

TTT4250 - Acoustical Measurement Techniques

Laboratory Exercise 3

Underwater Source Directivity

performed by

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Report by

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Summary

In this report the frequency response, beam pattern, directivity and directivity index of a transducer, used as a transmitter, is examined. The measurements were conducted in a fresh water tank. The frequency response were measured, at three distinct angles, and the beam pattern were measured, at three distinct frequencies. The transducer has a working frequency range of 8 kHz-16 kHz and was found to be most directional at 16 kHz, with a clear main lobe around 0°. These measurements are useful in order to check that the given specifications from the manufacturer are correct and at which frequency and incident angle, between the object and the transducer, the transducer is most directional. This has applications with for instance active sonar and echo sounding, which is a specialized form of sonar, and can also be useful in order to avoid noise pollution during long-term measurements underwater.

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1 Introduction

This report looks at the directivity of an underwater sound source. A transducer is used as a transmitter and produced the sound, which is then recorded by a hydrophone. From this the frequency response and the beam pattern of the transducer will be calculated. It will be examined at which angles and frequencies the source is most directiv. This is useful, in order to check that the given spesifications of the transducer coincide with the measured characteristics. This can be applied to for instance active sonars, which uses a directiv beam to determine distances to different objects underwater. These measurements are therefore useful, as they provide the frequency and the incident angle, between the object and the trasducer for which the transducer is most directional. For more extensive experiments and measurement, it may also be useful to know the directivity and reduce unwanted noise in other directions than the one of interest, in order to not cause harm to the surrounding environment.

2 Theory

Transducers are reciprocal and can be used as both transmitters and receivers . They are good transmitters because they can be modulated and can be constructed for both high and low frequencies.

When a piezoelectric transducer is used as a sound transmitter the sensitivity is a function of frequency and is defined as a ratio of the driving electric voltage to the sound pressure at a distance r in the far-field of the transducer. It is mathematically defined as

$$S_v = \left(\frac{P}{V} \right) \quad (2.1)$$

The acoustic pressure, sound pressure, created by the transmitter propagates in the free space and forms an intensity field in front of the transducer. For a transducer with a radiating surface with a circular geometry the shape of the waves forms a flashlight shaped beam. In order to achieve a strong focus, a larger aperture is required. Since the narrowness of the beam is a function of the ratio of the diameter of the radiating surface and the wavelength of the acoustic waves at the operating frequency. This means that the larger the diameter is, compared to the wavelength, the narrower the sound beam will become, resulting in better resolution. Wavelength is defined as

$$\lambda = \frac{v}{f}, \quad (2.2)$$

where f is the frequency of the wave and v is the speed of sound in the medium.

The directivity, or beam, pattern, which is unique to the transducer, is an important far-field characteristic. It consists of a main lobe and side lobes, as shown in Figure 2.1.

The beam pattern is a relative and dimensionless parameter that is a function of the spatial angle. The spatial angle is determined by many factors, including frequency of operation, size, shape and acoustic phase characteristics of the vibrating surface. For a plane circular piston the normalized directivity pattern can be determined analytically, for this the reader is advised to check [1]. For a cylindrical or spherical transducer no such simple expression exists and the directivity pattern must be measured.

From this measured normalized directivity pattern, $B(\theta)$, it is possible to determine the directivity D , which is defined as

$$D = \frac{4\pi}{\int_0^{2\pi} \int_0^{\pi/2} B(\theta)^2 \sin(\theta) d\theta d\phi} = \frac{2}{\int_0^{\pi/2} B(\theta)^2 \sin(\theta) d\theta} \quad (2.3)$$

From this it is now possible to determine the directivity index, DI, which is defined as

$$DI = 10 \log_{10}(D) \quad (2.4)$$

Furthermore, the Rayleigh distance needs to be calculated. The Rayleigh distance marks the transition between the near- and far-field. It is the range at which the main lobe becomes as wide as the aperture of the transducer.

The Rayleigh distance is defined as

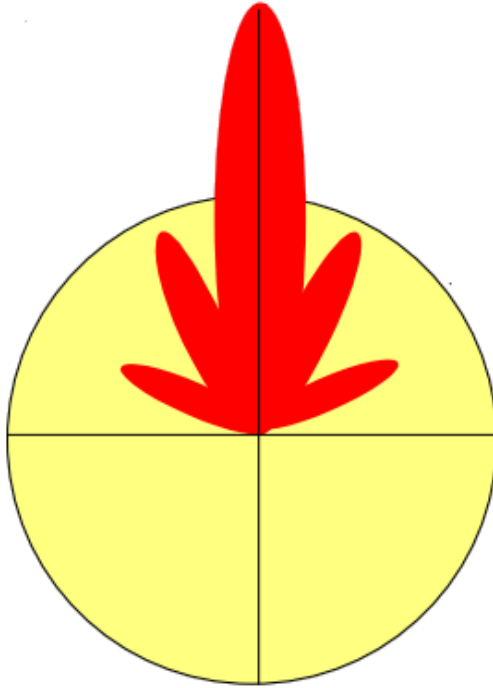


Figure 2.1: Example of how a beam pattern can look. We see the main lobe, which is the largest lobe, and the side lobes to each side of the main lobe.

$$R_R = \frac{L^2}{\lambda} (\text{line or rectangle}) \quad (2.5)$$

$$R_R = \frac{\pi a^2}{\lambda} (\text{circular disk}) \quad (2.6)$$

3 Method and Equipment

3.1 Set-up

The transducer used for this report is the SonoTube 008/D13-UW from Chelsea Technologies Group (CTG). It has a working frequency range of 8-16 kHz and is supposedly omnidirectional at 10 kHz. The hydrophone is a B&K 8105, which is omnidirectional and has a wider frequency band than the transmitting transducer. The tank in which the measurements were conducted in had the dimensions shown in Table 3.1.

Table 3.1: Measurements of the tank where the transducer and hydrophone was placed and the measurements were conducted in.

Parameter	Measurement [m]
Depth of water	1.6
Length of tank	3
Width of tank	2

The transducer and hydrophone was submerged a minimum of 30 minutes before the measurements began, as recommended in [3]. The sound is generated from an Arbitrary Waveform Generator (AWG) which can produce many different signal types for many different frequencies. The signal goes from this generator to a power amplifier that drives the underwater transducer, in this case a 9 V battery.

The transducer is placed in a rigid frame so that it is easy to change the placement in the tank, both in the vertical direction, the horizontal direction and the depth. It is also made easy to change the azimuthal angle at which the transducer is pointing.

The transmitted signal makes a acoustic wave in the water that is recorded by the hydrophone. The hydrophone is connected to an amplifier, where the output goes into an oscilloscope to read out the data. The oscilloscope also receives an electronic signal directly from AWG, so that it is possible to see both the transmitted signal and the received signal.

The pulse should be a minimum of three cycles long, in order to make sure the transducer has reached equilibrium with the driving pulse. If the signal is much longer the reflections from the short paths will arrive at the hydrophone and mix together with the direct path and it will be difficult to separate these.

The setup of the transducer and hydrophone in the tank is shown in Figure 3.1. The transducer was placed at x_2 , with the distance from x_2 to x_1 is 1.33 metre, and the distance between the hydrophone, x , and the transducer, x_2 , was 0.69 metre. The hydrophone was placed at x_3 and the distance from x_3 to x_4 was 1 metre. Both the hydrophone was placed in the center of the width-axis of the tank, 1 meter from the long walls. They were also 0.81 meters deep in the tank, which equals half of the height of the water in the tank.

It is also important to make sure we are in the far-field, and the Rayleigh distance was calculated to be

$$R_R = \begin{cases} 0.100\text{m} & \text{for } f = 8 \text{ kHz} \\ 0.050\text{m} & \text{for } f = 16 \text{ kHz} \end{cases} \quad (3.1)$$

with $a = 0.054$ m, as the diameter of the transducer is given as $d = 0.108$ m in [3]. This means that for the selected positions in the tank we are in the far-field, as desired.

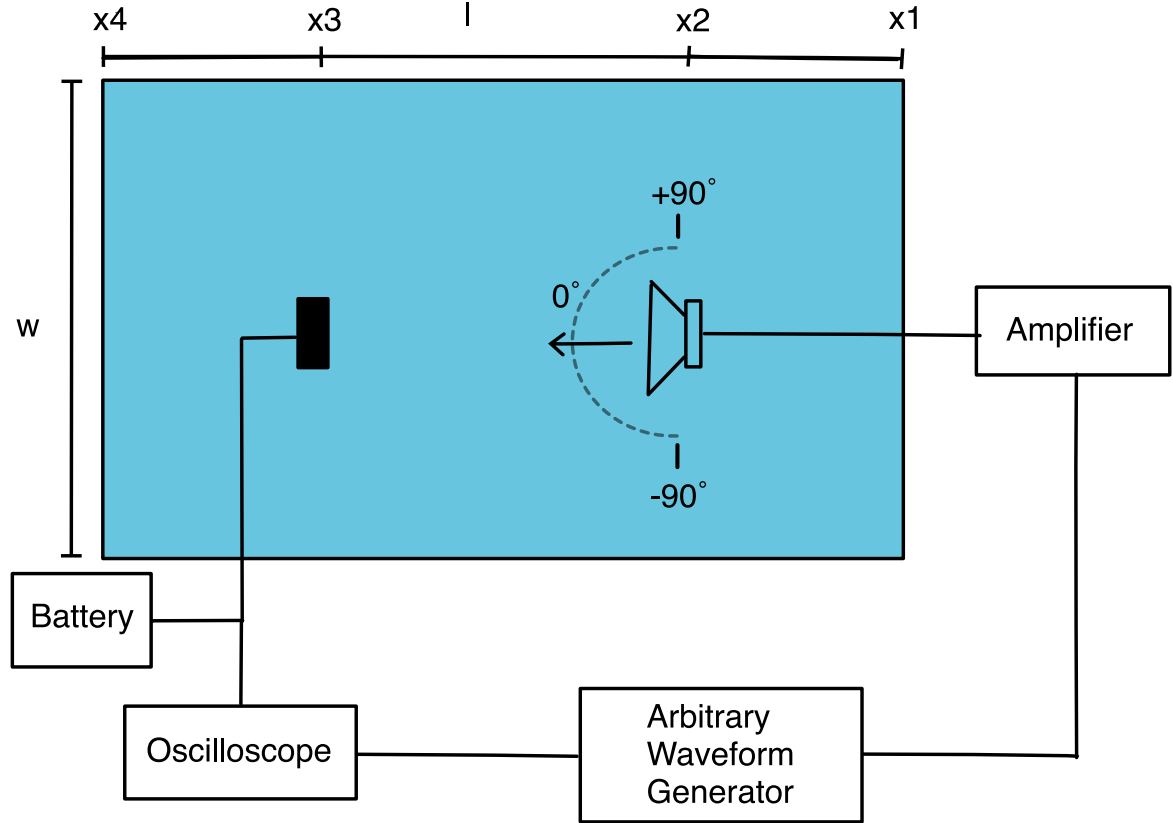


Figure 3.1: Setup of the measurement system. The width $w = 2\text{m}$, the length $l = 3\text{m}$ and the depth, not depicted, $d = 0.81\text{m}$. The hydrophone is the black box to the left and the transducer is to the right, with indications of where the positive/negative angles are. Furthermore, the battery, the oscilloscope, the AWG and the amplifier is shown, and the connections between them.

3.1.1 Part 1: Measuring the frequency at three distinct angles

The signal generator was selected to make a signal with an amplitude of 600 mV , 0 V in offset and the signal was three cycles long. The sweep was played, and recorded by the hydrophone and this was repeated from 8 kHz to 16 kHz , increasing the frequency with 1 kHz in between each measurement. The measurement data was read off the oscilloscope and saved to a file. This was first done for three different angles of the transducer relative to the hydrophone, $\theta_1 = 0^\circ$, then for $\theta_2 = 45^\circ$ and finally for $\theta_3 = 90^\circ$. Here an angle of 0° means that the transducer is pointed straight at the hydrophone, and a positive angle is in the clockwise direction (as seen when looking from the back of the transducer to the hydrophone), and a negative angle is in the counterclockwise direction.

For each measurement the peak-to-peak values of the received waves at the hydrophone was noted, for the two first waves received. The average of these two were calculated. The raw data can be seen in Appendix D.

3.1.2 Part 2: Measuring the beam pattern and directivity at three distinct frequencies

The signal generator had the same setup as for Part 1, but now the measurements were conducted for three specific frequencies, $f_1 = 8$ kHz, $f_2 = 12$ kHz and $f_3 = 16$ kHz. The measurements were conducted in the interval $\theta = [-90, 90]$ with an increase of 15° for each measurement. The measurements started at 90° and proceeded counterclockwise by 15° increments, until the last measurement at -90° .

For each measurement the peak-to-peak values of the received waves at the hydrophone was noted, for the two first waves received. The average of these two were calculated. The raw data can be seen in Appendix D.

3.1.3 Equipment

Table 3.2: Equipment list.

Equipment	Model number/type
Oscilloscope	Tektronix TDS 3012B Two channel color digital Phosphor Oscilloscope
Waveform generator	Keysight 33500B Arbitrary Waveform Generator
Transmission power amplifier	Made in-house
Hydrophone amplifier/filter	Brüel & Kjær Conditioning Amplifier Type 2626
Acoustic transducer	CTG-SonoTube008/D13-UW broadband transducer: 8-16 kHz
Hydrophone	Brüel & Kjær 8104 ITC 8095

4 Results

The frequency response for part 1 of the lab is shown in Figure 4.1. The plot is in Cartesian coordinates.

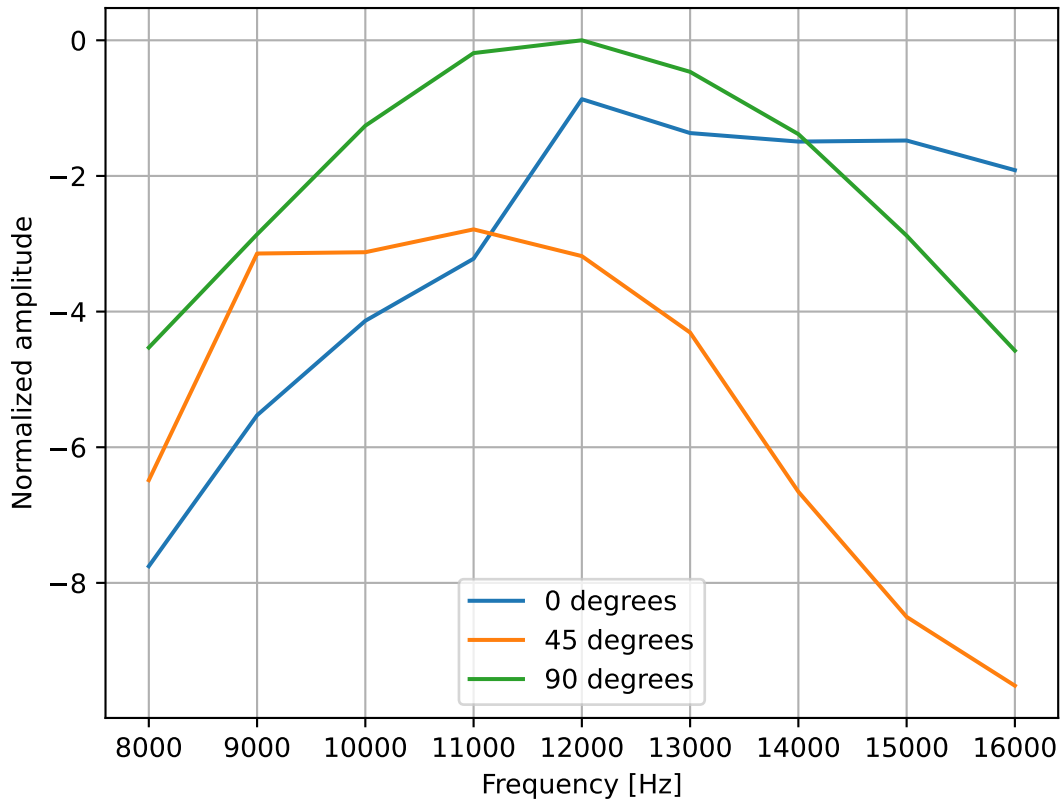


Figure 4.1: Frequency response for part 1 of the measurements, where the measurement started at 8 kHz and ended at 16 kHz, with increments of 1 kHz, for three different angles, $\theta = \{0^\circ, 45^\circ, 90^\circ\}$.

The graphs for the different angles have only discrete values for each frequency, $f = [8000, 16000]$, with an increase of 1 kHz between each measurement. This means the measured values are at 8000 Hz, 9000 Hz, 10000 Hz etc. From the plot it is clear that the 90° angle has the highest amplitudes. The values for the amplitudes have been normalized from the largest value of the measurement, which is here seen as it is the value of the 90° -graph that reaches 0 in amplitude. All the amplitudes are relative to this maximum.

The 90° graph has its maximum at 12 kHz, the 45° graph has its maximum at 11 kHz and the 0° graph has its maximum at 12 kHz.

The frequency response of part 2 of the lab is shown, in Cartesian coordinates, in Figure 4.2.

For this plot the amplitude values have been normalized in the same manner as for Figure 4.1.

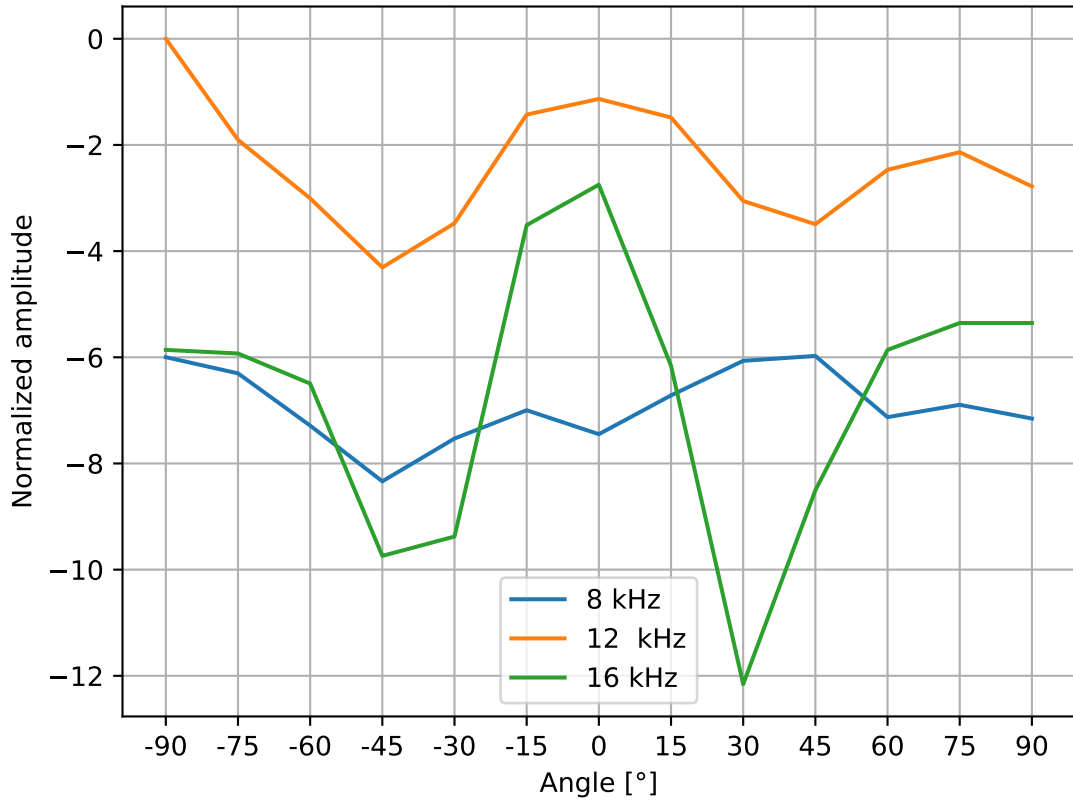


Figure 4.2: Frequency response for the second part of the measurements. The measurements were done for the frequencies $f = \{8 \text{ kHz}, 12 \text{ kHz}, 16 \text{ kHz}\}$. For each frequency it was measured for the angles between $\theta = [-90^\circ, 90^\circ]$, with increments of 15° .

The graphs for the different frequencies only have discrete values for each frequency, $\theta = [-90, 90]$, with an increase of 15° between each measurement. This means the measured values are at $90^\circ, 75^\circ, 60^\circ$ etc. The largest amplitude occurs at -90° and at 45° for 8 kHz, at -90° for 12 kHz and at 0° for 16 kHz.

In Figure 4.3 the beam pattern is plotted. This is a plot of amplitude versus angle, where the angle is marked on the circle, and the distance from the center of the circle indicates amplitude. The plot is in polar coordinates.

The directivity, D , was calculated from 2.3 and presented in Table 4.1. Quadrant 1 is $[-90^\circ, 0^\circ]$ and quadrant 2 is $[0^\circ, 90^\circ]$.

Frequency [Hz]	Directivity D	Quadrant
8000	2.3569	2
8000	9.3549	1
12000	3.6509	2
12000	12.3144	1
16000	4.8021	2
16000	16.0063	1

Table 4.1: Directivity

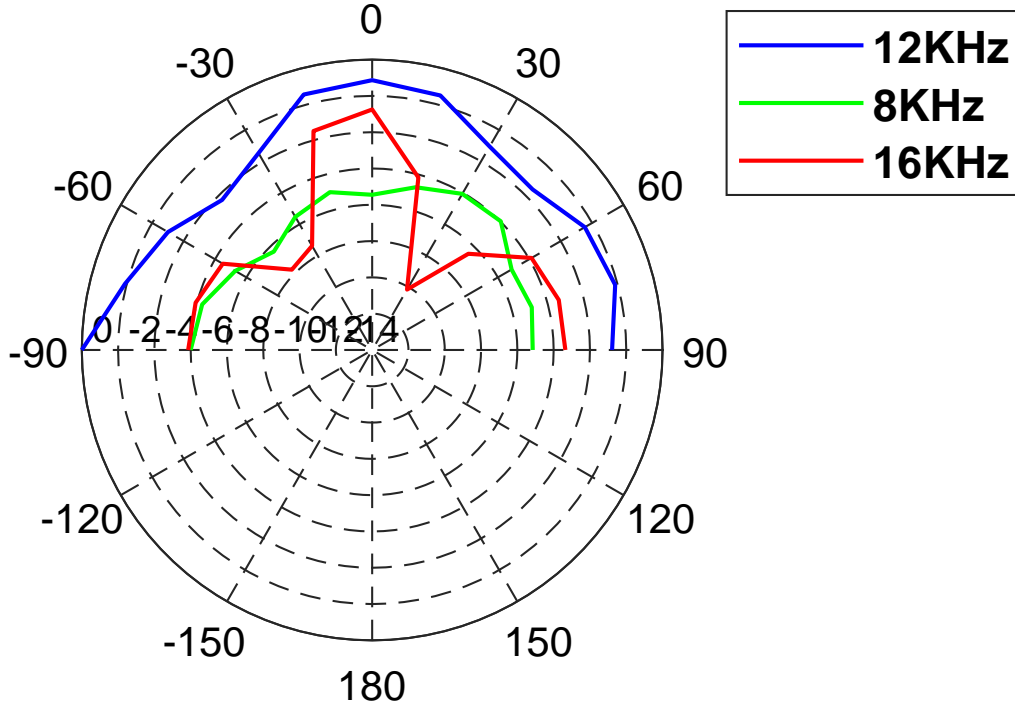


Figure 4.3: Beam pattern for the measurements in the second part, for the three frequencies $f = \{8 \text{ kHz}, 12 \text{ kHz}, 16 \text{ kHz}\}$. The distance from the center of the plot is the amplitude in dB, with the outer circle is 0 dB and the inner most circle is -12 dB.

Furthermore, the directivity index, DI, was calculated from 2.3, and presented in Table 4.2.

Frequency [Hz]	Directivity D	Quadrant
8000	9.3549	1
12000	12.3144	1
16000	16.0063	1
8000	8.5735	2
12000	12.9497	2
16000	15.6905	2

Table 4.2: Directivity Index, DI

5 Discussion

The directivity index is largest for 16 kHz, second largest for 12 kHz and smallest for 8 kHz. Given Figure 4.3, it seems like the results are reasonable. We see that for 16 kHz, there is a obvious mainlobe and smaller sidelobes. For 12 kHz, the main lobe is not very clear, but we get high amplitudes all around. For 8 kHz, it seems the largest amplitudes occur for 45° and -90° and there is no observable main lobe around 0° . All the values are also rather small and the amplitude does not have large variations, so it makes sense that the directivity index is smaller.

The fact that the 16 kHz-measurement has the highest directivity index is as expected. As mentioned in Chapter 2 the larger the diameter of the transducer is compared to the wavelength of the acoustic waves, the narrower the beam. 16 kHz is the largest frequency we operate with, and so it also has the smallest wavelength, as shown in Equation 2.2. Here $f = 16000$ Hz, $v = 1464$ m/s (for fresh water at $T = 288.15$ K). This gives the wavelength $\lambda = 0.0915$ m. The outer diameter of the transducer is 140 mm [2], and the actual diameter is 108 mm according to [3]. For the lowest frequency, $f = 8000$, the wavelength $\lambda = 0.183$ m. So this is according to theory.

Furthermore we can observe that for 12 kHz the beam pattern seems more or less equally strong for the different angles, and the transducer is supposedly omnidirectional for $10 \text{ kHz} \pm 2 \text{ dB}$ [2]. We observe the same for 8 kHz, that it is more omnidirectional, but with a spike at 45° . It is hard to determine why this is, but it could be due human error, in setting up the system both with regards to the angle of the transducer and the distance from the wall. The transducer was, as described, placed on a rigid frame with wheels that could slide along the length of the tank. The transducer was returned to the edge of the tank in order to manually change the angle. It is possible that the angles were not precise and that the transducer was not returned to the exact same position, but the position was marked with tape so it should be approximately the same position for each measurement. However, these effects on the results should be negligible.

The peak-to-peak values for the amplitudes were read manually off the oscilloscope. Also here there is some room for human error, in regards to how accurately the measurements were read off, and also whether the same top and bottoms of the waves were recorded for all the measurements. It was attempted to use the first and second wave, but it was not always so clear which of the received waves were first and second, and usually there was a bit of noise in the beginning, which may or may not have been the first wave. Still, the measurements were done within the three cycles that were transmitted and this should not affect the results much.

6 Conclusions

Two different measurements were conducted for this report, the frequency response of three distinct angles and the beam pattern for three distinct frequencies. It was conducted in a fresh water tank, where the transducer and hydrophone were carefully placed in order to ensure good measurements. The transducer was used as a transmitter and sent out a three cycle sine wave, that was recorded by a hydrophone. The peak-to-peak amplitude was noted and normalized before plotting.

The results seem reasonable, both with regards to the plots of the measured data, the specifications of the transducer and theory about transducers. The 16 kHz-measurement for the second part of the lab is the most directional with the clearest main lobe, as expected.

Bibliography

- [1] TTT4250 Acoustical Measurement Techniques Laboratory Exercise Compendium, Chapter 4
- [2] Chelsea Technologies Group Ltd, SonoTube 008/D13-DT Communications Transducers
- [3] Dong, H., Lab 3: Underwater transducer frequency response and beam pattern measurement.

A Appendix: Python script for averaging the data and plotting the frequency response

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4
5 filename = "method1.csv"
6 filename2 = "method2.csv"
7
8
9 def read_csv(filename, usecols): # np arange creates array from 1
    to 18, skipping the first row
10     return np.genfromtxt(filename, comments='*', delimiter=';',
        skip_header=1, usecols=usecols, dtype=
            float)
11
12
13 def rad_to_angle(arg):
14     return arg * 180 / np.pi
15
16 #
17
18 #####
19 # Method 1
20 #####
21
22 data = read_csv(filename, usecols=np.arange(1, 18 + 1))
23 data_resaped = data.reshape(data.shape[0], data.shape[1] // 2, 2)
24
25
26 twoer_average_data = np.average(data_resaped, axis=2)
27 twoer_average_data_transp = twoer_average_data.transpose()
28 twoer_max = np. max(twoer_average_data_transp)
29 twoer_decibel = 20 * np.log10(twoer_average_data_transp / twoer_max
    )
30
31
32 freq_axis = np.linspace(8000, 16000, 9).repeat(3).reshape(9, 3)
33 plt.plot(freq_axis, twoer_decibel, label=["0 degrees", "45 degrees"
    , "90 degrees"])
34 plt.legend(loc="lower center")
35 plt.grid(True)
```



```

36 plt.savefig("met1_freq_resp.pdf")
37 plt.show()
38
39 #####
40 # Method 2
41 #####
42
43 data_met2 = read_csv(filename2, usecols=np.arange(1, 26 + 1))
44 data_resaped_met2 = data_met2.reshape(data_met2.shape[0],
    data_met2.shape[1] // 2, 2)
45 twoer_average_data_met2 = np.average(data_resaped_met2, axis=2)
46
47
48 twoer_average_data_met2_transp = twoer_average_data_met2.transpose
    ()
49 twoer_max_met2 = np. max(twoer_average_data_met2_transp)
50 twoer_decibel_met2 = 20 * np.log10(twoer_average_data_met2_transp /
    twoer_max_met2)
51
52 angle_axis = np.linspace(-90, 90, 13)
53 angles = [-90, -75, -60, -45, -30, -15, 0, 15, 30, 45, 60, 75, 90]
54 angles = np.array(angles)
55 angle_rad = angles / (np.pi * 180)
56 x_ticks_angles = ["-90", "-75", "-60", "-45", "-30", "-15", "0", "
    15", "30", "45", "60", "75", "90"]
57
58
59 fig, ax = plt.subplots()
60 plt.plot(angle_axis, twoer_decibel_met2, label=["8 kHz", "12 kHz",
    "16 kHz"])
61 plt.legend(loc="lower center")
62 ax.set_xticks(angles)
63 ax.set_xticklabels(x_ticks_angles)
64 plt.grid(True)
65 plt.savefig("met2_freq_resp.pdf")
66 plt.show()

```

B Appendix: Matlab script for plotting of the beam pattern

```
1 data_met2_1 =[-5.99752981 -6.30232914 -7.28668959 -8.33650706
   -7.53061516 -6.99757126 -7.44854315 -6.71776732 -6.06692477
   -5.97452082 -7.12779694 -6.89477847 -7.15407801];
2 data_met2_2 = [0. -1.90361804 -3.00612417 -4.30704725 -3.47574608
   -1.42794324 -1.1334781 -1.48257165 -3.05524337 -3.49299711
   -2.46802819 -2.13681029 -2.78050396];
3 data_met2_3= [ -5.86038133 -5.92868488 -6.49535685 -9.74103512
   -9.37653872 -3.51028247 -2.74874564 -6.16032233 -12.15747853
   -8.50398777 -5.86038133 -5.35383614 -5.35383614];
4
5 theta_met2 = [-90,-75,-60,-45,-30,-15,0,15,30,45,60,75,90]
6
7 figure(11)
8 subplot(1,2,1)
9 second = polardb(theta_met2,data_met2_2, -15, 2, -b)
10 hold on
11 first = polardb(theta_met2, data_met2_1, -15, 2,-g)
12 third = polardb(theta_met2,data_met2_3, -15, 2, -r)
13 hold off
14 legend(, , , , ,12KHz, 8KHz, 16KHz, location, best);
15 set(gca,'fontsize',12,'fontweight','bold');
16
17 saveas(gcf,'beampattern.pdf')
```

C Appendix: Matlab script for calculating D and DI

```
1 row1 = [3.77, 3.64, 3.25, 2.88, 3.16, 3.36, 3.19, 3.47, 3.74, 3.78,
2       3.31, 3.4, 3.3];
3 row2 = [7.52, 6.04, 5.32, 4.58, 5.04, 6.38, 6.6, 6.34, 5.29, 5.03,
4       5.66, 5.88, 5.46];
5 row3 = [3.83, 3.8, 3.56, 2.45, 2.555, 5.02, 5.48, 3.7, 1.855,
6       2.825, 3.83, 4.06, 4.06];
7
8 row1_1quad = [3.77, 3.64, 3.25, 2.88, 3.16, 3.36, 3.19];
9 row1_2quad = [3.19, 3.47, 3.74, 3.78, 3.31, 3.4, 3.3];
10 row2_1quad = [7.52, 6.04, 5.32, 4.58, 5.04, 6.38, 6.6];
11 row2_2quad = [6.6, 6.34, 5.29, 5.03, 5.66, 5.88, 5.46];
12 row3_1quad = [3.83, 3.8, 3.56, 2.45, 2.555, 5.02, 5.48];
13 row3_2quad = [5.48, 3.7, 1.855, 2.825, 3.83, 4.06, 4.06];
14
15 row11_scaled = row1_1quad / max(row1);
16 row21_scaled = row2_1quad / max(row2);
17 row31_scaled = row3_1quad / max(row3);
18
19 row12_scaled = row1_2quad / max(row1);
20 row22_scaled = row2_2quad / max(row2);
21 row32_scaled = row3_2quad / max(row3);
22
23 rad_q2 = deg2rad(0:15:90)
24 rad_q1 = flip(rad_q2)
25
26
27 D8_q1 = 2 / (trapz(rad_q2, flip(row11_scaled) .^2 .* abs(sin(rad_q2
28     ))))
29 D12_q1 = 2 / (trapz(rad_q2, flip(row21_scaled) .^2 .* abs(sin(rad_q2
30     ))))
31 D16_q1 = 2 / (trapz(rad_q2, flip(row31_scaled) .^2 .* abs(sin(rad_q2
32     ))))
33
34 D8_q2 = 2 / (trapz(rad_q2, row12_scaled .^2 .* abs(sin(rad_q2))))
35 D12_q2 = 2 / (trapz(rad_q2, row22_scaled .^2 .* abs(sin(rad_q2))))
36 D16_q2 = 2 / (trapz(rad_q2, row32_scaled .^2 .* abs(sin(rad_q2))))
37
```

```
38 | DI_81 = 10*log(D8_q1)
39 | DI_121 = 10*log(D12_q1)
40 | DI_161 = 10*log(D16_q1)
41 |
42 | DI_82 = 10*log(D8_q2)
43 | DI_122 = 10*log(D12_q2)
44 | DI_162 = 10*log(D16_q2)
```

D Appendix: Raw data

Table D.1: Raw data from method 1.

Frequency response	0 degrees	45 degrees	90 degrees
8 kHz	25.	2.86	3.68
	2.84	3.32	4.06
9 kHz	3.04	4.18	4.28
	3.86	4.18	4.28
10 kHz	3.38	4.34	5.28
	4.72	4.92	6.48
11 kHz	3.68	4.34	5.28
	5.32	5.12	7.48
12 kHz	5.5	4.2	5.48
	6.3	4.84	7.56
13 kHz	4.6	3.82	5.44
	6.54	4.12	6.92
14 kHz	5.04	2.84	5.16
	5.94	3.22	5.96
15 kHz	6.08	2.68	4.76
	4.92	2.22	4.6
16 kHz	5.18	2.2	4.38
	5.28	2.16	3.32

Table D.2: Raw data for method 2.

Beam pattern	8 kHz	12 kHz	16 kHz
90 degrees	3.64	7.08	4.32
	3.9	7.96	3.34
75 degrees	3.38	5.16	4.44
	3.9	6.92	3.16
60 degrees	2.94	4.44	4.24
	3.56	6.2	2.88
45 degrees	2.56	3.54	3.2
	3.2	5.62	1.7
30 degrees	3.02	4.2	2.34
	3.3	5.88	2.77
15 degrees	3.26	5.2	3.62
	3.46	7.56	6.42
0 degrees	3.12	5.36	4
	3.26	7.84	6.96
-15 degrees	3.4	6	3.72
	3.54	6.68	3.68
-30 degrees	3.54	4.56	1.96
	3.94	6.02	1.75
-45 degrees	3.4	3.92	3.7
	4.16	6.14	1.95
-60 degrees	3.06	4.7	4.48
	3.56	6.62	3.18
-75 degrees	3.16	5.1	4.4
	3.64	6.66	3.72
-90 degrees	3.02	4.9	4.32
	3.58	6.02	3.8