust cycles. The extent to which credit supply shocks can drive quantitatively sizable housing booms in economic models is debated and depends on a variety of model details.[1] In the context of the U.S. Great Recession, a series of recent papers have developed mechanisms, where credit relaxation leads to a housing boom (Justiniano, Primiceri, and Tambalotti, 2015b; Favilukis, Ludvigson, and Van Nieuwerburgh, 2017; Greenwald, 2018).[2] Another line of work criticize these models for unrealistic assumptions, such as lack of rental markets, and argue that house prices were driven by (irrational) beliefs (Iacoviello and Neri, 2010; Kiyotaki, Michaelides, and Nikolov, 2011; Justiniano, Primiceri, and Tambalotti, 2015a; Kaplan, Mitman, and Violante, 2017).

An important feature of credit markets is their ability to affect productivity by changing the allocation of factor inputs, as productive but credit-constrained entrepreneurs get access to loans (see Restuccia and Rogerson (2017) and the references therein). For example, the U.S. credit relaxation preceding the Great Recession was associated with a sharp increase in returns to business capital, from 3.8% in Q1/2000 to 11.8% in Q4/2006, reaching the highest since 1975. Return to business capital increased throughout the early 2000s, even when excluding increasing capital gains

[1] In a series of influential papers Atif Mian and Amir Sufi relate the U.S. credit supply shock of early 2000s – associated with increased private label securitization of mortgages – to increased house prices, household debt accumulation and severity of the subsequent recession across U.S. zip codes; see Mian and Sufi (2009, 2010, 2014).

[2] U.S. household debt approximately doubled during 2000 – 2007, largely fueled by an increase in private-label securitization of mortgage backed securities (MBS), which contributed 46% of the MBS market in dollar volume in 2004, up from 22% in 2003 (Levitin and Wachter, 2011). Loan-to-value (LTV) ratios for new mortgage originations also rose from approximately 85 % in 2000 to 95% in 2005 (Duca, Muellbauer, and Murphy, 2011).

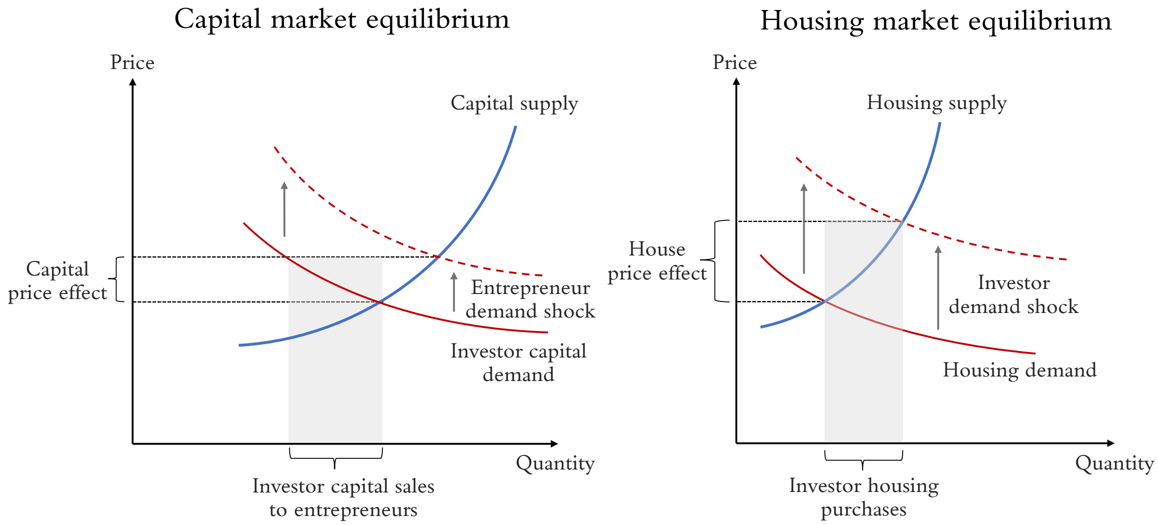


Figure 1: Credit relaxation allows productive entrepreneurs to purchase business capital increasing total capital demand (left). Capital price increases and investors' (*savers* in the model) demand is reduced along their demand curve (solid red). Investors' sales revenues (shaded) are reallocated to the housing market (right), observed as a housing demand shock.

during the period. (Gomme, Ravikumar, and Rupert, 2011)

I present a model, whereby an exogenous credit relaxation allows productive entrepreneurs with good investment opportunities, but limited access to external financing, to purchase new business capital. Capital productivity increases along with capital prices. At these higher prices, other investors find such investments less attractive and rebalance their portfolios towards other assets, such as housing.

Indeed, similarly to bonds, housing is special in that it provides uniform returns to all owners, contrary to capital with heterogeneous productivities across agents. Low-productivity investors disadvantaged at owning business capital increase their housing investments, when new entrepreneurs enter the market after a credit relaxation. For example, preceding the Great Recession, U.S. local and state government pension funds' average portfolio shares on U.S. real estate more than doubled from 3.1% in 2000 to 6.5% in 2008 (Pennacchi and Rastad, 2011).

Figure 1 illustrates this propagation mechanism from increased capital demand to increased house prices. The entrepreneurs' capital demand increases price of capital, and decreases other investors' capital demand along their demand schedule (left). These investors readjust their portfolios by selling some of their capital holdings to entrepreneurs, and use the sales revenues to buy housing (and bonds).

A key feature of this price mechanism is that due to their high productivity, the credit-constrained entrepreneurs choose to rent rather than own housing, and they are thus not marginal on the housing market. Hence, their increased demand for business capital is *not* associated with a decrease in housing demand. In a *static* set-up, by no-arbitrage the yield on business capital must match the housing rental yield. The increased output due to entrepreneurs' higher productivity, however, increases demand for all goods, including housing, which increases rents for residential real estate. While the increase in price-to-rent ratio matches the increase in bond prices and the price of business capital, due to higher rents, house prices increase disproportionally.

Moreover, in a fully *dynamic* model, the credit relaxation may lift the entrepreneurs off their credit constraints and lead to increasing house prices also after the initial shock. Depending on supply elasticities, house prices may appreciate faster than the price of business capital, in which case also price-to-rent ratios increase disproportionally compared to the price of capital.

Quantitatively, calibrating the housing supply elasticity with recent investment and rent data, and the credit shock with observed changes in loan-to-value ratios for new loan originations produces a 52.9% increase in house prices thus fully explaining the 40.0% price increase during the "house price bubble" in Q1/2003 – Q1/2007. This suggests that this house price mechanism emphasizing agents' portfolio incentives is quantitatively powerful, even if it was unlikely to be the only contributor to the house price boom.

The increased productivity underlying the price mechanism rationalizes the bubble; the boom is supported by fundamentals: "this time it's different" (Kindleberger and Aliber, 2011). While the house price "bubble" is fully rational, it doesn't, however, rule out a market crash. This is due to the nature of collateral constraints as famously discussed by Kiyotaki and Moore (1997) with "farmers" (entrepreneurs) having high productivity in using "land" (business capital):

> *"Intuitively, if the farmers' future landholdings are expected to be small, then currently the land price will be low, the farmers will have little net worth, and they will be unable to borrow much to buy land – which in turn justifies the expectation that their future landholdings will be small. Eventually, the economy returns to the unique steady state." (Kiyotaki and Moore, 1997, p. 227)*

Even if high house prices are supported by fundamentals, sunspot crashes – due to a coordinated change in (self-fulfilling) beliefs – are possible. Moreover, eventually the economy necessarily returns to a steady state, where entrepreneurs are credit constrained. This may produce a downturn even in the absence of a sunspot crash.

I develop the above price mechanism by considering housing as a second real asset in the Kiyotaki and Moore (1997) financial accelerator model with two types of agents, *entrepreneurs* and *savers* (pension and insurance funds), differing in their productivity and saving propensities. An exogenous loan-to-value relaxation lifts the entrepreneurs off their credit constraints and initiates the boom. In a steady-state, the entrepreneurs are, however, necessarily credit constrained, as in Kiyotaki and Moore (1997). Once hitting the credit limit, they begin to liquidate their business capital in order to finance consumption. This decreases average productivity and reverses the portfolio mechanism behind the housing boom: house price growth flattens and output begins to decrease generating a *bust*. The interest rate also drops in order to balance the returns on different assets.

The simultaneous drop in interest rate and output resembles a "credit crunch" studied by Guerrieri and Lorenzoni (2017). Despite these dynamics, in my model there is no exogenous or endogenous credit tightening. This corresponds to the empirical evidence by Mian and Sufi (2014), who argue that the bust was not driven by an exogenous credit shock reversal.[3] I suggest that, since my model mechanism is able to generate an entire boom-bust episode with a single one time shock in a rational expectations model, and without an explicit shock reversal, these types of mechanisms are a useful complement to existing explanations of the boom-bust episode.

My theoretical analysis also proposes a novel house price (q_{ht}) decomposition: for fixed supply of capital K and housing H, we may rewrite the $Y = C$ accounting identity as $q_{ht} = (Y_t/K)/(C_t/N_t) \times K/H \times (1 - \nu_t)$, where the first factor is the ratio of average productivity and consumption propensity out of net worth N_t, the second factor describes the amount of capital K relative to supply of housing H, and the last factor $(1 - \nu_t)$ denotes the value of the housing stock relative to the total wealth in the economy.

The economy-wide productive capacity Y_t relative to the supply of housing H affects house prices directly via the standard demand and supply channels. The consumption propensity C_t/N_t is a decreasing function of the economy-wide *saving rate* and corresponds to the proper discount rate in the economy: higher desired

[3] Mian and Sufi (2014) argue that since the employment reduction in 2007 – 2009 was concentrated on non-tradable sectors, whereas changes in credit supply should also have affected firms producing tradable goods, the crash was not due to a credit shock reversal.

savings lead to higher capital prices, holding capital supply constant. Finally, the last factor describes the relative valuations between housing and capital. Prices being endogenous, this is largely derived from preferences, but most importantly, adjusts for changes in the agents' portfolio incentives, which turn out important for explaining the price dynamics.

The main technical contribution of this paper is to analyze the non-linear and non-differentiable model dynamics. The model encompasses multiple equilibria with discontinuous interest rate paths, which may be challenging for typical computational methods. I discuss the intuition of my model mechanism by developing closed form characterizations for the dynamics of each of the components in the above pricing decomposition.

The paper is organized as follows. The next subsection discusses my contribution to the existing literature. Section 2 discusses testable predictions of my mechanism and presents a static model for discussing the price amplification. Section 3 represents the full dynamic model in general equilibrium. Section 4 includes the main theorems and pricing decompositions. Section 5 discusses the credit shock mechanism in a dynamic model. Section 6 produces the quantitative analysis with an estimate of the housing supply elasticity used in model calibration. Section 7 concludes.

Literature. This paper contributes to three broad literatures: (1) the literature on financial accelerators, (2) the literature on the housing boom-bust cycles, and (3) the literature on endogenous housing busts.

I propose a novel credit mechanism for explaining the boom-bust house price dynamics building on the financial accelerator literature (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Iacoviello, 2005; Brunnermeier and Sannikov, 2014). In this class of models, a negative shock is amplified by a collateral constraint, which forces additional negative adjustments by constrained agents.[4] In the context of the Great Recession, the effect of a credit crunch tightening the financing constraint has been explicitly considered by Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017), and in a quantitative housing model by Jones, Midrigan, and Philippon (2011), who study the role of tightening credit limits in explaining the consumption and employment patterns during the Great Recession.

In particular, my model extends earlier work by Iacoviello (2005) by considering rental markets and allowing productive capital be used by both the savers and entrepreneurs with different productivities. These additions contribute to an interesting

[4] I consider the opposite; i.e. a credit relaxation.

portfolio allocation problem, which is crucial for my price mechanism. I also analyze the nonlinear model dynamics along transition paths far from thee steady state and without using model linearization.

Another set of related papers consider the housing boom-bust in a quantitative macro model. Favilukis, Ludvigson, and Van Nieuwerburgh (2017) emphasize the role of aggregate business cycle risk combined with the observed steady influx of foreign capital lowering real interest rates during the pre-crisis years. Lower interest rates and, given an LTV credit relaxation, lower risk premium associated with shocks to potentially binding credit constraints raise house prices. Greenwald (2018) augments this type of model by allowing for long-term debt, which can violate the LTV limit after loan origination. He argues for the importance of the payment-to-income (PTI) limit with the LTV relaxation having little effect on house prices. Justiniano, Primiceri, and Tambalotti (2015b) in turn consider credit constraint on lenders to produce the house price boom along decreasing interest rates during the crisis.

An alternative view argues for exogenous beliefs having been the main driver of the housing boom (Iacoviello and Neri, 2010; Kiyotaki, Michaelides, and Nikolov, 2011; Justiniano, Primiceri, and Tambalotti, 2015a; Kaplan, Mitman, and Violante, 2017). Kaplan, Mitman, and Violante (2017) criticize the credit-based mechanisms for having unrealistic assumptions, such as (i) ignoring rental markets (Justiniano et al., 2015b; Favilukis et al., 2017; Greenwald, 2018), or (ii) relying on short-term non-defaultable debt (Favilukis, Ludvigson, and Van Nieuwerburgh, 2017). Similarly to my model, for them housing ownership is purely a portfolio choice problem, and demand for housing services can be met at the rental market. House prices do not respond much to credit shocks, since they are mostly absorbed by a change in the home ownership rate, but with little impact on total housing demand.

I augment this set-up by considering a small set of credit constrained wealthy hand-to-mouth entrepreneurs. While the credit relaxation doesn't increase their housing demand by much, their demand for business capital does increase, incentivizing other investors to rebalance their portfolios away from productive capital and towards housing.

Therefore, my mechanism explains the observed investor portfolio dynamics, and the observed increase in the productivity of business capital before the financial crisis. Moreover, the credit-based mechanism is in line with the large empirical literature connecting credit conditions to house prices (Mian and Sufi, 2009, 2010, 2017; Levitin and Wachter, 2011; Duca et al., 2011; Favara and Imbs, 2015; Di Maggio and Kermani, 2017).

Finally, the model also produces an endogenous *credit crunch* of decreasing output and decreasing interest rates as an outcome of the debt-financed boom. This relates to a growing literature on endogenous busts (Gorton and Ordonez, 2014; Kumhof et al., 2015; Justiniano et al., 2015b; Khorrami, 2018). In particular, my model shares several features with Khorrami (2018), who endogenizes the credit supply shock affecting the housing market as a consequence of an exogenous improvement in intermediaries' risk diversification capabilities. In his model, it is the intermediaries' leverage constraint, which produces the endogenous bust, whereas I focus on entrepreneur leverage.

2 Credit constraints and the housing boom

The price mechanism I propose has several testable predictions. An immediate implication of my theory is that increased access to business financing must lead to increased business activity. Figure 2 shows a correlation between the growth in number of small business loan originations and the growth in number of establishments across U.S. counties. While there is a growing literature documenting a *causal* effect from credit access to entrepreneurial activity, in the context of the U.S. housing boom the evidence is still limited.

Based on the U.S. Survey of Small Business Finances, Meisenzahl (2014) reports that for receiving a loan 52% of small businesses have to pledge collateral, 54% have to give personal guarantees, and 30% provide both with 29% collateralizing the entrepreneur's private residence. Differential house price growth rates, instrumented with the Saiz (2010) land topography instrument, therefore, provide plausibly exogenous variation in credit access for analyzing its causal impact on business activity.

Adelino, Schoar, and Severino (2015) use this strategy to document that increased house prices lead to increased employment in U.S. small businesses of less than 10 employees between 2002 and 2007. They show that this effect was particular pronounced in industries that need little startup capital, and argue for the *collateral channel*: small businesses using housing collateral to acquire external financing, explaining the increased employment. I augment their analysis by explicitly considering small business loans. I show that the causal effect of house prices to business activity is, indeed, fully explained by such loan originations: house price growth affects business activity, but only because it matters for credit availability. Credit access seems to have had a causal effect on U.S. entrepreneurial activity during the pre-crisis years.

Secondly, according to my price mechanism, entrepreneurs' increased demand

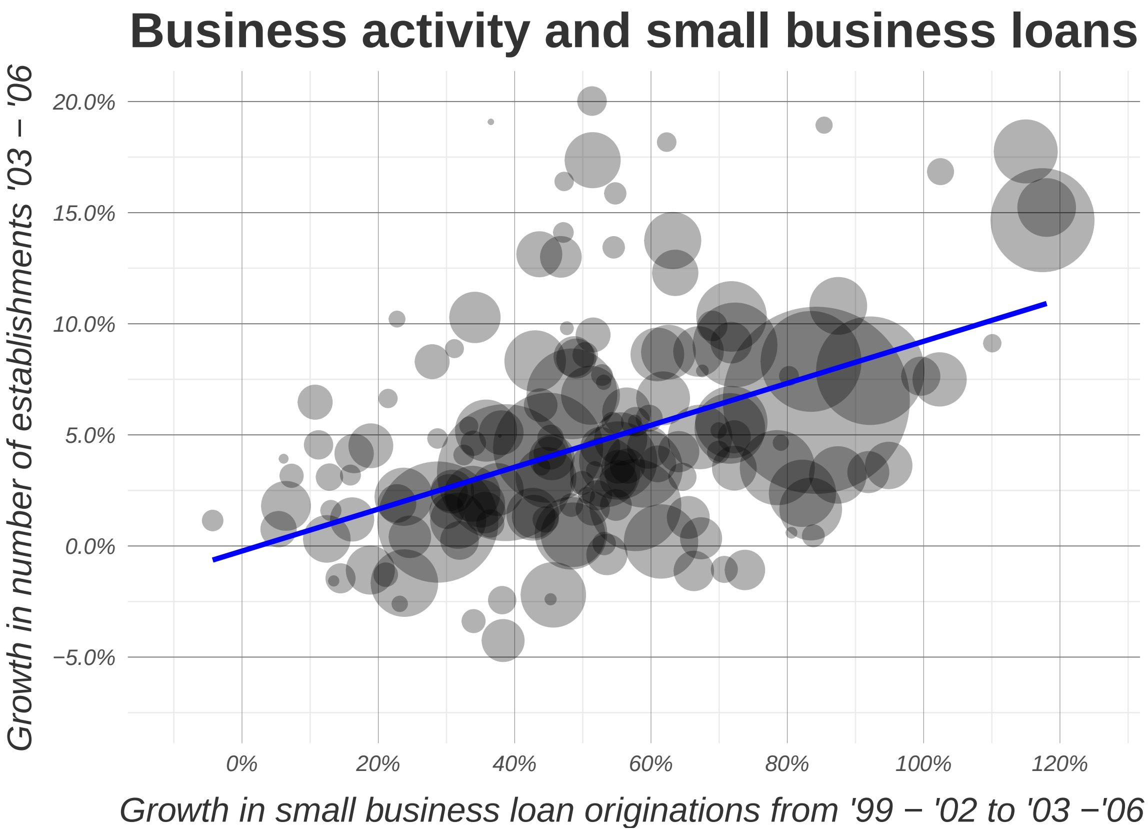


Figure 2: County level growth in number of establishments is plotted against growth in number of small business loans, for counties with at least 5,000 small business loan originations during '99 – '03. The size of the circles represent county-level loan origination counts.

Figure 3

for business capital incentivized other investors to rebalance their portfolios towards housing, leading to the housing boom. We should thus expect that this increased entrepreneurial activity, together with the underlying credit shock, is enough to produce a quantitatively sizable housing boom. I test this by regressing house price growth rate on the growth rate in the number of establishments across U.S. counties, using the Bartik (1991) instrument for establishment growth. I document a strong effect, and combining this in a naive back-of-the-envelope with the aforementioned causal feedback from house prices to business activity seems consistent with a substantial increase in house prices.

2.1 Small business loans and business activity

Using Community Reinvestment Act (CRA) data on small business loans, I regress the growth rate in number of establishments on house prices across U.S. metropolitan areas, while controlling for the growth in small business loans.[5] For robustness and additional details of the empirical analysis as well as for data description and summary statistics, I refer to Appendix A.

The main regression specification is given by

$$\begin{aligned}\text{Establishment Growth}_i &= \alpha_o + \beta_{\text{HP}\to\text{Est}}\,\text{HP growth}_i + \beta_{\text{Loan}\to\text{Est}}\,\text{Loan growth}_i \\ &\quad + \beta_{Interaction}\,\text{HP growth}_i \times \text{Loan growth}_i + \gamma_o X_i + \varepsilon_i,\end{aligned}$$

where $\beta_{Interaction}$ and to some extent $\beta_{\text{HP}\,\to\,\text{Est}}$ are the coefficients of interest and X_i includes MSA-level controls. House prices are instrumented with the Saiz (2010) land topography instrument using the first stage given by

$$\text{HP growth}_i = \alpha_1 + \beta_{\text{Elas}\to\text{HP growth}}\,\text{Housing supply elasticity}_i + \gamma_1 X_i + \varepsilon_i. \tag{2.1}$$

The results are reported in Table 1; the main regression of interest is that of Column (9), where I examine the interaction of house price and small business loan growth rates. Previous literature has documented positive effect from house prices to business activity; now it is completely loaded on the interaction term: one percentage point increase in house price and small business loan growth rates is associated with 0.26 percentage point increase in the growth of number of establishment. In fact, the pure house prices effect turns negative, which is consistent with a negative wealth effect on labor supply, given increased house prices.[6]

2.2 Growth in number of establishments and house prices

The feedback effect from increased business activity to house prices is analyzed by regressing house prices on the growth in number of establishments. For the first stage,

[5] Small business loans defined as business loans originated with principal value $\leq$ \$250,000 in the CRA data. The data on number of establishments is from County Business Patterns (CBP). All analyses exclude the construction sector.

[6] The result is even more drastic, when replacing establishment growth by employment growth as reported in Table 7 with one percentage point increase in house price growth leading to 0.83 percentage point decrease in employment growth during 2003 – 2006, when excluding the collateral effect, which more than compensates for the negative income effect on employment. Adelino et al. (2015) consider the effect of house price growth on employment especially in small businesses. In Table 8 I show that the business loan channel is pertinent also when only considering businesses with less than 20 employees.

Table 1: The impact of house price and small business loan growth on establishment growth

	Dependent variable: Growth in number of establishments '03-'06									
	Reduced form	*OLS*				*Instrumental variable (2SLS)*				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Housing supply elasticity	-0.01^{***}									
	(0.002)									
House price growth '03 - '06		0.07^{***}		0.06^{***}	0.07^{***}	0.06^{***}		-0.01	-0.17^{*}	-0.13^{*}
		(0.01)		(0.01)	(0.01)	(0.02)		(0.03)	(0.09)	(0.08)
Growth in small business loans			-0.001	-0.001^{*}	0.001		0.11^{***}	0.10^{***}	-0.01	-0.01
			(0.0004)	(0.001)	(0.002)		(0.03)	(0.02)	(0.03)	(0.03)
House price growth '03 - '06 × Growth in small business loans					-0.01				0.26^{**}	0.17^{*}
					(0.01)				(0.11)	(0.09)
Growth in establishments '99 - '02										0.69^{***}
										(0.10)
Constant	0.06^{***}	0.01^{***}	0.04^{***}	0.02^{***}	0.02^{***}	0.02^{***}	-0.03^{*}	-0.02^{**}	0.04^{*}	0.03^{*}
	(0.01)	(0.003)	(0.002)	(0.003)	(0.004)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
Observations	298	359	364	332	332	272	295	269	269	269
R^2	0.05	0.15	0.001	0.16	0.16	0.17	0.26	0.22	0.14	0.38
Adjusted R^2	0.05	0.15	-0.002	0.15	0.15	0.16	0.26	0.22	0.13	0.37

Note: *p<0.1; **p<0.05; ***p<0.001

Regression of MSA level growth in number of establishments during 2003 – 2006 on house price growth during the same period and on growth in number of small business loan originations ($\leq$ \$250,000 at origination) from 1999 – 2002 to 2003 – 2006 as well as on their interaction. House price growth is measured by Zillow MSA level price indices, and it is instrumented with housing supply elasticity as measured by Saiz (2010). In column (7), we also instrument the business loan origination with the same Saiz (2010) instrument. In column (10), pre-trend on growth in number of establishments during 1999 – 2002 is included as a control. Establishments data excludes construction sector. Standard errors, shown in parentheses, are Newey-West (1987) heteroskedastic robust.

I use Bartik (1991) instrument as

$$\text{Establishment growth}_i = \alpha_3 + \beta_{\text{Bartik}}\,\text{Bartik}_i + \gamma_3 X_i + \varepsilon_i,$$

whereas the second stage is given by

$$\text{HP growth}_i = \alpha_4 + \beta_{\text{Est}\to\text{HP}}\,\text{Establishment growth}_i + \gamma_3 X_i + \varepsilon_i,$$

where X_i is a vector of controls.

The results are shown in Table 2. Given one percentage point increase in the growth in number of establishments, the OLS and IV estimates, respectively, predict approximately 2.4 and 6.15 percentage point increases in house price growth rates. The strong response is not surprising and might reflect a standard demand channel. Increase in economic activity and output leads to higher demand both for non-durable goods and housing. Given fixed housing supply, at least in the short-run, this leads to a strong positive response in house prices. Note also that simultaneity bias typically leads to a higher IV estimate, if the true effect is strong enough, explaining the downward bias observed in the OLS estimate.

2.3 Aggregation in a back-of-the-envelope calculation

We may use these estimates to produce a back-of-the-envelope calculation for the house price effect, given an LTV collateral credit relaxation from 73% to 82% (An and Sanders, 2010). I look for a new post-shock equilibrium with house prices increased by x %. Given the shock, the total housing collateral has then increased by a factor of $1.123 \cdot (1 + x/100) - 1$. By column (6) of Table 6 this increase in housing collateral value leads to $0.77 \cdot (12.3 + 1.123x)$ % increase in the number of small business loans.

By the first row of column (9) in Table 1, the x % increase in house prices has an income effect, which reduces the number of establishments by $0.17 \cdot x$ %. On the other hand, by the third row, the increase in total collateral value – due to the credit relaxation and house price increase – increases the number of establishments by $0.26 \cdot 0.77 \cdot (12.3 + 1.123x)^2/100$ %. By column (5) of Table 2, this growth in number of establishments leads to $6.16 \cdot [\frac{1}{100} \cdot 0.26 \cdot 0.77 \cdot (12.3 + 1.123x)^2 - 0.17 \cdot x]$ % increase in house prices. For this to be consistent with the equilibrium, I look for an x such that

$$6.16 \cdot \left[\tfrac{1}{100} \cdot 0.26 \cdot 0.77 \cdot (12.3 + 1.123x)^2 - 0.17 \cdot x\right] = x.$$

Table 2: The effect of establishment growth on house price growth

	Dependent variable:							
	Establishment growth '03 - '06	House price growth '03 - '06						
	First stage	*Reduced form*	*OLS*		*Instrumental variable (2SLS)*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Etablishment growth Bartik	1.50***	8.65***						
	(0.28)	(1.52)						
Establishment growth '03 - '06			2.30***	2.44***	6.16***	6.15***	6.12***	6.14***
			(0.38)	(0.38)	(1.20)	(1.14)	(1.18)	(1.09)
House price growth '99 - '02				0.47***		0.55***		0.54***
				(0.09)		(0.09)		(0.10)
Number of establishments '03							0.0000***	0.0000
							(0.0000)	(0.0000)
Constant	−0.02	0.01	0.23***	0.13***	0.10*	−0.01	0.09*	−0.01
	(0.01)	(0.05)	(0.02)	(0.02)	(0.04)	(0.05)	(0.04)	(0.05)
Observations	391	359	359	359	359	359	359	359
R^2	0.11	0.10	0.15	0.23	−0.27	−0.16	−0.25	−0.15
Adjusted R^2	0.11	0.09	0.15	0.23	−0.28	−0.16	−0.25	−0.16

Note: *p<0.05; **p<0.01; ***p<0.001

Regression of MSA level house price growth during 2003 – 2006 on growth in number of establishments instrumented with the Bartik IV, constructed by summing the product of MSA level 2-digit industry shares and national industry growth rates over industries. Establishments data excludes construction sector. Columns (4), (6) and (8) control for pre-trends in house price growth during 1999 – 2002, and columns (7) and (8) control for the number of establishments in 2003. House prices are measured by Zillow MSA level price indices. Standard errors, shown in parentheses, are Newey-West (1987) heteroskedastic robust.

This is solved by $x = 108.6$ and $x = 1.1$. The multiplicity of possible solutions is not an artifact of this back-of-the-envelope argument, but rather an important feature of the mechanism I consider. I illustrate this amplification mechanism and multiplicity in more detail within a simple economic model.

2.4 House price amplification: static illustration

This paper proposes two inter-related mechanisms: one for the house price boom, and another one for the credit crunch. It is illustrative to first consider the amplification mechanism behind the boom in a simple one-page static model with two agents: a saver and an entrepreneur. Both are price-takers and do not internalize their actions effects on prices. The agents derive utility from non-durable consumption c and from housing services s with utility $u(c^\alpha s^{1-\alpha})$. The consumption good c, which is the numéraire in the economy, is produced with linear ak technology, where the saver's and the entrepreneur's productivity parameters are given, respectively by $\underline{a}$ and a. I assume that the entrepreneur is more productive: $a > \underline{a}$. The productive capital is in fixed supply K.

Housing services are traded at a free rental market with rental price r_h, and produced one-to-one by houses h, which are in fixed supply H. The prices of capital and housing are denoted by q_k and q_h, respectively. The entrepreneur and the saver enter the period with some portfolios (k_e, h_e, b_e) and (k_s, h_s, b_s), respectively. In the beginning of the period, the agents can trade on capital and housing as well as on a risk-free bond b, which pays interest r_f and is in zero supply $b_e + b_s = 0$. The trading is subject to short-selling constraints $k, h \geq 0$ and to a collateral constraint

$$-b \leq (1-\phi) q_k k, \tag{2.2}$$

where for simplicity I assume that only productive capital can be used as collateral. After production, possible debt positions are cleared, and consumption of both goods takes place.

For an initial portfolio $(k_i, h_i, b_i), i = s, e$, an agent's problem is

$$\begin{aligned} &\max_{c,s,k,h,b} \quad u(c^\alpha s^{1-\alpha}) \\ &\text{s.t.} \quad c + r_h s \leq ak + r_h h + (1+r_f)b. \\ &q_k k + q_h h + b = q_k k_i + q_h h_i + b_i, \end{aligned} \tag{2.3}$$

subject to the collateral constraint (2.2) and the short-selling constraints on h and k.

Concentrating on the case of a constrained entrepreneur, the model equilibria are characterized by the following proposition.

Proposition 2.1. *Suppose*

$$\frac{k_o}{K} + \frac{1-\alpha}{\alpha}\frac{h_o}{H} + \frac{b_o}{aK} < \phi. \tag{2.4}$$

Let $\psi_k = k_e/K$ *denote the ownership share of capital by the entrepreneur,* $A(\psi_k) = \psi_k k_e + (1-\psi_k)k_s$ *the average productivity of capital, and* $\nu = q_k K/(q_k K + q_h H)$ *the value of the capital shock relative to the total wealth in the economy. Then the model equilibria are characterized in terms of* ψ_k *and* ν *by*

$$\frac{\alpha}{\nu} = \alpha + (1-\alpha)\frac{A(\psi_k)}{\underline{a}}, \quad \textit{and} \tag{2.5}$$

$$0 = \left[\phi - \frac{h_o}{H}\frac{1-\alpha}{\alpha\underline{a}}[a-\underline{a}]\right]A(\psi_k)\psi_k - \left[\underline{a} + \frac{k_o}{K} + \frac{b_o}{K}\frac{1-\alpha}{\underline{a}}\right]A(\psi_k) - \frac{b_o}{K}\alpha, \tag{2.6}$$

In particular, $\nu = \nu(\psi_k)$ *is a decreasing function of* ψ_k *with* $\nu \le \nu(0) = \alpha$*, and for* $b_o \neq 0$*, equation (2.6) is a quadratic on* ψ_k*, possibly with two roots in* $(0,1)$ *corresponding to two different equilibria for a given set of parameters.*[7]

Note that assumption (2.4) implies a binding collateral constraint and $k < K$. Exactly as in Kiyotaki and Moore (1997) financial accelerator mechanism, given a binding collateral constraint, an increase in the price of capital allows the entrepreneur to take more debt to finance additional capital purchases. Due to her higher productivity, this drives up the price of capital, which depending on parameter values justifies the higher capital price in the first place, producing the multiplicity. The saver on the other side of the trade correspondingly increases her portfolio share on housing. Since she is marginal on both assets unlike the entrepreneur, her first order conditions determine the prices. Price-to-rent ratio q_h/r_h must match the price of capital:

$$\frac{q_k}{\underline{a}} = \frac{1}{1+r_f} = \frac{q_h}{r_h}.$$

Due to the increased productivity ($a > \underline{a}$), house prices, however, increase more than the price of capital, since higher productivity increases housing rents r_h, given inelastic housing supply. This corresponds to a *housing boom*, with house prices increasing disproportionately. More precisely, considering two possible roots $\psi_k^{\pm} \in (0,1)$, $\psi_k^- < \psi_k^+$ of (2.6), this translates by (2.5) to $\nu^+ < \nu^-$ for $\psi_k^+ > \psi_k^-$, and

[7] For instance, set $\phi = 0.2, h_o = 0, H = 1, k_o = 0.3, K = 1, \alpha = 0.8, \underline{a} = 0.1, a = 0.35$ and $b_o = -0.05$ to produce two roots $\psi_k^- = 0.23$ and $\psi_k^+ = 0.87$.

$q_h^+ > q_h^-$. Finally, since the entrepreneur is more productive than the saver, output is also higher, leading to higher non-durable consumption.

In the next Section this price amplification mechanism is embedded to a dynamic model, which also produces *disproportionally* increasing price-to-rent rations, a *credit crunch* and explains the decreasing interest rate and output dynamics following a debt-financed boom.

3 Dynamic model

This section discusses the full dynamic model with mechanisms producing the housing boom as well as the subsequent credit crunch.

3.1 Environment

Time is continuous and horizon is infinite. Two types of agents populate the economy: savers indexed by $j \in [0,1]$ and entrepreneurs indexed by $j \in (1,2]$. All agents maximize lifetime utility

$$\int_o^\infty e^{-\rho^j t} u(c_t^j, s_t^j)\, dt,$$

where c_t^j and s_t^j, respectively, denote non-durable consumption and housing services. Each agent produces the consumption good with a linear $a^j k_t$ production technology. Housing services s_t are produced using housing capital h_t and are traded freely on a rental market at a rental price r_{ht}. Housing and productive capital are in fixed supply H and K, respectively, and are both homogeneous.[8]

The agents can freely trade housing, productive capital as well as a risk-free bond b_t, which pays interest r_{ft}. However, they do face a short-selling constraints $k_t^j \geq 0$ and $h_t^j \geq 0$ as well as a collateral constraint given by

$$-b_t^j \leq (1-\phi) q_{kt} k_t^j, \quad \phi \in [0,1], \tag{3.1}$$

where q_{kt} denotes the price of capital. For simplicity, only productive capital can be used as collateral for loans, but all the results remain unaltered, even if allowing housing collateral.[9]

[8]The quantitative results are – up to the first-order – robust for also including housing investments, but for simplicity I only include them in my quantitative analysis in Section 6.

[9]Note that all debt is short-term; I concentrate on business loans, which typically have variable interest rates and many such loans have covenants allowing the lender to force loan pre-payment if a collateral condition is violated. For instance, Glennon and Nigro (2005) document that in their

The following two assumptions specify the type of heterogeneity I consider.

Assumption 3.1. Savers discount future at rate $\rho^j = \underline{\rho}$ for all $j \in [0,1]$, and entrepreneurs discount future at rate $\rho^j = \rho > \underline{\rho}$ for all $j \in (1,2]$.

Assumption 3.1 says that savers are more patient than entrepreneurs. Importantly, this implies that savers have larger saving rates. I abstract away from intermediaries and the banking sector, in general, and in equilibrium savers will provide the financing for entrepreneurs. The savers correspond to bondholders in the data: most of the wealth used to finance entrepreneurial activity is held by individuals in the top decile of the wealth distribution and insurance and pension funds.

Assumption 3.2. Savers have productivity $a^j = \underline{a}$ for all $j \in [0,1]$, and entrepreneurs have productivity $a^j = a > \underline{a}$ for all $j \in (1,2]$.

Assumption 3.2 says that entrepreneurs are more productive than savers in producing the consumption goods. Entrepreneurs are a set of wealthy hand-to-mouth individuals, who have good investment opportunities, but limited ability to finance all of them.[10]

3.2 Optimization problem

I denote an agent's net worth by $n_t^j = q_{kt}k_t^j + q_{ht}h_t^j + b_t^j$, where the agent chooses her portfolio (k_t^j, h_t^j, b_t^j) and q_{ht} is the house price. For simplicity I will drop the superscripts j and denote the portfolio shares of capital and housing, respectively, by

$$\theta_{kt} = \frac{q_{kt}k_t}{n_t} \quad \text{and} \quad \theta_{ht} = \frac{q_{ht}h_t}{n_t}.$$

With this notation I rewrite the collateral constraint (3.1) as

$$\theta_{kt} + \frac{\theta_{ht}}{\phi} \leq \frac{1}{\phi}. \tag{3.2}$$

sample of the Small Business Administration (SBA) 7(a) loan guarantee program 84% of loans had variable interest rate.

Moreover, Chodorow-Reich and Falato (2017) report that 37% of loans in the Shared National Credit Program data were terminated or reduced before maturity during the crisis years of 2008 – 2009, approximately matching the fraction of loans with a covenant violation. See also Falato and Liang (2016). This notwithstanding, my quantitative results are robust for considering long-term debt.

[10] An important implicit friction in this set-up is that savers cannot hire the entrepreneurs to operate their capital, but the actual ownership structure matters.

Using this notation, an agent's wealth evolves as

$$\frac{dn_t}{n_t} = -\frac{c_t}{n_t} - \frac{r_{ht}s_t}{n_t} + r_{ft} + \theta_{kt} \underbrace{\left[\frac{a}{q_{kt}} + \mu^q_{kt} - r_{ft}\right]}_{\text{Excess return on capital}} + \theta_{ht} \underbrace{\left[\frac{r_{ht}}{q_{kt}} + \mu^q_{ht} - r_{ft}\right]}_{\text{Excess return on housing}} dt, \tag{3.3}$$

where

$$\frac{dq_{kt}}{q_{kt}} =: \mu^q_{kt}\, dt \quad \text{and} \quad \frac{dq_{ht}}{q_{ht}} =: \mu^q_{ht}\, dt$$

denote capital and house price appreciation. The first two terms on the wealth evolution describe consumption of consumption goods and housing services. On the other hand, the agent's portfolio return is given by the risk-free rate and the excess returns on capital and housing investments.

I look for a recursive competitive equilibrium driven by an aggregate state vector $\mathbf{S}$. Observe that given the aggregate state, an agent's opportunity set is fully determined by her net worth n, which is the agent's state. Given capital and house prices $q_{kt} = q_k(\mathbf{S}_t), q_{ht} = q_h(\mathbf{S}_t)$, respectively, risk-free interest rate $r_{ft} = r_f(\mathbf{S}_t)$, rental price $r_{ht} = r_h(\mathbf{S}_t)$ and a law of motion $d\mathbf{S}_t$ for the aggregate state $\mathbf{S}_t$, agents' policies for non-durable consumption c_t, housing consumption s_t as well as portfolio shares on capital θ_{kt} and housing θ_{ht} solve the agent's problem; she maximizes her lifetime utility:

$$V(n; \mathbf{S}) = \max_{\{c_t, s_t, \theta_{kt}, \theta_{ht}\}_{t \geq \tau}} \mathbb{E}_\tau \left[\int_\tau^\infty e^{-\rho t} u(c_t, s_t)\, dt\right]$$

where the maximization is subject to the wealth evolution (3.3), the LTV constraint (3.2) and short-selling constraints $\theta_{kt} \geq 0$ and $\theta_{ht} \geq 0$, given initial states $n_\tau = n$ and $\mathbf{S}_\tau = \mathbf{S}$.

3.3 Equilibrium

With homothetic preferences the individual savers' and entrepreneurs' problems aggregate, respectively, to those of two representative agents, one for each type. Correspondingly, I denote the type-specific aggregate quantities for savers and entrepreneurs, respectively, by

$$\underline{n}_t = \int_0^1 n_t^j\, dj \quad \text{and} \quad n_t = \int_1^2 n_t^j\, dj,$$

and similarly for $\underline{k}_t, k_t, \underline{h}_t, h_t, \underline{b}_t, b_t, \underline{c}_t, c_t$ and $\underline{s}_t, s_t$.

Definition 3.3 (Recursive competitive equilibrium)**.** A recursive competitive equilibrium consists of

(i) value functions $V^i(n^i; \mathbf{S})$;

(ii) policy functions $c^i, s^i, \theta_h^i, \theta_k^i$ depending on n^i and $\mathbf{S}$;

(iii) prices $q_h(\mathbf{S}), q_k(\mathbf{S}), r_h(\mathbf{S}), r_f(\mathbf{S})$, and

(iv) aggregate state evolution $d\mathbf{S} = \boldsymbol{\mu}^S dt$

such that

1. $V^i(n^i; \mathbf{S})$ and the above associated policy functions solve the individual HJB equations (C.1) given prices and state evolution.

2. Markets clear

   - Goods market: $c_t + \underline{c}_t = ak_t + \underline{ak}_t$;
   - Rental market: $s_t + \underline{s}_t = H$;
   - Housing market: $h_t + \underline{h}_t = H$;
   - Capital market: $k_t + \underline{k}_t = K$;
   - Bond market: $b_t + \underline{b}_t = 0$.

3. The aggregate state evolution $d\mathbf{S}$ is consistent with agents' equilibrium choices, given policy functions and market clearing conditions.

I denote aggregate consumption by $C = c + \underline{c}$ and the total wealth in the economy by $N_t = \underline{n}_t + n_t = q_{kt}K + q_{ht}H$. Let

$$\eta_t = \frac{n_t}{\underline{n}_t + n_t} \quad \text{and} \quad \nu_t = \frac{q_{kt}K}{N_t}$$

denote, respectively, entrepreneurs' wealth share and the share of total wealth contributed to the productive capital. I look for a symmetric equilibrium, where entrepreneurs and savers have identical portfolio shares within a type, and with the aggregate state $\mathbf{S} = \eta$.

In the sequel, I denote the entrepreneurs' share of capital and housing ownerships, respectively, by

$$\psi_{kt} = \frac{k_t}{K} \quad \text{and} \quad \psi_{ht} = \frac{h_t}{H}.$$

4 Model dynamics

In order to illustrate the main feedback mechanism providing the house price amplification as well as the eventual bust, I first provide closed form decompositions for capital prices.

Proposition 4.1. *Price of productive capital can be expressed as*

$$q_k = \frac{A(\psi_k)}{\frac{c}{n}\eta + \frac{c}{\underline{n}}(1-\eta)}\nu$$

and house price q_h as

$$q_h = \frac{A(\psi_k)}{\frac{c}{n}\eta + \frac{c}{\underline{n}}(1-\eta)}\frac{K}{H}(1-\nu),$$

where $A(\psi_k) = a\psi_k + (1-\psi_k)\underline{a}$ is the average productivity of capital.

Moreover, for utility of the form $u(c^\alpha s^{1-\alpha})$, the housing rental rate is given by

$$r_h = A(\psi_k)\frac{K}{H}\frac{1-\alpha}{\alpha}.$$

First of all, note that by replacing the average productivity of capital $A(\psi_k)$ by Y/K, where Y denotes the total output, the pricing decompositions for q_k and q_h follow directly from the goods market clearing, and they are model-independent. The contribution of this Proposition is, therefore, not in establishing these decompositions, which is close to trivial, but rather developing pricing decompositions, which – under suitable modeling assumptions – allow for closed form representations for the dynamics of each of the components; see Theorem 4.2 and Appendix C. This enables a thorough model analysis without relying on computational methods, which risk losing model transparency; for instance, my analysis finds all symmetric model equilibria instead of computationally solving for only one of them.

Let us now examine in more detail the above formula for house price q_h, which is affected by four main components: (i) the output $A(\psi_k)$ per unit of capital represents the productivity of capital, which is determined by the resource allocation ψ_k: higher output increases demand for capital with a positive price effect; (ii) the consumption rate out of net worth

$$\frac{C}{N} = \frac{c}{n}\eta + \frac{c}{\underline{n}}(1-\eta). \tag{4.1}$$

describes – up to a constant, the negative of – economy-wide *saving rate* and corresponds to the proper discount rate in the economy: higher desired savings lead to higher capital prices, given fixed capital supply in the short-term; (iii) the ratio K/H represents the productive capacity – determining together with (i) the consumption and housing demand – relative to the supply of housing; and (iv) the last term $1-\nu$ describes the value of the housing stock relative to the value of the capital stock.

I make two observations. First, fluctuations in the savings rate occur due to two reasons. On one hand, individual consumption propensities fluctuate together with the economy. On the other hand, wealth-weighted composition of agents with different saving propensities may change. The latter is associated with a change in the wealth share η in (4.1). In typical model calibrations, such as the one in Section 6, individual consumption is smooth. Similarly to Mian, Straub, and Sufi (2020), with high enough heterogeneity in saving rates, changes in the wealth distribution dominate and thus largely determine fluctuations in the discount factor; see also Theorem 4.2.

Secondly, the factor $1-\nu$ describes the value of the housing stock relative to the total wealth in the economy. In case of unit elastic substitution between housing and non-durable consumption, with utility of the form $u(c^{\alpha}s^{1-\alpha})$, in the degenerate representative agent steady-state with only entrepreneurs or savers (i.e. $\eta=0$ or $\eta=1$), we have $\nu=\alpha$. The factor ν therefore is largely influenced by preferences. This notwithstanding, capital may also have collateral value, which is reflected in the value of ν outside of these steady-states, and ν also takes into account the expected future productivity of capital. Most importantly, however, it is affected by agent's portfolio incentives.

In case of a credit relaxation, constrained entrepreneurs use their new collateral capacity to purchase productive capital. The consequent increase in production increases all capital prices due to its effect on $A(\psi_k)$ in Proposition 4.1, but more importantly, the credit relaxation affects savers' portfolio choices: the new demand for capital by entrepreneurs incentivizes savers to sell productive capital and rebalance their portfolio towards higher housing ownership.

On the other hand, the saver return to business capital $\underline{a}/q_k+\mu_k^q$ must match the return $r_h/q_h+\mu_q^h$ on residential housing. Ignoring the possibly different rates of price appreciation, this would lead to price-to-rent ratios increasing exactly as much as the price of capital. However, the higher productivity $A(\psi_k)$ increases the demand for all consumption, including housing, and given fixed supply of housing, rents r_h must increase. Therefore, house prices increase more than the price of capital, even if price-to-rent ratios do not.

Regarding the rates of price appreciation, if the entrepreneurs are completely lift off their credit constraints, it is now the entrepreneurs' credit constraints, which determine the equilibrium, while the savers are constrained by the short-selling limit

$k \geq 0$:

$$\frac{a}{q_k} + \mu_k^q = \frac{r_h}{q_h} + \mu_q^h. \tag{4.2}$$

For $a > \underline{a}$, also price-to-rent ratio q_h/r_h increases disproportionally compared to the price of capital q_k. Indeed, after such a strong credit relaxation, in the long-run, entrepreneurs' lower saving propensity decreases their relative wealth compared to that of savers. During the boom they increase consumption, while also increasing their debt position: similarly to Kiyotaki and Moore (1997), the entrepreneurs eventually become credit constrained. Due to their higher production efficiency, they first liquidate all of their housing holdings before beginning to also sell some of their productive capital to savers. Once constrained, entrepreneurs offer savers attractive capital pricing for financing higher consumption by selling capital. Expecting lower capital prices in the future, the price of capital must decrease relative to that of housing during the boom: $\mu_k^q < \mu_h^q$, which further increases housing price-to-rent ratios after the original credit shock.

Once the capital liquidation begins, savers are incentivized to buy the capital due to reduced capital prices, and they (try to) finance their capital purchases partially by selling housing; in equilibrium all housing is still held by the savers, which lowers the relative price of housing compared to the price of capital during the *bust*: $\mu_k^q > \mu_h^q$. Savers increase their portfolio share on productive capital, which due to their lower productivity reduces output. Interest rate must also drop to balance the returns on different assets.

It is noteworthy to observe that with fixed supply of capital and housing, the rental rate is only determined by current productivity, provided unit elastic substitution between housing and non-durable consumption. Fluctuations in the rental yield (inverse of price-to rent ratio)

$$\frac{r_h}{q_h} = \frac{1-\alpha}{1-\nu}\frac{1}{\alpha}\left[\frac{c_t}{n_t}\eta_t + \frac{\underline{c}_t}{\underline{n}_t}(1-\eta_t)\right].$$

are thus mostly influenced by discount rates – that is, the economy-wide saving propensity – and the relative value of housing and capital.[11]

Let

$$\tau := \frac{a}{q_k} + \mu_k^q - r^f = \frac{\lambda}{\frac{\partial V}{\partial n} n}$$

[11]In the Appendix all of these results are generalized to include housing and capital investments: the main qualitative conclusions remain, although the formulas are more complicated.

denote an entrepreneur's wedge in her first order condition for capital portfolio choice. Here λ denotes the multiplier on the collateral constraint.[12] The next theorem describes the model dynamics for η and ν.

Theorem 4.2. *The evolution of the wealth distribution, characterized by η, and relative prices, characterized by ν, are given by*

$$\frac{d\eta}{dt} = \eta(1-\eta)\left[\frac{\underline{c}_t}{\underline{n}_t} - \frac{c_t}{n_t} + r_{ht}\left(\frac{\underline{s}_t}{\underline{n}_t} - \frac{s_t}{n_t}\right) + \frac{\tau}{\phi}\right]$$

$$\frac{d\nu}{dt} = \nu(1-\nu)\begin{cases} \frac{r_{ht}}{q_{ht}} - \frac{a}{q_{kt}}, & \text{if} \quad \eta < \phi\nu \\ \frac{r_{ht}}{q_{ht}} - \frac{a}{q_{kt}}, & \text{if} \quad \eta \geq \phi\nu. \end{cases}$$

Note that entrepreneurs are credit constrained if and only if $\eta < \phi\nu$. In the unconstrained region, where $\tau = 0$, the wealth evolution is determined by the relative average consumption propensities of the agents. Wealth is accumulated to the agents, who have higher saving rates. In the next Section, I show that under suitable additional assumptions, these dynamics fully characterize the model equilibria.

5 House price boom-bust in general equilibrium

My price amplification mechanism consists of two parts: (i) an exogenous relaxation of a collateral constraint leads to a large increase in house prices at the impact of the shock; (ii) the positive impulse response in house prices is then succeeded by a sustained house price boom: savers with high saving propensities accumulate wealth at a higher rate compared to the entrepreneurs, thus increasing aggregate demand for savings and increasing capital prices.

Similarly to Greenwald (2018), I demonstrate my mehcanism in a simplified set-up with logarithmic utility.[13]

Assumption 5.1. All agents derive utility from non-durable consumption c and housing services s with utility function

$$u(c,s) = \alpha \log c + (1-\alpha)\log s, \tag{5.1}$$

where $\alpha \in (0,1]$.

[12] With ak technology an entrepreneur may be credit constraint if and only if all entrepreneurs are credit constrained and $\tau^j = \tau$ for all $j \in (1,2]$; by definition, τ does not depend on j.

[13] Greenwald (2018) considers logarithmic utility over non-durable consumption and housing services, but contrary to my model, where output is produced by capital only, his utility specification includes a third term for disutility of working, as he considers a labor-only model.

Apart from analytic convenience, this utility specification has two main reasons. First, Piazzesi, Schneider, and Tuzel (2007), Davis and Ortalo-Magné (2011) and Aguiar and Hurst (2013) document that the elasticity of substitution between housing and non-durable consumption is close to 1. This argues for a utility specification of the form $u(c^{\alpha}s^{1-\alpha})$. Moreover, the model has no uncertainty and thus the logarithmic form corresponds to setting intertemporal elasticity of substitution to 1, which does not seem far-fetched relative to the reality.

For demonstrating the dynamics, logarithmic utility is convenient also because then the agents are PIH[14] consumers. We have the following Lemma.

Lemma 5.2. *Suppose all agents have logarithmic utility (5.1). Then*

$$\frac{c_t}{n_t} = \alpha\rho \quad \textit{and} \quad \frac{\underline{c}_t}{\underline{n}_t} = \alpha\underline{\rho} \quad \textit{as well as} \quad \frac{r_{ht}s_t}{n_t} = (1-\alpha)\rho \quad \textit{and} \quad \frac{r_{ht}\underline{s}_t}{\underline{n}_t} = (1-\alpha)\underline{\rho}$$

for all t. Moreover, $\nu = \alpha$, if $\eta = 0$ or $\eta = 1$.

This result has several important consequences. First, similarly to Berger, Guerrieri, Lorenzoni, and Vavra (2017), it follows that consumption response to a house price shock is given by

$$\frac{\partial c_t}{\partial q_{ht}/q_{ht}} = \mathrm{MPC} \cdot q_{ht}h_t, \tag{5.2}$$

when keeping all other prices and portfolios constant, but allowing net worth to evolve with the change in house prices: the consumption response for a house price shock by PIH agents is purely due to a wealth effect. In particular, given large increases in house prices, the model produces large consumption responses as documented empirically by Mian, Rao, and Sufi (2013).

Moreover, as discussed in the following Proposition, for highly levered agents, the consumption responses are relatively higher; this corresponds to empirical findings by Mian and Sufi (2014), who document strong and significant effect of leverage on marginal propensity to consume out of housing wealth.

Proposition 5.3. *Suppose Assumption 5.1 holds. Keeping all other prices and the agent's portfolio constant, but allowing consumption to react to a change in net worth, the consumption elasticities for capital price shocks are then given by the corresponding portfolio shares as*

$$\frac{\partial c_t/c_t}{\partial q_{ht}/q_{ht}} = \theta_{ht} \quad \textit{and} \quad \frac{\partial c_t/c_t}{\partial q_{kt}/q_{kt}} = \theta_{kt}. \tag{5.3}$$

[14]Permenant income hypothesis.

Moreover, the aggregate consumption response is given by

$$\frac{\partial C_t/C_t}{\partial q_t/q_t} = \eta_t^{APC}\frac{\partial c_t/c_t}{\partial q_t/q_t} + (1-\eta_t^{APC})\frac{\partial \underline{c}_t/\underline{c}_t}{\partial q_t/q_t},$$

where

$$\eta_t^{APC} = \frac{\eta_t APC}{\eta_t APC + (1-\eta_t)\underline{APC}} \tag{5.4}$$

is the average consumption propensity weighted entrepreneur wealth share.

Another important implication of Lemma 5.2 is that the rental yield may be written as

$$\frac{r_h}{q_h} = \frac{1-\alpha}{1-\nu}\big[\rho\eta + \underline{\rho}(1-\eta)\big].$$

In the unconstrained region $\dot{\nu} < 0$ and $\dot{\eta} < 0$ implying that the rental yield is therefore necessarily decreasing: price-to-rent ratio increases during the boom. A similar result holds for the risk-free interest rate. Indeed, we can characterize its behavior via the following Proposition.

Proposition 5.4. *Suppose Assumption 5.1 holds. Then in the unconstrained region, characterized by $\eta \geq \phi\nu$, the risk-free rate is given by*

$$r_{ft} = r_f(\eta) = \rho\eta^{APC} + \underline{\rho}(1-\eta^{APC}),$$

where η^{APC} is as in (5.4).

The risk-free interest rate is given by a properly weighted average of agents time discount rates. This is similar to Mian et al. (2020), where interest rate is driven by shifts wealth distribution of agents' with different saving propensities.

Recall that house prices are by Proposition 4.1 given by

$$q_{ht} = \frac{r_{ht} + \dot{q}_{ht}}{r_{ft}} = \frac{A(\psi_k)}{\alpha\big[\rho\eta + \underline{\rho}(1-\eta)\big]}\frac{K}{H}(1-\nu),$$

where

$$\psi_k = \begin{cases} 1, & \text{if } \eta \geq \phi\nu, \\ \frac{\eta}{\phi\nu}, & \text{if } \eta < \phi\nu. \end{cases}$$

For a large enough credit relaxation, i.e. reduction in the value of ϕ, entrepreneurs are lift off their credit constraints, after which $\psi_k = 1$ and $A(\psi_k) = a$. The shock is followed by a sustained house price boom, which is partially driven by decreasing interest rates: the risk free rate r_{ft} as well as the above discount factor $\rho\eta + \underline{\rho}(1-\eta)$

are decreasing, since wealth is being accumulated by the savers, who have high saving rates. High desired savings increase the prices of capital.

The exact long-run model dynamics depend on the model parametrization characterized in the following proposition.

Proposition 5.5. *The entrepreneur wealth share evolves with $\dot{\eta} > 0$ in the region, where $\eta > \phi\nu$. For $\eta \leq \phi\nu$, there are three possibilities:*

1. *Suppose*
$$\frac{a}{\underline{a}} - 1 < \phi\left[\frac{\rho}{\underline{\rho}} - 1\right].$$
Then the equilibrium entrepreneur wealth share η satisfies $\dot{\eta} \leq 0$ for all t, and the only steady-states are at $\eta = 0$ and $\eta = 1$ with $\nu = \alpha$. Only the steady-state at $\eta = 0$ is dynamically stable.

2. *Suppose*
$$\frac{a}{\underline{a}} - 1 > \phi\left[\frac{\rho}{\underline{\rho}} - 1\right].$$
Then, the model has an interior steady-state at
$$\eta_{ss} = \frac{\underline{\rho}\alpha\phi}{\underline{\rho} + (\rho - \underline{\rho})\phi(1-\alpha)} \quad \text{with} \quad \nu_{ss} = \frac{\eta_{ss}}{\phi}. \tag{5.5}$$
with dynamically stable dynamics both for $\eta < \phi\nu$ and $\eta > \phi\nu$.

3. *Suppose*
$$\frac{a}{\underline{a}} - 1 = \phi\left[\frac{\rho}{\underline{\rho}} - 1\right]. \tag{5.6}$$
Then, the loci $\dot{\eta} = 0$ and $\dot{\nu} = 0$ coincide in the region, where $\eta \leq \phi\nu$, constituting to a continuum of steady-states.

My model calibration concerns the first case in Proposition 5.5. Kumhof, Rancière, and Winant (2015) argue that the housing crisis was partially amplified due to a sequence of shocks, which increased income inequality and made the economy more vulnerable to large business fluctuations. While my mechanism is different from theirs, the empirical observation of increasing income and wealth inequality corresponds to the first case of Proposition 5.5 with non-stationary dynamics. For a phase diagram representing the model dynamics, see Figure 9 in the Appendix.

5.1 LTV collateral relaxation with an endogenous bust

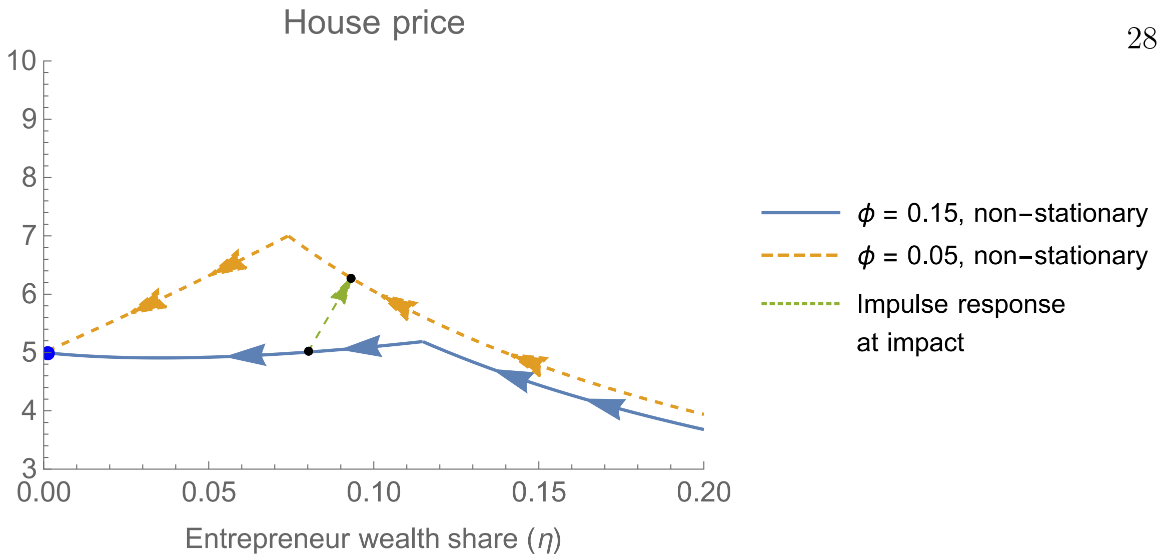
The main experiment I consider is an unanticipated shock decreasing the value of the leverage parameter ϕ. I begin by showing the house price dynamics for the case (a) of Proposition 5.5, which is depicted in panel (a) of Figure 4 for two different parameterizations: $\phi = 0.15$ and $\phi = 0.05$, keeping other parameters unaltered across these specifications. The model has two steady-states: $\eta = 0$ and $\eta = 1$, and I concentrate on $\eta = 0$, since it has a dynamically stable saddle path. The steady state is depicted as a blue dot.

The transition paths have a point of non-differentiability, when the path crosses the line $\eta = \phi\nu$. For $\eta > \phi\nu$ all of the productive capital is owned by the entrepreneurs; at the boundary $\eta = \phi\nu$, they begin to sell their productive capital to the savers at a positive rate: this produces a kink in the pricing schedules as well as a discontinuity for the risk-free interest rate. In particular, the collateral constraint is binding to the left of the kink in panel (a) of Figure 4. When an entrepreneur hits the collateral constraint, she sells her capital to the less productive savers at a positive rate. This deteriorates resource allocation and produces non-monotonic dynamics.

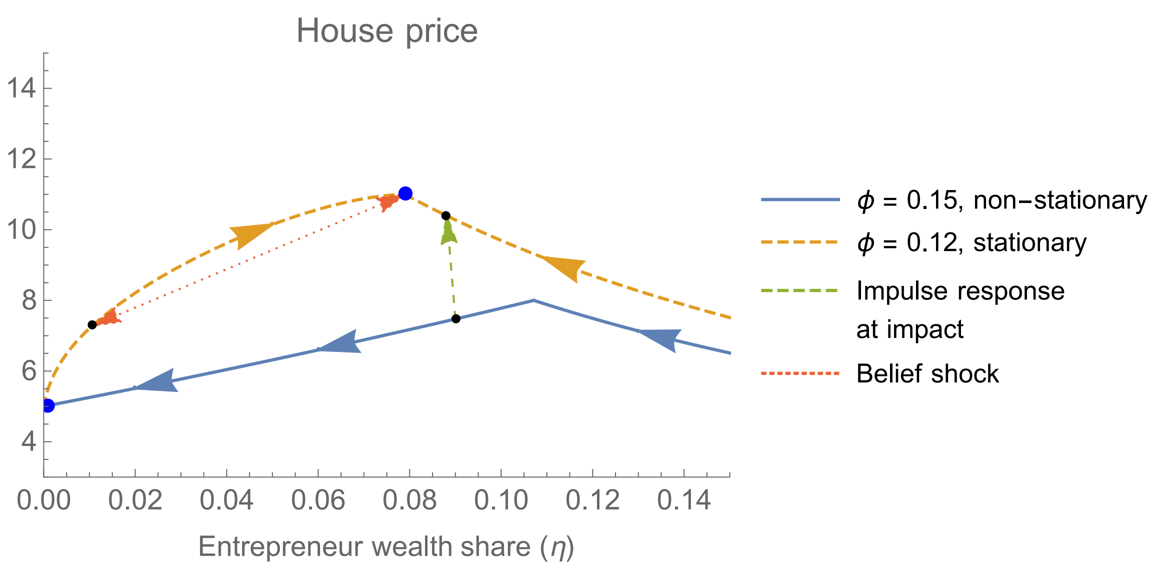
With both parameterizations depicted in panel (a) of Figure 4, the entrepreneurs thus eventually sell off all their housing stock and only hold productive capital in the long term. This is because of their lower saving rates compared to savers; entrepreneurs' relative wealth, and hence asset holdings, must decrease. Due to the entrepreneurs higher production efficiency, they always choose to first sell off their housing holdings before liquidating their income-generating capital. Consequently, only the productive capital may have any collateral value.

An unanticipated LTV shock affects the equilibrium quantities and, in particular, it moves the house price schedule. Concentrating first on panel (a), the original price schedule for $\phi = 0.15$ is depicted in solid blue line, whereas the new price schedule after an unanticipated shock $\phi = 0.15 \to 0.05$ is shown in a dashed yellow line. The initial impulse response at the impact of the shock is depicted with a dashed green arrow, given an initialization of the economy at $\eta = 0.08$.

The total price effect consists of three separate components. First, before the shock some of the productive capital was held by the savers, who are less efficient producers: the pre-shock wealth share located left of the point of non-differentiability, whereas after the shock, the economy moves to the right of this point in Figure 4. This reflects a change in the resource allocation increasing output and house prices. Secondly, even in the region, where the collateral constraint is not binding, the price schedule moves



(a) The picture depicts equilibrium price schedules as a function of the entrepreneur wealth share across the state space for $\eta \in [0, 0.2]$ corresponding to the case (a) of Proposition 5.5. For $\phi = 0.15$ and $\phi = 0.05$ the transitions are depicted in solid blue and dashed yellow lines, respectively. The economy is initialized at $\eta = 0.08$ and the impulse response at the impact of the shock is shown (dashed green), given an unanticipated exogenous shift in the value of ϕ from $\phi = 0.15$ to $\phi = 0.05$.



(b) The picture depicts equilibrium price schedules as a function of the entrepreneur wealth share across the state space for $\eta \in [0, 0.2]$. For $\phi = 0.15$ the non-stationary transition is shown (solid blue) corresponding to case (a) of Proposition 5.5. The economy is parametrized to produce the dynamics of case (b) of Proposition 5.5, given an exogenous shift in the value of $\phi = 0.15 \to 0.12$. The corresponding price schedule following the shock is depicted (dashed yellow) together with the impulse response (dashed green) for an initial state $\eta_o = 0.09$. The steady state following the LTV shock is vulnerable to a belief shock affecting the house price (double-sided, dotted red).

Figure 4

upwards. This is due to changes in the relative value of capital and housing, taking into account the forces explained in Section 4; most notably, the relative demand for housing and capital associated with the agents' portfolio decisions. Third, the shock produces a shift in the wealth distribution, since the agents have different portfolios. This affects the discount rate used for pricing assets. At the impact of the shock, this shift may mitigate the price effect, but in the long term the effect is necessarily overturned. In Section 6 (cf. Figure 6) each of these forces is studied quantitatively.

In panel (b) of Figure 4, I consider another parametrization, where the pre-shock dynamics are given by the non-stationary saddle path associated with case (a) of Proposition 5.5, but the exogenous relaxation of the LTV constraint from $\phi = 0.15$ to $\phi = 0.12$ moves the economy to case (b) of the Proposition. After the shock, the economy moves towards an interior steady state characterized by (5.5). The new steady-state as well as the associated transition path is, however, still vulnerable to belief shocks.

Starting from the steady-state, the model includes two equilibria corresponding to two different price paths: (i) one, where nothing happens and the economy stays at the steady-state, and (ii) another one, where the economy moves to another point at the transition path, leading to a house price crash. These equilibria are associated with different beliefs about the aggregate state evolution. In particular, a coordinated change in the agents' beliefs about the proper price level may relocate the economy to a different point of the state space, associated with different resource allocation and different prices. Belief shocks may thus have large price effects, for instance, producing a market crash. The strategic complementarity producing this multiplicity is exactly as in Section 2.4, and builds on the Kiyotaki and Moore (1997) financial accelerator mechanism.

6 Quantitative analysis

I now consider the quantitative implications of my model in greater detail and for this purpose, I introduce some extensions to the model set-up introduced in Section 3. In particular, I allow for investments such that for each individual j, her capital stock evolves according to

$$\frac{dk_t^j}{k_t^j} = \Phi_k(i_{kt}^j) - \delta_k \, dt$$

and housing stock according to

$$\frac{dh_t^j}{h_t^j} = \Phi_h(i_{ht}^j) - \delta_h \, dt,$$

where i_{kt}^j and i_{ht}^j are the capital and housing investment rates, respectively, δ_k and δ_h are the capital and housing depreciation rates, and

$$\Phi_k(i) = \frac{1}{\kappa_k}\log(1+\kappa_k i) \quad \text{and} \quad \Phi_h(i) = \frac{1}{\kappa_h}\log(1+\kappa_h i)$$

are the adjustment cost functions for capital and housing investments, respectively. Capital investments, $i_{kt}, \underline{i}_{kt}$ for entrepreneurs and savers, respectively, are paid for by production of non-durable consumption goods, given the market clearing equation

$$c_t + \underline{c}_t = (a - i_{kt})k_t + (\underline{a} - \underline{i}_{kt})\underline{k}_t.$$

For simplicity, I assume that housing investments are similarly produced by using housing services:

$$s_t + \underline{s}_t = (1 - i_{ht})h_t + (1 - \underline{i}_{ht})\underline{h}_t.$$

With this specification the return on holding capital is given by

$$\max_{i_{kt}^j} \frac{a^j - i_{kt}^j}{q_{kt}} + \Phi_k(i_{kt}^j) - \delta_k + \mu_{kt}^q.$$

and the return for holding housing is similarly given by

$$\max_{i_{ht}^j} \frac{1 - i_{ht}^j}{q_{kt}/r_{ht}} + \Phi_h(i_{ht}^j) - \delta_h + \mu_{ht}^q.$$

It follows that the supply schedule for investments of both types of capital take the forms

$$q_k = 1 + \kappa_k i_k \quad \text{and} \quad \frac{q_h}{r_h} = 1 + \kappa_h i_h. \tag{6.1}$$

In Table 4 I demonstrate that the latter model for price-to-rent ratios and housing investments is of empirical relevance, and produces a good fit for price and investment data. This observation motivates the use of an adjustment cost model for considering the effect of housing supply on house prices.

The response of housing supply to one percentage increase in housing price-to-rent ratio is now governed by the parameter κ_h as

$$\frac{d\Phi_h(i_h)}{\frac{d(q_h/r_h)}{q_h/r_h}} = \Phi_h'(i_h)\frac{di_h}{\frac{d(q_h/r_h)}{q_h/r_h}} = \frac{1}{\kappa_h}.$$

Much of the existing literature has assumed fixed housing supply, and this set-up is built to relax this assumption, without losing the model tractability.

6.1 Calibration

There are four crucial sets of parameters affecting my mechanism: (i) κ_h determines the housing supply elasticity and directly affects house price responses for economic shocks; (ii) the ratios $a/\underline{a}$ and $\rho/\underline{\rho}$ determine the extent of price amplification within the constrained region; see also Proposition 5.5; (iii) the evolution of K_t relative to H_t affects house prices and is governed by the investment technology parameters κ_k, δ_k and δ_h, in addition to κ_h; and (iv) the extent of the credit liberalization, which is the main experiment I consider, and is governed by the pre- and post-shock levels of ϕ. The parameter values I consider are shown in Table 3.

Table 3: Parameter values: baseline calibration

Parameter	Name	Value	Internal	Source/Target
		Experiment		
Capital LTV constraint	ϕ	$0.27 \to 0.18$	N	Shift in LTV ratios
		Parameter values		
Housing preferences	$1-\alpha$	0.183	N	Housing exp. share
saver time discount	$\underline{\rho}$	0.01	N	High saving propensity
Entrepreneur time discount	ρ	0.1	N	Low saving propensity
Entrepreneur productivity	a	1.23	N	Fagereng et al. (2020)
saver productivity	$\underline{a}$	1.0	N	Normalization
Housing adjustment cost	κ_h	1407	N	See Table 4
Capital adjustment cost	κ_k	17.1	Y	See text
Housing depreciation rate	δ_h	0.0	N	See text
Capital depreciation rate	δ_k	0.153	Y	Fraumeni (1997)

I use the linear relation in the second part of (6.1) to estimate κ_h. I construct price-to-rent ratios for 2014 – 2020 using rental and house price data from Zillow; unfortunately, rental data is available only starting 2014. I do not have data on repairs and maintenance, and I will exclude this component of housing investments and correspondingly set $\delta_h = 0$. Housing investment rate is measured by the ratio of real private residential investments and the value of the housing stock. I consider both contemporaneous and lagged effect of investments on price-to-rent ratios. The regressions are reported in Table 4.

The best explanatory power with $R^2 = 0.870$ is obtained with 15 months lag and coefficient estimate of 1664 in quarterly data. When forcing the regression intercept to 1, as indicated in (6.1), the quarterly coefficient estimates are relatively robust, slightly below 6000, independent of the lag. Using this specification arising from the model, I calibrate the value of housing supply elasticity at $\kappa_h = 1407$, corresponding to quarterly value of 5,628 in our annual model calibration and to R^2 of 0.996.[15]

The savers' productivity parameter is normalized to $\underline{a} = 1.0$. This is without loss of generality and corresponds to fixing the units of measurement for productive capital. I interpret the savers as bondholders, corresponding to pension and insurance funds as well as high net-worth individuals in the data. I use average returns on wealth for business owners (7.87%; high-achiever entrepreneurs) and for individuals in the top decile of the net worth distribution (6.39%) from Fagereng, Guiso, Malacrino, and Pistaferri (2020) to calibrate the productivity heterogeneity at $a = 1.23$. This represents a modest degree of heterogeneity in the agents' productivity, reflecting differences in the agents' ability for value-adding production with the assets (wealth) they own.

The time discount rates $\rho = 0.10$ and $\underline{\rho} = 0.01$ are chosen to produce ten-fold differences in marginal consumption propensities. Misra and Surico (2014) document that approximately half of the population have marginal consumption propensities close to zero, whereas another 20% consume over 50% out of unanticipated transitory income. My calibration produces substantial, but conservative heterogeneity in consumption propensities relative to observed extremes in the data, and yields pre-shock risk-free interest rate of 1.9%.

In order to concentrate on my house price mechanism, I shut down outside forces for economic growth, which might also affect house prices. Hence, the investment parameters $\kappa_k = 17.2$ and $\delta_k = 0.153$ are chosen within reasonable estimates provided in the literature to match zero growth rate in output.[16] This conservative growth is chosen to make sure that the house price effect during the boom is not driven merely by a large increase in output, reflected in a potential increase in K_t/H_t, which is ruled off by this calibration.[17]

Before the crisis most business loans were originated at LTVs close to 0.70 (Titman, Tompaidis, and Tsyplakov, 2005; Benmelech, Garmaise, and Moskowitz, 2005). I calibrate the LTV shock using the maximal commercial mortgage LTVs of approx-

[15]Producing the house price boom is robust for a more conservative calibration of $\kappa_h = 416$ corresponding to the 15 months lagged coefficient in Column (3) of Table 4.

[16]See Hall (2004) for estimates of κ_k and Fraumeni (1997) for BEA estimates for capital depreciation.

[17]See the formula for q_h in Proposition 4.1

Table 4: Price-to-rent ratio and residential investment

	Dependent variable:							
	Price-to-rent ratio				Price-to-rent ratio with forced intercept 1.0			
	Contemporaneous	*12 mos lag*	*15 mos lag*	*18 mos lag*	*Contemporaneous*	*12 mos lag*	*15 mos lag*	*18 mos lag*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Investment rate	2,100.2***	1,716.2***	1,664.1***	1,488.9***	5,628.1***	5,890.0***	5,971.3***	6,070.1***
	(197.6)	(197.1)	(160.7)	(189.7)	(72.2)	(113.1)	(123.2)	(147.0)
Constant	7.8***	8.8***	8.9***	9.3***	1.0	1.0	1.0	1.0
	(0.4)	(0.4)	(0.3)	(0.4)				
Observations	26	27	27	27	26	27	27	27
R^2	0.792	0.849	0.870	0.841	0.996	0.991	0.989	0.985
Adjusted R^2	0.783	0.843	0.864	0.835	0.996	0.990	0.989	0.984
Note:								*p<0.005; **p<0.001; ***p<5e-04

Regression of quarterly price-to-rent ratio on investment rate. Price-to-rent ratio is constructed from Zillow U.S. rental and price data. The price-to-rent ratio data is available since the beginning of 2014, and these time-series regressions are for price-to-rent ratios measured during 2014 – 2020. Investment rate is the growth rate of housing units in the U.S.A measure by the U.S. Census Bureau. Standard errors, shown in parentheses, are Newey-West (1987) heteroskedastic robust.

imately 0.73 in 2000 and 0.82 in 2004, reported by . These correspond to $\phi = 0.27$ and $\phi = 0.18$ before and after the credit shock, respectively. Note that mortgages constitute to a substantial part of small business loans, and calibrating the LTV shock according to commercial mortgages, therefore, seems appropriate.[18]

Finally, the preference parameter α is matched to the expenditure share of housing services: housing and utilities expenditure has been relatively stable at 18.3% of total GDP at least for the past 40 years.[19] The initial ratio of capital and housing stocks is normalized to $K_o/H_o = 1$, and the initial entrepreneur wealth share is set to $\eta_o = 0.19$ to match the 4-year duration of the boom from the credit relaxation in Q1/2003 lasting until Q1/2007.[20] Kaplan and Violante (2014) document that approximately one sixth of U.S. households are wealthy hand-to-mouth investors, who consume most of unexpected transitory income. The entrepreneurs constitute to such entrepreneurs in the data as they choose to use high leverage to finance their highly productive business projects, and the pre-shock calibration of η_o seems to be in line with this literature.

6.2 Results: impulse responses for an LTV relaxation

As shown in Figures 5 and 7, the credit relaxation leads to 52.9% increase in house prices, even though output increases only by 5.1% during the boom (when excluding any baseline growth in output, which is set to 0% before the shock). This more than explains the 40.0% house price increase between Q1/2003 and Q1/2007, during the "house price bubble"; Levitin and Wachter (2011) document that the "bubble" really formed only after 2003, as the preceding house price growth in the early 2000s was supported by increasing rents, and the price-to-rent ratios began to increase only later in 2003 – 2004.

At impact of the credit shock house prices are increased by 36.8% together with a similar change in the price-to-rent ratio. Entrepreneurs are lift off their credit constraints and they use the additional collateral capacity to purchase more productive

[18]For instance, in Q1/2002, the total value of U.S. proprietorship and partnership mortgages was $1.46 trillions, constituting 76% of their total debt holdings, whereas mortgages added up to 35 % of all debt for U.S. small businesses of less than 500 employees, which constitute to 99% of all U.S. firms and majority of total U.S. employment.

[19]Source: Personal Consumption Expenditures on Housing and utilities [DHUTRC1Q027SBEA] divided by total Personal Consumption Expenditures [PCEC], Federal Reserve Back of St. Louis.

[20]For timing the bubble see Levitin and Wachter (2011).

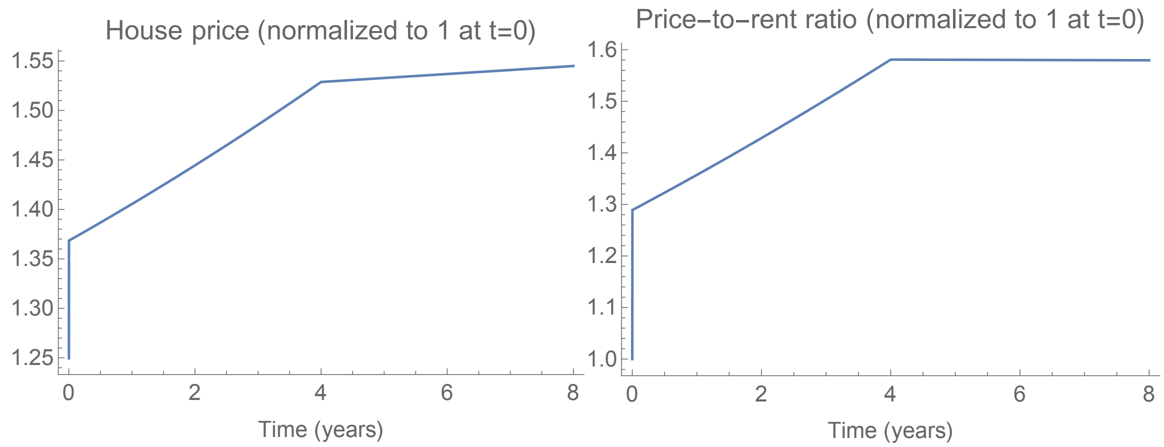


Figure 5: Impulse responses for house price and price-to-rent ratio. The shock lifts the entrepreneurs off their credit constraint; a non-differentiable change in the impulse responses occurs later, when the entrepreneurs hit the constraint again.

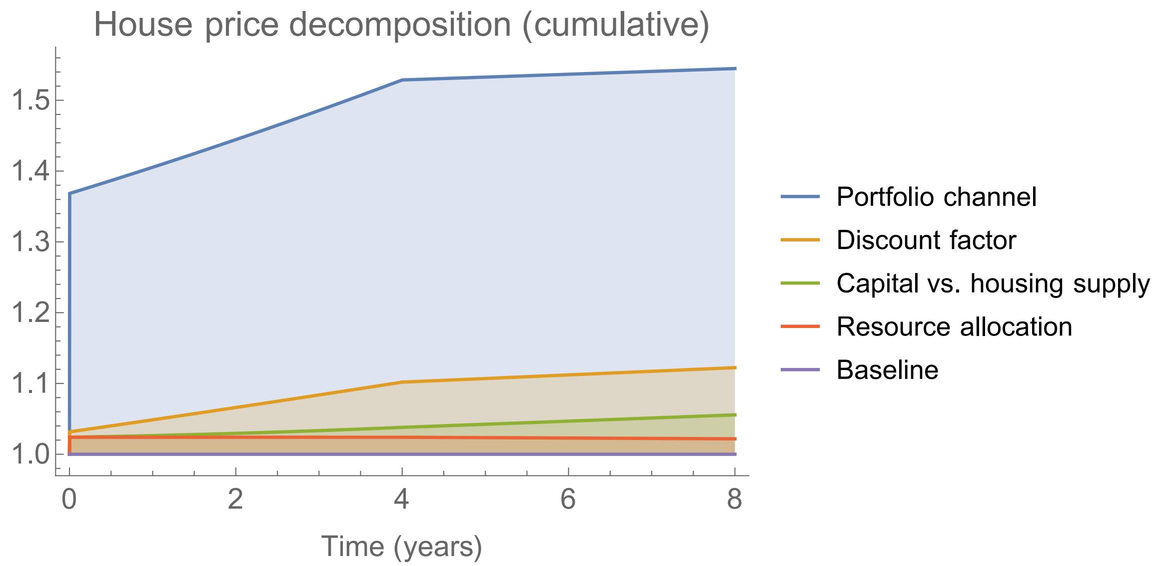
capital and to finance higher consumption during a sustained boom lasting for 4 years, after which they again hit the constraint and prices begin to decrease.

In Figure 6, I show, how the price impact is decomposed to the different components of Proposition 4.1: (i) increase in average productivity of capital, (ii) increase in the stock of productive capital relative to that of housing, (iii) change in the discount rate, and (iv) the *portfolio channel*.[21] Panels (a) and (b), respectively, discuss the price impact relative to the initial state and to the counterfactual price evolution; note that the model is not in steady-state at the time of the shock, since the interior of the state space does not have steady-states, as the model is non-stationary.

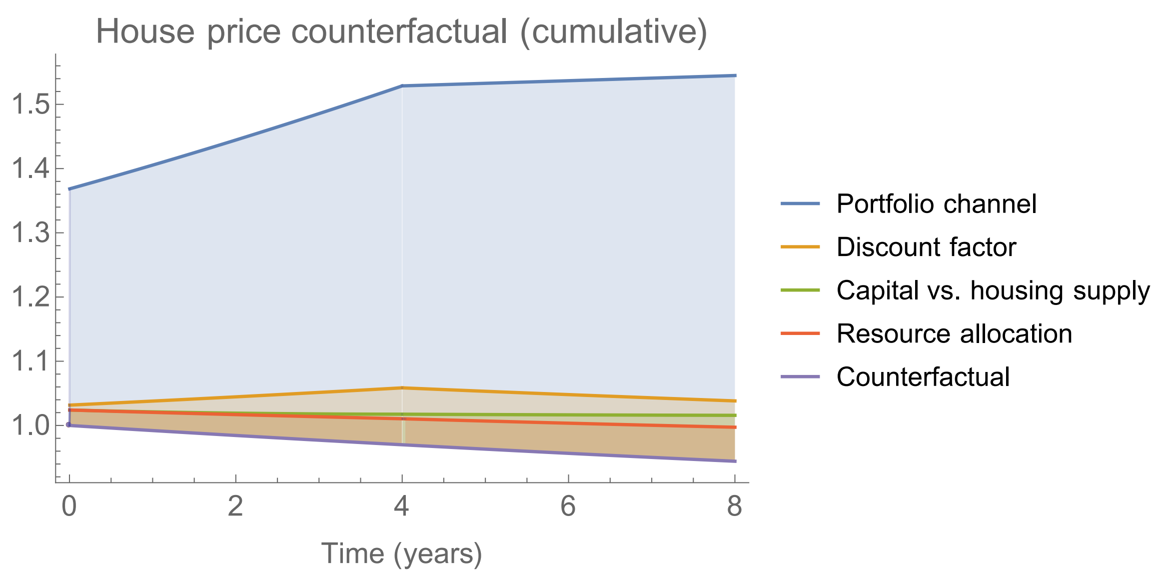
The components (i) and (ii) above also affect the price of productive capital; Figure 7 shows that the price increase in productive capital is substantially smaller compared to house prices. The large house price effect relative to price of capital is due to two reasons, both associated with the portfolio channel of (iv) above.

On one hand, the savers are willing to sell their capital to the entrepreneurs due to their differences in productivity; for housing investments savers remain marginal, and the demand for housing versus capital relative to the supply of each, therefore remains high: savers rebalance their portfolio towards housing, creating a housing boom. On the other hand, the entrepreneurs know that they will not hold the capital forever, but only for a few years during the boom, after which they prefer to liquidate it for financing higher future consumption. After the boom, capital is bought back by the

[21] For this quantitative analysis, Proposition 4.1 is extended to include capital investments; see Appendix D.



(a) The picture depicts the house price impulse response for a credit shock $\phi = 0.27 \to 0.18$, decomposing the price effect to its components according to Proposition 4.1.



(b) The picture depicts the house price impulse response for a credit shock $\phi = 0.27 \to 0.18$, decomposing the price effect to its components according to Proposition 4.1, relative to the counterfactual price evolution, where the credit shock never took place.

Figure 6

savers, who are not willing to pay a high price for it due to their lower productivity. This affects the entrepreneurs' valuation for capital them being forward-looking.

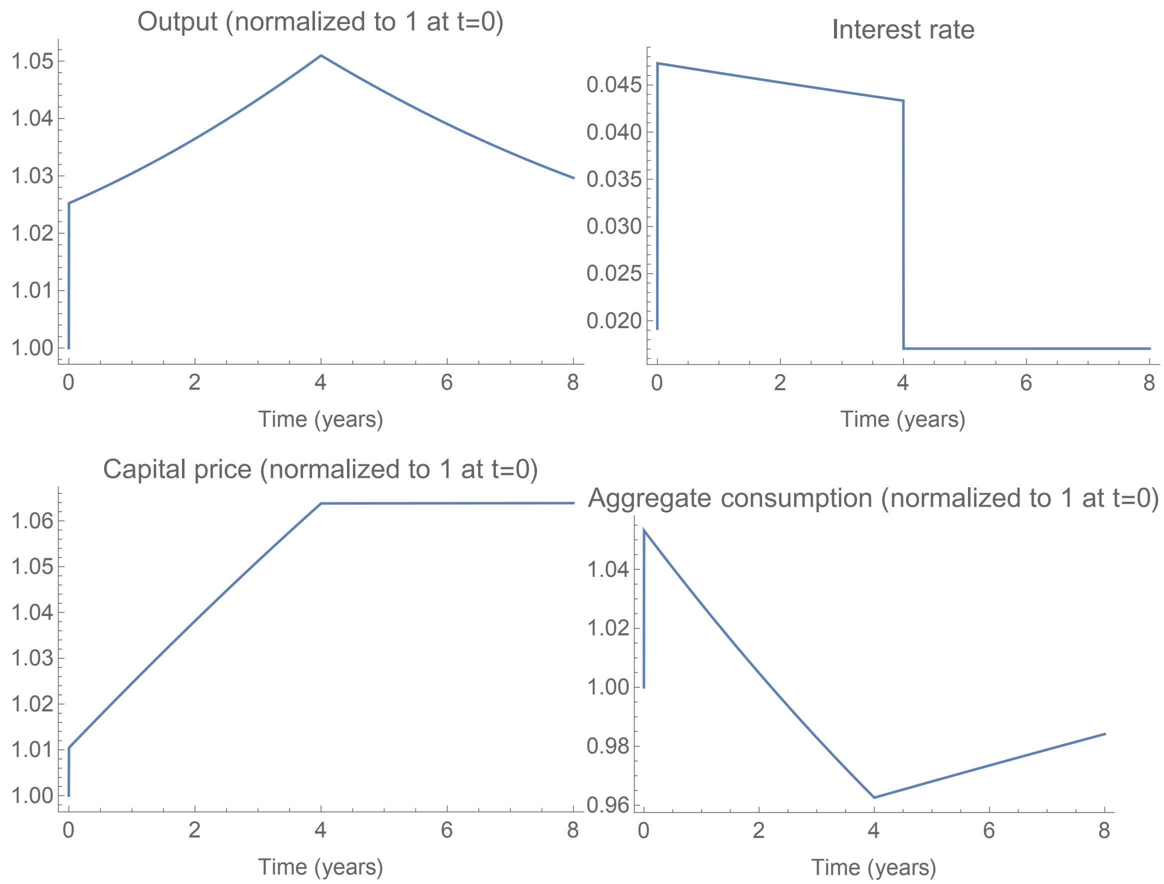


Figure 7: Impulse responses for real output, real interest rate, price of capital and aggregate consumption. The credit relaxation creates a strong boom in the affected sectors. Once the entrepreneurs hit the credit constraint, after being lift off the constraint at the impact of the shock, a decrease in the growth rate of real output is accompanied with a discontinuous downward jump in the real estate rate, resembling a credit crunch. Aggregate consumption increases together with the increased output at the impact of the shock, but the effect is dampened in the long run.

Once this liquidation begins, output decreases. House price growth flattens as savers' demand for housing is lowered, since now they are also marginal for capital investments. As the growth rates for capital and house prices decrease, the returns on these assets decrease, as well. Risk-free interest rate drops in order to balance the returns across different assets.

Note that the interest rate evolution follows the observed dynamics: during the PLS market expansion in 2003 – 2004, the long-running decrease in interest rates stopped, as the 30-year mortgage rate increased from 5.2% in June 2003 to 6.4% by August of the same year. Afterwards, the rates remained mostly flat until the crisis, when they dropped again from 6.4% in October 2008 to 4.8% in April 2009.

In conclusion, the bust in the model is created endogenously four years after the original credit supply shock. The simultaneous drop in output and real interest rate resembles a credit crunch studied by Guerrieri and Lorenzoni (2017), who produce similar dynamics as a consequence of an exogenous credit tightening, whereas in my model the credit crunch is an endogenous outcome of the preceding credit liberalization four years earlier. Note that the price of capital (cf. Figure 7), which is the collateral in the economy, never decreases; therefore, the difference between short- and long-term debt is irrelevant for the collateral constraint.

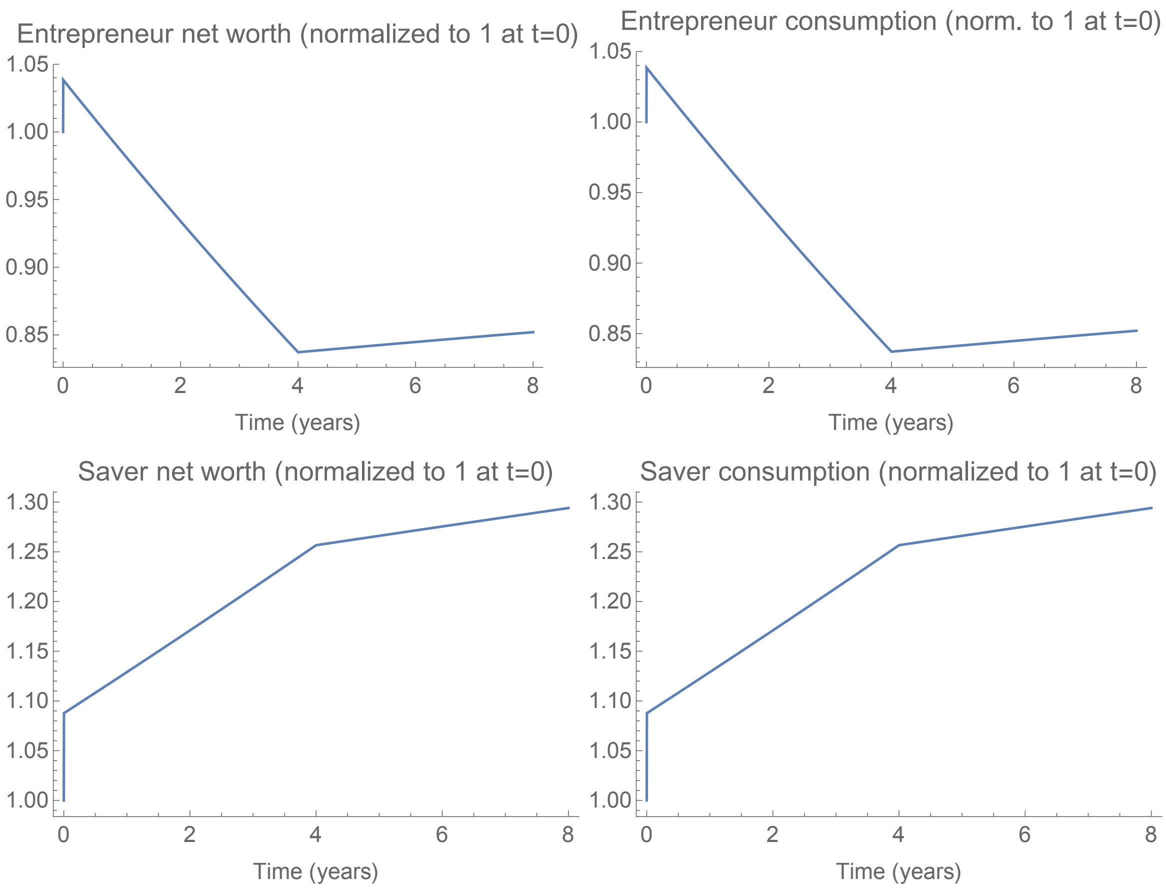


Figure 8: Impulse responses for aggregate, saver and entrepreneur consumption as well as for entrepreneur net worth. The credit relaxation creates a consumption boom, especially for the home owners (savers), who enjoy from a large house price appreciation.

The credit shock also produces a consumption boom, most notably for the homeowner savers, who enjoy from their increased housing wealth; see Figures 7 and 8. This corresponds to the evidence of Mian, Rao, and Sufi (2013), who document large consumption responses out of housing wealth, and the theoretical analysis of Berger et al. (2017), which I augment in Proposition 5.3. Savers' consumption increases approximately 25% during the boom, constituting to a large consumption elasticity close to 1 out of increasing housing wealth: savers' net worth also increases approximately 25% during the housing boom, and the increased consumption is thus due to a wealth effect. This additional consumption is partially produced by the additional production due to improved capital allocation, as entrepreneurs hold more of the productive capital stock after the shock. The entrepreneurs are, however, not driving the consumption boom, since their net worth is only increased by approximately 5% at the impact of the shock.

Moreover, while the entrepreneurs' net worth increases slightly at the impact of the shock, the effect is not long-lasting due to their relatively high consumption propensity. While the heterogeneity in the agents' productivity is crucial for my mechanism, the house price effect is produced by a shift in the savers' portfolio incentives. Entrepreneurs enjoy from their higher productivity, but the boom mostly benefits the savers, who own the houses.

7 Conclusion

I show that an exogenous relaxation of entrepreneur financing constraints may lead to a sustained real estate and GDP boom with a large consumption elasticity out of housing net worth shock, and to an endogenous credit crunch several years later. The price amplification mechanism required to produce large price responses emphasizes agents' portfolio decisions in a two-asset financial accelerator mechanism. Given an exogenous credit relaxation, the entrepreneurs increased ability to finance additional capital purchases affects the portfolio incentives of the existing capital holders. The model dynamics track the evolution of the U.S. housing boom-bust cycle before and during The Great Recession.

References

Manuel Adelino, Antoinette Schoar, and Felipe Severino. House prices, collateral, and self-employment. *Journal of Financial Economics*, 117(2):288–306, 2015.

Mark Aguiar and Erik Hurst. Deconstructing life cycle expenditure. *Journal of Political Economy*, 121(3):437–492, 2013.

Xudong An and Anthony B Sanders. Default of commercial mortgage loans during the financial crisis. In *46th Annual AREUEA Conference Paper*, 2010.

Timothy J Bartik. Who benefits from state and local economic development policies? 1991.

Efraim Benmelech, Mark J Garmaise, and Tobias J Moskowitz. Do liquidation values affect financial contracts? evidence from commercial loan contracts and zoning regulation. *The Quarterly Journal of Economics*, 120(3):1121–1154, 2005.

David Berger, Veronica Guerrieri, Guido Lorenzoni, and Joseph Vavra. House prices and consumer spending. *The Review of Economic Studies*, 85(3):1502–1542, 2017.

B Bernanke and M Gertler. Agency costs, net worth, and economic performance. *American Economic Review*, 79(1):14–31, 1989.

Markus K Brunnermeier and Yuliy Sannikov. A macroeconomic model with a financial sector. *American Economic Review*, 104(2):379–421, 2014.

Gabriel Chodorow-Reich and Antonio Falato. The loan covenant channel: How bank health transmits to the real economy. Technical report, National Bureau of Economic Research, 2017.

Morris A Davis and François Ortalo-Magné. Household expenditures, wages, rents. *Review of Economic Dynamics*, 14(2):248–261, 2011.

Marco Di Maggio and Amir Kermani. Credit-Induced Boom and Bust. *The Review of Financial Studies*, 30(11):3711–3758, 06 2017. ISSN 0893-9454. doi: 10.1093/rfs/hhx056. URL https://doi.org/10.1093/rfs/hhx056.

John V Duca, John Muellbauer, and Anthony Murphy. House prices and credit constraints: Making sense of the us experience. *The Economic Journal*, 121(552): 533–551, 2011.

Gauti B Eggertsson and Paul Krugman. Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach. *The Quarterly Journal of Economics*, 127(3):1469–1513, 2012.

Andreas Fagereng, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri. Heterogeneity and persistence in returns to wealth. *Econometrica*, 88(1):115–170, 2020.

Antonio Falato and Nellie Liang. Do creditor rights increase employment risk? evidence from loan covenants. *The Journal of Finance*, 71(6):2545–2590, 2016.

Giovanni Favara and Jean Imbs. Credit supply and the price of housing. *American Economic Review*, 105(3):958–92, March 2015. doi: 10.1257/aer.20121416. URL http://www.aeaweb.org/articles?id=10.1257/aer.20121416.

Jack Favilukis, Sydney C Ludvigson, and Stijn Van Nieuwerburgh. The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy*, 125(1):140–223, 2017.

Barbara Fraumeni. The measurement of depreciation in the us national income and product accounts. *Survey of current business-United States Department of Commerce*, 77:7–23, 1997.

Dennis Glennon and Peter Nigro. Measuring the default risk of small business loans: A survival analysis approach. *Journal of Money, Credit and Banking*, pages 923–947, 2005.

Paul Gomme, B Ravikumar, and Peter Rupert. The return to capital and the business cycle. *Review of Economic Dynamics*, 14(2):262–278, 2011.

Gary Gorton and Guillermo Ordonez. Collateral crises. *American Economic Review*, 104(2):343–78, 2014.

Daniel Greenwald. The mortgage credit channel of macroeconomic transmission. 2018.

Veronica Guerrieri and Guido Lorenzoni. Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467, 2017.

Robert E Hall. Measuring factor adjustment costs. *The Quarterly Journal of Economics*, 119(3):899–927, 2004.

Matteo Iacoviello. House prices, borrowing constraints, and monetary policy in the business cycle. *American economic review*, 95(3):739–764, 2005.

Matteo Iacoviello and Stefano Neri. Housing market spillovers: evidence from an estimated dsge model. *American Economic Journal: Macroeconomics*, 2(2):125–64, 2010.

Callum Jones, Virgiliu Midrigan, and Thomas Philippon. Household leverage and the recession. Technical report, National Bureau of Economic Research, 2011.

Alejandro Justiniano, Giorgio E Primiceri, and Andrea Tambalotti. Household leveraging and deleveraging. *Review of Economic Dynamics*, 18(1):3–20, 2015a.

Alejandro Justiniano, Giorgio E Primiceri, and Andrea Tambalotti. Credit supply and the housing boom. Technical report, National Bureau of Economic Research, 2015b.

Greg Kaplan and Giovanni L Violante. A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239, 2014.

Greg Kaplan, Kurt Mitman, and Giovanni L Violante. The housing boom and bust: Model meets evidence. Technical report, National Bureau of Economic Research, 2017.

Paymon Khorrami. The risk of risk-sharing: Diversification and boom-bust cycles. 2018.

Charles P Kindleberger and Robert Z Aliber. *Manias, panics and crashes: a history of financial crises*. Palgrave Macmillan, 2011.

Nobuhiro Kiyotaki and John Moore. Credit cycles. *Journal of political economy*, 105 (2):211–248, 1997.

Nobuhiro Kiyotaki, Alexander Michaelides, and Kalin Nikolov. Winners and losers in housing markets. *Journal of Money, Credit and Banking*, 43(2-3):255–296, 2011.

Michael Kumhof, Romain Rancière, and Pablo Winant. Inequality, leverage, and crises. *American Economic Review*, 105(3):1217–45, 2015.

Adam J Levitin and Susan M Wachter. Explaining the housing bubble. *Geo. LJ*, 100: 1177, 2011.

Ralf R Meisenzahl. Verifying the state of financing constraints: Evidence from us business credit contracts. *Journal of Economic Dynamics and Control*, 43:58–77, 2014.

Atif Mian and Amir Sufi. The consequences of mortgage credit expansion: Evidence from the us mortgage default crisis. *The Quarterly Journal of Economics*, 124(4): 1449–1496, 2009.

Atif Mian and Amir Sufi. Household leverage and the recession of 2007–09. *IMF Economic Review*, 58(1):74–117, 2010.

Atif Mian and Amir Sufi. What explains the 2007–2009 drop in employment? *Econometrica*, 82(6):2197–2223, 2014.

Atif Mian and Amir Sufi. Household debt and defaults from 2000 to 2010: The credit supply view. *Evidence and Innovation in Housing Law and Policy*, pages 257–288, 2017.

Atif Mian, Kamalesh Rao, and Amir Sufi. Household balance sheets, consumption, and the economic slump. *The Quarterly Journal of Economics*, 128(4):1687–1726, 2013.

Atif R Mian, Ludwig Straub, and Amir Sufi. Indebted demand. Technical report, National Bureau of Economic Research, 2020.

Kanishka Misra and Paolo Surico. Consumption, income changes, and heterogeneity: Evidence from two fiscal stimulus programs. *American Economic Journal: Macroeconomics*, 6(4):84–106, 2014.

George Pennacchi and Mahdi Rastad. Portfolio allocation for public pension funds. *Journal of Pension Economics & Finance*, 10(2):221–245, 2011.

Monika Piazzesi, Martin Schneider, and Selale Tuzel. Housing, consumption and asset pricing. *Journal of Financial Economics*, 83(3):531–569, 2007.

Diego Restuccia and Richard Rogerson. The causes and costs of misallocation. *Journal of Economic Perspectives*, 31(3):151–74, 2017.

Albert Saiz. The geographic determinants of housing supply. *The Quarterly Journal of Economics*, 125(3):1253–1296, 2010.

Sheridan Titman, Stathis Tompaidis, and Sergey Tsyplakov. Determinants of credit spreads in commercial mortgages. *Real Estate Economics*, 33(4):711–738, 2005.

A The empirics of the collateral channel: additional details and robustness

A.1 Data and summary statistics

The employment and establishment counts are from the County Business Patterns (CBP) dataset published by the U.S. Census Bureau. The data includes employment by county, industry and establishment size (measured in number of employees) between 1998 until 2010 as of March of the reported year. In all of the analyses I use 2-digit National American Industry Classification System (NAICS) level and exclude the construction sector to ensure my results are not driven by the specifics of the pre-crisis housing market. All results are robust for including construction.

In some small MSAs the employment data is not disclosed for all industries due to privacy reasons and only a size class is provided. In such cases I use the midpoint of the category, except for "over 100,000 employees", where 200,000 employees is used as a proxy for the employment count. The data also includes the full MSA-level employment and establishment counts, but in my regression specifications I will only use the industry level data, which allows for removing the construction sector.

Similarly, I use the midpoint of the category for small business analyses, where the breakdown of establishments by the number of employees is provided in categories of 1 to 4 employees, 5 to 9 employees and 10 to 19 employees, which are the only three categories I consider: I restrict my analyses of small business employment on entrepreneurs with less than 20 employees.

The small business loan data is from Community Reinvestment Act (CRA) dataset provided by the Federal Financial Institutions Examination Council (FFIEC). Community Reinvestment Act was passed in 1977 in order to reduce discriminatory lending practices towards low-income neighborhoods. Several revisions on the law have since been enacted and the loan data has been made available since 1996. The data provides county-level information on CRA small business loans, including number and value of loans by borrower income and by loan size at origination – categorized as less than $100,000, between $100,000 and $250,000 and over $250,000 but less than $1,000,000 – as well as total number and value of loans provided for entrepreneurs with gross annual revenue below one million dollars. I aggregate the loan data over 1999 – 2002 and 2003 – 2006 to produce growth rates for number and value of business loans originated at value of less than $250,000 between the two time periods.

House prices are measured by the MSA-level Zillow Home Value Index (ZHVI).

Table 5: Summary Statistics

	N	Mean	Median	SD
Panel A: County Business Patterns MSA-level data				
Growth in number of establishments ’03 – ’06	391	0.034	0.028	0.042
Growth in number of establishments ’99 – ’02	363	0.027	0.022	0.032
Employment growth ’03 – ’06	391	0.042	0.036	0.089
Employment growth ’99 – ’02	363	0.012	0.010	0.064
Employment growth in entrepreneurs with $\leq$ 20 employees ’03 – ’06	391	0.035	0.031	0.040
Panel B: Community Reinvestment Act Small Business Loan MSA-level data				
Average loan value ($) for entrepreneurs with revenue $\leq$ $1M in ’03 – ’06	388	40,821	40,226	12,010
Average loan value ($) for entrepreneurs with revenue $\leq$ $1M in ’99 – ’02	385	47,129	47,125	12,640
Growth in average loan value from ’99 – ’02 to ’03 – ’06	385	-0.13	-0.14	0.14
Growth in # of loans originated at $\leq$ $250,000	385	0.97	0.60	2.62
Growth in value of loans originated at $\leq$ $250,000	385	0.75	0.35	2.90
Growth in # of loans for entrepreneurs with revenue $\leq$ $1M	385	0.98	0.52	3.17
Growth in value of loans for entrepreneurs with revenue $\leq$ $1M	385	0.73	0.31	3.05
Panel C: House Price MSA-level data				
Saiz (2010) housing supply elasticity	314	2.57	2.29	1.41
House price growth ’03 – ’06	374	0.32	0.23	0.25
House price growth ’99 – ’02	374	0.21	0.18	0.16

In Table 5, I provide summary statistics for the data considered in my analysis. The average house price growth across the U.S. MSAs was 32 percent from 2003 to 2006 and 21 percent from 1999 to 2002. The growth rates for number of establishments were 3.4 percent ('03 - '06) and 2.7 percent ('99 - '02) as well as 4.2 percent ('03 - '06) and 1.2 percent ('99 - '02) for employment, respectively.

Interestingly, the average value of a business loans extended for entrepreneurs with gross annual revenue less than one million dollars decreased by 13 percent between the study periods of 1999 – 2002 and 2003 – 2006, while the total value of loans granted increased by 73 percent during the same time period. This means that most of the increase in lending activity took place on the extensive margin increasing the number of loans, while decreasing the loan size. This might be an indication of an increased access to financing by small entrepreneurs, for which the access to collateral is particularly crucial, and which were traditionally denied credit.

As the extensive margin seems important, I use the number of business loans originated at values less than $250,000 as the main explanatory variable for the business loan analyses. The results are generally robust for replacing this variable by number of all small business loan originations extended to entrepreneurs with gross annual revenue of less than $1,000,000 or by the total dollar value of loans instead of using the number of all loans.

Finally, in order to evaluate the small business's potential for creating aggregate effects, note that according to the Small Business Administration (SBA) 99 percent of all U.S. entrepreneurs were classified as small businesses ($\leq$ 500 employees) in 2002, and the 1998 Survey of Small Business Finances (SSBF) estimates that over 91 percent of these businesses had fewer than 20 employees.[22] Adelino et al. (2015) report that such entrepreneurs contributed 30.0 percent of all employment in the U.S.A. in 2002, and they calculate that the collateral channel affecting small businesses with less than 50 employees constituted to 10 - 25 percent of the aggregate employment growth during the pre-crisis years, even when excluding the growth beyond the small business categories after entrepreneurs grow out of the small business status.

A.2 The effect of house prices on business loan origination

Adelino et al. (2015) argue that increases in real estate prices affect employment growth by facilitating the creation or expansion of small business due to a collat-

[22] Report to the Congress on the Availability of Credit to Small Businesses, Board of Governors of the Federal Reserve System

eral channel. They consider the differential effect of house prices on the expansion of employment between establishments of different sizes: assuming small business financing is more likely to be reliant on real estate collateral, house prices should have a differential impact on small business expansion compared to large corporations. In order to reason for a causal effect, they follow existing literature by instrumenting house prices by the housing supply elasticity, measured by land topography differences by Saiz (2010). While arguing to the same effect, I augment their analysis by explicitly considering small business loan originations in the CRA data.

I first estimate the effect of house prices on the MSA-level growth rate in the number of business loans originated at values less than \$250,000 as

$$\text{Business Loan Growth}_i = \alpha_o + \beta_{\text{HP}\to\text{Loan}}\,\text{HP growth}_i + \gamma_o X_i + \varepsilon_i,$$

where β_{HP} is the coefficient of interest and X_i includes the MSA-level employment growth as a control. House price growth is instrumented with housing supply elasticity and the first stage is thus specified as

$$\text{HP growth}_i = \alpha_1 + \beta_{\text{Elas}\to\text{HP growth}}\,\text{Housing supply elasticity}_i + \gamma_1 X_i + \varepsilon_i. \tag{A.1}$$

The results are reported in Table 6. One percentage point increase in house prices during 2003 – 2006 predicted approximately 0.38 percentage point increase in number of small business loans ($\leq$ \$250,000 at origination) in the OLS estimate, and approximately 0.75 percentage point increase in the IV estimate.

Later I estimate the impact of establishment growth on house prices, and find very large effect, possibly due to a standard demand channel (cf. Table 2). Next I also document that business loans have a positive impact on establishment growth, and thus due to the simultaneity bias, one should not be surprised to observe a higher IV estimate than the OLS coefficient in Table 6.

A.3 The effect of house prices and small business loans on employment

In Table 7, I document the effect of house price and business loan growth on employment growth. The regression specification is given by

$$\begin{aligned}\text{Employment Growth}_i = \alpha_2 &+ \beta_{\text{HP}\to\text{Est}}\,\text{HP growth}_i + \beta_{\text{Loan}\to\text{Est}}\,\text{Loan growth}_i \\ &+ \beta_{Interaction}\,\text{HP growth}_i \times \text{Loan growth}_i + \gamma_2 X_i + \varepsilon_i,\end{aligned}$$

Table 6: The impact of house price growth on small business loan growth

	Dependent variable:					
	House price growth '03 - '06	Growth in number of small business loans ($\leq$ \$250K at origination)				
	First stage	*Reduced form*	*OLS*		*Instrumental variable (2SLS)*	
	(1)	(2)	(3)	(4)	(5)	(6)
Housing supply elasticity	-0.10^{***}	-0.06^{***}				
	(0.01)	(0.01)				
House price growth '03 - '06			0.39^{***}	0.37^{***}	0.72^{***}	0.77^{***}
			(0.05)	(0.06)	(0.09)	(0.11)
Employment growth '03 - '06				0.06		0.04
				(0.14)		(0.18)
Constant	0.56^{***}	0.77^{***}	0.58^{***}	0.58^{***}	0.42^{***}	0.41^{***}
	(0.04)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)
Observations	283	311	347	332	280	269
R^2	0.22	0.15	0.04	0.03	0.14	0.02
Adjusted R^2	0.22	0.15	0.04	0.03	0.13	0.02
Note:						*p<0.05; **p<0.01; ***p<0.001

Regression of MSA level growth in number of small business loans from 1999 – 2002 to 2003 – 2006, originated at values of less than \$250,000, on house price growth during 2003 – 2006 instrumented with housing supply elasticity as measured by Saiz (2010). Columns (4) and (6) control for growth in employment during 2003 – 2006. Employment data excludes construction sector. House prices are measured by Zillow MSA level price indices. Standard errors, shown in parentheses, are Newey-West (1987) heteroskedastic robust. Regressions are weighted by the number of small business loan originations during 1999 – 2002.

where house price growth is instrumented by the housing supply elasticity with first stage given by (A.1).

The results are very similar to those of Table 1, when considering the growth in number of establishment instead of employment growth, although the negative effect of house prices on employment in this interaction regression is more drastic. Based on Column (9) of Table 7, one percentage point increase in house price growth leading to 0.83 percentage point decrease in employment growth during 2003 – 2006, when excluding the collateral effect, which more than compensates for the negative income effect on employment. The results are similar also, when considering only small businesses, see Table 8.[23]

B Details for the static model of Section 2.4

Let's denote the Lagrangian associated with (2.3) by

$$\begin{aligned}\mathcal{L} = u(c^{\alpha}s^{1-\alpha}) &+ \lambda_1[ak + r_h h + (1+r_f)b - c - r_h s] \\ &+ \lambda_2[q_k(k_o - k) + q_h(h_o - h) + b_o - b] \\ &+ \lambda_3[(1-\phi)q_k k + b] + \lambda_4 k + \lambda_5 h.\end{aligned}$$

The first order conditions for (2.3) are given by

$$\begin{aligned}0 &= \frac{\partial \mathcal{L}}{\partial c} = \alpha u' c^{\alpha-1}s^{1-\alpha} - \lambda_1, \\ 0 &= \frac{\partial \mathcal{L}}{\partial s} = (1-\alpha)u' c^{\alpha}s^{-\alpha} - \lambda_1 r_h, \\ 0 &= \frac{\partial \mathcal{L}}{\partial k} = \lambda_1 a - \lambda_2 q_k + \lambda_3(1-\phi)q_k + \lambda_4, \\ 0 &= \frac{\partial \mathcal{L}}{\partial h} = \lambda_1 r_h - \lambda_2 q_h + \lambda_5, \\ 0 &= \frac{\partial \mathcal{L}}{\partial b} = \lambda_1(1+r_f) - \lambda_2 + \lambda_3.\end{aligned}$$

The first two equations now yield

$$r_h s = \frac{1-\alpha}{\alpha}c.$$

[23] Adelino et al. (2013) consider the effect of house price growth on employment especially in small businesses. In Table 8 I show that the business loan channel is pertinent also when only considering businesses with less than 20 employees.

Table 7: The impact of house price and business loan growth on employment

	Dependent variable: Employment growth '03 - '06									
	Reduced form	*OLS*				*Instrumental variable (2SLS)*				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Housing supply elasticity	0.01									
	(0.01)									
House price growth '03 - '06		0.03		0.04	0.03	−0.06		−0.09	-0.83^{***}	-0.83^{*}
		(0.04)		(0.04)	(0.05)	(0.06)		(0.05)	(0.26)	(0.34)
Growth in small business loans			−0.001	−0.001	−0.004		−0.08	0.13^{*}	−0.19	−0.19
			(0.002)	(0.002)	(0.01)		(0.08)	(0.05)	(0.14)	(0.14)
House price growth '03 - '06 × Growth in small business loans					0.01				0.96^{**}	0.96^{*}
					(0.04)				(0.35)	(0.40)
Employment growth '99 - '02										−0.01
										(0.87)
Constant	0.01	0.004	0.02	0.001	0.004	0.04	0.08	−0.02	0.18^{*}	0.18^{*}
	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.06)	(0.03)	(0.08)	(0.08)
Observations	298	359	364	332	332	272	295	269	269	269
R^2	0.005	0.01	0.0001	0.01	0.01	−0.07	−0.08	−0.25	−0.08	−0.08
Adjusted R^2	0.001	0.01	−0.003	0.01	0.003	−0.08	−0.08	−0.26	−0.09	−0.10

Note: $^{*}p<0.05$; $^{**}p<0.01$; $^{***}p<0.005$

Regression of MSA level employment growth during 2003 – 2006 on house price growth during the same period and on growth in number of small business loan originations (≤ $250,000 at origination) from 1999 – 2002 to 2003 – 2006 as well as on their interaction. House price growth is measured by Zillow MSA level price indices, and it is instrumented with housing supply elasticity as measured by Saiz (2010). In column (7), we also instrument the business loan origination with the same Saiz (2010) instrument. In column (10), pre-trend on employment growth during 1999 – 2002 is included as a control. Employment data excludes construction sector. Standard errors, shown in parentheses, are Newey-West (1987) heteroskedastic robust. Regressions are weighted by employment in 2003.

Table 8: The impact of house price and small business loan growth on small business employment

	Dependent variable: Small business employment growth '03 - '06								
	Reduced form	*OLS*				*Instrumental variable (2SLS)*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Housing supply elasticity	-0.01^{***}								
	(0.002)								
House price growth '03 - '06		0.06^{***}		0.06^{***}	0.06^{***}	0.05^{***}		-0.03	-0.16
		(0.01)		(0.01)	(0.01)	(0.02)		(0.04)	(0.09)
Growth in small business loans			0.0002	-0.0002	0.001		0.09^{***}	0.10^{***}	0.01
			(0.0003)	(0.0003)	(0.002)		(0.03)	(0.03)	(0.03)
House price growth '03 - '06 × Growth in small business loans					-0.01				0.22^{*}
					(0.01)				(0.11)
Constant	0.05^{***}	0.02^{***}	0.04^{***}	0.02^{***}	0.02^{***}	0.02^{***}	-0.02	-0.02	0.03
	(0.01)	(0.003)	(0.002)	(0.003)	(0.004)	(0.005)	(0.02)	(0.01)	(0.02)
Observations	298	359	364	332	332	272	295	269	269
R^2	0.03	0.13	0.0001	0.13	0.13	0.13	0.21	0.14	0.12
Adjusted R^2	0.03	0.13	-0.003	0.12	0.12	0.13	0.21	0.13	0.11
Note:							*p<0.05; **p<0.01; ***p<0.005		

Regression of MSA level small business ($\leq$ 20 employees) employment growth during 2003 – 2006 on house price growth during the same period and on growth in number of small business loan originations ($\leq$ \$250,000 at origination) from 1999 – 2002 to 2003 – 2006 as well as on their interaction. House price growth is measured by Zillow MSA level price indices, and it is instrumented with housing supply elasticity as measured by Saiz (2010). In column (7), we also instrument the business loan origination with the same Saiz (2010) instrument. Employment data excludes construction sector. Standard errors, shown in parentheses, are Newey-West (1987) heteroskedastic robust. Regressions are not weighted contrary to Table 7.

Plugging this into an agent's budget constraint gives the Marshallian demand functions

$$c^* = \alpha[ak + r_h h + (1 + r_f)b] \quad \text{and} \quad r_h s^* = (1 - \alpha)[ak + r_h h + (1 + r_f)b]. \tag{B.1}$$

Summing the demand functions c^* over the agents, by goods market clearing we have

$$A(\psi_k)K = ak + \underline{ak} = \alpha[A(\psi_k)K + r_h H],$$

which implies

$$r_h = A(\psi_k)\frac{K}{H}\frac{1-\alpha}{\alpha}. \tag{B.2}$$

Eventually all trades must be paid for by consumption, which the agents care about. Since consumption is the numéraire, this dictates that the total net worth in the economy is given by

$$N = q_k K + q_h H = C + r_h H = A(\psi_k)K + r_h H = \frac{A(\psi_k)K}{\alpha}.$$

It follows that

$$\frac{C}{N} = 1 - \frac{r_h H}{N} = 1 - (1 - \alpha) = \alpha.$$

By the goods market clearing this implies

$$q_k = \frac{A(\psi_k)}{C/N}\frac{q_k K}{N} = \frac{A(\psi_k)}{\alpha}\nu \quad \text{and} \quad q_h = q_k \frac{K}{H}\frac{1-\nu}{\nu}, \tag{B.3}$$

where I have denoted

$$\nu := \frac{q_k K}{N}.$$

Let us rewrite the first order conditions as

$$1 + r_f = \frac{\lambda_2}{\lambda_1} - \frac{\lambda_3}{\lambda_1}$$

and

$$\frac{\lambda_2}{\lambda_1} = \frac{a}{q_k} + \frac{\lambda_3}{\lambda_1}(1 - \phi) + \frac{\lambda_4}{\lambda_1 q_k}.$$

and

$$\frac{r_h}{q_h} = \frac{\lambda_2}{\lambda_1} - \frac{\lambda_5}{\lambda_2 q_h}$$

which imply

$$\frac{\lambda_2}{\lambda_1} = 1 + r_f + \frac{\lambda_3}{\lambda_1} = \frac{r_h}{q_h} + \frac{\lambda_5}{\lambda_2 q_h} = \frac{a}{q_k} + \frac{\lambda_3}{\lambda_1}(1-\phi) + \frac{\lambda_4}{\lambda_1 q_k}$$

Therefore

$$1 + r_f = \frac{a}{q_k} - \frac{\lambda_3}{\lambda_1}\phi + \frac{\lambda_4}{\lambda_1 q_k},$$

and

$$1 + r_f = \frac{A(\psi_k)}{q_k} - \psi_k \frac{\lambda_3}{\lambda_1}\phi.$$

Suppose $\lambda_3 = 0$ for both agents. Then necessarily $\underline{\lambda}_4 > 0$ and $k = K$. This gives

$$\frac{a}{q_k} = 1 + r_f = \frac{r_h}{q_h}$$

and

$$\frac{\alpha}{\nu} = \frac{a}{q_k} = 1 + r_f = \frac{r_h}{q_h} = \frac{a}{q_h}\frac{K}{H}\frac{1-\alpha}{\alpha} \implies q_h = \frac{a}{\alpha}\nu\frac{K}{H}\frac{1-\alpha}{\alpha}.$$

We know that

$$\frac{aK}{\alpha} = N = q_k K + q_h H = aK\frac{\nu}{\alpha} + \frac{aK}{\alpha}\frac{1-\alpha}{\alpha}\nu.$$

This implies

$$\frac{1}{\alpha} = \nu\left[\frac{1}{\alpha} + \frac{1-\alpha}{\alpha^2}\right] = \nu\frac{1}{\alpha^2}.$$

Altogether

$$\lambda_3 = 0 \implies \nu = \alpha, \quad q_k = a, \quad r_f = 0 \quad \text{and} \quad q_h = r_h = \frac{aK}{H}\frac{1-\alpha}{\alpha}. \tag{B.4}$$

It also follows that entrepreneur net worth satisfies

$$a$$

Claim: Assumption 2.4 implies that the entrepreneur capital purchases satisfy $k < K$. Suppose to the contrary. Then $q_k = a$ and

$$ak_o + a\frac{K}{H}\frac{1-\alpha}{\alpha}h_o + b_o = q_k k + q_h h + b \geq aK - (1-\phi)aK = \phi aK,$$

which yields contradiction with Assumption 2.4 by dividing this by aK.

Suppose now that $\lambda_3 > 0$ for the entrepreneur, and $\psi_k \in (0,1)$. It follows that $\lambda_5 > 0$ and $h = 0$ as well as $\lambda_4 = 0$ for both agents. We obtain

$$\frac{\lambda_3}{\lambda_1} = \frac{1}{\phi}\frac{a - \underline{a}}{q_k} = \frac{1}{\phi}\frac{a - \underline{a}}{A(\psi_k)}\frac{\alpha}{\nu}.$$

as well as

$$\begin{aligned} q_k(1-\psi_k)K + q_h H &= -(1-\phi)q_k\psi_k K + q_k\underline{k}_o + q_h\underline{h}_o + \underline{b}_o \\ &= -(1-\phi)q_k\psi_k K + q_k K\left[1 - \frac{k_o}{K} + \left(1 - \frac{h_o}{H}\right)\frac{1-\nu}{\nu}\right] - b_o. \end{aligned}$$

This implies

$$1 - \psi_k + \frac{1-\nu}{\nu} = -(1-\phi)\psi_k + 1 - \frac{k_o}{K} + \left(1 - \frac{h_o}{H}\right)\frac{1-\nu}{\nu} - \frac{b_o}{A(\psi_k)K}\frac{\alpha}{\nu},$$

which can be rewritten as

$$0 = \phi\psi_k - \frac{k_o}{K} - \frac{h_o}{H}\frac{1-\nu}{\nu} - \frac{b_o}{A(\psi_k)K}\frac{\alpha}{\nu},$$

Moreover

$$\phi\frac{A(\psi_k)\psi_k K}{\alpha}\nu = \phi q_k\psi_k K = \phi q_k k = q_k k_o + q_h h_o + b_o = \frac{A(\psi_k)K}{\alpha}\nu\left[\frac{k_o}{K} + \frac{h_o}{H}\frac{1-\nu}{\nu}\right] + b_o.$$

$$1 + r_f = \frac{\underline{a}}{q_k} = \frac{\underline{\lambda}_2}{\underline{\lambda}_1} = \frac{\lambda_2}{\lambda_1} - \frac{\lambda_3}{\lambda_1}.$$

$$\frac{1-\alpha}{1-\nu} = \frac{r_h}{q_h} = \frac{\underline{\lambda}_2}{\underline{\lambda}_1} = \frac{\underline{a}}{q_k} = \frac{\underline{a}}{A(\psi_k)}\frac{\alpha}{\nu},$$

which implies

$$\underline{a} + (a - \underline{a})\psi_k = A(\psi_k) = \underline{a}\frac{\alpha}{1-\alpha}\frac{1-\nu}{\nu} \implies \frac{1-\nu}{\nu} = \frac{1-\alpha}{\alpha\underline{a}}A(\psi_k) \implies \nu \leq \alpha.$$

and

$$\left[A(\psi_k) + \underline{a}\frac{\alpha}{1-\alpha}\right]\nu = \underline{a}\frac{\alpha}{1-\alpha}.$$

This gives

$$\frac{\alpha}{\nu} = \frac{1-\alpha}{\underline{a}}\left[A(\psi_k) + \underline{a}\frac{\alpha}{1-\alpha}\right] = \alpha + (1-\alpha)\frac{A(\psi_k)}{\underline{a}}. \tag{B.5}$$

We obtain

$$\begin{aligned} 0 &= \phi\psi_k - \frac{k_o}{K} - \frac{h_o}{H}\frac{1-\alpha}{\alpha\underline{a}}A(\psi_k) - \frac{b_o}{A(\psi_k)K}\frac{1-\alpha}{\underline{a}}\left[A(\psi_k) + \underline{a}\frac{\alpha}{1-\alpha}\right] \\ &= \phi\psi_k - \frac{h_o}{H}\frac{1-\alpha}{\alpha\underline{a}}A(\psi_k) - \frac{k_o}{K} - \frac{b_o}{K}\frac{1-\alpha}{\underline{a}} - \frac{b_o}{A(\psi_k)K}\alpha, \end{aligned}$$

which gives a quadratic equation on ψ_k:

$$
\begin{aligned}
0 &= \left[\phi - \frac{h_o}{H}\frac{1-\alpha}{\alpha\underline{a}}[a-\underline{a}]\right] A(\psi_k)\psi_k - \left[\underline{a} + \frac{k_o}{K} + \frac{b_o}{K}\frac{1-\alpha}{\underline{a}}\right] A(\psi_k) - \frac{b_o}{K}\alpha \\
&= \left[\phi - \frac{h_o}{H}\frac{1-\alpha}{\alpha\underline{a}}[a-\underline{a}]\right] A(\psi_k)\psi_k - \left[\underline{a} + \frac{k_o}{K} + \frac{b_o}{K}\frac{1-\alpha}{\underline{a}}\right] (a-\underline{a})\psi_k - \frac{b_o}{K} - \left[\underline{a} + \frac{k_o}{K}\right] \underline{a}
\end{aligned}
$$

Collateral constraint at the boundary $k = K, h = 0$ can be also written as

$$\phi q_k K = q_k K + b = n \implies \eta = \phi\nu.$$

C Dynamic model

C.1 Notation

$$\psi_{ht} = \frac{h_t}{h_t + \underline{h}_t} \quad \text{and} \quad \psi_{kt} = \frac{k_t}{k_t + \underline{k}_t}$$

and

$$\nu_t = \frac{q_{kt}K}{q_{kt}K + q_{ht}H}$$

Net worth

$$\underline{n}_t = q_{ht}\underline{h}_t + q_{kt}\underline{k}_t + \underline{b}_t \quad \text{and} \quad n_t = q_{ht}h_t + q_{kt}k_t + b_t,$$

Portfolio shares

$$\underline{\theta}_{ht} = \frac{q_{ht}\underline{h}_t}{\underline{n}_t} \quad \text{and} \quad \underline{\theta}_{kt} = \frac{q_{kt}\underline{k}_t}{\underline{n}_t}.$$

Saver wealth share

$$\eta_t = \frac{n_t}{n_t + \underline{n}_t}.$$

C.2 Asset market clearing

$$
\begin{aligned}
\underline{\theta}_{ht} &= \frac{1-\psi_{ht}}{1-\eta_t}(1-\nu_t) \\
\underline{\theta}_{kt} &= \frac{1-\psi_{kt}}{1-\eta_t}\nu_t \\
\theta_{ht} &= \frac{\psi_{ht}}{\eta_t}(1-\nu_t) \\
\theta_{kt} &= \frac{\psi_{kt}}{\eta_t}\nu_t.
\end{aligned}
$$

LTV constraint:
$$\frac{\psi_{ht}}{\eta_t}(1-\nu_t)+\frac{\phi_k}{\phi_h}\frac{\psi_{kt}}{\eta_t}\nu_t \le \frac{1}{\phi_h}.$$
The boundary of the binding set is characterized by $\eta = \phi_k \nu$.

C.3 Hamilton-Jacobi-Bellman equation

The Hamilton-Jacobi-Bellman equation associated with this problem is given by

$$\begin{aligned}
&\rho V(n;\mathbf{S}) \\
&= \max_{c,s,\theta_k,\theta_h} u(c,s) + \nabla_{\mathbf{S}} V \cdot d\mathbf{S} \\
&\quad + \frac{\partial V}{\partial n} n \left[-\frac{c}{n} - \frac{r_h s}{n} + r_f + \theta_k \left(\frac{a}{q_k} + \mu_k^q - r_f \right) + \theta_{ht} \left(\frac{r_h}{q_k} + \mu_h^q - r_f \right) \right] \\
&\text{s.t.} \quad \theta_k + \frac{\theta_h}{\phi} \le \frac{1}{\phi}, \quad \theta_k \ge 0, \quad \theta_h \ge 0.
\end{aligned} \tag{C.1}$$

where $\nabla_{\mathbf{S}} V$ denotes the gradient of V with respect to $\mathbf{S}$.

C.4 Proofs

Denote the multiplier on the collateral constraint by λ^{LTV} and the multipliers on the nonnegativity constraints by λ^k and λ^h, respectively. I suppress the aggregate state variable from the value function and denote $V'(n) := \frac{\partial V}{\partial n}$, The FOCs are

$$\begin{aligned}
\frac{a}{q_k} + \mu_k^q &= r^f + \frac{\lambda^{LTV} - \lambda^k}{V'(n)n}, \\
\frac{r_h}{q_h} + \mu_h^q &= r^f + \frac{\lambda^{LTV}/\phi - \lambda^h}{V'(n)n}, \\
u_c &= V'(n), \\
u_s &= V'(n) r_h.
\end{aligned}$$

It immediately follows that
$$\frac{u_s}{u_c} = r_h.$$
For the utility specification $u(c^\alpha s^{1-\alpha})$ this gives
$$\frac{1-\alpha}{\alpha} c = r_h s \implies r_h = A(\psi_k) \frac{K}{H} \frac{1-\alpha}{\alpha}, \tag{C.2}$$
where I used the goods market clearing $C = A(\psi_k)K$.

Moreover, by the goods market clearing we also have

$$C = A(\psi_k)K \implies \frac{C}{N} = \frac{A(\psi_k)}{q_k} \frac{q_k K}{N},$$

which gives

$$q_k = \frac{A(\psi_k)}{C/N}\nu \implies q_h = \frac{A(\psi_k)}{C/N}\frac{K}{H}(1-\nu). \tag{C.3}$$

This implies that the rental yield is given by

$$\frac{r_h}{q_h} = \frac{1-\alpha}{1-\nu}\frac{1}{\alpha}\left[\frac{c_t}{n_t}\eta_t + \frac{\underline{c}_t}{\underline{n}_t}(1-\eta_t)\right].$$

Now

$$\mu_k^q = \frac{\dot{q}_k}{q_k} = \frac{1}{q_k}\left[\frac{d}{dt}\left[\frac{A(\psi_k)}{C/N}\right]\nu + \frac{A(\psi_k)}{C/N}\dot{\nu}\right]$$

and

$$\mu_h^q = \frac{\dot{q}_k}{q_k} = \frac{1}{q_h}\left[\frac{d}{dt}\left[\frac{A(\psi_k)}{C/N}\right]\frac{K}{H}(1-\nu) - \frac{A(\psi_k)}{C/N}\frac{K}{H}\dot{\nu}\right],$$

which implies

$$\mu_k^q - \mu_h^q = \frac{\dot{\nu}}{\nu} + \frac{\dot{\nu}}{1-\nu} = \frac{\dot{\nu}}{\nu(1-\nu)}$$

at points of differentiability.

The dynamics for ν in Theorem 4.2 follow. Moreover, in the unconstrained region

$$\begin{aligned}\frac{r_h}{q_h} - \frac{a}{q_k} &= \left[\frac{1/\alpha - 1}{1-\nu} - \frac{1}{\nu}\right]\left[\frac{c_t}{n_t}\eta_t + \frac{\underline{c}_t}{\underline{n}_t}(1-\eta_t)\right] \\ &= \frac{\frac{\nu}{\alpha}-1}{\nu(1-\nu)}\left[\frac{c_t}{n_t}\eta_t + \frac{\underline{c}_t}{\underline{n}_t}(1-\eta_t)\right]\end{aligned}$$

and in the constrained region

$$\begin{aligned}\frac{r_h}{q_h} - \frac{\underline{a}}{q_k} &= \left[\frac{1/\alpha - 1}{1-\nu} - \frac{\underline{a}}{A(\psi_k)\nu}\right]\left[\frac{c_t}{n_t}\eta_t + \frac{\underline{c}_t}{\underline{n}_t}(1-\eta_t)\right] \\ &= \left[\frac{1/\alpha - 1}{1-\nu} - \frac{1 - \frac{a-\underline{a}}{A(\psi_k)}\psi_k}{\nu}\right]\left[\frac{c_t}{n_t}\eta_t + \frac{\underline{c}_t}{\underline{n}_t}(1-\eta_t)\right] \\ &= \left[\frac{\nu/\alpha - 1 + \frac{a-\underline{a}}{A(\psi_k)}\psi_k(1-\nu)}{\nu(1-\nu)}\right]\left[\frac{c_t}{n_t}\eta_t + \frac{\underline{c}_t}{\underline{n}_t}(1-\eta_t)\right].\end{aligned}$$

It follows that necessarily $\nu < \alpha$ along the saddle path in the constrained region, if $\nu_t \to \alpha$ as $t \to \infty$.

Moreover,

$$dn_t = -c_t - r_{ht}s_t + r_{ft}n_t + \frac{\lambda^{LTV}}{\phi}n_t$$

and similarly for $d\underline{n}_t$. It follows that

$$\frac{d(n+\underline{n})}{n+\underline{n}} = \eta\frac{dn}{n} + (1-\eta)\frac{d\underline{n}}{\underline{n}}\,dt = \eta\left[r_{ft} - \frac{c_t}{n_t} - \frac{r_{ht}s_t}{n_t} + \frac{\lambda^{LTV}}{\phi V'(n)n}\right] + (1-\eta)\left[r_{ft} - \frac{\underline{c}_t}{\underline{n}_t} - \frac{r_{ht}\underline{s}_t}{\underline{n}_t}\right].$$

We obtain

$$\frac{d\eta}{\eta} = \frac{d\left(\frac{n}{n+\underline{n}}\right)}{\frac{n}{n+\underline{n}}} = r_{ft} - \frac{c_t}{n_t} - \frac{r_{ht}s_t}{n_t} + \frac{\tau}{\phi} - \frac{d(n+\underline{n})}{n+\underline{n}}$$
$$= (1-\eta)\left[\frac{\underline{c}_t}{\underline{n}_t} - \frac{c_t}{n_t} + \frac{\tau}{\phi}\right] + (1-\eta)\left[\frac{r_{ht}\underline{s}_t}{\underline{n}_t} - \frac{r_{ht}s_t}{n_t}\right].$$

C.5 Log utility

Guess and verify

$$V(n;\boldsymbol{S}) = \frac{1}{\rho}\log\left[\omega(\boldsymbol{S})n\right].$$

which implies

$$V'(n) = \frac{1}{\rho n} \implies c = \alpha\rho n, \quad r_h s = (1-\alpha)\rho n.$$

This implies that

$$\dot{\eta} = \eta(1-\eta)\left[\underline{\rho} - \rho + \frac{\tau}{\phi}\right],$$

which fully determines the model dynamics in the unconstrained region $\eta > \phi_k\nu$.

C.5.1 The constrained region: $\eta < \phi_k\nu$.

Now by the collateral constraint

$$\psi_k = \frac{\eta}{\phi_k\nu}$$

and by the FOCs

$$\tau = \frac{a-\underline{a}}{q_k}.$$

These together fully determine the dynamics in the constrained region.

D Modeling details for the Quantitative Analysis of Section 6

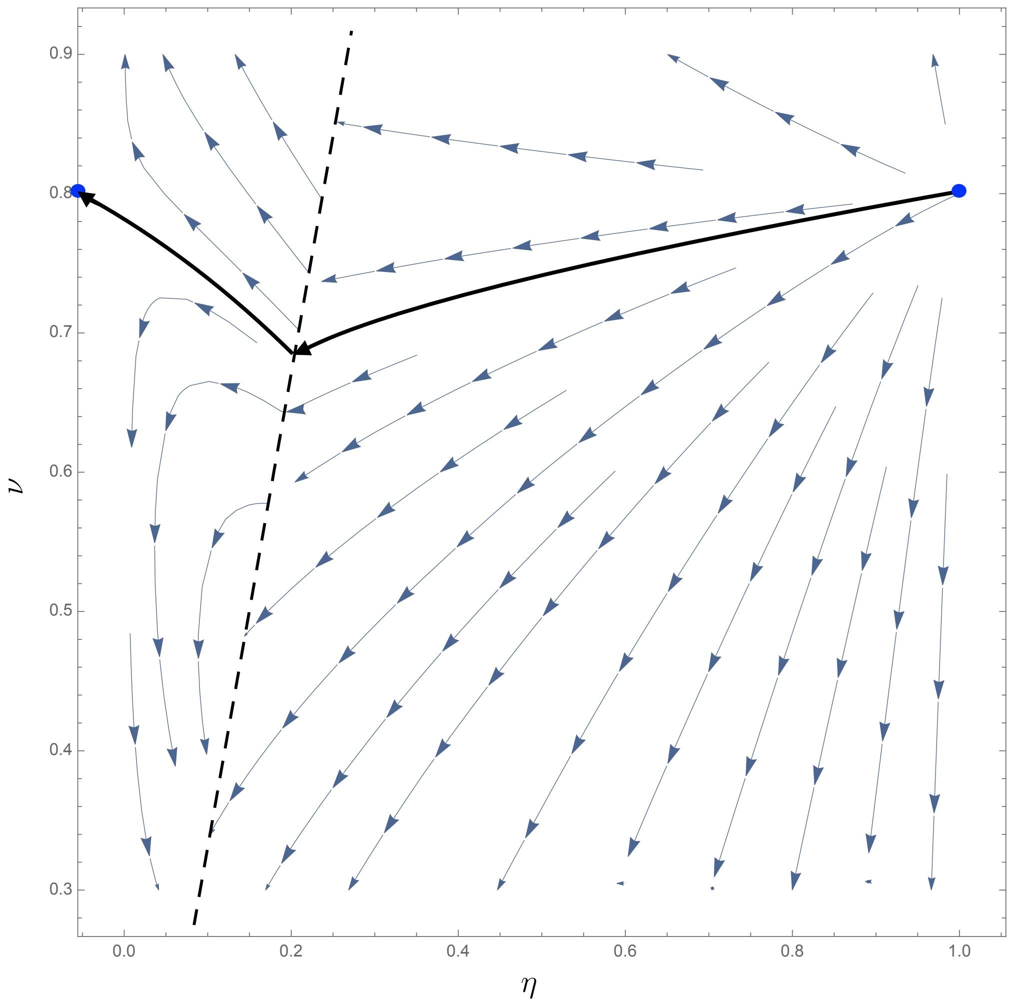
By the goods market clearing we have

$$C = A(\psi_k)K - i_kK = \frac{A(\psi_k)}{q_k}q_kK - \frac{q_k-1}{\kappa_k}K = \frac{A(\psi_k)}{q_k}q_kK + \frac{q_kK}{\kappa_kq_k} - \frac{q_kK}{\kappa_k}.$$

This implies which gives

$$q_k = \frac{1+\kappa_kA(\psi_k)}{\nu+\kappa_kC/N}\nu \tag{D.1}$$

Figure 9: Phase diagram



The picture depicts the (η, ν) phase diagram for the case (a) of Proposition 5.5. The solid black line shows the saddle path, which connects the non-stable steady-state at $(\eta, \nu) = (1, \alpha)$ to the dynamically stable one at $(\eta, \nu) = (0, \alpha)$. The steady-states are shown in blue dots. The dashed line separates the two regions $\{\eta < \phi\nu\}$ and $\{\eta \geq \phi\nu\}$.

as well as

$$q_h = \frac{1+\kappa_k A(\psi_k)}{\nu+\kappa_k C/N}\frac{K}{H}(1-\nu) \quad \text{and} \quad \frac{q_h}{r_h} = \frac{1+\kappa_h}{1-\nu+\kappa_h \frac{r_h S}{N}}(1-\nu).$$

The investment rates are given by

$$i_k = \frac{A(\psi_k)\nu - C/N}{\nu+\kappa_k C/N} \quad \text{and} \quad i_h = \frac{1-\nu-\frac{r_h S}{N}}{1-\nu+\kappa_h \frac{r_h S}{N}}.$$

Denote the multiplier on the collateral constraint by λ^{LTV} and the multipliers on the nonnegativity constraints by λ^k and λ^h, respectively. I suppress the state variable from the value function and denote $V'(n) := \frac{\partial V}{\partial n}$, The FOCs are

$$\begin{aligned}
\frac{a-i_k}{q_k} + g_k + \mu_k^q &= r^f + \frac{\lambda^{LTV}-\lambda^k}{V'(n)n}, \\
\frac{1-i_h}{q_h/r_h} + g_h + \mu_h^q &= r^f + \frac{\lambda^{LTV}/\phi-\lambda^h}{V'(n)n}, \\
u_c &= V'(n), \\
u_s &= V'(n)r_h,
\end{aligned}$$

where

$$g_k := \Phi_k(i_k) - \delta_k \quad \text{and} \quad g_h = \Phi_h(i_h) - \delta_h.$$

It immediately follows that

$$\frac{u_s}{u_c} = r_h.$$

For the utility specification $u(c^\alpha s^{1-\alpha})$ this gives

$$\frac{1-\alpha}{\alpha}c = r_h s. \tag{D.2}$$

By (D.2) we also have

$$\begin{aligned}
r_h = \frac{A(\psi_k)-i_k}{1-i_h}\frac{K}{H}\frac{1-\alpha}{\alpha} &= \frac{\frac{\left[1+\kappa_k A(\psi_k)\right]C/N}{\nu+\kappa_k C/N}}{\frac{[1+\kappa_h]\frac{r_h S}{N}}{1-\nu+\kappa_h\frac{r_h S}{N}}}\frac{K}{H}\frac{1-\alpha}{\alpha} \\
&= \frac{\left[1+\kappa_k A(\psi_k)\right]C/N}{[1+\kappa_h]\frac{r_h S}{N}}\frac{1-\nu+\kappa_h\frac{r_h S}{N}}{\nu+\kappa_k C/N}\frac{K}{H}\frac{1-\alpha}{\alpha} \\
&= \frac{1+\kappa_k A(\psi_k)}{1+\kappa_h}\frac{1-\nu+\kappa_h\frac{r_h S}{N}}{\nu+\kappa_k C/N}\frac{K}{H},
\end{aligned}$$