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DsixTools

The Standard Model Effective Field Theory Toolkit

Alejandro Celis[*,a], Javier Fuentes-Martín[†,b], Avelino Vicente[†,c], Javier Virto[‡,d]

[*] *Ludwig-Maximilians-Universität München, Fakultät für Physik, Arnold Sommerfeld Center for Theoretical Physics, 80333 München, Germany.*

[†] *Instituto de Física Corpuscular, Universitat de València - CSIC, E-46071 València, Spain.*

[‡] *Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, CH-3012 Bern, Switzerland.*

Abstract

We present `DsixTools`, a `Mathematica` package for the handling of the Standard Model Effective Field Theory with operators up to dimension six. Among other features, `DsixTools` allows the user to perform the full one-loop Renormalization Group Evolution of the Wilson coefficients in the Warsaw basis. This is achieved thanks to the `SMEFTrunner` module, which implements the full one-loop anomalous dimension matrix previously derived in the literature. In addition, `DsixTools` also contains modules devoted to the matching to the $\Delta B = \Delta S = 1,2$ and $\Delta B = \Delta C = 1$ operators of the Weak Effective Theory at the electroweak scale, and their QCD and QED Renormalization Group Evolution below the electroweak scale.

[a]Email: Alejandro.Celis@physik.uni-muenchen.de
[b]Email: Javier.Fuentes@ific.uv.es
[c]Email: Avelino.Vicente@ific.uv.es
[d]Email: jvirto@mit.edu

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1 Introduction

The experimental success of the Standard Model (SM) of particle physics and the absence of new physics (NP) signals so far seem to hint at the presence of a mass gap between the SM degrees of freedom and the new dynamics. In this case, departures from the SM at energies much smaller than the new physics scale can be described using effective field theory (EFT) methods. The so-called Standard Model EFT (SMEFT) parametrizes possible deviations from the SM caused by heavy degrees of freedom in a model independent way.

The SMEFT Lagrangian is organized as an expansion in powers of $1/\Lambda$, where Λ represents the new physics scale and compensates the canonical dimension of the effective operators. The leading order of the SMEFT corresponds to the renormalizable SM Lagrangian. Dominant new physics contributions to most processes of interest are expected to be encoded in the Wilson Coefficients (WC) of effective operators of canonical dimension five and six [1].

In recent years, a non-redundant basis for the dimension-six SMEFT operators was derived [2]. This basis is currently known as the *Warsaw basis*. The complete one-loop anomalous dimension matrix (ADM) for the dimension-six operators has been calculated very recently in this basis [3–6]. These advances, together with simultaneous theoretical developments occurring in the field (such as in the matching of specific models to the SMEFT at one loop [7–12, 14, 15] and in the automatization of calculations within the SMEFT [13]), pave the way to the systematic use of EFT methods in the analysis of new physics models. The power of the SMEFT approach is that it allows to relate physics at disparate energy scales, in our case properties of the high-energy dynamics at the scale Λ, with measurements that take place at low energies, while performing an expansion in $1/\Lambda$ that allows to keep leading new physics effects in a consistent manner. Here we present DsixTools, a Mathematica[1] package that aims to facilitate such enterprise.

Given some initial conditions for the Warsaw basis at the high-energy scale Λ, obtained from the matching of a UV model to the SMEFT, DsixTools allows the user to perform the full one-loop Renormalization Group Evolution (RGE) down to the electroweak scale. Furthermore, when the physics of interest lies well below the electroweak scale, it is useful to perform a matching of the dimension-six basis to a low-energy EFT in the broken electroweak phase. In the so-called Weak Effective Theory (WET), the SM heavy degrees of freedom (top quark, Higgs, $W^\pm$ and Z) have been integrated out. DsixTools implements the tree-level matching of the Warsaw basis to $\Delta B = \Delta S = 1,2$ and $\Delta B = \Delta C = 1$ operators of the WET based on the results obtained in [17]. Last but not least, the relevant effects due to QCD and QED running from the electroweak scale down to the b-quark mass scale in the WET is also implemented in DsixTools, using the results in [18].

DsixTools is structured into different modules, each of them taking care of a specific task. Their functionality is described in Sec. 2. Sec. 3 explains how to download and load DsixTools whereas a detailed review of DsixTools and its modules is given in Sec. 4. A summary is given in Sec. 5. Finally, this manual includes several appendices to document all the features present in DsixTools: A contains details of the conventions used for the implementation of the SMEFT, B contains the SMEFT RGEs, C describes relevant aspects of the WET, D contains a detailed description of the DsixTools routines, E is devoted to compile the SMEFT and WET parameters list and F gives results for the SMEFT operators in the fermion mass basis.

2 DsixTools in a nutshell

DsixTools is a modular package, with each module performing an independent task related to effective operators up to dimension six. The different modules can be easily communicated with each other, since one module's output can be used as input for another module. In practice, this means that a particular project with DsixTools can involve all modules or just a few, depending on the user's goals. Fig. 1 shows the global flowchart of DsixTools.

The current version of DsixTools contains three independent modules, called: SMEFTrunner, EWmatcher and WETrunner. The **SMEFTrunner** module implements:

- The SM contribution to the one-loop RGEs of the SM parameters [19–22].
- The one-loop RGEs for the dimension-six operators in the Warsaw basis from Refs. [3–5].[2]
- The one-loop RGEs for the dimension-six Baryon-number-violating operators using the results in [6].

[1] Mathematica is a product from Wolfram Research, Inc. [16].

[2] We have taken into account the errata published in http://einstein.ucsd.edu/smeft/.

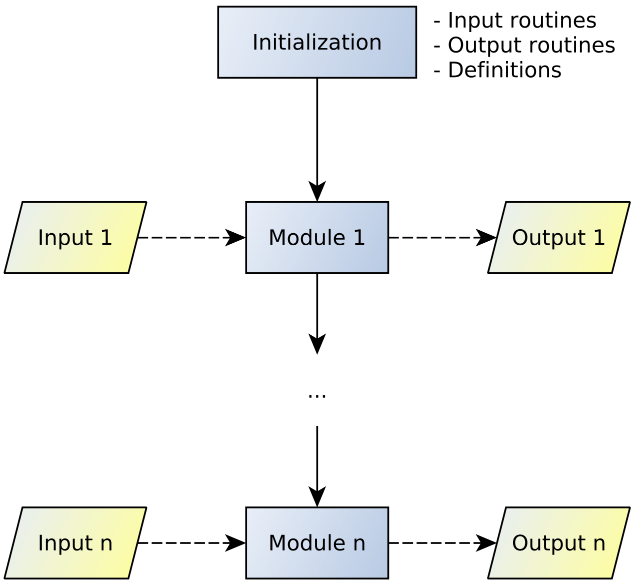

Figure 1: `DsixTools`' global flowchart. The user can provide input to each module or use the results obtained with the previous one.

- The one-loop RGEs for the dimension-five Lepton-number-violating operator using the results in [23].

The **EWmatcher** module implements:

- The tree-level matching of the Warsaw basis to the $\Delta B = \Delta S = 1, 2$ and $\Delta B = \Delta C = 1$ operators of the WET at the electroweak scale, using the results in [17].

The **WETrunner** module implements:

- QCD and QED RGEs in the WET from the electroweak scale down to the low-energy scale $\Lambda_{\rm IR}$. The `WETrunner` module fixes by default $\Lambda_{\rm IR} = m_b$, the b-quark mass scale, and uses the RGEs derived in [18].

The functionality of the `DsixTools` modules is illustrated in Fig. 2, where one can also see how they relate to the different energy ranges and effective theories. Relevant details of the SMEFT and WET implementations are given in A to C, where our conventions are also presented.

3 Downloading and loading DsixTools

`DsixTools` is free software under the copyleft of the GNU General Public License. It can be downloaded from the web page [24]:

https://dsixtools.github.io

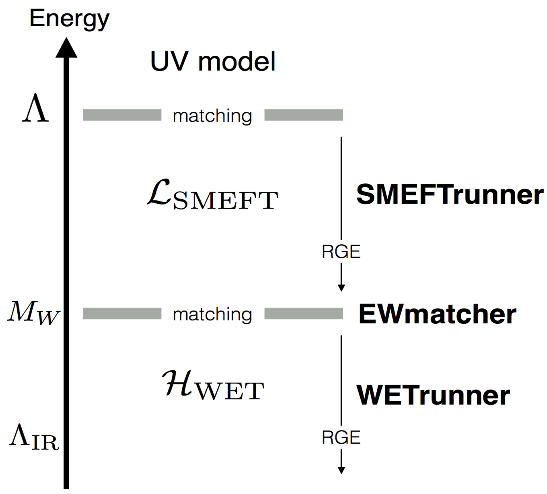

Figure 2: Functionality of `DsixTools` modules.

After placing the `DsixTools` folder in the *Applications* folder of the `Mathematica` base directory, the user can load `DsixTools` with the command

```
Needs["DsixTools`"]
```

Alternatively, the user can place the `DsixTools` folder in a given directory and call the package by specifying its location via

```
pathtoDsixTools = "<directory>";
AppendTo[$Path, pathtoDsixTools];
```

before executing the `Needs` command. We also recommend to use

```
SetDirectory[NotebookDirectory[]];
```

at the beginning of all projects with `DsixTools`.

4 Using DsixTools

In this Section we describe how to use `DsixTools` in detail. Once the package has been loaded, the user can already execute some basic I/O functions and routines. Several global variables are also introduced at this stage. Here we list the general `DsixTools` routines:

- `LoadModule[moduleName]`: Loads the `DsixTools` module `moduleName`.
- `MyPrint[string]`: Prints the message `string`. It can be switched on and off by using the `DsixTools` routines `TurnOnMessages` and `TurnOffMessages`.
- `TurnOnMessages`: Turns on the messages written by `DsixTools`.
- `TurnOffMessages`: Turns off the messages written by `DsixTools`.

- ReadInputFiles[options_file, {WCsInput_file}, {SMInput_file}]: Reads all the input files. The second and third arguments are optional. The routine should be called with only one argument when it is used to read the options file only. The routine should be called with two arguments when it is used to read WET input.

- WriteInputFiles[options_file, WCsInput_file, {SMInput_file},data]: Creates input files with the parameter values in data. The third argument is optional and should be absent when the routine is used to write WET input.

- WriteAndReadInputFiles[options_file,WCsInput_file,{SMInput_file}]: Writes data into new input files and then reads them. The third argument is optional and should be absent when the routine is used to write and read WET input.

- WCXFtoSLHA[WCXF_file,SLHA_file,HIGHSCALE]: translates the SMEFT WCs in WCxf format given in the file WCXF_file into SLHA format, and writes them in the file SLHA_file.

- SLHAtoWCXF[SLHA_file,WCXF_file,CPV,SCALE,HIGHSCALE]: translates the SMEFT WCs file in SLHA format given in the file SLHA_file into WCxf format, and writes them in the file WCXF_file.

- NewInput[parameter,newvalue,dispatch]: Replaces the current input (contained in the Mathematica dispatch dispatch) by a new one in which parameter takes the value newvalue.[3]

- NewScale[scale,newvalue]: Replaces the current value of scale by newvalue. Here scale can be either "high" or "low".

- H[mat]: Returns the Hermitian conjugate of the matrix mat. If the option CPV has been set to 0 then it returns the transpose.

- CC[x]: Returns the complex conjugate of x. If the option CPV is set to 0 then it returns x.

A more detailed description of these DsixTools routines can be found in D.1. Once DsixTools is loaded the user can already perform some basic operations. However, the real power of DsixTools resides in its modules. A project with DsixTools can involve one or many modules, depending on the user's goals. For instance, if one is interested in the running of WET operators between the electroweak scale and the b-quark mass scale, the WETrunner module suffices for the task. But if one wants to study the RGE evolution of the SMEFT operators and their matching to WET ones at the electroweak scale, both the SMEFTrunner and EWmatcher modules must be included in the project. In the following we proceed to describe the existing modules, the routines they include and how they can be combined together in a practical DsixTools project.

[3]For those not familiar with Mathematica dispatch tables, we clarify that these are optimized representations of lists of replacement rules. In practice they work in exactly the same way as replacement rules, but their execution time is much lower when the list of replacements is long.

4.1 SMEFTrunner module

The SMEFTrunner module takes care of the one-loop RGE of the SMEFT between the high-energy scale Λ and the electroweak scale. The module is based on the anomalous dimension matrices computed in [3–5] (for the Baryon-number-conserving operators), [6] (for the Baryon-number-violating operators) and [23] (for the dimension-five Weinberg operator). This is the list of routines implemented in this module:

- InitializeSMEFTrunnerInput: Initializes the input for the SMEFTrunner module.
- FindParameterSMEFT[parameter]: Returns the position in which parameter is located within the Parameters list.
- GetBeta: Computes the SMEFT β functions.
- LoadBetaFunctions: Constructs the SMEFT β functions or reads them from a file.
- RunRGEsSMEFT: Runs the SMEFT RGEs.
- RunRGEsSMEFTpython[options_file, WCsInput_file, SMInput_file]: Reads the input files and runs the SMEFT RGEs using the external python package python-smeftrunner by Xuanyou Pan and David Straub [25].
- ExportSMEFTrunner: Exports the SMEFTrunner results to an output file.
- WriteSMEFTrunnerOutputFile[Output_file,data]: Exports the SMEFTrunner results in data to Output_file.

More details about these routines can be found in D.2.

The general flowchart of the SMEFTrunner module can be seen in Fig. 3. The first step is to read the input. This not only includes the numerical values of the SMEFT WCs at the high-energy scale Λ, but also the numerical values of the SM parameters and some options. The default input and output format of DsixTools is inspired by the Supersymmetry Les Houches Accord (SLHA) [26, 27]. A complete *options card* would read:

Options.dat

```
Block SCALES
1 10000    # UV scale [GeV]
2 80.385   # EW scale [GeV]
Block OPTIONS
1 0      # CPV=0 : all parameters and WCs are assumed to be real
2 1      # ReadRGEs : 0 (RGEs reconstructed) or 1 (RGEs read ↪
   from a file)
3 1      # Method to solve RGEs : 1 (NDSolve) or 2 (leading log)
4 0      # Export RGEs
```

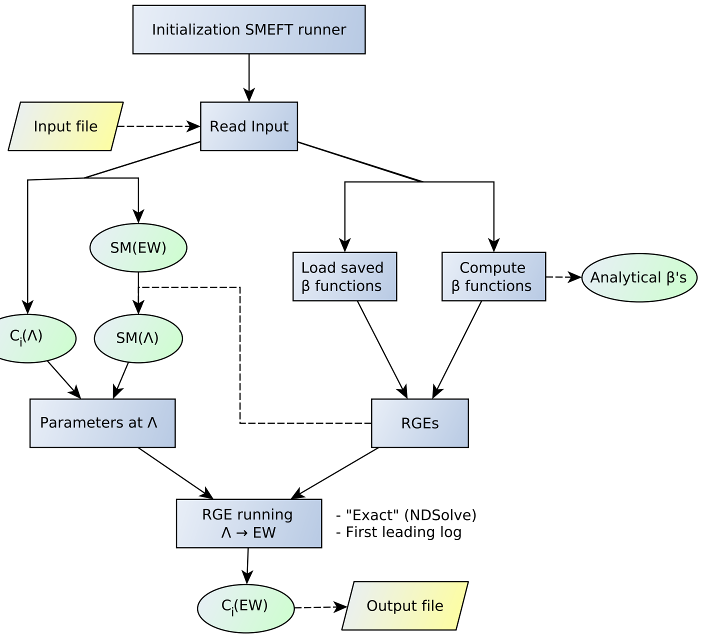

Figure 3: `SMEFTrunner` module flowchart.

```
5 1      # Use SM RGEs to compute SM parameters at the ↪
    high-energy scale
6 0      # Export SMEFTrunner results
7 0      # Export EWmatcher results
8 0      # Export WETrunner results
9 1      # Type of input WCs: 1 (SMEFT) or 2 (WET)
```

For this particular example we have chosen the default options. In fact, if a flag is not present in the options file, its value will be taken as in this example. The user defines the high-energy and electroweak scales in the first block. We see that in this example the electroweak scale is fixed to 80.385 GeV, the W-boson mass, whereas the high-energy scale is $\Lambda = 10$ TeV. Then the user defines several options. For instance, in this case CP-violation has been switched off by setting `CPV` to 0, but the user can also consider complex parameters setting `CPV` to 1. The comments accompanying the other options are self-explanatory, but we will review them below. A sample of a typical *input card* for the SM parameters is as follows:

SMInput.dat

```
Block GAUGE
1 0.629787       # g
2 0.345367       # gp
3 1.218232       # gs
Block SCALAR
```

```
1 0.257736         # lambda
2 7812.500000      # m2 [GeV^2]
Block GU
1 1 1.23231E-5     # Gamma_u(1,1)
...
3 3 0.994858       # Gamma_u(3,3)
Block IMGU
1 1 0              # Gamma_u(1,1)
...
3 3 0              # Gamma_u(3,3)
...
Block THETA
1 0                # theta
2 0                # thetap
3 0                # thetas
```

Again, the input values are distributed in blocks, each devoted to a set of parameters. For instance, the SM gauge couplings are given in the block `GAUGE`. We note that the Yukawa matrices (and in general any complex parameter) are given in two blocks. For the up-quark Yukawas these are `GU` and `IMGU`: the first one is used for the real parts and the second for the imaginary ones. By default, the SM parameters are assumed to be given at the electroweak scale, where their experimental values are known. Then, before running down from Λ to the electroweak scale they must be computed at Λ. This is done by running up from the electroweak scale using pure SM RGEs, hence neglecting possible deviations caused by non-zero SMEFT WCs.[4] However, in case the user prefers to give the SM parameters directly at high-energy scale Λ, this can be done by setting the `UseRGEsSM` option to 0 or, equivalently, by introducing a 0 in flag number 5 of the options file,

`Options.dat`

```
5 0      # Use SM RGEs to compute SM parameters at the ↪
    high-energy scale
```

This choice is recommended when the user wants to use the *First leading log* method to solve the RGEs, see below. A standard *input card* for the SMEFT WCs reads:

`WCsInput.dat`

```
Block WC1
1 0.000000         # G
2 0.000000         # G tilde
```

[4] The user can check the validity of this approximation by using the `SMEFTrunner` routines, for instance by checking whether the resulting values for the SM parameters at the electroweak scale (after running down) do not match their initial values. This can be fixed by readjusting the SM parameters at Λ. We note, however, that one should take into account NP corrections to the standard electroweak parameters induced by non-zero SMEFT WCs.

```
3 0.000000        # W
4 0.000000        # W tilde
Block WC2
1 0.000000        # phi
Block WC3
1 0.000000        # phiBox
2 0.000000        # phiD
Block WC4
1 0.000000        # phiG
...
8 0.000000        # phiWtildeB
Block WCUPHI
1 1 0.000000      # uphi(1,1)
1 2 0.000000      # uphi(1,2)
...
3 3 0.000000      # uphi(3,3)
...
```

The notation for the operators is self-explanatory:

BlockWC1

$$Q_G = f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$$
$$Q_{\widetilde{G}} = f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$$
$$Q_W = \varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$$
$$Q_{\widetilde{W}} = \varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$$

BlockWC2

$$Q_\varphi = \left(\varphi^\dagger \varphi\right)^3$$

BlockWC3

$$Q_{\varphi\Box} = \left(\varphi^\dagger \varphi\right) \Box \left(\varphi^\dagger \varphi\right)$$
$$Q_{\varphi D} = \left(\varphi^\dagger D^\mu \varphi\right)^* \left(\varphi^\dagger D_\mu \varphi\right)$$

...

BlockWCUPHI

$$Q_{u\varphi}[1,1] = \left(\varphi^\dagger \varphi\right) (\bar{q}_1 u_1 \widetilde{\varphi})$$

$$Q_{u\varphi}[1,2] = \left(\varphi^\dagger \varphi\right)(\bar{q}_1 u_2 \widetilde{\varphi})$$

$$\ldots$$

$$Q_{u\varphi}[3,3] = \left(\varphi^\dagger \varphi\right)(\bar{q}_3 u_3 \widetilde{\varphi})$$

It is important to note that all WCs are assumed to vanish by default. Therefore, it suffices to include the non-zero WCs (and only these) in the input card. The rest can be absent.

Additionally, `DsixTools` can also read WCs input files in WCxf format [28], a standard data exchange format for numerical values of Wilson coefficients. In this case, the WCs input card can be a `JSON` or `YAML` file. Note however that reading `YAML` input files requires previous installation of a `YAML` importer for Mathematica [29]. A standard `JSON` input card reads:

WCsInput.json

```
{
  "eft": "SMEFT",
  "basis": "Warsaw",
  "scale": 10000.0,
  "values": {
    "phiBtilde": 0.0,
    "dphi_11": {
      "Re": 0.0,
      "Im": 0.0
    },
    "dphi_12": {
      "Re": 0.0,
      "Im": 0.0
    },
  ...
  }
}
```

Similarly, a standard `YAML` input card reads:

WCsInput.yaml

```
eft: SMEFT
basis: Warsaw
scale: 10000.0
values:
  phiBtilde: 0.0
  dphi_11:
    Re: 0.0
```

```
    Im: 0.0
  dphi_12:
    Re: 0.0
    Im: 0.0
  ...
```

Like in the case of the SLHA format, it suffices to include the non-zero WCs in the input card. Notice that the EFT, the basis and the high-energy scale Λ at which the WCs are generated are indicated in these files. The elements of the WCs carrying flavor indices are provided individually, explicitly writing the values of the specific indices. For real WCs one must provide only a single number, whereas for the complex ones the real and imaginary parts must be given separately. For more details about the WCxf format, such as the specific fermion basis that is implicitly assumed, we refer to [28].

In addition to getting input values at Λ, the `SMEFTrunner` module constructs the SMEFT RGEs and prepares a routine for their evaluation. There are two ways to do this: (i) to "compute" the RGEs, or (ii) to read them from a file. In the first case, `DsixTools` takes their saved forms and computes all flavor traces and expands over all possible indices. In the second case, `DsixTools` simply reads them from a file (already present in the `DsixTools` folder). In both cases, the command to give this step is `LoadBetaFunctions`. The user can choose between these two methods in the options file using flag number 2. Method (i) is more time consuming but provides nice-looking β functions in `Mathematica` format. This would be very useful for analytical studies. Moreover, the user can obtain these equations at any moment by using the routine `GetBeta`. Method (ii) is much faster and produces the RGEs in the format required for their immediate evaluation with the `SMEFTrunner` routines (in particular with `RunRGEsSMEFT`).

Once both the initial conditions (input values at Λ) and the RGEs are completely built, the user can apply the RGE to obtain the goal of `SMEFTrunner`: the SMEFT WCs at the electroweak scale. This is done with the `RunRGEsSMEFT` routine. We have implemented two different methods for the resolution of the RGEs:

- *"Exact"*: This method applies the `Mathematica` internal command `NDSolve` for the numerical resolution of differential equations. Given the large number of differential equations involved in this case (several thousand), this might be time consuming, with each evaluation requiring a few (< 10) seconds, the exact number depending on the particular case and computer.

- *First leading log*: This approximate method might be sufficient for many phenomenological studies, in particular when Λ is not too far from the electroweak scale. The solution of the RGEs is obtained as
$$C_i(\mu) = C_i(\Lambda) + \frac{\beta_i}{16\pi^2} \log\left(\frac{\mu}{\Lambda}\right) , \tag{4.1}$$
where C_i is any of the running parameters, μ is the renormalization scale and β_i is the β function for the C_i evaluated at $\mu = \Lambda$. This method is much faster but neglects subleading effects.

The user chooses between these two methods by setting the global option `RGEsMethod` to 0 (for the `NDSolve` method) or 1 (for the first leading log approximate method). This is also done via flag number 3 in the options card. After running, the results are saved in the array `outSMEFTrunner` as a function of $t = \log_{10}\mu$. The ordering of the parameters is the same as in the `Parameters` global array, see E.[5] Moreover, two important values of t are predefined: `tLOW` ($= \log_{10}\mu_{\rm EW}$) and `tHIGH` ($= \log_{10}\Lambda$). Therefore, for instance, one can obtain the value of the $\left[C_{e\varphi}\right]_{12}$ at the electroweak scale by evaluating

```
outSMEFTrunner[[70]]/.t->tLOW
```

Alternatively, the user can also solve the RGEs by means of the external `python` package `python-smeftrunner` by Xuanyou Pan and David Straub [25]. This is done with the routine `RunRGEsSMEFTpython`. In contrast to `RunRGEsSMEFT`, the resulting `outSMEFTrunner` array is not a function of $t = \log_{10}\mu$, but is just evaluated at `tLOW`. The method employed for the resolution of the RGEs is purely numerical, equivalent to the *"Exact"* method described above. The numerical agreement between the results obtained with `RunRGEsSMEFT` and those obtained with `RunRGEsSMEFTpython` is very good, but `RunRGEsSMEFTpython` might be preferable for users interested in speeding up the calculation.

Finally, the output of `SMEFTrunner` can be exported to a text file. This is done by running `ExportSMEFTrunner`. The file `Output_SMEFTrunner.dat` is then generated, with an SLHA format, completely analogous to the SMEFT WCs input card. If the RGEs have been solved with `RunRGEsSMEFTpython`, the file `Output_SMEFTrunner.dat` is automatically generated and this step is not necessary.

4.2 `EWmatcher` module

The `EWmatcher` module applies the tree-level matching of the Warsaw basis of the SMEFT to the $\Delta B = \Delta S = 1,2$ and $\Delta B = \Delta C = 1$ operators of the WET. For this purpose we make use of the analytical results obtained in [17]. This is the list of routines implemented in this module:

- `InitializeEWmatcherInput`: Initializes the input for the `EWmatcher` module.
- `FindParameterWET[parameter]`: Returns the position where `parameter` is located within the `WETParameters` list.
- `Biunitary[mat,dim]`: Applies a biunitary transformation that diagonalizes the `dim` $\times$ `dim` matrix `mat`.
- `RotateToMassBasis`: Transforms the SMEFT WCs to the fermion mass basis.
- `RotateToWCXFBasis`: Transforms the SMEFT WCs to the fermion WCxf basis.
- `ApplyEWmatching`: Matches the SMEFT WCs onto the WET WCs.

[5] The position of a specific parameter can also be obtained by using the `FindParameterSMEFT` function, see D.

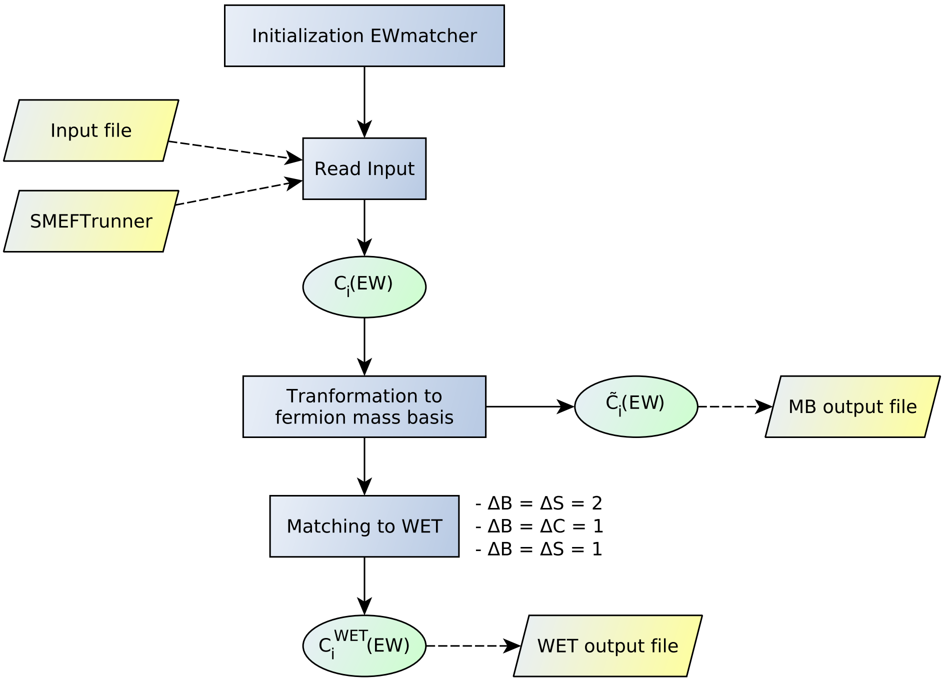

Figure 4: `EWmatcher` module flowchart.

- `Match[WC]`: Returns the value of the WET Wilson coefficient `WC` after matching it to the SMEFT.
- `MatchAnalytical[WC]`: Returns the analytical expression of the WET Wilson coefficient `WC` after matching it to the SMEFT.
- `WriteWCsOutputFile[Output_file,basis,format]`: Exports the SMEFT WCs in the fermion basis `basis` to the output file `Output_file`, written in the format `format`.
- `ExportEWmatcher`: exports the `EWmatcher` results to an output file.
- `WriteEWmatcherOutputFile[Output_file,data]`: Exports the `EWmatcher` results in `data` to `Output_file`.

Appendix D.3 contains more details about these routines and functions.

The `EWmatcher` module flowchart can be seen in Fig. 4. The first step is, as usual, to determine the input of the module. If the `SMEFTrunner` module has preceded the `EWmatcher`, the input is automatically created using the output of the former. Otherwise, the user can also provide an input file and start the `DsixTools` project with the `EWmatcher` module as first step.

In what concerns the input card, the same `WCsInput.dat` file that is read by the `SMEFTrunner` module can also be used as WCs input for the `EWmatcher` module. Instead of interpreting the input values as given at $\mu = \Lambda$, the `EWmatcher` module will interpret them as given at $\mu = \mu_{\mathrm{EW}}$.

Next, the `EWmatcher` transforms the SMEFT WCs to the fermion mass basis, applying the required biunitary transformations to the fermion mass matrices. This necessary step is done by means of the `RotateToMassBasis` routine and gives, as a result, the SMEFT WCs in the fermion mass basis, generally denoted as $\widetilde{C}_i$.[6] In case the user is particularly interested in these coefficients, they can be exported to an external text file by running

```
WriteWCsOutputFile[Output_file,"MassBasis","SLHA"]
```

Here we have required the resulting file to be written in SLHA format, but the third argument of this routine can also be `"JSON"` or `"YAML"`. Furthermore, `RotateToMassBasis` also creates the replacements array `ToMassBasis`, which can be used to print the numerical values of the SMEFT WCs in the fermion mass basis. For example,

```
LL[1, 1, 2, 2]/.ToMassBasis
```

would print the numerical value of the coefficient $\left[\widetilde{C}_{\ell\ell}\right]_{1122}$.

The user can also export the results using the WCxf format, which assumes the WCs to be given in the WCXF basis [28]. In this basis the down-quark and charged lepton mass matrices are diagonal, whereas the up-quark and neutrino mass matrices are given by $V^\dagger m_u^{\text{diag}}$ and $U^* m_\nu^{\text{diag}} U^\dagger$, respectively, with V and U the CKM and PMNS matrices. One can get the SMEFT WCs in the fermion WCXF basis by using analogous commands to those used to go to the fermion mass basis. First, the SMEFT WCs are computed in the WCXF basis by using the `RotateToWCXFBasis` routine. The result can be exported to an external file by running

```
WriteWCsOutputFile[Output_file,"WCXF","JSON"]
```

Finally, `RotateToWCXFBasis` also creates the replacements array `ToWCXFBasis`, which can then be used obtain the values of specific Wilson coefficients in the WCXF basis. For instance,

```
LQ1[2, 2, 2, 3] /. ToWCXFBasis
```

would give the numerical value of the $\left[C_{\ell q}^{(1)}\right]_{2223}$ coefficient in this basis. We emphasize again that all the input and output files in WCxf format assume that all WCs are given in the WCXF basis. However, the matching to the WET operators must be done in the mass basis.

The next step after computing the WCs in the fermion mass basis is the matching to the WET operators. For this the user must execute

```
ApplyEWmatching
```

This creates the arrays `BS2`, `BC1`, `BS1Hunprimed`, `BS1Hprimed`, `BS1GB`, `BS1SLunprimed` and `BS1SLprimed`, containing the numerical values of the WET WCs at the electroweak scale. While

[6] See F for the complete list of $\widetilde{C}_i$ coefficients in the mass basis, including Baryon- and Lepton-number-violating ones and taking into account rotations in the lepton sector (relevant in the presence of the dimension-five operator). The Baryon-number-conserving coefficients in the mass basis were computed in [17] without including rotations in the lepton sector.

BS2 and BC1 contain the results for the $\Delta B = \Delta S = 2$ and $\Delta B = \Delta C = 1$ coefficients, respectively, the other arrays contain the results for the $\Delta B = \Delta S = 2$ ones, split into several sub-arrays with hadronic (H), semileptonic (SL) and magnetic, or gauge boson (GB) involving, WCs. They can also be accessed individually thanks to the function Match. For example, CBS1[e][1] // Match would print the numerical value of C_1^{bsee}. If one is interested in the analytical expression after matching the function to use is MatchAnalytical, which only replaces the energy scales and the SM parameters by their numerical values.

Finally, the output of EWmatcher can be exported to a text file. This is done by running ExportEWmatcher, which generates the file Output_EWmatcher.dat with a compilation of all results obtained with this module.

4.3 WETrunner module

The WETrunner module is devoted to the RGE of the WET WCs between the electroweak scale and the infrared scale Λ_{IR}. In the current version of DsixTools, Λ_{IR} is set by default at b-quark mass scale, $m_b = 4.18$ GeV, and the RGE is based on the analytical results obtained in Ref. [18]. This is the list of routines implemented in this module:

- InitializeWETrunnerInput: Initializes the input for the WETrunner module.
- RunRGEsWET: Runs the WET RGEs.
- ExportWETrunner: Exports the WETrunner results to an output file.
- WriteWETrunnerOutputFile[Output_file,data,scale]: Exports the WETrunner results in data to Output_file after evaluating them at $\mu =$ scale.

For more details about these routines see Appendix D.4.

The general flowchart of the WETrunner module can be seen in Fig. 5. As for the EWmatcher module, the input for the WETrunner can be obtained directly (and automatically) from the previous module (EWmatcher in this case) if this is run before. Otherwise, the user can also provide new input values and begin a DsixTools project in this step of the chain. In this case, the input card for the WET WCs, which we will call WCsInput.dat as for the SMEFT ones, also follows a simple SLHA-inspired format:

WCsInput.dat

```
Block SCALES
2 80.385   # EW scale [GeV]
Block BS2
1 0.000000       # C1sb
2 0.000000       # C2sb
3 0.000000       # C3sb
4 1.000000       # C4sb
```

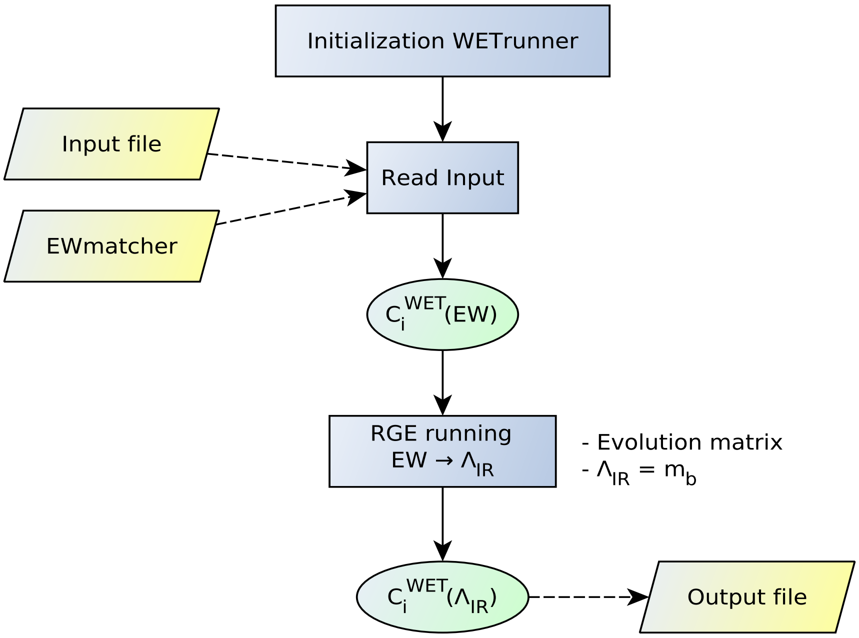

Figure 5: `WETrunner` module flowchart.

```
5 0.000000        # C5sb
6 0.000000        # C1sb'
7 0.000000        # C2sb'
8 0.000000        # C3sb'
Block BC1
1 0.000000        # CV(e)
2 0.000000        # CV(mu)
3 0.000000        # CV(tau)
4 0.000000        # CS(e)
5 0.000000        # CS(mu)
6 0.000000        # CS(tau)
7 0.000000        # CT(e)
8 0.000000        # CT(mu)
9 0.000000        # CT(tau)
10 0.000000       # CV(e)'
11 0.000000       # CV(mu)'
12 0.000000       # CV(tau)'
13 0.000000       # CS(e)'
14 0.000000       # CS(mu)'
15 0.000000       # CS(tau)'
Block BS1H
1 0.000000        # C1(sbuu)
```

```
...
```

We see that the first block gives the input value for the electroweak scale while the other blocks give the input values for the WET WCs. In this particular example the only non-vanishing WET WC is $C_4^{sbsb} = 1$. It is important to note, however, that `DsixTools` must know that the input WCs are of WET type. This is done via the option `inputWCsType`, which must be set to 2 (the default value is 1, which indicates SMEFT WCs) before reading the input file with the `ReadInputFiles` routine. This option choice can also be achieved by using the flag number 9 in the options card:

`Options.dat`

```
9 2      # Type of input WCs: 1 (SMEFT) or 2 (WET)
```

Once the input has been read (or taken from the `EWmatcher` module) one can proceed to the RGE of the WET WCs. The `WETrunner` module implements an evolution matrix formalism for this step, based on the results of Ref. [18]. This can be performed with the `RunRGEsWET` which creates the numerical arrays `BS2Low`, `BC1Low`, `BS1unprimedLow` and `BS1primedLow`, all of them functions of $t = \log_{10} \mu$. Therefore, in order to find the values of the WET WCs at the b-quark mass scale one can evaluate commands like

```
BS2Low[[4]] /. t -> Log[10, mb]
```

where `mb` is the global `DsixTools` variable containing the b-quark mass (in GeV). This command would print the numerical value of $C_4^{sbsb}(m_b)$. Finally, the results can be exported to an output file with the routine `ExportWETrunner`. This creates the text file `Output_WETrunner.dat` with the values of all WET WCs at the infrared scale.

5 Summary

We have presented `DsixTools`, a `Mathematica` package for the handling of the SMEFT and WET. `DsixTools` facilitates the treatment of these two effective theories in a systematic and complete manner.

`DsixTools` is a modular code. In the current version, it includes three independent modules, designed for specific tasks related to the SMEFT and WET. The `SMEFTrunner` performs the one-loop RGE from the UV scale Λ to the electroweak scale, the `EWmatcher` matches the SMEFT Wilson coefficients to the $\Delta B = \Delta S = 1, 2$ and $\Delta B = \Delta C = 1$ operators of the WET, and the `WETrunner` runs these down to an IR scale, in this case the b-quark mass scale.

The structure of `DsixTools` allows for an easy implementation of new modules. Therefore, the current content of the package is expected to grow substantially with future improvements, including additional tools and features. The final outcome of this endevour will be a complete and powerful framework for the systematic exploration of new physics models using the language of Effective Field Theories.

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A Standard Model Effective Field Theory

The Lagrangian for the SMEFT can be written as

$$\mathscr{L} = \mathscr{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathscr{O}\left(\frac{1}{\Lambda^3}\right) . \tag{A.1}$$

Here Λ is the new physics scale suppressing higher dimensional operators, assumed to be much larger than the electroweak scale. A non-redundant basis of dimension-six operators was introduced in [2] and is known as the *Warsaw basis*. We list these operators in Tables 2, 3 and 4. Barring flavor structure, and assuming Baryon number conservation, there are 59 operators, some of which are non-Hermitian, yielding in total 76 real coefficients. Taking into account flavour indices, the dimension-six Lagrangian contains 1350 CP-even and 1149 CP-odd operators, for a total of 2499 hermitian operators [5]. The complete set of independent dimension-6 Baryon number violating operators were identified in [31]. Barring flavor structure, there are only 4 Baryon-number-violating operators. These are listed in Table 5. Finally, there is only one operator of dimension five, the so-called Weinberg operator that gives a Majorana mass term for the neutrinos after spontaneous symmetry breaking [30]. This operator is shown in Table 6.

The implementation of the SMEFT in `DsixTools` follows the conventions used in [2].[7] The SM renormalizable Lagrangian $\mathscr{L}_{\rm SM}^{(4)}$ is given by

$$\begin{aligned}\mathscr{L}_{\rm SM}^{(4)} = &-\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \left(D_\mu\varphi\right)^\dagger\left(D^\mu\varphi\right) + m^2\varphi^\dagger\varphi - \frac{\lambda}{2}\left(\varphi^\dagger\varphi\right)^2 \\ &+ i\left(\bar\ell \not{D}\ell + \bar e\not{D}e + \bar q\not{D}q + \bar u\not{D}u + \bar d\not{D}d\right) - \left(\bar\ell\Gamma_e e\varphi + \bar q\Gamma_u u\widetilde\varphi + \bar q\Gamma_d d\varphi + \text{h.c.}\right) .\end{aligned} \tag{A.2}$$

Here $A = 1\ldots 8$ and $I = 1\ldots 3$ denote gauge indices associated to $SU(3)_c$ and $SU(2)_L$. The fields ℓ and q correspond to the lepton and quark $SU(2)_L$ doublets of the SM, while e,u,d are the right-handed fields. The Higgs $SU(2)_L$ doublet is denoted by φ. The Yukawa couplings $\Gamma_{e,u,d}$ are 3×3 matrices in flavor space. The covariant derivative is generically defined as

$$D_\mu = \partial_\mu + ig_s T^A G_\mu^A + igT^I W_\mu^I + ig'YB_\mu \,, \tag{A.3}$$

where $\{g_s, g, g'\}$ and $\{G,W,B\}$ are, respectively, the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ gauge couplings and gauge fields. T^A and T^I are the corresponding gauge group generators. The hypercharge assignments for the matter fields are given in Table 1. `DsixTools` also implements the running of the θ terms

$$\mathscr{L}_\theta = \frac{\theta' g'^2}{32\pi^2}\widetilde B_{\mu\nu}B^{\mu\nu} + \frac{\theta g^2}{32\pi^2}\widetilde W_{\mu\nu}^I W_I^{\mu\nu} + \frac{\theta_s g_s^2}{32\pi^2}\widetilde G_{\mu\nu}^A G_A^{\mu\nu} \,, \tag{A.4}$$

calculated in [3]. The dual tensors are defined as $\widetilde X = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$ (with $\varepsilon_{0123} = +1$).

[7] The reader should keep in mind that these conventions differ from those used in [3–5]. The differences appear in the normalization of λ and m, the definition of the Yukawa matrices, the name of the gauge couplings, and in whether the NP scale is introduced into the definition of the WCs.

Table 1: Hypercharge assignments.

Field	ℓ_L	e_R	q_L	u_R	d_R	φ
Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Table 2: Purely bosonic operators.

X^3		$X^2\varphi^2$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi G}$	$\varphi^\dagger\varphi G^A_{\mu\nu}G^{A\mu\nu}$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi B}$	$\varphi^\dagger\varphi B_{\mu\nu}B^{\mu\nu}$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi W}$	$\varphi^\dagger\varphi W^I_{\mu\nu}W^{I\mu\nu}$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi W^I_{\mu\nu}B^{\mu\nu}$
φ^6		$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$
Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\widetilde{B}_{\mu\nu}B^{\mu\nu}$
φ^4D^2		$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$
$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$
$Q_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^*(\varphi^\dagger D_\mu\varphi)$		

B SMEFT Renormalization Group Equations

At leading order, the RGEs governing the energy evolution of the SMEFT Wilson coefficients C_i can be written as

$$\frac{dC_i}{d\log\mu} = \frac{1}{16\pi^2}\sum_j \gamma_{ij}C_j \equiv \frac{1}{16\pi^2}\beta_i\,. \tag{B.1}$$

Here μ is the renormalization scale, γ is the anomalous dimensions matrix and β_i are the β functions. The complete anomalous dimension matrix for the dimension-six SMEFT operators has been recently computed in [3–6] whereas the result for the dimension-five operator has been known for some time [23]. We collect here the resulting β functions, adapted to our notation and conventions.

Table 3: Mixed operators involving bosons and fermions.

$\psi^2\varphi^3$		$\psi^2\varphi^2 D$	
$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}u\widetilde{\varphi})$	$Q_{\varphi\ell}^{(1)}$	$\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)(\bar{\ell}\gamma^\mu\ell)$
$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}d\varphi)$	$Q_{\varphi\ell}^{(3)}$	$\left(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi\right)(\bar{\ell}\tau^I\gamma^\mu\ell)$
$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{\ell}e\varphi)$	$Q_{\varphi e}$	$\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)(\bar{e}\gamma^\mu e)$
$\psi^2 X\varphi$		$Q_{\varphi q}^{(1)}$	$\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)(\bar{q}\gamma^\mu q)$
Q_{eW}	$(\bar{\ell}\sigma^{\mu\nu}e)\,\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$\left(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi\right)(\bar{q}\tau^I\gamma^\mu q)$
Q_{eB}	$(\bar{\ell}\sigma^{\mu\nu}e)\,\varphi B_{\mu\nu}$	$Q_{\varphi u}$	$\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)(\bar{u}\gamma^\mu u)$
Q_{uG}	$(\bar{q}\sigma^{\mu\nu}T^A u)\,\widetilde{\varphi}G^A_{\mu\nu}$	$Q_{\varphi d}$	$\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)(\bar{d}\gamma^\mu d)$
Q_{uW}	$(\bar{q}\sigma^{\mu\nu}u)\,\tau^I\widetilde{\varphi}W^I_{\mu\nu}$	$Q_{\varphi ud}$	$(\widetilde{\varphi}^\dagger iD_\mu\varphi)(\bar{u}\gamma^\mu d)$
Q_{uB}	$(\bar{q}\sigma^{\mu\nu}u)\,\widetilde{\varphi}B_{\mu\nu}$		
Q_{dG}	$(\bar{q}\sigma^{\mu\nu}T^A d)\,\varphi G^A_{\mu\nu}$		
Q_{dW}	$(\bar{q}\sigma^{\mu\nu}d)\,\tau^I\varphi W^I_{\mu\nu}$		
Q_{dB}	$(\bar{q}\sigma^{\mu\nu}d)\,\varphi B_{\mu\nu}$		

Table 4: Purely fermionic operators which preserve Baryon number.

$(\bar{L}L)(\bar{L}L)$	
$Q_{\ell\ell}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma^\mu \ell)$
$Q_{qq}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$
$Q_{qq}^{(3)}$	$(\bar{q}\gamma_\mu \tau^I q)(\bar{q}\gamma^\mu \tau^I q)$
$Q_{\ell q}^{(1)}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma^\mu q)$
$Q_{\ell q}^{(3)}$	$(\bar{\ell}\gamma_\mu \tau^I \ell)(\bar{q}\gamma^\mu \tau^I q)$

$(\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$
Q_{uu}	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$
Q_{dd}	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$
Q_{eu}	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$
Q_{ed}	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$
$Q_{ud}^{(1)}$	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$
$Q_{ud}^{(8)}$	$(\bar{u}\gamma_\mu T^A u)(\bar{d}\gamma^\mu T^A d)$

$(\bar{L}L)(\bar{R}R)$	
$Q_{\ell e}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{e}\gamma^\mu e)$
$Q_{\ell u}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{u}\gamma^\mu u)$
$Q_{\ell d}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma^\mu d)$
Q_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}\gamma^\mu e)$
$Q_{qu}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma^\mu u)$
$Q_{qu}^{(8)}$	$(\bar{q}\gamma_\mu T^A q)(\bar{u}\gamma^\mu T^A u)$
$Q_{qd}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma^\mu d)$
$Q_{qd}^{(8)}$	$(\bar{q}\gamma_\mu T^A q)(\bar{d}\gamma^\mu T^A d)$

$(\bar{L}R)(\bar{R}L)$	
$Q_{\ell edq}$	$(\bar{\ell}^j e)(\bar{d} q^j)$

$(\bar{L}R)(\bar{L}R)$	
$Q_{quqd}^{(1)}$	$(\bar{q}^j u)\,\varepsilon_{jk}\,(\bar{q}^k d)$
$Q_{quqd}^{(8)}$	$(\bar{q}^j T^A u)\,\varepsilon_{jk}\,(\bar{q}^k T^A d)$
$Q_{\ell equ}^{(1)}$	$(\bar{\ell}^j e)\,\varepsilon_{jk}\,(\bar{q}^k u)$
$Q_{\ell equ}^{(3)}$	$(\bar{\ell}^j \sigma_{\mu\nu} e)\,\varepsilon_{jk}\,(\bar{q}^k \sigma^{\mu\nu} u)$

Table 5: Baryon-number-violating operators. We use C to denote the Dirac charge conjugation matrix.

Baryon-number-violating	
$Q_{duq\ell}$	$(d^T C u)(q^T C \ell)$
Q_{qque}	$(q^T C q)(u^T C e)$
$Q_{qqq\ell}$	$\varepsilon_{il}\varepsilon_{jk}\,(q_i^T C q_j)(q_k^T C \ell_l)$
Q_{duue}	$(d^T C u)(u^T C e)$

Table 6: Dimension-five lepton-number-violating operator. We use C to denote the Dirac charge conjugation matrix.

Dimension-five	
$Q_{\ell\ell\varphi\varphi}$	$\left(\widetilde{\varphi}^{\dagger}\ell\right)^{T} C\left(\widetilde{\varphi}^{\dagger}\ell\right)$

First, we give some definitions that turn out to be useful in order to simplify the β functions:

$$\eta_1=\frac{1}{2}\left[3\,\mathrm{Tr}\left(C_{u\varphi}\Gamma_u^{\dagger}\right)+3\,\mathrm{Tr}\left(C_{d\varphi}\Gamma_d^{\dagger}\right)+\mathrm{Tr}\left(C_{e\varphi}\Gamma_e^{\dagger}\right)+\text{c.c.}\right], \tag{B.2}$$

$$\eta_2=-6\,\mathrm{Tr}\left(C_{\varphi q}^{(3)}\Gamma_u\Gamma_u^{\dagger}\right)-6\,\mathrm{Tr}\left(C_{\varphi q}^{(3)}\Gamma_d\Gamma_d^{\dagger}\right)-2\,\mathrm{Tr}\left(C_{\varphi\ell}^{(3)}\Gamma_e\Gamma_e^{\dagger}\right)+3\left[\mathrm{Tr}\left(C_{\varphi ud}\Gamma_d^{\dagger}\Gamma_u\right)+\text{c.c.}\right], \tag{B.3}$$

$$\begin{aligned}\eta_3=&\,3\,\mathrm{Tr}\left(C_{\varphi q}^{(1)}\Gamma_d\Gamma_d^{\dagger}\right)-3\,\mathrm{Tr}\left(C_{\varphi q}^{(1)}\Gamma_u\Gamma_u^{\dagger}\right)+9\,\mathrm{Tr}\left(C_{\varphi q}^{(3)}\Gamma_d\Gamma_d^{\dagger}\right)+9\,\mathrm{Tr}\left(C_{\varphi q}^{(3)}\Gamma_u\Gamma_u^{\dagger}\right)\\&+3\,\mathrm{Tr}\left(C_{\varphi u}\Gamma_u^{\dagger}\Gamma_u\right)-3\,\mathrm{Tr}\left(C_{\varphi d}\Gamma_d^{\dagger}\Gamma_d\right)-3\left[\mathrm{Tr}\left(C_{\varphi ud}\Gamma_d^{\dagger}\Gamma_u\right)+\text{c.c.}\right]\\&+\mathrm{Tr}\left(C_{\varphi\ell}^{(1)}\Gamma_e\Gamma_e^{\dagger}\right)+3\,\mathrm{Tr}\left(C_{\varphi\ell}^{(3)}\Gamma_e\Gamma_e^{\dagger}\right)-\mathrm{Tr}\left(C_{\varphi e}\Gamma_e^{\dagger}\Gamma_e\right),\end{aligned} \tag{B.4}$$

$$\begin{aligned}\eta_4=&\,12\,\mathrm{Tr}\left(C_{\varphi q}^{(1)}\Gamma_d\Gamma_d^{\dagger}\right)-12\,\mathrm{Tr}\left(C_{\varphi q}^{(1)}\Gamma_u\Gamma_u^{\dagger}\right)\\&+12\,\mathrm{Tr}\left(C_{\varphi u}\Gamma_u^{\dagger}\Gamma_u\right)-12\,\mathrm{Tr}\left(C_{\varphi d}\Gamma_d^{\dagger}\Gamma_d\right)+6\left[\mathrm{Tr}\left(C_{\varphi ud}\Gamma_d^{\dagger}\Gamma_u\right)+\text{c.c.}\right]\\&+4\,\mathrm{Tr}\left(C_{\varphi\ell}^{(1)}\Gamma_e\Gamma_e^{\dagger}\right)-4\,\mathrm{Tr}\left(C_{\varphi e}\Gamma_e^{\dagger}\Gamma_e\right),\end{aligned} \tag{B.5}$$

$$\eta_5=\frac{3}{2}i\left[\mathrm{Tr}\left(\Gamma_d C_{d\varphi}^{\dagger}\right)-\text{c.c.}\right]-\frac{3}{2}i\left[\mathrm{Tr}\left(\Gamma_u C_{u\varphi}^{\dagger}\right)-\text{c.c.}\right]+\frac{1}{2}i\left[\mathrm{Tr}\left(\Gamma_e C_{e\varphi}^{\dagger}\right)-\text{c.c.}\right], \tag{B.6}$$

as well as

$$\xi_B=\frac{2}{3}\left(C_{\varphi\Box}+C_{\varphi D}\right)+\frac{8}{3}\left(-\mathrm{Tr}C_{\varphi\ell}^{(1)}+\mathrm{Tr}C_{\varphi q}^{(1)}-\mathrm{Tr}C_{\varphi e}+2\mathrm{Tr}C_{\varphi u}-\mathrm{Tr}C_{\varphi d}\right), \tag{B.7}$$

and the wavefunction renormalization terms

$$\gamma_H^{(Y)}=\mathrm{Tr}\left(3\Gamma_u\Gamma_u^{\dagger}+3\Gamma_d\Gamma_d^{\dagger}+\Gamma_e\Gamma_e^{\dagger}\right), \tag{B.8}$$

and

$$\left[\gamma_q^{(Y)}\right]_{rs}=\frac{1}{2}\left[\Gamma_u\Gamma_u^{\dagger}+\Gamma_d\Gamma_d^{\dagger}\right]_{rs}, \tag{B.9}$$

$$\left[\gamma_u^{(Y)}\right]_{rs}=\left[\Gamma_u^{\dagger}\Gamma_u\right]_{rs}, \tag{B.10}$$

$$\left[\gamma_d^{(Y)}\right]_{rs}=\left[\Gamma_d^{\dagger}\Gamma_d\right]_{rs}, \tag{B.11}$$

$$\left[\gamma_\ell^{(Y)}\right]_{rs} = \frac{1}{2}\left[\Gamma_e \Gamma_e^\dagger\right]_{rs}, \tag{B.12}$$

$$\left[\gamma_e^{(Y)}\right]_{rs} = \left[\Gamma_e^\dagger \Gamma_e\right]_{rs}. \tag{B.13}$$

Finally we also use the following definitions

$$[\xi_e]_{pt} = 2\,[C_{\ell e}]_{prst}\,[\Gamma_e]_{rs} - 3\left[C_{\ell edq}\right]_{ptsr}[\Gamma_d]_{rs} + 3\left[C^{(1)}_{lequ}\right]_{ptsr}[\Gamma_u]^*_{sr}, \tag{B.14}$$

$$\begin{aligned}[\xi_d]_{pt} = {} & 2\left[C^{(1)}_{qd} + \frac{4}{3}C^{(8)}_{qd}\right]_{prst}[\Gamma_d]_{rs} - \left(3\left[C^{(1)}_{quqd}\right]_{srpt} + \frac{1}{2}\left[C^{(1)}_{quqd} + \frac{4}{3}C^{(8)}_{quqd}\right]_{prst}\right)[\Gamma_u]^*_{sr} \\ & - \left[C_{ledq}\right]^*_{srtp}[\Gamma_e]_{sr}, \end{aligned} \tag{B.15}$$

$$\begin{aligned}[\xi_u]_{pt} = {} & 2\left[C^{(1)}_{qu} + \frac{4}{3}C^{(8)}_{qu}\right]_{prst}[\Gamma_u]_{rs} - \left(3\left[C^{(1)}_{quqd}\right]_{ptsr} + \frac{1}{2}\left[C^{(1)}_{quqd} + \frac{4}{3}C^{(8)}_{quqd}\right]_{stpr}\right)[\Gamma_d]^*_{sr} \\ & + \left[C^{(1)}_{lequ}\right]_{srpt}[\Gamma_e]^*_{sr}. \end{aligned} \tag{B.16}$$

$\boxed{X^3}$

$$\beta_G = 15\,g_s^2\,C_G, \tag{B.17}$$

$$\beta_{\widetilde{G}} = 15\,g_s^2\,C_{\widetilde{G}}, \tag{B.18}$$

$$\beta_W = \frac{29}{2}\,g^2\,C_W, \tag{B.19}$$

$$\beta_{\widetilde{W}} = \frac{29}{2}\,g^2\,C_{\widetilde{W}}. \tag{B.20}$$

$\boxed{\varphi^6}$

$$\begin{aligned}\beta_\varphi = {} & -\frac{9}{2}\left(3g^2 + g'^2\right)C_\varphi + \lambda\left[\frac{20}{3}g^2 C_{\varphi\Box} + 3\left(g'^2 - g^2\right)C_{\varphi D}\right] - \frac{3}{4}\left(g^2 + g'^2\right)^2 C_{\varphi D} \\ & + 6\lambda\left(3g^2 C_{\varphi W} + g'^2 C_{\varphi B} + g g' C_{\varphi WB}\right) - 3\left(g^2 g'^2 + 3g^4\right)C_{\varphi W} - 3\left(g'^4 + g^2 g'^2\right)C_{\varphi B} \\ & - 3\left(g g'^3 + g^3 g'\right)C_{\varphi WB} + \frac{8}{3}\lambda g^2\left(\mathrm{Tr}\,C^{(3)}_{\varphi\ell} + 3\,\mathrm{Tr}\,C^{(3)}_{\varphi q}\right) + 54\lambda C_\varphi - 40\lambda^2 C_{\varphi\Box} + 12\lambda^2 C_{\varphi D} \\ & + 4\lambda\left(\eta_1 + \eta_2\right) - 4\left[3\,\mathrm{Tr}\left(C_{u\varphi}\Gamma_u^\dagger \Gamma_u \Gamma_u^\dagger\right) + 3\,\mathrm{Tr}\left(C_{d\varphi}\Gamma_d^\dagger \Gamma_d \Gamma_d^\dagger\right) + \mathrm{Tr}\left(C_{e\varphi}\Gamma_e^\dagger \Gamma_e \Gamma_e^\dagger\right) + \text{c.c.}\right] \\ & + 6\gamma_H^{(Y)} C_\varphi. \end{aligned} \tag{B.21}$$

$\boxed{\varphi^4 D^2}$

$$\beta_{\varphi\Box} = -\left(4g^2 + \frac{4}{3}g'^2\right) C_{\varphi\Box} + \frac{5}{3}g'^2 C_{\varphi D} + 2g^2\left(\mathrm{Tr}\, C^{(3)}_{\varphi\ell} + 3\,\mathrm{Tr}\, C^{(3)}_{\varphi q}\right)$$
$$+ \frac{2}{3}g'^2\left(2\,\mathrm{Tr}\, C_{\varphi u} - \mathrm{Tr}\, C_{\varphi d} - \mathrm{Tr}\, C_{\varphi e} + \mathrm{Tr}\, C^{(1)}_{\varphi q} - \mathrm{Tr}\, C^{(1)}_{\varphi\ell}\right) + 12\lambda\, C_{\varphi\Box} - 2\,\eta_3 + 4\gamma_H^{(Y)} C_{\varphi\Box}\,, \tag{B.22}$$

$$\beta_{\varphi D} = \frac{20}{3}g'^2 C_{\varphi\Box} + \left(\frac{9}{2}g^2 - \frac{5}{6}g'^2\right) C_{\varphi D} + \frac{8}{3}g'^2\left(2\,\mathrm{Tr}\, C_{\varphi u} - \mathrm{Tr}\, C_{\varphi d} - \mathrm{Tr}\, C_{\varphi e} + \mathrm{Tr}\, C^{(1)}_{\varphi q} - \mathrm{Tr}\, C^{(1)}_{\varphi\ell}\right)$$
$$+ 6\lambda\, C_{\varphi D} - 2\,\eta_4 + 4\gamma_H^{(Y)} C_{\varphi D}\,. \tag{B.23}$$

$\boxed{X^2\varphi^2}$

$$\beta_{\varphi G} = \left(-\frac{3}{2}g'^2 - \frac{9}{2}g^2 - 14g_s^2\right) C_{\varphi G} + 6\,\lambda C_{\varphi G}$$
$$- 2g_s\left[\mathrm{Tr}\left(C_{uG}\Gamma_u^\dagger\right) + \mathrm{Tr}\left(C_{dG}\Gamma_d^\dagger\right) + \text{c.c.}\right] + 2\gamma_H^{(Y)} C_{\varphi G}\,, \tag{B.24}$$

$$\beta_{\varphi B} = \left(\frac{85}{6}g'^2 - \frac{9}{2}g^2\right) C_{\varphi B} + 3gg' C_{\varphi WB} + 6\,\lambda C_{\varphi B}$$
$$+ g'\left[-5\,\mathrm{Tr}\left(C_{uB}\Gamma_u^\dagger\right) + \mathrm{Tr}\left(C_{dB}\Gamma_d^\dagger\right) + 3\,\mathrm{Tr}\left(C_{eB}\Gamma_e^\dagger\right) + \text{c.c.}\right] + 2\gamma_H^{(Y)} C_{\varphi B}\,, \tag{B.25}$$

$$\beta_{\varphi W} = \left(-\frac{3}{2}g'^2 - \frac{53}{6}g^2\right) C_{\varphi W} + gg' C_{\varphi WB} - 15g^3 C_W + 6\,\lambda C_{\varphi W}$$
$$- g\left[3\,\mathrm{Tr}\left(C_{uW}\Gamma_u^\dagger\right) + 3\,\mathrm{Tr}\left(C_{dW}\Gamma_d^\dagger\right) + \mathrm{Tr}\left(C_{eW}\Gamma_e^\dagger\right) + \text{c.c.}\right] + 2\gamma_H^{(Y)} C_{\varphi W}\,, \tag{B.26}$$

$$\beta_{\varphi WB} = \left(\frac{19}{3}g'^2 + \frac{4}{3}g^2\right) C_{\varphi WB} + 2gg'\left(C_{\varphi B} + C_{\varphi W}\right) + 3g^2 g' C_W + 2\,\lambda C_{\varphi WB}$$
$$+ g\left[3\,\mathrm{Tr}\left(C_{uB}\Gamma_u^\dagger\right) - 3\,\mathrm{Tr}\left(C_{dB}\Gamma_d^\dagger\right) - \mathrm{Tr}\left(C_{eB}\Gamma_e^\dagger\right) + \text{c.c.}\right]$$
$$+ g'\left[5\,\mathrm{Tr}\left(C_{uW}\Gamma_u^\dagger\right) + \mathrm{Tr}\left(C_{dW}\Gamma_d^\dagger\right) + 3\,\mathrm{Tr}\left(C_{eW}\Gamma_e^\dagger\right) + \text{c.c.}\right] + 2\gamma_H^{(Y)} C_{\varphi WB}\,, \tag{B.27}$$

$$\beta_{\varphi\widetilde{G}} = \left(-\frac{3}{2}g'^2 - \frac{9}{2}g^2 - 14g_s^2\right) C_{\varphi\widetilde{G}} + 6\,\lambda C_{\varphi\widetilde{G}}$$
$$+ 2ig_s\left[\mathrm{Tr}\left(C_{uG}\Gamma_u^\dagger\right) + \mathrm{Tr}\left(C_{dG}\Gamma_d^\dagger\right) - \text{c.c.}\right] + 2\gamma_H^{(Y)} C_{\varphi\widetilde{G}}\,, \tag{B.28}$$

$$\beta_{\varphi\widetilde{B}} = \left(\frac{85}{6}g'^2 - \frac{9}{2}g^2\right) C_{\varphi\widetilde{B}} + 3gg' C_{\varphi\widetilde{W}B} + 6\,\lambda C_{\varphi\widetilde{B}}$$
$$- ig'\left[-5\,\mathrm{Tr}\left(C_{uB}\Gamma_u^\dagger\right) + \mathrm{Tr}\left(C_{dB}\Gamma_d^\dagger\right) + 3\,\mathrm{Tr}\left(C_{eB}\Gamma_e^\dagger\right) - \text{c.c.}\right] + 2\gamma_H^{(Y)} C_{\varphi\widetilde{B}}\,, \tag{B.29}$$

$$\beta_{\varphi\widetilde{W}} = \left(-\frac{3}{2}g'^2 - \frac{53}{6}g^2\right)C_{\varphi\widetilde{W}} + gg'C_{\varphi\widetilde{W}B} - 15g^3 C_{\widetilde{W}} + 6\lambda C_{\varphi\widetilde{W}}$$
$$+ ig\left[3\,\mathrm{Tr}\left(C_{uW}\Gamma_u^\dagger\right) + 3\,\mathrm{Tr}\left(C_{dW}\Gamma_d^\dagger\right) + \mathrm{Tr}\left(C_{eW}\Gamma_e^\dagger\right) - \text{c.c.}\right] + 2\gamma_H^{(Y)}C_{\varphi\widetilde{W}}\,, \tag{B.30}$$

$$\beta_{\varphi\widetilde{W}B} = \left(\frac{19}{3}g'^2 + \frac{4}{3}g^2\right)C_{\varphi\widetilde{W}B} + 2gg'\left(C_{\varphi\widetilde{B}} + C_{\varphi\widetilde{W}}\right) + 3g^2 g' C_{\widetilde{W}} + 2\lambda C_{\varphi\widetilde{W}B}$$
$$- ig\left[3\,\mathrm{Tr}\left(C_{uB}\Gamma_u^\dagger\right) - 3\,\mathrm{Tr}\left(C_{dB}\Gamma_d^\dagger\right) - \mathrm{Tr}\left(C_{eB}\Gamma_e^\dagger\right) - \text{c.c.}\right]$$
$$- ig'\left[5\,\mathrm{Tr}\left(C_{uW}\Gamma_u^\dagger\right) + \mathrm{Tr}\left(C_{dW}\Gamma_d^\dagger\right) + 3\,\mathrm{Tr}\left(C_{eW}\Gamma_e^\dagger\right) - \text{c.c.}\right] + 2\gamma_H^{(Y)}C_{\varphi\widetilde{W}B}\,. \tag{B.31}$$

$\boxed{\psi^2\varphi^3}$

$$\left[\beta_{u\varphi}\right]_{rs} = \Bigg[\frac{10}{3}g^2 C_{\varphi\Box} + \frac{3}{2}\left(g'^2 - g^2\right)C_{\varphi D} + 32g_s^2\left(C_{\varphi G} + iC_{\varphi\widetilde{G}}\right) + 9g^2\left(C_{\varphi W} + iC_{\varphi\widetilde{W}}\right)$$
$$+ \frac{17}{3}g'^2\left(C_{\varphi B} + iC_{\varphi\widetilde{B}}\right) - gg'\left(C_{\varphi WB} + iC_{\varphi\widetilde{W}B}\right) + \frac{4}{3}g^2\left(\mathrm{Tr}\,C_{\varphi\ell}^{(3)} + 3\,\mathrm{Tr}\,C_{\varphi q}^{(3)}\right)\Bigg]\left[\Gamma_u\right]_{rs}$$
$$- \left(\frac{35}{12}g'^2 + \frac{27}{4}g^2 + 8g_s^2\right)\left[C_{u\varphi}\right]_{rs} - g'\left(5g'^2 - 3g^2\right)\left[C_{uB}\right]_{rs} + g\left(5g'^2 - 9g^2\right)\left[C_{uW}\right]_{rs}$$
$$- \left(3g^2 - g'^2\right)\left[\Gamma_u C_{\varphi u}\right]_{rs} + 3g^2\left[\Gamma_d C_{\varphi ud}^\dagger\right]_{rs} + 4g'^2\left[C_{\varphi q}^{(1)}\Gamma_u\right]_{rs} - 4g'^2\left[C_{\varphi q}^{(3)}\Gamma_u\right]_{rs}$$
$$- 5g'\left[C_{uB}\Gamma_u^\dagger\Gamma_u + \Gamma_u\Gamma_u^\dagger C_{uB}\right]_{rs} - 3g\left[C_{uW}\Gamma_u^\dagger\Gamma_u - \Gamma_u\Gamma_u^\dagger C_{uW}\right]_{rs}$$
$$- 16g_s\left[C_{uG}\Gamma_u^\dagger\Gamma_u + \Gamma_u\Gamma_u^\dagger C_{uG}\right]_{rs} - 12g\left[\Gamma_d\Gamma_d^\dagger C_{uW}\right]_{rs} - 6g\left[C_{dW}\Gamma_d^\dagger\Gamma_u\right]_{rs}$$
$$+ \lambda\Bigg(12\left[C_{u\varphi}\right]_{rs} - 2\left[C_{\varphi q}^{(1)}\Gamma_u\right]_{rs} + 6\left[C_{\varphi q}^{(3)}\Gamma_u\right]_{rs} + 2\left[\Gamma_u C_{\varphi u}\right]_{rs} - 2\left[\Gamma_d C_{\varphi ud}^\dagger\right]_{rs}$$
$$- 2C_{\varphi\Box}\left[\Gamma_u\right]_{rs} + C_{\varphi D}\left[\Gamma_u\right]_{rs} - 4\left[C_{qu}^{(1)}\right]_{rpts}\left[\Gamma_u\right]_{pt} - \frac{16}{3}\left[C_{qu}^{(8)}\right]_{rpts}\left[\Gamma_u\right]_{pt}$$
$$- 2\left[C_{\ell equ}^{(1)}\right]_{ptrs}\left[\Gamma_e\right]_{pt}^* + 6\left[C_{quqd}^{(1)}\right]_{rspt}\left[\Gamma_d\right]_{pt}^* + \left[C_{quqd}^{(1)}\right]_{psrt}\left[\Gamma_d\right]_{pt}^* + \frac{4}{3}\left[C_{quqd}^{(8)}\right]_{psrt}\left[\Gamma_d\right]_{pt}^*\Bigg)$$
$$+ 2\left(\eta_1 + \eta_2 - i\eta_5\right)\left[\Gamma_u\right]_{rs} + \left(C_{\varphi D} - 6C_{\varphi\Box}\right)\left[\Gamma_u\Gamma_u^\dagger\Gamma_u\right]_{rs} - 2\left[C_{\varphi q}^{(1)}\Gamma_u\Gamma_u^\dagger\Gamma_u\right]_{rs}$$
$$+ 6\left[C_{\varphi q}^{(3)}\Gamma_d\Gamma_d^\dagger\Gamma_u\right]_{rs} + 2\left[\Gamma_u\Gamma_u^\dagger\Gamma_u C_{\varphi u}\right]_{rs} - 2\left[\Gamma_d\Gamma_d^\dagger\Gamma_d C_{\varphi ud}^\dagger\right]_{rs}$$
$$+ 8\left[C_{qu}^{(1)} + \frac{4}{3}C_{qu}^{(8)}\right]_{rpts}\left[\Gamma_u\Gamma_u^\dagger\Gamma_u\right]_{pt} - 2\left[C_{quqd}^{(1)} + \frac{4}{3}C_{quqd}^{(8)}\right]_{tsrp}\left[\Gamma_d^\dagger\Gamma_d\Gamma_d^\dagger\right]_{pt}$$
$$- 12\left[C_{quqd}^{(1)}\right]_{rstp}\left[\Gamma_d^\dagger\Gamma_d\Gamma_d^\dagger\right]_{pt} + 4\left[C_{\ell equ}^{(1)}\right]_{tprs}\left[\Gamma_e^\dagger\Gamma_e\Gamma_e^\dagger\right]_{pt} + 4\left[C_{u\varphi}\Gamma_u^\dagger\Gamma_u\right]_{rs}$$
$$+ 5\left[\Gamma_u\Gamma_u^\dagger C_{u\varphi}\right]_{rs} - 2\left[\Gamma_d C_{d\varphi}^\dagger\Gamma_u\right]_{rs} - \left[C_{d\varphi}\Gamma_d^\dagger\Gamma_u\right]_{rs} - 2\left[\Gamma_d\Gamma_d^\dagger C_{u\varphi}\right]_{rs}$$
$$+ 3\gamma_H^{(Y)}\left[C_{u\varphi}\right]_{rs} + \left[\gamma_q^{(Y)}C_{u\varphi}\right]_{rs} + \left[C_{u\varphi}\gamma_u^{(Y)}\right]_{rs}\,, \tag{B.32}$$

$$
\begin{aligned}
[\beta_{d\varphi}]_{rs} = {} & \Big[\frac{10}{3} g^2 C_{\varphi\Box} + \frac{3}{2}\left(g'^2 - g^2\right) C_{\varphi D} + 32 g_s^2 \left(C_{\varphi G} + i C_{\varphi\widetilde{G}}\right) + 9 g^2 \left(C_{\varphi W} + i C_{\varphi\widetilde{W}}\right) \\
& + \frac{5}{3} g'^2 \left(C_{\varphi B} + i C_{\varphi\widetilde{B}}\right) + g g' \left(C_{\varphi WB} + i C_{\varphi\widetilde{W}B}\right) + \frac{4}{3} g^2 \left(\mathrm{Tr}\, C_{\varphi\ell}^{(3)} + 3\, \mathrm{Tr}\, C_{\varphi q}^{(3)}\right)\Big] [\Gamma_d]_{rs} \\
& - \left(\frac{23}{12} g'^2 + \frac{27}{4} g^2 + 8 g_s^2\right) [C_{d\varphi}]_{rs} - g' \left(3 g^2 - g'^2\right) [C_{dB}]_{rs} - g \left(9 g^2 - g'^2\right) [C_{dW}]_{rs} \\
& + \left(3 g^2 + g'^2\right) [\Gamma_d C_{\varphi d}]_{rs} + 3 g^2 [\Gamma_u C_{\varphi ud}]_{rs} - 2 g'^2 \left[C_{\varphi q}^{(1)} \Gamma_d\right]_{rs} - 2 g'^2 \left[C_{\varphi q}^{(3)} \Gamma_d\right]_{rs} \\
& + g' \left[C_{dB} \Gamma_d^\dagger \Gamma_d + \Gamma_d \Gamma_d^\dagger C_{dB}\right]_{rs} - 3 g \left[C_{dW} \Gamma_d^\dagger \Gamma_d - \Gamma_d \Gamma_d^\dagger C_{dW}\right]_{rs} \\
& - 16 g_s \left[C_{dG} \Gamma_d^\dagger \Gamma_d + \Gamma_d \Gamma_d^\dagger C_{dG}\right]_{rs} - 12 g \left[\Gamma_u \Gamma_u^\dagger C_{dW}\right]_{rs} - 6 g \left[C_{uW} \Gamma_u^\dagger \Gamma_d\right]_{rs} \\
& + \lambda \Big(12 [C_{d\varphi}]_{rs} + 2 \left[C_{\varphi q}^{(1)} \Gamma_d\right]_{rs} + 6 \left[C_{\varphi q}^{(3)} \Gamma_d\right]_{rs} - 2 [\Gamma_d C_{\varphi d}]_{rs} - 2 [\Gamma_u C_{\varphi ud}]_{rs} \\
& - 2 C_{\varphi\Box} [\Gamma_d]_{rs} + C_{\varphi D} [\Gamma_d]_{rs} - 4 \left[C_{qd}^{(1)}\right]_{rpts} [\Gamma_d]_{pt} - \frac{16}{3} \left[C_{qd}^{(8)}\right]_{rpts} [\Gamma_d]_{pt} \\
& + 2 \left[C_{\ell edq}^*\right]_{ptsr} [\Gamma_e]_{pt} + 6 \left[C_{quqd}^{(1)}\right]_{ptrs} [\Gamma_u]_{pt}^* + \left[C_{quqd}^{(1)}\right]_{rtps} [\Gamma_u]_{pt}^* + \frac{4}{3} \left[C_{quqd}^{(8)}\right]_{rtps} [\Gamma_u]_{pt}^* \Big) \\
& + 2 (\eta_1 + \eta_2 + i \eta_5) [\Gamma_d]_{rs} + \left(C_{\varphi D} - 6 C_{\varphi\Box}\right) \left[\Gamma_d \Gamma_d^\dagger \Gamma_d\right]_{rs} + 2 \left[C_{\varphi q}^{(1)} \Gamma_d \Gamma_d^\dagger \Gamma_d\right]_{rs} \\
& + 6 \left[C_{\varphi q}^{(3)} \Gamma_u \Gamma_u^\dagger \Gamma_d\right]_{rs} - 2 \left[\Gamma_d \Gamma_d^\dagger \Gamma_d C_{\varphi d}\right]_{rs} - 2 \left[\Gamma_u \Gamma_u^\dagger \Gamma_u C_{\varphi ud}\right]_{rs} \\
& + 8 \left[C_{qd}^{(1)} + \frac{4}{3} C_{qd}^{(8)}\right]_{rpts} \left[\Gamma_d \Gamma_d^\dagger \Gamma_d\right]_{pt} - 2 \left[C_{quqd}^{(1)} + \frac{4}{3} C_{quqd}^{(8)}\right]_{rpts} \left[\Gamma_u^\dagger \Gamma_u \Gamma_u^\dagger\right]_{pt} \\
& - 12 \left[C_{quqd}^{(1)}\right]_{tprs} \left[\Gamma_u^\dagger \Gamma_u \Gamma_u^\dagger\right]_{pt} - 4 \left[C_{\ell edq}^*\right]_{ptsr} \left[\Gamma_e \Gamma_e^\dagger \Gamma_e\right]_{pt} + 4 \left[C_{d\varphi} \Gamma_d^\dagger \Gamma_d\right]_{rs} \\
& + 5 \left[\Gamma_d \Gamma_d^\dagger C_{d\varphi}\right]_{rs} - 2 \left[\Gamma_u C_{u\varphi}^\dagger \Gamma_d\right]_{rs} - \left[C_{u\varphi} \Gamma_u^\dagger \Gamma_d\right]_{rs} - 2 \left[\Gamma_u \Gamma_u^\dagger C_{d\varphi}\right]_{rs} \\
& + 3 \gamma_H^{(Y)} [C_{d\varphi}]_{rs} + \left[\gamma_q^{(Y)} C_{d\varphi}\right]_{rs} + \left[C_{d\varphi} \gamma_d^{(Y)}\right]_{rs},
\end{aligned}
\tag{B.33}
$$

$$
\begin{aligned}
[\beta_{e\varphi}]_{rs} = {} & \Big[\frac{10}{3} g^2 C_{\varphi\Box} + \frac{3}{2}\left(g'^2 - g^2\right) C_{\varphi D} + 9 g^2 \left(C_{\varphi W} + i C_{\varphi\widetilde{W}}\right) + 15 g'^2 \left(C_{\varphi B} + i C_{\varphi\widetilde{B}}\right) \\
& - 3 g g' \left(C_{\varphi WB} + i C_{\varphi\widetilde{W}B}\right) + \frac{4}{3} g^2 \left(\mathrm{Tr}\, C_{\varphi\ell}^{(3)} + 3\, \mathrm{Tr}\, C_{\varphi q}^{(3)}\right)\Big] [\Gamma_e]_{rs} - \frac{3}{4}\left(7 g'^2 + 9 g^2\right) [C_{e\varphi}]_{rs} \\
& - 3 g' \left(g^2 - 3 g'^2\right) [C_{eB}]_{rs} - 9 g \left(g^2 - g'^2\right) [C_{eW}]_{rs} + 3 \left(g^2 - g'^2\right) [\Gamma_e C_{\varphi e}]_{rs} - 6 g'^2 \left[C_{\varphi\ell}^{(1)} \Gamma_e\right]_{rs} \\
& - 6 g'^2 \left[C_{\varphi\ell}^{(3)} \Gamma_e\right]_{rs} + 9 g' \left[C_{eB} \Gamma_e^\dagger \Gamma_e + \Gamma_e \Gamma_e^\dagger C_{eB}\right]_{rs} - 3 g \left[C_{eW} \Gamma_e^\dagger \Gamma_e - \Gamma_e \Gamma_e^\dagger C_{eW}\right]_{rs} \\
& + \lambda \Big(12 [C_{e\varphi}]_{rs} + 2 \left[C_{\varphi\ell}^{(1)} \Gamma_e\right]_{rs} + 6 \left[C_{\varphi\ell}^{(3)} \Gamma_e\right]_{rs} - 2 [\Gamma_e C_{\varphi e}]_{rs} - 2 C_{\varphi\Box} [\Gamma_e]_{rs} + C_{\varphi D} [\Gamma_e]_{rs} \\
& - 4 [C_{\ell e}]_{rpts} [\Gamma_e]_{pt} + 6 [C_{\ell edq}]_{rspt} [\Gamma_d]_{tp} - 6 \left[C_{\ell equ}^{(1)}\right]_{rspt} [\Gamma_u]_{pt}^* \Big) + 2 (\eta_1 + \eta_2 + i \eta_5) [\Gamma_e]_{rs} \\
& + \left(C_{\varphi D} - 6 C_{\varphi\Box}\right) \left[\Gamma_e \Gamma_e^\dagger \Gamma_e\right]_{rs} + 2 \left[C_{\varphi\ell}^{(1)} \Gamma_e \Gamma_e^\dagger \Gamma_e\right]_{rs} - 2 \left[\Gamma_e \Gamma_e^\dagger \Gamma_e C_{\varphi e}\right]_{rs}
\end{aligned}
$$

$$+8\left[C_{\ell e}\right]_{rpts}\left[\Gamma_e\Gamma_e^\dagger\Gamma_e\right]_{pt}-12\left[C_{\ell edq}\right]_{rspt}\left[\Gamma_d\Gamma_d^\dagger\Gamma_d\right]_{tp}+12\left[C_{\ell equ}^{(1)}\right]_{rstp}\left[\Gamma_u^\dagger\Gamma_u\Gamma_u^\dagger\right]_{pt}$$
$$+4\left[C_{e\varphi}\Gamma_e^\dagger\Gamma_e\right]_{rs}+5\left[\Gamma_e\Gamma_e^\dagger C_{e\varphi}\right]_{rs}+3\gamma_H^{(Y)}\left[C_{e\varphi}\right]_{rs}+\left[\gamma_\ell^{(Y)}C_{e\varphi}\right]_{rs}+\left[C_{e\varphi}\gamma_e^{(Y)}\right]_{rs}. \tag{B.34}$$

$\boxed{\psi^2 X\varphi}$

$$\begin{aligned}
\left[\beta_{eW}\right]_{rs} &= \frac{1}{12}\left(3g'^2-11g^2\right)\left[C_{eW}\right]_{rs}-\frac{1}{2}gg'\left[C_{eB}\right]_{rs} \\
&-\left[g\left(C_{\varphi W}+iC_{\varphi\widetilde{W}}\right)-\frac{3}{2}g'\left(C_{\varphi WB}+iC_{\varphi\widetilde{W}B}\right)\right]\left[\Gamma_e\right]_{rs} \\
&-6g\left[C_{\ell equ}^{(3)}\right]_{rspt}\left[\Gamma_u\right]_{pt}^*+\left[C_{eW}\Gamma_e^\dagger\Gamma_e\right]_{rs}+\gamma_H^{(Y)}\left[C_{eW}\right]_{rs}+\left[\gamma_\ell^{(Y)}C_{eW}\right]_{rs}+\left[C_{eW}\gamma_e^{(Y)}\right]_{rs},
\end{aligned} \tag{B.35}$$

$$\begin{aligned}
\left[\beta_{eB}\right]_{rs} &= \frac{1}{4}\left(\frac{151}{3}g'^2-9g^2\right)\left[C_{eB}\right]_{rs}-\frac{3}{2}gg'\left[C_{eW}\right]_{rs} \\
&-\left[\frac{3}{2}g\left(C_{\varphi WB}+iC_{\varphi\widetilde{W}B}\right)-3g'\left(C_{\varphi B}+iC_{\varphi\widetilde{B}}\right)\right]\left[\Gamma_e\right]_{rs} \\
&+10g'\left[C_{\ell equ}^{(3)}\right]_{rspt}\left[\Gamma_u\right]_{pt}^*+\left[C_{eB}\Gamma_e^\dagger\Gamma_e\right]_{rs}+2\left[\Gamma_e\Gamma_e^\dagger C_{eB}\right]_{rs} \\
&+\gamma_H^{(Y)}\left[C_{eB}\right]_{rs}+\left[\gamma_\ell^{(Y)}C_{eB}\right]_{rs}+\left[C_{eB}\gamma_e^{(Y)}\right]_{rs},
\end{aligned} \tag{B.36}$$

$$\begin{aligned}
\left[\beta_{uG}\right]_{rs} &= -\frac{1}{36}\left(81g^2+19g'^2+204g_s^2\right)\left[C_{uG}\right]_{rs}+6gg_s\left[C_{uW}\right]_{rs}+\frac{10}{3}g'g_s\left[C_{uB}\right]_{rs} \\
&-g_s\left[4\left(C_{\varphi G}+iC_{\varphi\widetilde{G}}\right)-9g_s\left(C_G+iC_{\widetilde{G}}\right)\right]\left[\Gamma_u\right]_{rs} \\
&-g_s\left(\left[C_{quqd}^{(1)}\right]_{psrt}-\frac{1}{6}\left[C_{quqd}^{(8)}\right]_{psrt}\right)\left[\Gamma_d\right]_{pt}^*+2\left[\Gamma_u\Gamma_u^\dagger C_{uG}\right]_{rs}-2\left[\Gamma_d\Gamma_d^\dagger C_{uG}\right]_{rs} \\
&-\left[C_{dG}\Gamma_d^\dagger\Gamma_u\right]_{rs}+\left[C_{uG}\Gamma_u^\dagger\Gamma_u\right]_{rs}+\gamma_H^{(Y)}\left[C_{uG}\right]_{rs}+\left[\gamma_q^{(Y)}C_{uG}\right]_{rs}+\left[C_{uG}\gamma_u^{(Y)}\right]_{rs},
\end{aligned} \tag{B.37}$$

$$\begin{aligned}
\left[\beta_{uW}\right]_{rs} &= -\frac{1}{36}\left(33g^2+19g'^2-96g_s^2\right)\left[C_{uW}\right]_{rs}+\frac{8}{3}gg_s\left[C_{uG}\right]_{rs}-\frac{1}{6}gg'\left[C_{uB}\right]_{rs} \\
&-\left[g\left(C_{\varphi W}+iC_{\varphi\widetilde{W}}\right)-\frac{5}{6}g'\left(C_{\varphi WB}+iC_{\varphi\widetilde{W}B}\right)\right]\left[\Gamma_u\right]_{rs} \\
&+\frac{g}{4}\left(\left[C_{quqd}^{(1)}\right]_{psrt}+\frac{4}{3}\left[C_{quqd}^{(8)}\right]_{psrt}\right)\left[\Gamma_d\right]_{pt}^*-2g\left[C_{\ell equ}^{(3)}\right]_{ptrs}\left[\Gamma_e\right]_{pt}^*+2\left[\Gamma_d\Gamma_d^\dagger C_{uW}\right]_{rs} \\
&-\left[C_{dW}\Gamma_d^\dagger\Gamma_u\right]_{rs}+\left[C_{uW}\Gamma_u^\dagger\Gamma_u\right]_{rs}+\gamma_H^{(Y)}\left[C_{uW}\right]_{rs}+\left[\gamma_q^{(Y)}C_{uW}\right]_{rs}+\left[C_{uW}\gamma_u^{(Y)}\right]_{rs},
\end{aligned} \tag{B.38}$$

$$\left[\beta_{uB}\right]_{rs} = -\frac{1}{36}\left(81g^2-313g'^2-96g_s^2\right)\left[C_{uB}\right]_{rs}+\frac{40}{9}g'g_s\left[C_{uG}\right]_{rs}-\frac{1}{2}gg'\left[C_{uW}\right]_{rs}$$

$$
\begin{aligned}
&-\left[-\frac{3}{2}g\left(C_{\varphi WB}+iC_{\varphi\widetilde{W}B}\right)+\frac{5}{3}g'\left(C_{\varphi B}+iC_{\varphi\widetilde{B}}\right)\right][\Gamma_u]_{rs}\\
&+\frac{g'}{12}\left(\left[C^{(1)}_{quqd}\right]_{psrt}+\frac{4}{3}\left[C^{(8)}_{quqd}\right]_{psrt}\right)[\Gamma_d]^*_{pt}-6g'\left[C^{(3)}_{\ell equ}\right]_{ptrs}[\Gamma_e]^*_{pt}+2\left[\Gamma_u\Gamma_u^\dagger C_{uB}\right]_{rs}\\
&-2\left[\Gamma_d\Gamma_d^\dagger C_{uB}\right]_{rs}-\left[C_{dB}\Gamma_d^\dagger\Gamma_u\right]_{rs}+\left[C_{uB}\Gamma_u^\dagger\Gamma_u\right]_{rs}+\gamma_H^{(Y)}[C_{uB}]_{rs}\\
&+\left[\gamma_q^{(Y)}C_{uB}\right]_{rs}+\left[C_{uB}\gamma_u^{(Y)}\right]_{rs}, \qquad \text{(B.39)}
\end{aligned}
$$

$$
\begin{aligned}
[\beta_{dG}]_{rs}=&-\frac{1}{36}\left(81g^2+31g'^2+204g_s^2\right)[C_{dG}]_{rs}+6gg_s[C_{dW}]_{rs}-\frac{2}{3}g'g_s[C_{dB}]_{rs}\\
&-g_s\left[4\left(C_{\varphi G}+iC_{\varphi\widetilde{G}}\right)-9g_s\left(C_G+iC_{\widetilde{G}}\right)\right][\Gamma_d]_{rs}\\
&-g_s\left(\left[C^{(1)}_{quqd}\right]_{rtps}-\frac{1}{6}\left[C^{(8)}_{quqd}\right]_{rtps}\right)[\Gamma_u]^*_{pt}-2\left[\Gamma_u\Gamma_u^\dagger C_{dG}\right]_{rs}+2\left[\Gamma_d\Gamma_d^\dagger C_{dG}\right]_{rs}\\
&-\left[C_{uG}\Gamma_u^\dagger\Gamma_d\right]_{rs}+\left[C_{dG}\Gamma_d^\dagger\Gamma_d\right]_{rs}+\gamma_H^{(Y)}[C_{dG}]_{rs}+\left[\gamma_q^{(Y)}C_{dG}\right]_{rs}+\left[C_{dG}\gamma_d^{(Y)}\right]_{rs}, \qquad \text{(B.40)}
\end{aligned}
$$

$$
\begin{aligned}
[\beta_{dW}]_{rs}=&-\frac{1}{36}\left(33g^2+31g'^2-96g_s^2\right)[C_{dW}]_{rs}+\frac{8}{3}gg_s[C_{dG}]_{rs}+\frac{5}{6}gg'[C_{dB}]_{rs}\\
&-\left[g\left(C_{\varphi W}+iC_{\varphi\widetilde{W}}\right)-\frac{1}{6}g'\left(C_{\varphi WB}+iC_{\varphi\widetilde{W}B}\right)\right][\Gamma_d]_{rs}\\
&+\frac{g}{4}\left(\left[C^{(1)}_{quqd}\right]_{rtps}+\frac{4}{3}\left[C^{(8)}_{quqd}\right]_{rtps}\right)[\Gamma_u]^*_{pt}+2\left[\Gamma_u\Gamma_u^\dagger C_{dW}\right]_{rs}\\
&-\left[C_{uW}\Gamma_u^\dagger\Gamma_d\right]_{rs}+\left[C_{dW}\Gamma_d^\dagger\Gamma_d\right]_{rs}+\gamma_H^{(Y)}[C_{dW}]_{rs}+\left[\gamma_q^{(Y)}C_{dW}\right]_{rs}+\left[C_{dW}\gamma_d^{(Y)}\right]_{rs}, \qquad \text{(B.41)}
\end{aligned}
$$

$$
\begin{aligned}
[\beta_{dB}]_{rs}=&-\frac{1}{36}\left(81g^2-253g'^2-96g_s^2\right)[C_{dB}]_{rs}-\frac{8}{9}g'g_s[C_{dG}]_{rs}+\frac{5}{2}gg'[C_{dW}]_{rs}\\
&-\left[\frac{3}{2}g\left(C_{\varphi WB}+iC_{\varphi\widetilde{W}B}\right)-\frac{1}{3}g'\left(C_{\varphi B}+iC_{\varphi\widetilde{B}}\right)\right][\Gamma_d]_{rs}\\
&-\frac{5g'}{12}\left(\left[C^{(1)}_{quqd}\right]_{rtps}+\frac{4}{3}\left[C^{(8)}_{quqd}\right]_{rtps}\right)[\Gamma_u]^*_{pt}-2\left[\Gamma_u\Gamma_u^\dagger C_{dB}\right]_{rs}+2\left[\Gamma_d\Gamma_d^\dagger C_{dB}\right]_{rs}\\
&-\left[C_{uB}\Gamma_u^\dagger\Gamma_d\right]_{rs}+\left[C_{dB}\Gamma_d^\dagger\Gamma_d\right]_{rs}+\gamma_H^{(Y)}[C_{dB}]_{rs}+\left[\gamma_q^{(Y)}C_{dB}\right]_{rs}+\left[C_{dB}\gamma_d^{(Y)}\right]_{rs}. \qquad \text{(B.42)}
\end{aligned}
$$

$\boxed{\psi^2\varphi^2 D}$

$$
\begin{aligned}
\left[\beta^{(1)}_{\varphi\ell}\right]_{rs}=&-\frac{1}{4}\xi_B g'^2\delta_{rs}+\frac{1}{3}g'^2\left[C^{(1)}_{\varphi\ell}\right]_{rs}-\frac{2}{3}g'^2\Big([C_{\ell d}]_{rstt}+[C_{\ell e}]_{rstt}+[C_{\ell\ell}]_{rstt}+\frac{1}{2}[C_{\ell\ell}]_{rtts}\\
&+\frac{1}{2}[C_{\ell\ell}]_{tsrt}+[C_{\ell\ell}]_{ttrs}-\left[C^{(1)}_{\ell q}\right]_{rstt}-2[C_{\ell u}]_{rstt}\Big)-\frac{1}{2}\left(C_{\varphi\Box}+C_{\varphi D}\right)\left[\Gamma_e\Gamma_e^\dagger\right]_{rs}\\
&-\left[\Gamma_e C_{\varphi e}\Gamma_e^\dagger\right]_{rs}+\frac{3}{2}\left(\left[\Gamma_e\Gamma_e^\dagger C^{(1)}_{\varphi\ell}\right]_{rs}+\left[C^{(1)}_{\varphi\ell}\Gamma_e\Gamma_e^\dagger\right]_{rs}+3\left[\Gamma_e\Gamma_e^\dagger C^{(3)}_{\varphi\ell}\right]_{rs}+3\left[C^{(3)}_{\varphi\ell}\Gamma_e\Gamma_e^\dagger\right]_{rs}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+2\,[C_{\ell e}]_{rspt}\left[\Gamma_e^\dagger\Gamma_e\right]_{tp}-\Big(2\,[C_{\ell\ell}]_{rspt}+2\,[C_{\ell\ell}]_{ptrs}+[C_{\ell\ell}]_{rtps}+[C_{\ell\ell}]_{psrt}\Big)\left[\Gamma_e\Gamma_e^\dagger\right]_{tp}\\
&-6\left[C_{\ell q}^{(1)}\right]_{rspt}\left[\Gamma_d\Gamma_d^\dagger\right]_{tp}+6\left[C_{\ell q}^{(1)}\right]_{rspt}\left[\Gamma_u\Gamma_u^\dagger\right]_{tp}-6\,[C_{\ell u}]_{rspt}\left[\Gamma_u^\dagger\Gamma_u\right]_{tp}\\
&+6\,[C_{\ell d}]_{rspt}\left[\Gamma_d^\dagger\Gamma_d\right]_{tp}+2\gamma_H^{(Y)}\left[C_{\varphi\ell}^{(1)}\right]_{rs}+\left[\gamma_\ell^{(Y)}C_{\varphi\ell}^{(1)}\right]_{rs}+\left[C_{\varphi\ell}^{(1)}\gamma_\ell^{(Y)}\right]_{rs}, \qquad \text{(B.43)}
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta_{\varphi\ell}^{(3)}\right]_{rs}=&\;\frac{2}{3}g^2\left(\frac{1}{4}C_{\varphi\Box}+\mathrm{Tr}\,C_{\varphi\ell}^{(3)}+3\,\mathrm{Tr}\,C_{\varphi q}^{(3)}\right)\delta_{rs}-\frac{17}{3}g^2\left[C_{\varphi\ell}^{(3)}\right]_{rs}+\frac{1}{3}g^2\Big([C_{\ell\ell}]_{rtts}+[C_{\ell\ell}]_{tsrt}\Big)\\
&+2g^2\left[C_{\ell q}^{(3)}\right]_{rstt}-\frac{1}{2}C_{\varphi\Box}\left[\Gamma_e\Gamma_e^\dagger\right]_{rs}+\frac{1}{2}\Big(3\left[\Gamma_e\Gamma_e^\dagger C_{\varphi\ell}^{(1)}\right]_{rs}+3\left[C_{\varphi\ell}^{(1)}\Gamma_e\Gamma_e^\dagger\right]_{rs}+\left[\Gamma_e\Gamma_e^\dagger C_{\varphi\ell}^{(3)}\right]_{rs}\\
&+\left[C_{\varphi\ell}^{(3)}\Gamma_e\Gamma_e^\dagger\right]_{rs}\Big)-\Big([C_{\ell\ell}]_{rtps}+[C_{\ell\ell}]_{psrt}\Big)\left[\Gamma_e\Gamma_e^\dagger\right]_{tp}-6\left[C_{\ell q}^{(3)}\right]_{rspt}\left[\Gamma_u\Gamma_u^\dagger+\Gamma_d\Gamma_d^\dagger\right]_{tp}\\
&+2\gamma_H^{(Y)}\left[C_{\varphi\ell}^{(3)}\right]_{rs}+\left[\gamma_\ell^{(Y)}C_{\varphi\ell}^{(3)}\right]_{rs}+\left[C_{\varphi\ell}^{(3)}\gamma_\ell^{(Y)}\right]_{rs}, \qquad \text{(B.44)}
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta_{\varphi e}\right]_{rs}=&-\frac{1}{2}\xi_B g'^2\delta_{rs}+\frac{1}{3}g'^2\left[C_{\varphi e}\right]_{rs}-\frac{2}{3}g'^2\Big([C_{ed}]_{rstt}+[C_{ee}]_{rstt}+[C_{ee}]_{rtts}+[C_{ee}]_{tsrt}+[C_{ee}]_{ttrs}\\
&-2\,[C_{eu}]_{rstt}+[C_{\ell e}]_{ttrs}-\left[C_{qe}\right]_{ttrs}\Big)+\left(C_{\varphi\Box}+C_{\varphi D}\right)\left[\Gamma_e^\dagger\Gamma_e\right]_{rs}-2\left[\Gamma_e^\dagger C_{\varphi\ell}^{(1)}\Gamma_e\right]_{rs}\\
&+3\left(\left[\Gamma_e^\dagger\Gamma_e C_{\varphi e}\right]_{rs}+\left[C_{\varphi e}\Gamma_e^\dagger\Gamma_e\right]_{rs}\right)-2\,[C_{\ell e}]_{ptrs}\left[\Gamma_e\Gamma_e^\dagger\right]_{tp}-6\,[C_{eu}]_{rspt}\left[\Gamma_u^\dagger\Gamma_u\right]_{tp} \qquad \text{(B.45)}\\
&+2\Big([C_{ee}]_{rspt}+[C_{ee}]_{ptrs}+[C_{ee}]_{rtps}+[C_{ee}]_{psrt}\Big)\left[\Gamma_e^\dagger\Gamma_e\right]_{tp}+6\,[C_{ed}]_{rspt}\left[\Gamma_d^\dagger\Gamma_d\right]_{tp}\\
&-6\left[C_{qe}\right]_{ptrs}\left[\Gamma_d\Gamma_d^\dagger\right]_{tp}+6\left[C_{qe}\right]_{ptrs}\left[\Gamma_u\Gamma_u^\dagger\right]_{tp}+2\gamma_H^{(Y)}\left[C_{\varphi e}\right]_{rs}+\left[\gamma_e^{(Y)}C_{\varphi e}\right]_{rs}+\left[C_{\varphi e}\gamma_e^{(Y)}\right]_{rs},
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta_{\varphi q}^{(1)}\right]_{rs}=&\;\frac{1}{12}\xi_B g'^2\delta_{rs}+\frac{1}{3}g'^2\left[C_{\varphi q}^{(1)}\right]_{rs}-\frac{2}{3}g'^2\Big(\left[C_{\ell q}^{(1)}\right]_{ttrs}+\left[C_{qd}^{(1)}\right]_{rstt}-2\left[C_{qu}^{(1)}\right]_{rstt}+\left[C_{qe}\right]_{rstt}\\
&-\left[C_{qq}^{(1)}\right]_{rstt}-\frac{1}{6}\left[C_{qq}^{(1)}\right]_{rtts}-\frac{1}{6}\left[C_{qq}^{(1)}\right]_{tsrt}-\left[C_{qq}^{(1)}\right]_{ttrs}-\frac{1}{2}\left[C_{qq}^{(3)}\right]_{rtts}-\frac{1}{2}\left[C_{qq}^{(3)}\right]_{tsrt}\Big)\\
&+\frac{1}{2}\left(C_{\varphi\Box}+C_{\varphi D}\right)\left(\left[\Gamma_u\Gamma_u^\dagger\right]_{rs}-\left[\Gamma_d\Gamma_d^\dagger\right]_{rs}\right)-\left[\Gamma_u C_{\varphi u}\Gamma_u^\dagger\right]_{rs}-\left[\Gamma_d C_{\varphi d}\Gamma_d^\dagger\right]_{rs}\\
&+2\left[C_{qe}\right]_{rspt}\left[\Gamma_e^\dagger\Gamma_e\right]_{tp}-2\left[C_{\ell q}^{(1)}\right]_{ptrs}\left[\Gamma_e\Gamma_e^\dagger\right]_{tp}+\frac{3}{2}\Big(\left[\Gamma_d\Gamma_d^\dagger C_{\varphi q}^{(1)}\right]_{rs}+\left[\Gamma_u\Gamma_u^\dagger C_{\varphi q}^{(1)}\right]_{rs}\\
&+\left[C_{\varphi q}^{(1)}\Gamma_d\Gamma_d^\dagger\right]_{rs}+\left[C_{\varphi q}^{(1)}\Gamma_u\Gamma_u^\dagger\right]_{rs}+3\left[\Gamma_d\Gamma_d^\dagger C_{\varphi q}^{(3)}\right]_{rs}-3\left[\Gamma_u\Gamma_u^\dagger C_{\varphi q}^{(3)}\right]_{rs}+3\left[C_{\varphi q}^{(3)}\Gamma_d\Gamma_d^\dagger\right]_{rs}\\
&-3\left[C_{\varphi q}^{(3)}\Gamma_u\Gamma_u^\dagger\right]_{rs}\Big)-\Big(6\left[C_{qq}^{(1)}\right]_{rspt}+6\left[C_{qq}^{(1)}\right]_{ptrs}+\left[C_{qq}^{(1)}\right]_{rtps}+\left[C_{qq}^{(1)}\right]_{psrt}\\
&+3\left[C_{qq}^{(3)}\right]_{rtps}+3\left[C_{qq}^{(3)}\right]_{psrt}\Big)\left(\left[\Gamma_d\Gamma_d^\dagger\right]_{tp}-\left[\Gamma_u\Gamma_u^\dagger\right]_{tp}\right)-6\left[C_{qu}^{(1)}\right]_{rspt}\left[\Gamma_u^\dagger\Gamma_u\right]_{tp}\\
&+6\left[C_{qd}^{(1)}\right]_{rspt}\left[\Gamma_d^\dagger\Gamma_d\right]_{tp}+2\gamma_H^{(Y)}\left[C_{\varphi q}^{(1)}\right]_{rs}+\left[\gamma_q^{(Y)}C_{\varphi q}^{(1)}\right]_{rs}+\left[C_{\varphi q}^{(1)}\gamma_q^{(Y)}\right]_{rs}, \qquad \text{(B.46)}
\end{aligned}
$$

$$
\left[\beta_{\varphi q}^{(3)}\right]_{rs}=\frac{2}{3}g^2\left(\frac{1}{4}C_{\varphi\Box}+\mathrm{Tr}\,C_{\varphi\ell}^{(3)}+3\,\mathrm{Tr}\,C_{\varphi q}^{(3)}\right)\delta_{rs}-\frac{17}{3}g^2\left[C_{\varphi q}^{(3)}\right]_{rs}+\frac{1}{3}g^2\Big(2\left[C_{\ell q}^{(3)}\right]_{ttrs}
$$

$$
\begin{aligned}
&+\left[C_{qq}^{(1)}\right]_{rtts}+\left[C_{qq}^{(1)}\right]_{tsrt}+6\left[C_{qq}^{(3)}\right]_{rstt}-\left[C_{qq}^{(3)}\right]_{rtts}-\left[C_{qq}^{(3)}\right]_{tsrt}+6\left[C_{qq}^{(3)}\right]_{ttrs}\Big)\\
&-\frac{1}{2}C_{\varphi\Box}\left(\left[\Gamma_u\Gamma_u^\dagger\right]_{rs}+\left[\Gamma_d\Gamma_d^\dagger\right]_{rs}\right)+\frac{1}{2}\Big(3\left[\Gamma_d\Gamma_d^\dagger C_{\varphi q}^{(1)}\right]_{rs}-3\left[\Gamma_u\Gamma_u^\dagger C_{\varphi q}^{(1)}\right]_{rs}\\
&+3\left[C_{\varphi q}^{(1)}\Gamma_d\Gamma_d^\dagger\right]_{rs}-3\left[C_{\varphi q}^{(1)}\Gamma_u\Gamma_u^\dagger\right]_{rs}+\left[\Gamma_d\Gamma_d^\dagger C_{\varphi q}^{(3)}\right]_{rs}+\left[\Gamma_u\Gamma_u^\dagger C_{\varphi q}^{(3)}\right]_{rs}\\
&+\left[C_{\varphi q}^{(3)}\Gamma_d\Gamma_d^\dagger\right]_{rs}+\left[C_{\varphi q}^{(3)}\Gamma_u\Gamma_u^\dagger\right]_{rs}\Big)-\Big(6\left[C_{qq}^{(3)}\right]_{rspt}+6\left[C_{qq}^{(3)}\right]_{ptrs}+\left[C_{qq}^{(1)}\right]_{rtps}\\
&+\left[C_{qq}^{(1)}\right]_{psrt}-\left[C_{qq}^{(3)}\right]_{rtps}-\left[C_{qq}^{(3)}\right]_{psrt}\Big)\left(\left[\Gamma_d\Gamma_d^\dagger\right]_{tp}+\left[\Gamma_u\Gamma_u^\dagger\right]_{tp}\right)\\
&-2\left[C_{\ell q}^{(3)}\right]_{ptrs}\left[\Gamma_e\Gamma_e^\dagger\right]_{tp}+2\gamma_H^{(Y)}\left[C_{\varphi q}^{(3)}\right]_{rs}+\left[\gamma_q^{(Y)}C_{\varphi q}^{(3)}\right]_{rs}+\left[C_{\varphi q}^{(3)}\gamma_q^{(Y)}\right]_{rs},
\end{aligned}
\tag{B.47}
$$

$$
\begin{aligned}
\left[\beta_{\varphi u}\right]_{rs}=&\ \frac{1}{3}\xi_B g'^2\delta_{rs}+\frac{1}{3}g'^2\left[C_{\varphi u}\right]_{rs}-\frac{2}{3}g'^2\Big(\left[C_{eu}\right]_{ttrs}+\left[C_{\ell u}\right]_{ttrs}-\left[C_{qu}^{(1)}\right]_{ttrs}+\left[C_{ud}^{(1)}\right]_{rstt}\\
&-2\left[C_{uu}\right]_{rstt}-\frac{2}{3}\left[C_{uu}\right]_{rtts}-\frac{2}{3}\left[C_{uu}\right]_{tsrt}-2\left[C_{uu}\right]_{ttrs}\Big)-\left(C_{\varphi\Box}+C_{\varphi D}\right)\left[\Gamma_u^\dagger\Gamma_u\right]_{rs}\\
&-2\left[\Gamma_u^\dagger C_{\varphi q}^{(1)}\Gamma_u\right]_{rs}+3\left(\left[\Gamma_u^\dagger\Gamma_u C_{\varphi u}\right]_{rs}+\left[C_{\varphi u}\Gamma_u^\dagger\Gamma_u\right]_{rs}\right)+\left[\Gamma_u^\dagger\Gamma_d C_{\varphi ud}^\dagger\right]_{rs}+\left[C_{\varphi ud}\Gamma_d^\dagger\Gamma_u\right]_{rs}\\
&-2\Big(3\left[C_{uu}\right]_{rspt}+3\left[C_{uu}\right]_{ptrs}+\left[C_{uu}\right]_{rtps}+\left[C_{uu}\right]_{psrt}\Big)\left[\Gamma_u^\dagger\Gamma_u\right]_{tp}+2\left[C_{eu}\right]_{ptrs}\left[\Gamma_e^\dagger\Gamma_e\right]_{tp}\\
&-2\left[C_{\ell u}\right]_{ptrs}\left[\Gamma_e\Gamma_e^\dagger\right]_{tp}+6\left[C_{ud}^{(1)}\right]_{rspt}\left[\Gamma_d^\dagger\Gamma_d\right]_{tp}-6\left[C_{qu}^{(1)}\right]_{ptrs}\left[\Gamma_d\Gamma_d^\dagger\right]_{tp}\\
&+6\left[C_{qu}^{(1)}\right]_{ptrs}\left[\Gamma_u\Gamma_u^\dagger\right]_{tp}+2\gamma_H^{(Y)}\left[C_{\varphi u}\right]_{rs}+\left[\gamma_u^{(Y)}C_{\varphi u}\right]_{rs}+\left[C_{\varphi u}\gamma_u^{(Y)}\right]_{rs},
\end{aligned}
\tag{B.48}
$$

$$
\begin{aligned}
\left[\beta_{\varphi d}\right]_{rs}=&-\frac{1}{6}\xi_B g'^2\delta_{rs}+\frac{1}{3}g'^2\left[C_{\varphi d}\right]_{rs}-\frac{2}{3}g'^2\Big(\left[C_{dd}\right]_{rstt}+\frac{1}{3}\left[C_{dd}\right]_{rtts}+\frac{1}{3}\left[C_{dd}\right]_{tsrt}+\left[C_{dd}\right]_{ttrs}\\
&+\left[C_{ed}\right]_{ttrs}+\left[C_{\ell d}\right]_{ttrs}-\left[C_{qd}^{(1)}\right]_{ttrs}-2\left[C_{ud}^{(1)}\right]_{ttrs}\Big)+\left(C_{\varphi\Box}+C_{\varphi D}\right)\left[\Gamma_d^\dagger\Gamma_d\right]_{rs}\\
&-2\left[\Gamma_d^\dagger C_{\varphi q}^{(1)}\Gamma_d\right]_{rs}+3\left(\left[\Gamma_d^\dagger\Gamma_d C_{\varphi d}\right]_{rs}+\left[C_{\varphi d}\Gamma_d^\dagger\Gamma_d\right]_{rs}\right)-\left[\Gamma_d^\dagger\Gamma_u C_{\varphi ud}\right]_{rs}-\left[C_{\varphi ud}^\dagger\Gamma_u^\dagger\Gamma_d\right]_{rs}\\
&+2\Big(3\left[C_{dd}\right]_{rspt}+3\left[C_{dd}\right]_{ptrs}+\left[C_{dd}\right]_{rtps}+\left[C_{dd}\right]_{psrt}\Big)\left[\Gamma_d^\dagger\Gamma_d\right]_{tp}+2\left[C_{ed}\right]_{ptrs}\left[\Gamma_e^\dagger\Gamma_e\right]_{tp}\\
&-2\left[C_{\ell d}\right]_{ptrs}\left[\Gamma_e\Gamma_e^\dagger\right]_{tp}-6\left[C_{ud}^{(1)}\right]_{ptrs}\left[\Gamma_u^\dagger\Gamma_u\right]_{tp}-6\left[C_{qd}^{(1)}\right]_{ptrs}\left[\Gamma_d\Gamma_d^\dagger\right]_{tp}\\
&+6\left[C_{qd}^{(1)}\right]_{ptrs}\left[\Gamma_u\Gamma_u^\dagger\right]_{tp}+2\gamma_H^{(Y)}\left[C_{\varphi d}\right]_{rs}+\left[\gamma_d^{(Y)}C_{\varphi d}\right]_{rs}+\left[C_{\varphi d}\gamma_d^{(Y)}\right]_{rs},
\end{aligned}
\tag{B.49}
$$

$$
\begin{aligned}
\left[\beta_{\varphi ud}\right]_{rs}=&-3g'^2\left[C_{\varphi ud}\right]_{rs}+\left(2C_{\varphi\Box}-C_{\varphi D}\right)\left[\Gamma_u^\dagger\Gamma_d\right]_{rs}-2\left[\Gamma_u^\dagger\Gamma_d C_{\varphi d}\right]_{rs}+2\left[C_{\varphi u}\Gamma_u^\dagger\Gamma_d\right]_{rs}\\
&+4\left(\left[C_{ud}^{(1)}\right]_{rtps}+\frac{4}{3}\left[C_{ud}^{(8)}\right]_{rtps}\right)\left[\Gamma_u^\dagger\Gamma_d\right]_{tp}+2\left[\Gamma_u^\dagger\Gamma_u C_{\varphi ud}\right]_{rs}+2\left[C_{\varphi ud}\Gamma_d^\dagger\Gamma_d\right]_{rs}\\
&+2\gamma_H^{(Y)}\left[C_{\varphi ud}\right]_{rs}+\left[\gamma_u^{(Y)}C_{\varphi ud}\right]_{rs}+\left[C_{\varphi ud}\gamma_d^{(Y)}\right]_{rs}.
\end{aligned}
\tag{B.50}
$$

$\boxed{(\bar{L}L)\,(\bar{L}L)}$

$$
\begin{aligned}
[\beta_{\ell\ell}]_{prst} = \Bigg[& -\frac{1}{6}g'^2 \left[C_{\varphi\ell}^{(1)}\right]_{st}\delta_{pr} - \frac{1}{6}g^2\left(\left[C_{\varphi\ell}^{(3)}\right]_{st}\delta_{pr} - 2\left[C_{\varphi\ell}^{(3)}\right]_{sr}\delta_{pt}\right) \\
& + \frac{1}{3}g'^2\left(2\,[C_{\ell\ell}]_{prww} + [C_{\ell\ell}]_{pwwr}\right)\delta_{st} - \frac{1}{3}g^2\,[C_{\ell\ell}]_{pwwr}\,\delta_{st} + \frac{2}{3}g^2\,[C_{\ell\ell}]_{swwr}\,\delta_{pt} \\
& - \frac{1}{3}g'^2\left[C_{\ell q}^{(1)}\right]_{prww}\delta_{st} - g^2\left[C_{\ell q}^{(3)}\right]_{prww}\delta_{st} + 2g^2\left[C_{\ell q}^{(3)}\right]_{ptww}\delta_{rs} \\
& + \frac{1}{3}g'^2\left(-2\,[C_{\ell u}]_{prww} + [C_{\ell d}]_{prww} + [C_{\ell e}]_{prww}\right)\delta_{st} - \frac{1}{2}\left[\Gamma_e\Gamma_e^\dagger\right]_{pr}\left[C_{\varphi\ell}^{(1)} - C_{\varphi\ell}^{(3)}\right]_{st} \\
& - \left[\Gamma_e\Gamma_e^\dagger\right]_{pt}\left[C_{\varphi\ell}^{(3)}\right]_{sr} - \frac{1}{2}\,[\Gamma_e]_{sv}\,[\Gamma_e]^*_{tw}\,[C_{\ell e}]_{prvw} + \left[\gamma_\ell^{(Y)}\right]_{pv}[C_{\ell\ell}]_{vrst} \\
& + [C_{\ell\ell}]_{pvst}\left[\gamma_\ell^{(Y)}\right]_{vr} + (pr)\leftrightarrow(st)\Bigg] + 6g^2\,[C_{\ell\ell}]_{ptsr} + 3\left(g'^2 - g^2\right)[C_{\ell\ell}]_{prst}\,,
\end{aligned}
\tag{B.51}
$$

$$
\begin{aligned}
\left[\beta_{qq}^{(1)}\right]_{prst} = \Bigg[& \frac{1}{18}g'^2\left[C_{\varphi q}^{(1)}\right]_{st}\delta_{pr} - \frac{1}{9}g'^2\left[C_{\ell q}^{(1)}\right]_{wwst}\delta_{pr} \\
& + \frac{1}{9}g'^2\left(2\left[C_{qq}^{(1)}\right]_{prww} + \frac{1}{3}\left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{pwwr}\right)\delta_{st} + \frac{1}{3}g_s^2\left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{swwr}\delta_{pt} \\
& - \frac{2}{9}g_s^2\left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{pwwr}\delta_{st} + \frac{2}{9}g'^2\left[C_{qu}^{(1)}\right]_{prww}\delta_{st} - \frac{1}{9}g'^2\left[C_{qd}^{(1)}\right]_{prww}\delta_{st} \\
& + \frac{1}{12}g_s^2\left(\left[C_{qu}^{(8)}\right]_{srww} + \left[C_{qd}^{(8)}\right]_{srww}\right)\delta_{pt} - \frac{1}{18}g_s^2\left(\left[C_{qu}^{(8)}\right]_{prww} + \left[C_{qd}^{(8)}\right]_{prww}\right)\delta_{st} \\
& - \frac{1}{9}g'^2\,[C_{qe}]_{prww}\,\delta_{st} + \frac{1}{2}\left[\Gamma_u\Gamma_u^\dagger - \Gamma_d\Gamma_d^\dagger\right]_{pr}\left[C_{\varphi q}^{(1)}\right]_{st} - \frac{1}{2}\,[\Gamma_u]_{pv}\,[\Gamma_u]^*_{rw}\left[C_{qu}^{(1)} - \frac{1}{6}C_{qu}^{(8)}\right]_{stvw} \\
& - \frac{1}{2}\,[\Gamma_d]_{pv}\,[\Gamma_d]^*_{rw}\left[C_{qd}^{(1)} - \frac{1}{6}C_{qd}^{(8)}\right]_{stvw} \\
& - \frac{1}{8}\left([\Gamma_u]_{pv}\,[\Gamma_u]^*_{tw}\left[C_{qu}^{(8)}\right]_{srvw} + [\Gamma_d]_{pv}\,[\Gamma_d]^*_{tw}\left[C_{qd}^{(8)}\right]_{srvw}\right) \\
& - \frac{1}{8}\,[\Gamma_d]^*_{tw}\,[\Gamma_u]^*_{rv}\left[C_{quqd}^{(1)} - \frac{1}{6}C_{quqd}^{(8)}\right]_{pvsw} - \frac{1}{8}\,[\Gamma_d]_{sw}\,[\Gamma_u]_{pv}\left[C_{quqd}^{(1)} - \frac{1}{6}C_{quqd}^{(8)}\right]^*_{rvtw} \\
& + \frac{1}{16}\left([\Gamma_d]^*_{tw}\,[\Gamma_u]^*_{rv}\left[C_{quqd}^{(8)}\right]_{svpw} + [\Gamma_d]_{sw}\,[\Gamma_u]_{pv}\left[C_{quqd}^{(8)}\right]^*_{tvrw}\right) \\
& + \left[\gamma_q^{(Y)}\right]_{pv}\left[C_{qq}^{(1)}\right]_{vrst} + \left[C_{qq}^{(1)}\right]_{pvst}\left[\gamma_q^{(Y)}\right]_{vr} + (pr)\leftrightarrow(st)\Bigg] \\
& + 9g^2\left[C_{qq}^{(3)}\right]_{prst} - 2\left(g_s^2 - \frac{1}{6}g'^2\right)\left[C_{qq}^{(1)}\right]_{prst} + 3g_s^2\left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{ptsr}\,,
\end{aligned}
\tag{B.52}
$$

$$
\begin{aligned}
\left[\beta_{qq}^{(3)}\right]_{prst} = & \Bigg[\frac{1}{6}g^2\left[C_{\varphi q}^{(3)}\right]_{st}\delta_{pr} + \frac{1}{3}g^2\left[C_{\ell q}^{(3)}\right]_{wwst}\delta_{pr} + \frac{1}{3}g^2\left[C_{qq}^{(1)} - C_{qq}^{(3)}\right]_{pwwr}\delta_{st} \\
& + 2g^2\left[C_{qq}^{(3)}\right]_{prww}\delta_{st} + \frac{1}{3}g_s^2\left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{swwr}\delta_{pt} + \frac{1}{12}g_s^2\left[C_{qu}^{(8)} + C_{qd}^{(8)}\right]_{srww}\delta_{pt} \\
& - \frac{1}{2}\left[\Gamma_u\Gamma_u^\dagger + \Gamma_d\Gamma_d^\dagger\right]_{pr}\left[C_{\varphi q}^{(3)}\right]_{st} - \frac{1}{8}\left([\Gamma_u]_{pv}[\Gamma_u]_{tw}^*\left[C_{qu}^{(8)}\right]_{srvw} + [\Gamma_d]_{pv}[\Gamma_d]_{tw}^*\left[C_{qd}^{(8)}\right]_{srvw}\right) \\
& + \frac{1}{8}[\Gamma_d]_{tw}^*[\Gamma_u]_{rv}^*\left[C_{quqd}^{(1)} - \frac{1}{6}C_{quqd}^{(8)}\right]_{pvsw} + \frac{1}{8}[\Gamma_d]_{sw}[\Gamma_u]_{pv}\left[C_{quqd}^{(1)} - \frac{1}{6}C_{quqd}^{(8)}\right]_{rvtw}^* \\
& - \frac{1}{16}\left([\Gamma_d]_{tw}^*[\Gamma_u]_{rv}^*\left[C_{quqd}^{(8)}\right]_{svpw} + [\Gamma_d]_{sw}[\Gamma_u]_{pv}\left[C_{quqd}^{(8)}\right]_{tvrw}^*\right) + \left[\gamma_q^{(Y)}\right]_{pv}\left[C_{qq}^{(3)}\right]_{vrst} \\
& + \left[C_{qq}^{(3)}\right]_{pvst}\left[\gamma_q^{(Y)}\right]_{vr} + (pr) \leftrightarrow (st)\Bigg] + 3g_s^2\left[C_{qq}^{(1)} - C_{qq}^{(3)}\right]_{ptsr} \\
& - 2\left(g_s^2 + 3g^2 - \frac{1}{6}g'^2\right)\left[C_{qq}^{(3)}\right]_{prst} + 3g^2\left[C_{qq}^{(1)}\right]_{prst}\,, \qquad \text{(B.53)}
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta_{\ell q}^{(1)}\right]_{prst} = & -\frac{1}{3}g'^2\left[C_{\varphi q}^{(1)}\right]_{st}\delta_{pr} + \frac{1}{9}g'^2\left[C_{\varphi \ell}^{(1)}\right]_{pr}\delta_{st} - \frac{2}{9}g'^2\left(2[C_{\ell\ell}]_{prww} + [C_{\ell\ell}]_{pwwr}\right)\delta_{st} \\
& + \frac{2}{9}g'^2\left[C_{\ell q}^{(1)}\right]_{prww}\delta_{st} + \frac{2}{3}g'^2\left[C_{\ell q}^{(1)}\right]_{wwst}\delta_{pr} - g'^2\left[C_{\ell q}^{(1)}\right]_{prst} + 9g^2\left[C_{\ell q}^{(3)}\right]_{prst} \\
& - \frac{2}{9}g'^2\left(6\left[C_{qq}^{(1)}\right]_{stww} + \left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{swwt}\right)\delta_{pr} - [\Gamma_u]_{sv}[\Gamma_u]_{tw}^*[C_{\ell u}]_{prvw} \\
& - \frac{2}{3}g'^2\left(2\left[C_{qu}^{(1)}\right]_{stww} - \left[C_{qd}^{(1)}\right]_{stww} - \left[C_{qe}\right]_{stww}\right)\delta_{pr} - \left[\Gamma_e\Gamma_e^\dagger\right]_{pr}\left[C_{\varphi q}^{(1)}\right]_{st} \\
& + \frac{2}{9}g'^2\left(2[C_{\ell u}]_{prww} - [C_{\ell d}]_{prww} - [C_{\ell e}]_{prww}\right)\delta_{st} + \left[\Gamma_u\Gamma_u^\dagger - \Gamma_d\Gamma_d^\dagger\right]_{st}\left[C_{\varphi \ell}^{(1)}\right]_{pr} \\
& + \frac{1}{4}\left([\Gamma_u]_{tw}^*[\Gamma_e]_{rv}^*\left[C_{\ell equ}^{(1)} - 12C_{\ell equ}^{(3)}\right]_{pvsw} + [\Gamma_u]_{sw}[\Gamma_e]_{pv}\left[C_{\ell equ}^{(1)} - 12C_{\ell equ}^{(3)}\right]_{rvtw}^*\right) \\
& - [\Gamma_d]_{sv}[\Gamma_d]_{tw}^*[C_{\ell d}]_{prvw} - [\Gamma_e]_{pv}[\Gamma_e]_{rw}^*[C_{qe}]_{stvw} \\
& + \frac{1}{4}\left([\Gamma_d]_{sw}[\Gamma_e]_{rv}^*[C_{\ell edq}]_{pvwt} + [\Gamma_e]_{pv}[\Gamma_d]_{tw}^*[C_{\ell edq}]_{rvws}^*\right) \\
& + \left[\gamma_\ell^{(Y)}\right]_{pv}\left[C_{\ell q}^{(1)}\right]_{vrst} + \left[\gamma_q^{(Y)}\right]_{sv}\left[C_{\ell q}^{(1)}\right]_{prvt} + \left[C_{\ell q}^{(1)}\right]_{pvst}\left[\gamma_\ell^{(Y)}\right]_{vr} + \left[C_{\ell q}^{(1)}\right]_{prsv}\left[\gamma_q^{(Y)}\right]_{vt}\,, \qquad \text{(B.54)}
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta_{\ell q}^{(3)}\right]_{prst} = & \frac{1}{3}g^2\left(\left[C_{\varphi q}^{(3)}\right]_{st}\delta_{pr} + \left[C_{\varphi \ell}^{(3)}\right]_{pr}\delta_{st}\right) + \frac{2}{3}g^2\left(3\left[C_{\ell q}^{(3)}\right]_{prww}\delta_{st} + \left[C_{\ell q}^{(3)}\right]_{wwst}\delta_{pr}\right) \\
& + \frac{2}{3}g^2\left(6\left[C_{qq}^{(3)}\right]_{stww} + \left[C_{qq}^{(1)} - C_{qq}^{(3)}\right]_{swwt}\right)\delta_{pr} + \frac{2}{3}g^2[C_{\ell\ell}]_{pwwr}\delta_{st} + 3g^2\left[C_{\ell q}^{(1)}\right]_{prst} \\
& - \left(6g^2 + g'^2\right)\left[C_{\ell q}^{(3)}\right]_{prst} - \left[\Gamma_e\Gamma_e^\dagger\right]_{pr}\left[C_{\varphi q}^{(3)}\right]_{st} - \left[\Gamma_u\Gamma_u^\dagger + \Gamma_d\Gamma_d^\dagger\right]_{st}\left[C_{\varphi \ell}^{(3)}\right]_{pr}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{4}\left([\Gamma_u]^*_{tw}[\Gamma_e]^*_{rv}\left[C^{(1)}_{\ell equ}-12C^{(3)}_{\ell equ}\right]_{pvsw}+[\Gamma_u]_{sw}[\Gamma_e]_{pv}\left[C^{(1)}_{\ell equ}-12C^{(3)}_{\ell equ}\right]^*_{rvtw}\right)\\
&+\frac{1}{4}\left([\Gamma_d]_{sw}[\Gamma_e]^*_{rv}\left[C_{\ell edq}\right]_{pvwt}+[\Gamma_e]_{pv}[\Gamma_d]^*_{tw}\left[C_{\ell edq}\right]^*_{rvws}\right)\\
&+\left[\gamma^{(Y)}_\ell\right]_{pv}\left[C^{(3)}_{\ell q}\right]_{vrst}+\left[\gamma^{(Y)}_q\right]_{sv}\left[C^{(3)}_{\ell q}\right]_{prvt}+\left[C^{(3)}_{\ell q}\right]_{pvst}\left[\gamma^{(Y)}_\ell\right]_{vr}+\left[C^{(3)}_{\ell q}\right]_{prsv}\left[\gamma^{(Y)}_q\right]_{vt}.
\end{aligned}
\tag{B.55}
$$

$\boxed{(\bar{R}R)(\bar{R}R)}$

$$
\begin{aligned}
[\beta_{ee}]_{prst}=\Bigg[&-\frac{1}{3}g'^2\left[C_{\varphi e}\right]_{st}\delta_{pr}+\frac{2}{3}g'^2\Big([C_{\ell e}]_{wwpr}-\left[C_{qe}\right]_{wwpr}-2\left[C_{eu}\right]_{prww}+\left[C_{ed}\right]_{prww}\\
&+4\left[C_{ee}\right]_{prww}\Big)\delta_{st}+6g'^2\left[C_{ee}\right]_{prst}+\left[\Gamma_e^\dagger\Gamma_e\right]_{pr}\left[C_{\varphi e}\right]_{st}-[\Gamma_e]_{wr}[\Gamma_e]^*_{vp}\left[C_{\ell e}\right]_{vwst}\\
&+\left[\gamma^{(Y)}_e\right]_{pv}\left[C_{ee}\right]_{vrst}+\left[C_{ee}\right]_{pvst}\left[\gamma^{(Y)}_e\right]_{vr}+(pr)\leftrightarrow(st)\Bigg],
\end{aligned}
\tag{B.56}
$$

$$
\begin{aligned}
[\beta_{uu}]_{prst}=\Bigg[&\frac{2}{9}g'^2\left[C_{\varphi u}\right]_{st}\delta_{pr}-\frac{1}{9}g_s^2\left(\left[C^{(8)}_{qu}\right]_{wwst}\delta_{pr}-3\left[C^{(8)}_{qu}\right]_{wwsr}\delta_{pt}\right)+\frac{2}{3}g_s^2\left[C_{uu}\right]_{pwwt}\delta_{rs}\\
&-\frac{4}{9}g'^2\left(\left[C_{eu}\right]_{wwst}+\left[C_{\ell u}\right]_{wwst}-\left[C^{(1)}_{qu}\right]_{wwst}-4\left[C_{uu}\right]_{wwst}-\frac{4}{3}\left[C_{uu}\right]_{swwt}\right)\delta_{pr}+3g_s^2\left[C_{uu}\right]_{ptsr}\\
&-\frac{2}{9}g_s^2\left[C_{uu}\right]_{swwt}\delta_{pr}-\frac{4}{9}g'^2\left[C^{(1)}_{ud}\right]_{stww}\delta_{pr}-\frac{1}{18}g_s^2\left(\left[C^{(8)}_{ud}\right]_{stww}\delta_{pr}-3\left[C^{(8)}_{ud}\right]_{srww}\delta_{pt}\right)\\
&+\left(\frac{8}{3}g'^2-g_s^2\right)\left[C_{uu}\right]_{prst}-\left[\Gamma_u^\dagger\Gamma_u\right]_{pr}\left[C_{\varphi u}\right]_{st}-[\Gamma_u]_{wr}[\Gamma_u]^*_{vp}\left(\left[C^{(1)}_{qu}\right]_{vwst}-\frac{1}{6}\left[C^{(8)}_{qu}\right]_{vwst}\right)\\
&-\frac{1}{2}[\Gamma_u]_{wr}[\Gamma_u]^*_{vs}\left[C^{(8)}_{qu}\right]_{vwpt}+\left[\gamma^{(Y)}_u\right]_{pv}\left[C_{uu}\right]_{vrst}+\left[C_{uu}\right]_{pvst}\left[\gamma^{(Y)}_u\right]_{vr}+(pr)\leftrightarrow(st)\Bigg],
\end{aligned}
\tag{B.57}
$$

$$
\begin{aligned}
[\beta_{dd}]_{prst}=\Bigg[&-\frac{1}{9}g'^2\left[C_{\varphi d}\right]_{st}\delta_{pr}-\frac{1}{9}g_s^2\left(\left[C^{(8)}_{qd}\right]_{wwst}\delta_{pr}-3\left[C^{(8)}_{qd}\right]_{wwsr}\delta_{pt}\right)+\frac{2}{3}g_s^2\left[C_{dd}\right]_{pwwt}\delta_{rs}\\
&+\frac{2}{9}g'^2\left(\left[C_{ed}\right]_{wwst}+\left[C_{\ell d}\right]_{wwst}-\left[C^{(1)}_{qd}\right]_{wwst}+2\left[C_{dd}\right]_{wwst}+\frac{2}{3}\left[C_{dd}\right]_{swwt}\right)\delta_{pr}+3g_s^2\left[C_{dd}\right]_{ptsr}\\
&-\frac{2}{9}g_s^2\left[C_{dd}\right]_{swwt}\delta_{pr}-\frac{4}{9}g'^2\left[C^{(1)}_{ud}\right]_{wwst}\delta_{pr}-\frac{1}{18}g_s^2\left(\left[C^{(8)}_{ud}\right]_{wwst}\delta_{pr}-3\left[C^{(8)}_{ud}\right]_{wwsr}\delta_{pt}\right)\\
&+\left(\frac{2}{3}g'^2-g_s^2\right)\left[C_{dd}\right]_{prst}+\left[\Gamma_d^\dagger\Gamma_d\right]_{pr}\left[C_{\varphi d}\right]_{st}-[\Gamma_d]_{wr}[\Gamma_d]^*_{vp}\left(\left[C^{(1)}_{qd}\right]_{vwst}-\frac{1}{6}\left[C^{(8)}_{qd}\right]_{vwst}\right)
\end{aligned}
$$

$$
-\frac{1}{2}\left[\Gamma_d\right]_{wr}\left[\Gamma_d\right]^*_{vs}\left[C^{(8)}_{qd}\right]_{vwpt}+\left[\gamma^{(Y)}_d\right]_{pv}\left[C_{dd}\right]_{vrst}+\left[C_{dd}\right]_{pvst}\left[\gamma^{(Y)}_d\right]_{vr}+(pr)\leftrightarrow(st)\Bigg],
\tag{B.58}
$$

$$
\begin{aligned}
\left[\beta_{eu}\right]_{prst} =& -\frac{2}{3}g'^2\left(\left[C_{\varphi u}\right]_{st}+2\left[C^{(1)}_{qu}-C_{\ell u}+4C_{uu}-C_{eu}\right]_{wwst}-2\left[C^{(1)}_{ud}\right]_{stww}+\frac{8}{3}\left[C_{uu}\right]_{swwt}\right)\delta_{pr}\\
&+\frac{4}{9}g'^2\left(\left[C_{\varphi e}\right]_{pr}+2\left[C_{qe}-C_{\ell e}-4C_{ee}\right]_{wwpr}-2\left[C_{ed}-2C_{eu}\right]_{prww}\right)\delta_{st}-8g'^2\left[C_{eu}\right]_{prst}\\
&+2\left[\Gamma^\dagger_e\Gamma_e\right]_{pr}\left[C_{\varphi u}\right]_{st}-2\left[\Gamma^\dagger_u\Gamma_u\right]_{st}\left[C_{\varphi e}\right]_{pr}+\left[\Gamma_e\right]^*_{vp}\left[\Gamma_u\right]^*_{ws}\left[C^{(1)}_{\ell equ}-12C^{(3)}_{\ell equ}\right]_{vrwt}\\
&+\left[\Gamma_e\right]_{vr}\left[\Gamma_u\right]_{wt}\left[C^{(1)}_{\ell equ}-12C^{(3)}_{\ell equ}\right]^*_{vpws}-2\left[\Gamma_e\right]^*_{vp}\left[\Gamma_e\right]_{wr}\left[C_{\ell u}\right]_{vwst}-2\left[\Gamma_u\right]^*_{vs}\left[\Gamma_u\right]_{wt}\left[C_{qe}\right]_{vwpr}\\
&+\left[\gamma^{(Y)}_e\right]_{pv}\left[C_{eu}\right]_{vrst}+\left[\gamma^{(Y)}_u\right]_{sv}\left[C_{eu}\right]_{prvt}+\left[C_{eu}\right]_{pvst}\left[\gamma^{(Y)}_e\right]_{vr}+\left[C_{eu}\right]_{prsv}\left[\gamma^{(Y)}_u\right]_{vt},
\end{aligned}
\tag{B.59}
$$

$$
\begin{aligned}
\left[\beta_{ed}\right]_{prst} =& -\frac{2}{3}g'^2\left(\left[C_{\varphi d}\right]_{st}+2\left[C^{(1)}_{qd}-C_{\ell d}-2C_{dd}-C_{ed}+2C^{(1)}_{ud}\right]_{wwst}-\frac{4}{3}\left[C_{dd}\right]_{swwt}\right)\delta_{pr}\\
&-\frac{2}{9}g'^2\left(\left[C_{\varphi e}\right]_{pr}+2\left[C_{qe}-C_{\ell e}-4C_{ee}\right]_{wwpr}-2\left[C_{ed}-2C_{eu}\right]_{prww}\right)\delta_{st}+4g'^2\left[C_{ed}\right]_{prst}\\
&+2\left[\Gamma^\dagger_e\Gamma_e\right]_{pr}\left[C_{\varphi d}\right]_{st}+2\left[\Gamma^\dagger_d\Gamma_d\right]_{st}\left[C_{\varphi e}\right]_{pr}-2\left[\Gamma_e\right]^*_{vp}\left[\Gamma_e\right]_{wr}\left[C_{\ell d}\right]_{vwst}\\
&-2\left[\Gamma_d\right]^*_{vs}\left[\Gamma_d\right]_{wt}\left[C_{qe}\right]_{vwpr}+\left[\Gamma_e\right]^*_{vp}\left[\Gamma_d\right]_{wt}\left[C_{\ell edq}\right]_{vrsw}+\left[\Gamma_e\right]_{vr}\left[\Gamma_d\right]^*_{ws}\left[C_{\ell edq}\right]^*_{vptw}\\
&+\left[\gamma^{(Y)}_e\right]_{pv}\left[C_{ed}\right]_{vrst}+\left[\gamma^{(Y)}_d\right]_{sv}\left[C_{ed}\right]_{prvt}+\left[C_{ed}\right]_{pvst}\left[\gamma^{(Y)}_e\right]_{vr}+\left[C_{ed}\right]_{prsv}\left[\gamma^{(Y)}_d\right]_{vt},
\end{aligned}
\tag{B.60}
$$

$$
\begin{aligned}
\left[\beta^{(1)}_{ud}\right]_{prst} =& \frac{4}{9}g'^2\left(\left[C_{\varphi d}\right]_{st}+2\left[C^{(1)}_{qd}-C_{\ell d}-2C_{dd}+2C^{(1)}_{ud}-C_{ed}\right]_{wwst}-\frac{4}{3}\left[C_{dd}\right]_{swwt}\right)\delta_{pr}\\
&-\frac{2}{9}g'^2\left(\left[C_{\varphi u}\right]_{pr}+2\left[C^{(1)}_{qu}-C_{\ell u}+4C_{uu}-C_{eu}\right]_{wwpr}-2\left[C^{(1)}_{ud}\right]_{prww}+\frac{8}{3}\left[C_{uu}\right]_{pwwr}\right)\delta_{st}\\
&-\frac{8}{3}\left(g'^2\left[C^{(1)}_{ud}\right]_{prst}-g_s^2\left[C^{(8)}_{ud}\right]_{prst}\right)-2\left[\Gamma^\dagger_u\Gamma_u\right]_{pr}\left[C_{\varphi d}\right]_{st}+2\left[\Gamma^\dagger_d\Gamma_d\right]_{st}\left[C_{\varphi u}\right]_{pr}\\
&+\frac{2}{3}\left[\Gamma^\dagger_d\Gamma_u\right]_{sr}\left[C_{\varphi ud}\right]_{pt}+\frac{2}{3}\left[\Gamma^\dagger_u\Gamma_d\right]_{pt}\left[C_{\varphi ud}\right]^*_{rs}-\left[\Gamma_d\right]^*_{ws}\left[\Gamma_u\right]^*_{vp}\left[C^{(1)}_{quqd}\right]_{vrwt}\\
&+\frac{1}{3}\left(\left[\Gamma_d\right]^*_{vs}\left[\Gamma_u\right]^*_{wp}\left[C^{(1)}_{quqd}+\frac{4}{3}C^{(8)}_{quqd}\right]_{vrwt}+\left[\Gamma_d\right]_{vt}\left[\Gamma_u\right]_{wr}\left[C^{(1)}_{quqd}+\frac{4}{3}C^{(8)}_{quqd}\right]^*_{vpws}\right)
\end{aligned}
\tag{B.61}
$$

$$
\begin{aligned}
&-\left[\Gamma_d\right]_{wt}\left[\Gamma_u\right]_{vr}\left[C^{(1)}_{quqd}\right]^*_{vpws}-2\left[\Gamma_u\right]^*_{vp}\left[\Gamma_u\right]_{wr}\left[C^{(1)}_{qd}\right]_{vwst}-2\left[\Gamma_d\right]^*_{vs}\left[\Gamma_d\right]_{wt}\left[C^{(1)}_{qu}\right]_{vwpr}\\
&+\left[\gamma^{(Y)}_u\right]_{pv}\left[C^{(1)}_{ud}\right]_{vrst}+\left[\gamma^{(Y)}_d\right]_{sv}\left[C^{(1)}_{ud}\right]_{prvt}+\left[C^{(1)}_{ud}\right]_{pvst}\left[\gamma^{(Y)}_u\right]_{vr}+\left[C^{(1)}_{ud}\right]_{prsv}\left[\gamma^{(Y)}_d\right]_{vt},
\end{aligned}
$$

$$\begin{aligned}
\left[\beta_{ud}^{(8)}\right]_{prst} &= \frac{8}{3}g_s^2\,[C_{uu}]_{pwwr}\,\delta_{st} + \frac{8}{3}g_s^2\,[C_{dd}]_{swwt}\,\delta_{pr} + \frac{4}{3}g_s^2\left[C_{qu}^{(8)}\right]_{wwpr}\delta_{st} + \frac{4}{3}g_s^2\left[C_{qd}^{(8)}\right]_{wwst}\delta_{pr} \\
&+ \frac{2}{3}g_s^2\left[C_{ud}^{(8)}\right]_{prww}\delta_{st} + \frac{2}{3}g_s^2\left[C_{ud}^{(8)}\right]_{wwst}\delta_{pr} - 4\left(\frac{2}{3}g'^2 + g_s^2\right)\left[C_{ud}^{(8)}\right]_{prst} + 12g_s^2\left[C_{ud}^{(1)}\right]_{prst} \\
&+ 4\left[\Gamma_d^\dagger\Gamma_u\right]_{sr}\left[C_{\varphi ud}\right]_{pt} + 4\left[\Gamma_u^\dagger\Gamma_d\right]_{pt}\left[C_{\varphi ud}\right]^*_{rs} - 2\,[\Gamma_u]^*_{vp}\,[\Gamma_u]_{wr}\left[C_{qd}^{(8)}\right]_{vwst} \\
&+ 2\left([\Gamma_d]^*_{vs}\,[\Gamma_u]^*_{wp}\left[C_{quqd}^{(1)} - \frac{1}{6}C_{quqd}^{(8)}\right]_{vrwt} + [\Gamma_d]_{vt}\,[\Gamma_u]_{wr}\left[C_{quqd}^{(1)} - \frac{1}{6}C_{quqd}^{(8)}\right]^*_{vpws}\right) \qquad \text{(B.62)} \\
&- 2\,[\Gamma_d]^*_{vs}\,[\Gamma_d]_{wt}\left[C_{qu}^{(8)}\right]_{vwpr} - \left([\Gamma_d]^*_{ws}\,[\Gamma_u]^*_{vp}\left[C_{quqd}^{(8)}\right]_{vrwt} + [\Gamma_d]_{wt}\,[\Gamma_u]_{vr}\left[C_{quqd}^{(8)}\right]^*_{vpws}\right) \\
&+ \left[\gamma_u^{(Y)}\right]_{pv}\left[C_{ud}^{(8)}\right]_{vrst} + \left[\gamma_d^{(Y)}\right]_{sv}\left[C_{ud}^{(8)}\right]_{prvt} + \left[C_{ud}^{(8)}\right]_{pvst}\left[\gamma_u^{(Y)}\right]_{vr} + \left[C_{ud}^{(8)}\right]_{prsv}\left[\gamma_d^{(Y)}\right]_{vt}.
\end{aligned}$$

$\boxed{(\bar{L}L)\,(\bar{R}R)}$

$$\begin{aligned}
[\beta_{\ell e}]_{prst} &= -\frac{1}{3}g'^2\,[C_{\varphi e}]_{st}\,\delta_{pr} - \frac{2}{3}g'^2\left[C_{\varphi\ell}^{(1)}\right]_{pr}\delta_{st} + \frac{8}{3}g'^2\,[C_{\ell\ell}]_{prww}\,\delta_{st} + \frac{4}{3}g'^2\,[C_{\ell\ell}]_{pwwr}\,\delta_{st} \\
&- \frac{4}{3}g'^2\left[C_{\ell q}^{(1)}\right]_{prww}\delta_{st} - \frac{2}{3}g'^2\,[C_{qe}]_{wwst}\,\delta_{pr} + \frac{4}{3}g'^2\,[C_{\ell e}]_{prww}\,\delta_{st} + \frac{2}{3}g'^2\,[C_{\ell e}]_{wwst}\,\delta_{pr} \\
&- \frac{8}{3}g'^2\,[C_{\ell u}]_{prww}\,\delta_{st} + \frac{4}{3}g'^2\,[C_{\ell d}]_{prww}\,\delta_{st} - \frac{4}{3}g'^2\,[C_{eu}]_{stww}\,\delta_{pr} + \frac{2}{3}g'^2\,[C_{ed}]_{stww}\,\delta_{pr} \\
&+ \frac{8}{3}g'^2\,[C_{ee}]_{wwst}\,\delta_{pr} - 6g'^2\,[C_{\ell e}]_{prst} + [\Gamma_e]^*_{rs}\,[\xi_e]_{pt} + [\Gamma_e]_{pt}\,[\xi_e]^*_{rs} - \left[\Gamma_e\Gamma_e^\dagger\right]_{pr}[C_{\varphi e}]_{st} \\
&+ 2\left[\Gamma_e^\dagger\Gamma_e\right]_{st}\left[C_{\varphi\ell}^{(1)}\right]_{pr} - 4\,[\Gamma_e]_{pv}\,[\Gamma_e]^*_{rw}\,[C_{ee}]_{vtsw} + [\Gamma_e]_{pw}\,[\Gamma_e]^*_{vs}\,[C_{\ell e}]_{vrwt} \\
&- 2\,[\Gamma_e]_{wt}\,[\Gamma_e]^*_{vs}\,[C_{\ell\ell}]_{pwvr} - 4\,[\Gamma_e]_{wt}\,[\Gamma_e]^*_{vs}\,[C_{\ell\ell}]_{prvw} + [\Gamma_e]_{vt}\,[\Gamma_e]^*_{rw}\,[C_{\ell e}]_{pvsw} \\
&+ \left[\gamma_\ell^{(Y)}\right]_{pv}[C_{\ell e}]_{vrst} + \left[\gamma_e^{(Y)}\right]_{sv}[C_{\ell e}]_{prvt} + [C_{\ell e}]_{pvst}\left[\gamma_\ell^{(Y)}\right]_{vr} + [C_{\ell e}]_{prsv}\left[\gamma_e^{(Y)}\right]_{vt}, \qquad \text{(B.63)}
\end{aligned}$$

$$\begin{aligned}
[\beta_{\ell u}]_{prst} &= -\frac{1}{3}g'^2\,[C_{\varphi u}]_{st}\,\delta_{pr} + \frac{4}{9}g'^2\left[C_{\varphi\ell}^{(1)}\right]_{pr}\delta_{st} - \frac{16}{9}g'^2\,[C_{\ell\ell}]_{prww}\,\delta_{st} - \frac{8}{9}g'^2\,[C_{\ell\ell}]_{pwwr}\,\delta_{st} \\
&+ \frac{8}{9}g'^2\left[C_{\ell q}^{(1)}\right]_{prww}\delta_{st} - \frac{2}{3}g'^2\left[C_{qu}^{(1)}\right]_{wwst}\delta_{pr} + \frac{16}{9}g'^2\,[C_{\ell u}]_{prww}\,\delta_{st} + \frac{2}{3}g'^2\,[C_{\ell u}]_{wwst}\,\delta_{pr} \\
&- \frac{8}{9}g'^2\,[C_{\ell d}]_{prww}\,\delta_{st} - \frac{8}{9}g'^2\,[C_{\ell e}]_{prww}\,\delta_{st} + \frac{2}{3}g'^2\left[C_{ud}^{(1)}\right]_{stww}\delta_{pr} + \frac{2}{3}g'^2\,[C_{eu}]_{wwst}\,\delta_{pr} \\
&- \frac{8}{3}g'^2\,[C_{uu}]_{stww}\,\delta_{pr} - \frac{8}{9}g'^2\,[C_{uu}]_{swwt}\,\delta_{pr} + 4g'^2\,[C_{\ell u}]_{prst} - \left[\Gamma_e\Gamma_e^\dagger\right]_{pr}[C_{\varphi u}]_{st} \\
&- 2\left[\Gamma_u^\dagger\Gamma_u\right]_{st}\left[C_{\varphi\ell}^{(1)}\right]_{pr} - \frac{1}{2}\,[\Gamma_e]^*_{rv}\,[\Gamma_u]^*_{ws}\left[C_{\ell equ}^{(1)} + 12C_{\ell equ}^{(3)}\right]_{pvwt} - 2\,[\Gamma_u]^*_{vs}\,[\Gamma_u]_{wt}\left[C_{\ell q}^{(1)}\right]_{prvw} \\
&- \frac{1}{2}\,[\Gamma_e]_{pv}\,[\Gamma_u]_{wt}\left[C_{\ell equ}^{(1)} + 12C_{\ell equ}^{(3)}\right]^*_{rvws} - [\Gamma_e]^*_{rw}\,[\Gamma_e]_{pv}\,[C_{eu}]_{vwst} + \left[\gamma_\ell^{(Y)}\right]_{pv}[C_{\ell u}]_{vrst}
\end{aligned}$$

$$
+\left[\gamma_u^{(Y)}\right]_{sv}[C_{\ell u}]_{prvt}+[C_{\ell u}]_{pvst}\left[\gamma_\ell^{(Y)}\right]_{vr}+[C_{\ell u}]_{prsv}\left[\gamma_u^{(Y)}\right]_{vt}, \tag{B.64}
$$

$$
\begin{aligned}
[\beta_{\ell d}]_{prst} ={}& -\frac{1}{3}g'^2\,[C_{\varphi d}]_{st}\,\delta_{pr}-\frac{2}{9}g'^2\left[C_{\varphi\ell}^{(1)}\right]_{pr}\delta_{st}+\frac{8}{9}g'^2\,[C_{\ell\ell}]_{prww}\,\delta_{st}+\frac{4}{9}g'^2\,[C_{\ell\ell}]_{pwwr}\,\delta_{st}\\
&-\frac{4}{9}g'^2\left[C_{\ell q}^{(1)}\right]_{prww}\delta_{st}-\frac{2}{3}g'^2\left[C_{qd}^{(1)}\right]_{wwst}\delta_{pr}+\frac{4}{9}g'^2\,[C_{\ell d}]_{prww}\,\delta_{st}+\frac{2}{3}g'^2\,[C_{\ell d}]_{wwst}\,\delta_{pr}\\
&-\frac{8}{9}g'^2\,[C_{\ell u}]_{prww}\,\delta_{st}+\frac{4}{9}g'^2\,[C_{\ell e}]_{prww}\,\delta_{st}-\frac{4}{3}g'^2\left[C_{ud}^{(1)}\right]_{wwst}\delta_{pr}+\frac{2}{3}g'^2\,[C_{ed}]_{wwst}\,\delta_{pr}\\
&+\frac{4}{3}g'^2\,[C_{dd}]_{stww}\,\delta_{pr}+\frac{4}{9}g'^2\,[C_{dd}]_{swwt}\,\delta_{pr}-2g'^2\,[C_{\ell d}]_{prst}-\left[\Gamma_e\Gamma_e^\dagger\right]_{pr}[C_{\varphi d}]_{st}\\
&+2\left[\Gamma_d^\dagger\Gamma_d\right]_{st}\left[C_{\varphi\ell}^{(1)}\right]_{pr}-\frac{1}{2}[\Gamma_e]^*_{rv}[\Gamma_d]_{wt}\,[C_{\ell edq}]_{pvsw}-\frac{1}{2}[\Gamma_e]_{pv}[\Gamma_d]^*_{ws}\,[C_{\ell edq}]^*_{rvtw}\\
&-2[\Gamma_d]^*_{vs}[\Gamma_d]_{wt}\left[C_{\ell q}^{(1)}\right]_{prvw}-[\Gamma_e]^*_{rw}[\Gamma_e]_{pv}[C_{ed}]_{vwst}+\left[\gamma_\ell^{(Y)}\right]_{pv}[C_{\ell d}]_{vrst}\\
&+\left[\gamma_d^{(Y)}\right]_{sv}[C_{\ell d}]_{prvt}+[C_{\ell d}]_{pvst}\left[\gamma_\ell^{(Y)}\right]_{vr}+[C_{\ell d}]_{prsv}\left[\gamma_d^{(Y)}\right]_{vt},
\end{aligned} \tag{B.65}
$$

$$
\begin{aligned}
[\beta_{qe}]_{prst} ={}& \frac{1}{9}g'^2\,[C_{\varphi e}]_{st}\,\delta_{pr}-\frac{2}{3}g'^2\left[C_{\varphi q}^{(1)}\right]_{pr}\delta_{st}-\frac{8}{3}g'^2\left[C_{qq}^{(1)}\right]_{prww}\delta_{st}\\
&-\frac{4}{9}g'^2\left[C_{qq}^{(1)}+3C_{qq}^{(3)}\right]_{pwwr}\delta_{st}+\frac{4}{3}g'^2\left[C_{\ell q}^{(1)}\right]_{wwpr}\delta_{st}-\frac{2}{9}g'^2\,[C_{\ell e}]_{wwst}\,\delta_{pr}\\
&+\frac{4}{3}g'^2\,[C_{qe}]_{prww}\,\delta_{st}+\frac{2}{9}g'^2\,[C_{qe}]_{wwst}\,\delta_{pr}-\frac{8}{3}g'^2\left[C_{qu}^{(1)}\right]_{prww}\delta_{st}+\frac{4}{3}g'^2\left[C_{qd}^{(1)}\right]_{prww}\delta_{st}\\
&+\frac{4}{9}g'^2\,[C_{eu}]_{stww}\,\delta_{pr}-\frac{2}{9}g'^2\,[C_{ed}]_{stww}\,\delta_{pr}-\frac{8}{9}g'^2\,[C_{ee}]_{wwst}\,\delta_{pr}+2g'^2\,[C_{qe}]_{prst}\\
&+\left[\Gamma_u\Gamma_u^\dagger\right]_{pr}[C_{\varphi e}]_{st}-\left[\Gamma_d\Gamma_d^\dagger\right]_{pr}[C_{\varphi e}]_{st}+2\left[\Gamma_e^\dagger\Gamma_e\right]_{st}\left[C_{\varphi q}^{(1)}\right]_{pr}\\
&-\frac{1}{2}[\Gamma_d]_{pw}[\Gamma_e]^*_{vs}\,[C_{\ell edq}]_{vtwr}-\frac{1}{2}[\Gamma_e]_{vt}[\Gamma_d]^*_{rw}\,[C_{\ell edq}]^*_{vswp}-2[\Gamma_e]^*_{vs}[\Gamma_e]_{wt}\left[C_{\ell q}^{(1)}\right]_{vwpr}\\
&-\frac{1}{2}[\Gamma_u]^*_{rw}[\Gamma_e]^*_{vs}\left[C_{\ell equ}^{(1)}+12C_{\ell equ}^{(3)}\right]_{vtpw}-\frac{1}{2}[\Gamma_u]_{pw}[\Gamma_e]_{vt}\left[C_{\ell equ}^{(1)}+12C_{\ell equ}^{(3)}\right]^*_{vsrw}\\
&-[\Gamma_d]^*_{rw}[\Gamma_d]_{pv}[C_{ed}]_{stvw}-[\Gamma_u]^*_{rw}[\Gamma_u]_{pv}[C_{eu}]_{stvw}+\left[\gamma_q^{(Y)}\right]_{pv}[C_{qe}]_{vrst}\\
&+\left[\gamma_e^{(Y)}\right]_{sv}[C_{qe}]_{prvt}+[C_{qe}]_{pvst}\left[\gamma_q^{(Y)}\right]_{vr}+[C_{qe}]_{prsv}\left[\gamma_e^{(Y)}\right]_{vt},
\end{aligned} \tag{B.66}
$$

$$
\begin{aligned}
\left[\beta_{qu}^{(1)}\right]_{prst} ={}& \frac{1}{9}g'^2\,[C_{\varphi u}]_{st}\,\delta_{pr}+\frac{4}{9}g'^2\left[C_{\varphi q}^{(1)}\right]_{pr}\delta_{st}+\frac{16}{9}g'^2\left[C_{qq}^{(1)}\right]_{prww}\delta_{st}\\
&+\frac{8}{27}g'^2\left[C_{qq}^{(1)}+3C_{qq}^{(3)}\right]_{pwwr}\delta_{st}-\frac{8}{9}g'^2\left[C_{\ell q}^{(1)}\right]_{wwpr}\delta_{st}-\frac{8}{9}g'^2\,[C_{qe}]_{prww}\,\delta_{st}\\
&-\frac{8}{9}g'^2\left[C_{qd}^{(1)}\right]_{prww}\delta_{st}+\frac{16}{9}g'^2\left[C_{qu}^{(1)}\right]_{prww}\delta_{st}+\frac{2}{9}g'^2\left[C_{qu}^{(1)}\right]_{wwst}\delta_{pr}\\
&-\frac{2}{9}g'^2\,[C_{\ell u}]_{wwst}\,\delta_{pr}-\frac{2}{9}g'^2\,[C_{eu}]_{wwst}\,\delta_{pr}-\frac{2}{9}g'^2\left[C_{ud}^{(1)}\right]_{stww}\delta_{pr}+\frac{8}{9}g'^2\,[C_{uu}]_{stww}\,\delta_{pr}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{8}{27}g'^2\left[C_{uu}\right]_{swwt}\delta_{pr}-\frac{4}{3}g'^2\left[C^{(1)}_{qu}\right]_{prst}-\frac{8}{3}g_s^2\left[C^{(8)}_{qu}\right]_{prst}+\frac{1}{3}\left[\Gamma_u\right]^*_{rs}\left[\xi_u\right]_{pt}\\
&+\frac{1}{3}\left[\Gamma_u\right]_{pt}\left[\xi_u\right]^*_{rs}+\left[\Gamma_u\Gamma_u^\dagger-\Gamma_d\Gamma_d^\dagger\right]_{pr}\left[C_{\varphi u}\right]_{st}-2\left[\Gamma_u^\dagger\Gamma_u\right]_{st}\left[C^{(1)}_{\varphi q}\right]_{pr}\\
&+\frac{1}{3}\left[\Gamma_u\right]_{pw}\left[\Gamma_u\right]^*_{vs}\left[C^{(1)}_{qu}+\frac{4}{3}C^{(8)}_{qu}\right]_{vrwt}+\frac{1}{3}\left[\Gamma_u\right]_{vt}\left[\Gamma_u\right]^*_{rw}\left[C^{(1)}_{qu}+\frac{4}{3}C^{(8)}_{qu}\right]_{pvsw}\\
&+\frac{1}{3}\left[\Gamma_d\right]^*_{rw}\left[\Gamma_u\right]^*_{vs}\left[C^{(1)}_{quqd}+\frac{4}{3}C^{(8)}_{quqd}\right]_{ptvw}+\frac{1}{3}\left[\Gamma_d\right]_{pw}\left[\Gamma_u\right]_{vt}\left[C^{(1)}_{quqd}+\frac{4}{3}C^{(8)}_{quqd}\right]^*_{rsvw}\\
&+\frac{1}{2}\left[\Gamma_d\right]^*_{rw}\left[\Gamma_u\right]^*_{vs}\left[C^{(1)}_{quqd}\right]_{vtpw}+\frac{1}{2}\left[\Gamma_d\right]_{pw}\left[\Gamma_u\right]_{vt}\left[C^{(1)}_{quqd}\right]^*_{vsrw}\\
&-\frac{2}{3}\left[\Gamma_u\right]_{vt}\left[\Gamma_u\right]^*_{ws}\left[C^{(1)}_{qq}+3C^{(3)}_{qq}\right]_{pvwr}-4\left[\Gamma_u\right]_{wt}\left[\Gamma_u\right]^*_{vs}\left[C^{(1)}_{qq}\right]_{prvw} \qquad \text{(B.67)}\\
&-\frac{2}{3}\left[\Gamma_u\right]_{pv}\left[\Gamma_u\right]^*_{rw}\left[C_{uu}\right]_{vtsw}-2\left[\Gamma_u\right]_{pv}\left[\Gamma_u\right]^*_{rw}\left[C_{uu}\right]_{vwst}-\left[\Gamma_d\right]_{pv}\left[\Gamma_d\right]^*_{rw}\left[C^{(1)}_{ud}\right]_{stvw}\\
&+\left[\gamma_q^{(Y)}\right]_{pv}\left[C^{(1)}_{qu}\right]_{vrst}+\left[\gamma_u^{(Y)}\right]_{sv}\left[C^{(1)}_{qu}\right]_{prvt}+\left[C^{(1)}_{qu}\right]_{pvst}\left[\gamma_q^{(Y)}\right]_{vr}+\left[C^{(1)}_{qu}\right]_{prsv}\left[\gamma_u^{(Y)}\right]_{vt},
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta^{(1)}_{qd}\right]_{prst}=&\frac{1}{9}g'^2\left[C_{\varphi d}\right]_{st}\delta_{pr}-\frac{2}{9}g'^2\left[C^{(1)}_{\varphi q}\right]_{pr}\delta_{st}-\frac{8}{9}g'^2\left[C^{(1)}_{qq}\right]_{prww}\delta_{st}\\
&-\frac{4}{27}g'^2\left[C^{(1)}_{qq}+3C^{(3)}_{qq}\right]_{pwwr}\delta_{st}+\frac{4}{9}g'^2\left[C^{(1)}_{\ell q}\right]_{wwpr}\delta_{st}+\frac{4}{9}g'^2\left[C_{qe}\right]_{prww}\delta_{st}\\
&-\frac{8}{9}g'^2\left[C^{(1)}_{qu}\right]_{prww}\delta_{st}+\frac{4}{9}g'^2\left[C^{(1)}_{qd}\right]_{prww}\delta_{st}+\frac{2}{9}g'^2\left[C^{(1)}_{qd}\right]_{wwst}\delta_{pr}\\
&-\frac{2}{9}g'^2\left[C_{\ell d}\right]_{wwst}\delta_{pr}-\frac{2}{9}g'^2\left[C_{ed}\right]_{wwst}\delta_{pr}+\frac{4}{9}g'^2\left[C^{(1)}_{ud}\right]_{wwst}\delta_{pr}-\frac{4}{9}g'^2\left[C_{dd}\right]_{stww}\delta_{pr}\\
&-\frac{4}{27}g'^2\left[C_{dd}\right]_{swwt}\delta_{pr}+\frac{2}{3}g'^2\left[C^{(1)}_{qd}\right]_{prst}-\frac{8}{3}g_s^2\left[C^{(8)}_{qd}\right]_{prst}+\frac{1}{3}\left[\Gamma_d\right]^*_{rs}\left[\xi_d\right]_{pt}\\
&+\frac{1}{3}\left[\Gamma_d\right]_{pt}\left[\xi_d\right]^*_{rs}+\left[\Gamma_u\Gamma_u^\dagger-\Gamma_d\Gamma_d^\dagger\right]_{pr}\left[C_{\varphi d}\right]_{st}+2\left[\Gamma_d^\dagger\Gamma_d\right]_{st}\left[C^{(1)}_{\varphi q}\right]_{pr}\\
&+\frac{1}{3}\left[\Gamma_d\right]_{pw}\left[\Gamma_d\right]^*_{vs}\left[C^{(1)}_{qd}+\frac{4}{3}C^{(8)}_{qd}\right]_{vrwt}+\frac{1}{3}\left[\Gamma_d\right]_{vt}\left[\Gamma_d\right]^*_{rw}\left[C^{(1)}_{qd}+\frac{4}{3}C^{(8)}_{qd}\right]_{pvsw}\\
&+\frac{1}{3}\left[\Gamma_u\right]^*_{rw}\left[\Gamma_d\right]^*_{vs}\left[C^{(1)}_{quqd}+\frac{4}{3}C^{(8)}_{quqd}\right]_{vwpt}+\frac{1}{3}\left[\Gamma_u\right]_{pw}\left[\Gamma_d\right]_{vt}\left[C^{(1)}_{quqd}+\frac{4}{3}C^{(8)}_{quqd}\right]^*_{vwrs}\\
&+\frac{1}{2}\left[\Gamma_d\right]^*_{ws}\left[\Gamma_u\right]^*_{rv}\left[C^{(1)}_{quqd}\right]_{pvwt}+\frac{1}{2}\left[\Gamma_u\right]_{pv}\left[\Gamma_d\right]_{wt}\left[C^{(1)}_{quqd}\right]^*_{rvws}\\
&-\frac{2}{3}\left[\Gamma_d\right]_{vt}\left[\Gamma_d\right]^*_{ws}\left[C^{(1)}_{qq}+3C^{(3)}_{qq}\right]_{pvwr}-4\left[\Gamma_d\right]_{wt}\left[\Gamma_d\right]^*_{vs}\left[C^{(1)}_{qq}\right]_{prvw} \qquad \text{(B.68)}\\
&-\frac{2}{3}\left[\Gamma_d\right]_{pv}\left[\Gamma_d\right]^*_{rw}\left[C_{dd}\right]_{vtsw}-2\left[\Gamma_d\right]_{pv}\left[\Gamma_d\right]^*_{rw}\left[C_{dd}\right]_{vwst}-\left[\Gamma_u\right]_{pv}\left[\Gamma_u\right]^*_{rw}\left[C^{(1)}_{ud}\right]_{vwst}\\
&+\left[\gamma_q^{(Y)}\right]_{pv}\left[C^{(1)}_{qd}\right]_{vrst}+\left[\gamma_d^{(Y)}\right]_{sv}\left[C^{(1)}_{qd}\right]_{prvt}+\left[C^{(1)}_{qd}\right]_{pvst}\left[\gamma_q^{(Y)}\right]_{vr}+\left[C^{(1)}_{qd}\right]_{prsv}\left[\gamma_d^{(Y)}\right]_{vt},
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta_{qu}^{(8)}\right]_{prst} &= \frac{8}{3} g_s^2 \left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{pwwr} \delta_{st} + \frac{2}{3} g_s^2 \left[C_{qu}^{(8)}\right]_{prww} \delta_{st} + \frac{2}{3} g_s^2 \left[C_{qd}^{(8)}\right]_{prww} \delta_{st} \\
&+ \frac{4}{3} g_s^2 \left[C_{qu}^{(8)}\right]_{wwst} \delta_{pr} + \frac{2}{3} g_s^2 \left[C_{ud}^{(8)}\right]_{stww} \delta_{pr} + \frac{8}{3} g_s^2 \left[C_{uu}\right]_{swwt} \delta_{pr} \\
&- \left(\frac{4}{3} g'^2 + 14 g_s^2\right) \left[C_{qu}^{(8)}\right]_{prst} - 12 g_s^2 \left[C_{qu}^{(1)}\right]_{prst} + 2 \left[\Gamma_u\right]_{rs}^* \left[\xi_u\right]_{pt} + 2 \left[\Gamma_u\right]_{pt} \left[\xi_u\right]_{rs}^* \\
&+ 2 \left[\Gamma_u\right]_{pw} \left[\Gamma_u\right]_{vs}^* \left[C_{qu}^{(1)} - \frac{1}{6} C_{qu}^{(8)}\right]_{vrwt} + 2 \left[\Gamma_u\right]_{vt} \left[\Gamma_u\right]_{rw}^* \left[C_{qu}^{(1)} - \frac{1}{6} C_{qu}^{(8)}\right]_{pvsw} \\
&+ 2 \left[\Gamma_d\right]_{rw}^* \left[\Gamma_u\right]_{vs}^* \left[C_{quqd}^{(1)} - \frac{1}{6} C_{quqd}^{(8)}\right]_{ptvw} + 2 \left[\Gamma_d\right]_{pw} \left[\Gamma_u\right]_{vt} \left[C_{quqd}^{(1)} - \frac{1}{6} C_{quqd}^{(8)}\right]_{rsvw}^* \\
&+ \frac{1}{2} \left[\Gamma_u\right]_{vs}^* \left[\Gamma_d\right]_{rw}^* \left[C_{quqd}^{(8)}\right]_{vtpw} + \frac{1}{2} \left[\Gamma_u\right]_{vt} \left[\Gamma_d\right]_{pw} \left[C_{quqd}^{(8)}\right]_{vsrw}^* - 4 \left[\Gamma_u\right]_{vt} \left[\Gamma_u\right]_{ws}^* \left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{pvwr} \\
&- 4 \left[\Gamma_u\right]_{pv} \left[\Gamma_u\right]_{rw}^* \left[C_{uu}\right]_{vtsw} - \left[\Gamma_d\right]_{pv} \left[\Gamma_d\right]_{rw}^* \left[C_{ud}^{(8)}\right]_{stvw} + \left[\gamma_q^{(Y)}\right]_{pv} \left[C_{qu}^{(8)}\right]_{vrst} \\
&+ \left[\gamma_u^{(Y)}\right]_{sv} \left[C_{qu}^{(8)}\right]_{prvt} + \left[C_{qu}^{(8)}\right]_{pvst} \left[\gamma_q^{(Y)}\right]_{vr} + \left[C_{qu}^{(8)}\right]_{prsv} \left[\gamma_u^{(Y)}\right]_{vt}, \qquad \text{(B.69)}
\end{aligned}
$$

$$
\begin{aligned}
\left[\beta_{qd}^{(8)}\right]_{prst} &= \frac{8}{3} g_s^2 \left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{pwwr} \delta_{st} + \frac{2}{3} g_s^2 \left[C_{qu}^{(8)}\right]_{prww} \delta_{st} + \frac{2}{3} g_s^2 \left[C_{qd}^{(8)}\right]_{prww} \delta_{st} \\
&+ \frac{4}{3} g_s^2 \left[C_{qd}^{(8)}\right]_{wwst} \delta_{pr} + \frac{2}{3} g_s^2 \left[C_{ud}^{(8)}\right]_{wwst} \delta_{pr} + \frac{8}{3} g_s^2 \left[C_{dd}\right]_{swwt} \delta_{pr} \\
&- \left(-\frac{2}{3} g'^2 + 14 g_s^2\right) \left[C_{qd}^{(8)}\right]_{prst} - 12 g_s^2 \left[C_{qd}^{(1)}\right]_{prst} + 2 \left[\Gamma_d\right]_{rs}^* \left[\xi_d\right]_{pt} + 2 \left[\Gamma_d\right]_{pt} \left[\xi_d\right]_{rs}^* \\
&+ 2 \left[\Gamma_d\right]_{pw} \left[\Gamma_d\right]_{vs}^* \left[C_{qd}^{(1)} - \frac{1}{6} C_{qd}^{(8)}\right]_{vrwt} + 2 \left[\Gamma_d\right]_{vt} \left[\Gamma_d\right]_{rw}^* \left[C_{qd}^{(1)} - \frac{1}{6} C_{qd}^{(8)}\right]_{pvsw} \\
&+ 2 \left[\Gamma_u\right]_{rw}^* \left[\Gamma_d\right]_{vs}^* \left[C_{quqd}^{(1)} - \frac{1}{6} C_{quqd}^{(8)}\right]_{vwpt} + 2 \left[\Gamma_u\right]_{pw} \left[\Gamma_d\right]_{vt} \left[C_{quqd}^{(1)} - \frac{1}{6} C_{quqd}^{(8)}\right]_{vwrs}^* \\
&+ \frac{1}{2} \left[\Gamma_d\right]_{vs}^* \left[\Gamma_u\right]_{rw}^* \left[C_{quqd}^{(8)}\right]_{pwvt} + \frac{1}{2} \left[\Gamma_d\right]_{vt} \left[\Gamma_u\right]_{pw} \left[C_{quqd}^{(8)}\right]_{rwvs}^* \\
&- 4 \left[\Gamma_d\right]_{vt} \left[\Gamma_d\right]_{ws}^* \left[C_{qq}^{(1)} + 3C_{qq}^{(3)}\right]_{pvwr} \\
&- 4 \left[\Gamma_d\right]_{pv} \left[\Gamma_d\right]_{rw}^* \left[C_{dd}\right]_{vtsw} - \left[\Gamma_u\right]_{pv} \left[\Gamma_u\right]_{rw}^* \left[C_{ud}^{(8)}\right]_{vwst} + \left[\gamma_q^{(Y)}\right]_{pv} \left[C_{qd}^{(8)}\right]_{vrst} \\
&+ \left[\gamma_d^{(Y)}\right]_{sv} \left[C_{qd}^{(8)}\right]_{prvt} + \left[C_{qd}^{(8)}\right]_{pvst} \left[\gamma_q^{(Y)}\right]_{vr} + \left[C_{qd}^{(8)}\right]_{prsv} \left[\gamma_d^{(Y)}\right]_{vt}, \qquad \text{(B.70)}
\end{aligned}
$$

$\boxed{(\bar{L}R)\,(\bar{R}L)}$

$$
\begin{aligned}
\left[\beta_{\ell edq}\right]_{prst} &= -\left(\frac{8}{3} g'^2 + 8 g_s^2\right) \left[C_{\ell edq}\right]_{prst} - 2 \left[\Gamma_d\right]_{ts}^* \left[\xi_e\right]_{pr} - 2 \left[\Gamma_e\right]_{pr} \left[\xi_d\right]_{ts}^* + 2 \left[\Gamma_e\right]_{pv} \left[\Gamma_d\right]_{tw}^* \left[C_{ed}\right]_{vrsw} \\
&- 2 \left[\Gamma_e\right]_{vr} \left[\Gamma_d\right]_{tw}^* \left[C_{\ell d}\right]_{pvsw} + 2 \left[\Gamma_e\right]_{vr} \left[\Gamma_d\right]_{ws}^* \left[C_{\ell q}^{(1)}\right]_{pvwt} + 6 \left[\Gamma_e\right]_{vr} \left[\Gamma_d\right]_{ws}^* \left[C_{\ell q}^{(3)}\right]_{pvwt}
\end{aligned}
$$

$$- 2\,[\Gamma_e]_{pw}\,[\Gamma_d]^*_{vs}\,\left[C_{qe}\right]_{vtwr} + 2\,[\Gamma_d]^*_{vs}\,[\Gamma_u]^*_{tw}\,\left[C^{(1)}_{\ell equ}\right]_{prvw} + \left[\gamma^{(Y)}_{\ell}\right]_{pv}\,\left[C_{\ell edq}\right]_{vrst}$$

$$+ \left[\gamma^{(Y)}_{d}\right]_{sv}\,\left[C_{\ell edq}\right]_{prvt} + \left[C_{\ell edq}\right]_{pvst}\,\left[\gamma^{(Y)}_{e}\right]_{vr} + \left[C_{\ell edq}\right]_{prsv}\,\left[\gamma^{(Y)}_{q}\right]_{vt}\,, \tag{B.71}$$

$\boxed{(\bar{L}R)\,(\bar{L}R)}$

$$\begin{aligned}
\left[\beta^{(1)}_{quqd}\right]_{prst} &= \frac{10}{3}g'\,[C_{dB}]_{st}\,[\Gamma_u]_{pr} - 6g\,[C_{dW}]_{st}\,[\Gamma_u]_{pr} - \frac{20}{9}g'\,[C_{dB}]_{pt}\,[\Gamma_u]_{sr} + 4g\,[C_{dW}]_{pt}\,[\Gamma_u]_{sr} \\
&- \frac{64}{9}g_s\,[C_{dG}]_{pt}\,[\Gamma_u]_{sr} - \frac{2}{3}g'\,[C_{uB}]_{pr}\,[\Gamma_d]_{st} - 6g\,[C_{uW}]_{pr}\,[\Gamma_d]_{st} + \frac{4}{9}g'\,[C_{uB}]_{sr}\,[\Gamma_d]_{pt} \\
&+ 4g\,[C_{uW}]_{sr}\,[\Gamma_d]_{pt} - \frac{64}{9}g_s\,[C_{uG}]_{sr}\,[\Gamma_d]_{pt} - \frac{1}{2}\left(\frac{11}{9}g'^2 + 3g^2 + 32g_s^2\right)\left[C^{(1)}_{quqd}\right]_{prst} \\
&- \frac{1}{3}\left(-\frac{5}{9}g'^2 - 3g^2 + \frac{64}{3}g_s^2\right)\left[C^{(1)}_{quqd}\right]_{srpt} - \frac{4}{9}\left(-\frac{5}{9}g'^2 - 3g^2 + \frac{28}{3}g_s^2\right)\left[C^{(8)}_{quqd}\right]_{srpt} \\
&+ \frac{16}{9}g_s^2\left[C^{(8)}_{quqd}\right]_{prst} - 2\,[\Gamma_u]_{pr}\,[\xi_d]_{st} - 2\,[\Gamma_d]_{st}\,[\xi_u]_{pr} + \frac{4}{3}\Bigg([\Gamma_u]_{vr}\,[\Gamma_d]_{pw}\left[C^{(1)}_{qd} + \frac{4}{3}C^{(8)}_{qd}\right]_{svwt} \\
&+ [\Gamma_d]_{vt}\,[\Gamma_u]_{sw}\left[C^{(1)}_{qu} + \frac{4}{3}C^{(8)}_{qu}\right]_{pvwr} + [\Gamma_d]_{pw}\,[\Gamma_u]_{sv}\left[C^{(1)}_{ud} + \frac{4}{3}C^{(8)}_{ud}\right]_{vrwt}\Bigg) \\
&+ \frac{8}{3}\,[\Gamma_d]_{wt}\,[\Gamma_u]_{vr}\left(\left[C^{(1)}_{qq} - 3\,C^{(3)}_{qq}\right]_{svpw} - 3\left[C^{(1)}_{qq} - 3C^{(3)}_{qq}\right]_{swpv}\right) \\
&- 4\,[\Gamma_d]_{sw}\,[\Gamma_u]_{pv}\left[C^{(1)}_{ud}\right]_{vrwt} + \left[\gamma^{(Y)}_{q}\right]_{pv}\left[C^{(1)}_{quqd}\right]_{vrst} + \left[\gamma^{(Y)}_{q}\right]_{sv}\left[C^{(1)}_{quqd}\right]_{prvt} \\
&+ \left[C^{(1)}_{quqd}\right]_{pvst}\left[\gamma^{(Y)}_{u}\right]_{vr} + \left[C^{(1)}_{quqd}\right]_{prsv}\left[\gamma^{(Y)}_{d}\right]_{vt}\,,
\end{aligned} \tag{B.72}$$

$$\begin{aligned}
\left[\beta^{(8)}_{quqd}\right]_{prst} &= 8g_s\,[C_{dG}]_{st}\,[\Gamma_u]_{pr} - \frac{40}{3}g'\,[C_{dB}]_{pt}\,[\Gamma_u]_{sr} + 24g\,[C_{dW}]_{pt}\,[\Gamma_u]_{sr} + \frac{16}{3}g_s\,[C_{dG}]_{pt}\,[\Gamma_u]_{sr} \\
&+ 8g_s\,[C_{uG}]_{pr}\,[\Gamma_d]_{st} + \frac{8}{3}g'\,[C_{uB}]_{sr}\,[\Gamma_d]_{pt} + 24g\,[C_{uW}]_{sr}\,[\Gamma_d]_{pt} + \frac{16}{3}g_s\,[C_{uG}]_{sr}\,[\Gamma_d]_{pt} \\
&+ 8g_s^2\left[C^{(1)}_{quqd}\right]_{prst} + \left(\frac{10}{9}g'^2 + 6g^2 + \frac{16}{3}g_s^2\right)\left[C^{(1)}_{quqd}\right]_{srpt} \\
&+ \left(-\frac{11}{18}g'^2 - \frac{3}{2}g^2 + \frac{16}{3}g_s^2\right)\left[C^{(8)}_{quqd}\right]_{prst} - \frac{1}{3}\left(\frac{5}{9}g'^2 + 3g^2 + \frac{44}{3}g_s^2\right)\left[C^{(8)}_{quqd}\right]_{srpt} \\
&+ 8\Bigg([\Gamma_u]_{vr}\,[\Gamma_d]_{pw}\left[C^{(1)}_{qd} - \frac{1}{6}C^{(8)}_{qd}\right]_{svwt} + [\Gamma_d]_{vt}\,[\Gamma_u]_{sw}\left[C^{(1)}_{qu} - \frac{1}{6}C^{(8)}_{qu}\right]_{pvwr} \\
&+ [\Gamma_d]_{pw}\,[\Gamma_u]_{sv}\left[C^{(1)}_{ud} - \frac{1}{6}C^{(8)}_{ud}\right]_{vrwt}\Bigg) + 16\,[\Gamma_d]_{wt}\,[\Gamma_u]_{vr}\left[C^{(1)}_{qq} - 3C^{(3)}_{qq}\right]_{svpw} \\
&- 4\,[\Gamma_d]_{sw}\,[\Gamma_u]_{pv}\left[C^{(8)}_{ud}\right]_{vrwt} + \left[\gamma^{(Y)}_{q}\right]_{pv}\left[C^{(8)}_{quqd}\right]_{vrst}
\end{aligned}$$

$$
+\left[\gamma_q^{(Y)}\right]_{sv}\left[C^{(8)}_{quqd}\right]_{prvt}+\left[C^{(8)}_{quqd}\right]_{pvst}\left[\gamma_u^{(Y)}\right]_{vr}+\left[C^{(8)}_{quqd}\right]_{prsv}\left[\gamma_d^{(Y)}\right]_{vt}\,, \tag{B.73}
$$

$$
\begin{aligned}
\left[\beta^{(1)}_{\ell equ}\right]_{prst} =& -\left(\frac{11}{3}g'^2+8g_s^2\right)\left[C^{(1)}_{\ell equ}\right]_{prst}+\left(30g'^2+18g^2\right)\left[C^{(3)}_{\ell equ}\right]_{prst}\\
&+2\left[\Gamma_u\right]_{st}\left[\xi_e\right]_{pr}+2\left[\Gamma_e\right]_{pr}\left[\xi_u\right]_{st}+2\left[\Gamma_d\right]_{sv}\left[\Gamma_u\right]_{wt}\left[C_{\ell edq}\right]_{prvw}+2\left[\Gamma_e\right]_{pv}\left[\Gamma_u\right]_{sw}\left[C_{eu}\right]_{vrwt}\\
&+2\left[\Gamma_e\right]_{vr}\left[\Gamma_u\right]_{wt}\left[C^{(1)}_{\ell q}\right]_{pvsw}-6\left[\Gamma_e\right]_{vr}\left[\Gamma_u\right]_{wt}\left[C^{(3)}_{\ell q}\right]_{pvsw}-2\left[\Gamma_e\right]_{vr}\left[\Gamma_u\right]_{sw}\left[C_{\ell u}\right]_{pvwt}\\
&-2\left[\Gamma_e\right]_{pw}\left[\Gamma_u\right]_{vt}\left[C_{qe}\right]_{svwr}+\left[\gamma_\ell^{(Y)}\right]_{pv}\left[C^{(1)}_{\ell equ}\right]_{vrst}+\left[\gamma_q^{(Y)}\right]_{sv}\left[C^{(1)}_{\ell equ}\right]_{prvt}\\
&+\left[C^{(1)}_{\ell equ}\right]_{pvst}\left[\gamma_e^{(Y)}\right]_{vr}+\left[C^{(1)}_{\ell equ}\right]_{prsv}\left[\gamma_u^{(Y)}\right]_{vt}\,,
\end{aligned} \tag{B.74}
$$

$$
\begin{aligned}
\left[\beta^{(3)}_{\ell equ}\right]_{prst} =& \frac{5}{6}g'\left[C_{eB}\right]_{pr}\left[\Gamma_u\right]_{st}-\frac{3}{2}g\left[C_{uW}\right]_{st}\left[\Gamma_e\right]_{pr}-\frac{3}{2}g'\left[C_{uB}\right]_{st}\left[\Gamma_e\right]_{pr}-\frac{3}{2}g\left[C_{eW}\right]_{pr}\left[\Gamma_u\right]_{st}\\
&+\left(\frac{2}{9}g'^2-3g^2+\frac{8}{3}g_s^2\right)\left[C^{(3)}_{\ell equ}\right]_{prst}+\frac{1}{8}\left(5g'^2+3g^2\right)\left[C^{(1)}_{\ell equ}\right]_{prst}\\
&-\frac{1}{2}\left[\Gamma_u\right]_{sw}\left[\Gamma_e\right]_{pv}\left[C_{eu}\right]_{vrwt}-\frac{1}{2}\left[\Gamma_e\right]_{vr}\left[\Gamma_u\right]_{wt}\left[C^{(1)}_{\ell q}\right]_{pvsw}+\frac{3}{2}\left[\Gamma_e\right]_{vr}\left[\Gamma_u\right]_{wt}\left[C^{(3)}_{\ell q}\right]_{pvsw}\\
&-\frac{1}{2}\left[\Gamma_e\right]_{vr}\left[\Gamma_u\right]_{sw}\left[C_{\ell u}\right]_{pvwt}-\frac{1}{2}\left[\Gamma_e\right]_{pw}\left[\Gamma_u\right]_{vt}\left[C_{qe}\right]_{svwr}+\left[\gamma_\ell^{(Y)}\right]_{pv}\left[C^{(3)}_{\ell equ}\right]_{vrst}\\
&+\left[\gamma_q^{(Y)}\right]_{sv}\left[C^{(3)}_{\ell equ}\right]_{prvt}+\left[C^{(3)}_{\ell equ}\right]_{pvst}\left[\gamma_e^{(Y)}\right]_{vr}+\left[C^{(3)}_{\ell equ}\right]_{prsv}\left[\gamma_u^{(Y)}\right]_{vt}\,.
\end{aligned} \tag{B.75}
$$

Baryon-number-violating

$$
\begin{aligned}
\left[\beta_{duq\ell}\right]_{prst} =& -\left(\frac{9}{2}g^2+\frac{11}{6}g'^2+4g_s^2\right)\left[C_{duq\ell}\right]_{prst}-\left[C_{duq\ell}\right]_{vrwt}\left[\Gamma_d\right]^*_{sv}\left[\Gamma_d\right]_{wp}\\
&-\left[C_{duq\ell}\right]_{pvwt}\left[\Gamma_u\right]^*_{sv}\left[\Gamma_u\right]_{wr}+\left(2\left[C_{duue}\right]_{prwv}+\left[C_{duue}\right]_{pwrv}\right)\left[\Gamma_e\right]^*_{tv}\left[\Gamma_u\right]^*_{sw}\\
&+\left(4\left[C_{qqq\ell}\right]_{vwst}+4\left[C_{qqq\ell}\right]_{wvst}-\left[C_{qqq\ell}\right]_{vswt}-\left[C_{qqq\ell}\right]_{wsvt}\right)\left[\Gamma_d\right]_{vp}\left[\Gamma_u\right]_{wr}\\
&+2\left[C_{qque}\right]_{wsrv}\left[\Gamma_d\right]_{wp}\left[\Gamma_e\right]^*_{tv}+\left[C_{duq\ell}\right]_{vrst}\left[\Gamma_d^\dagger\Gamma_d\right]_{vp}+\left[C_{duq\ell}\right]_{pvst}\left[\Gamma_u^\dagger\Gamma_u\right]_{vr}\\
&+\frac{1}{2}\left[C_{duq\ell}\right]_{prvt}\left[\Gamma_u\Gamma_u^\dagger+\Gamma_d\Gamma_d^\dagger\right]_{vs}+\frac{1}{2}\left[C_{duq\ell}\right]_{prsv}\left[\Gamma_e\Gamma_e^\dagger\right]_{vt}\,,
\end{aligned} \tag{B.76}
$$

$$
\begin{aligned}
\left[\beta_{qque}\right]_{prst} =& -\left(\frac{9}{2}g^2+\frac{23}{6}g'^2+4g_s^2\right)\left[C_{qque}\right]_{prst}+\Bigg[-\left[C_{qque}\right]_{pwvt}\left[\Gamma_u\right]^*_{rv}\left[\Gamma_u\right]_{ws}\\
&+\frac{1}{2}\left[C_{duq\ell}\right]_{vspw}\left[\Gamma_e\right]_{wt}\left[\Gamma_d\right]^*_{rv}-\frac{1}{2}\left(2\left[C_{duue}\right]_{vwst}+\left[C_{duue}\right]_{vswt}\right)\left[\Gamma_d\right]^*_{pv}\left[\Gamma_u\right]^*_{rw}\\
&+\frac{1}{2}\left(-2\left[C_{qqq\ell}\right]_{prwv}+\left[C_{qqq\ell}\right]_{pwrv}-2\left[C_{qqq\ell}\right]_{wprv}\right)\left[\Gamma_u\right]_{ws}\left[\Gamma_e\right]_{vt}
\end{aligned}
$$

$$+\frac{1}{2}\left[C_{qque}\right]_{vrst}\left[\Gamma_u\Gamma_u^\dagger+\Gamma_d\Gamma_d^\dagger\right]_{vp}+p\leftrightarrow r\Bigg]$$
$$+\left[C_{qque}\right]_{prvt}\left[\Gamma_u^\dagger\Gamma_u\right]_{vs}+\left[C_{qque}\right]_{prsv}\left[\Gamma_e^\dagger\Gamma_e\right]_{vt}, \tag{B.77}$$

$$\begin{aligned}\left[\beta_{qqq\ell}\right]_{prst} &= -\left(3g^2+\frac{1}{3}g'^2+4g_s^2\right)\left[C_{qqq\ell}\right]_{prst}-4g^2\left(\left[C_{qqq\ell}\right]_{rpst}+\left[C_{qqq\ell}\right]_{srpt}+\left[C_{qqq\ell}\right]_{psrt}\right)\\
&-4\left[C_{qque}\right]_{prwv}\left[\Gamma_e\right]_{tv}^*\left[\Gamma_u\right]_{sw}^*+2\left[C_{duq\ell}\right]_{vwst}\left(\left[\Gamma_d\right]_{pv}^*\left[\Gamma_u\right]_{rw}^*+\left[\Gamma_d\right]_{rv}^*\left[\Gamma_u\right]_{pw}^*\right)\\
&+\frac{1}{2}\left[C_{qqq\ell}\right]_{vrst}\left[\Gamma_u\Gamma_u^\dagger+\Gamma_d\Gamma_d^\dagger\right]_{vp}+\frac{1}{2}\left[C_{qqq\ell}\right]_{pvst}\left[\Gamma_u\Gamma_u^\dagger+\Gamma_d\Gamma_d^\dagger\right]_{vr}\\
&+\frac{1}{2}\left[C_{qqq\ell}\right]_{prvt}\left[\Gamma_u\Gamma_u^\dagger+\Gamma_d\Gamma_d^\dagger\right]_{vs}+\frac{1}{2}\left[C_{qqq\ell}\right]_{prsv}\left[\Gamma_e\Gamma_e^\dagger\right]_{vt},\end{aligned} \tag{B.78}$$

$$\begin{aligned}\left[\beta_{duue}\right]_{prst} &= -\left(2g'^2+4g_s^2\right)\left[C_{duue}\right]_{prst}-\frac{20}{3}g'^2\left[C_{duue}\right]_{psrt}\\
&+4\left[C_{duq\ell}\right]_{prwv}\left[\Gamma_u\right]_{ws}\left[\Gamma_e\right]_{vt}-8\left[C_{qque}\right]_{vwst}\left[\Gamma_d\right]_{vp}\left[\Gamma_u\right]_{wr}+\left[C_{duue}\right]_{vrst}\left[\Gamma_d^\dagger\Gamma_d\right]_{vp}\\
&+\left[C_{duue}\right]_{pvst}\left[\Gamma_u^\dagger\Gamma_u\right]_{vr}+\left[C_{duue}\right]_{prvt}\left[\Gamma_u^\dagger\Gamma_u\right]_{vs}+\left[C_{duue}\right]_{prsv}\left[\Gamma_e^\dagger\Gamma_e\right]_{vt}.\end{aligned} \tag{B.79}$$

Dimension-5

$$\left[\beta_{\ell\ell\varphi\varphi}\right]_{rs}=\left(2\lambda-3g^2+2\gamma_H^{(Y)}\right)\left[C_{\ell\ell\varphi\varphi}\right]_{rs}-\frac{3}{2}\left(\left[C_{\ell\ell\varphi\varphi}\Gamma_e\Gamma_e^\dagger\right]_{rs}+\left[\Gamma_e^*\Gamma_e^T C_{\ell\ell\varphi\varphi}\right]_{rs}\right). \tag{B.80}$$

SM parameters

The RGEs for the SM parameters get also modified in the presence of the dim-6 operators. Using an analogous notation for their β functions

$$\frac{dX}{dt}\equiv\frac{1}{16\pi^2}\beta_X\,. \tag{B.81}$$

these are given by

$$\beta_g=-\frac{19}{6}g^3-8\,g\,\frac{m^2}{\Lambda^2}C_{\varphi W}\,, \tag{B.82}$$

$$\beta_{g'}=\frac{41}{6}g'^3-8\,g'\,\frac{m^2}{\Lambda^2}C_{\varphi B}\,, \tag{B.83}$$

$$\beta_{g_s}=-7g_s^3-8\,g_s\,\frac{m^2}{\Lambda^2}C_{\varphi G}\,, \tag{B.84}$$

$$\beta_\lambda=12\lambda^2+\frac{3}{4}g'^4+\frac{3}{2}g'^2g^2+\frac{9}{4}g^4-3\left(g'^2+3g^2\right)\lambda+4\lambda\,\gamma_H^{(Y)}$$

$$
\begin{aligned}
&-4\left(3\,\mathrm{Tr}\,\Gamma_d\Gamma_d^\dagger\Gamma_d\Gamma_d^\dagger+3\,\mathrm{Tr}\,\Gamma_u\Gamma_u^\dagger\Gamma_u\Gamma_u^\dagger+\mathrm{Tr}\,\Gamma_e\Gamma_e^\dagger\Gamma_e\Gamma_e^\dagger\right)\\
&+4\frac{m^2}{\Lambda^2}\Bigg[12C_\varphi+\left(-16\lambda+\frac{10}{3}g^2\right)C_{\varphi\Box}+\left(6\lambda+\frac{3}{2}(g'^2-g^2)\right)C_{\varphi D}+2\left(\eta_1+\eta_2\right)\\
&+9g^2C_{\varphi W}+3g'^2C_{\varphi B}+3gg'C_{\varphi WB}+\frac{4}{3}g^2\left(\mathrm{Tr}\,C^{(3)}_{\varphi\ell}+3\,\mathrm{Tr}\,C^{(3)}_{\varphi q}\right)\Bigg],
\end{aligned}
\tag{B.85}
$$

$$
\beta_{m^2}=m^2\left[6\lambda-\frac{9}{2}g^2-\frac{3}{2}g'^2+2\gamma_H^{(Y)}+4\frac{m^2}{\Lambda^2}\left(C_{\varphi D}-2C_{\varphi\Box}\right)\right],
\tag{B.86}
$$

$$
\begin{aligned}
[\beta_{\Gamma_u}]_{rs}=&\ \frac{3}{2}\left(\left[\Gamma_u\Gamma_u^\dagger\Gamma_u\right]_{rs}-\left[\Gamma_d\Gamma_d^\dagger\Gamma_u\right]_{rs}\right)+\left(\gamma_H^{(Y)}-\frac{9}{4}g^2-\frac{17}{12}g'^2-8g_s^2\right)[\Gamma_u]_{rs}\\
&+2\frac{m^2}{\Lambda^2}\Bigg[3\left[C_{u\varphi}\right]_{rs}+\frac{1}{2}\left(C_{\varphi D}-2C_{\varphi\Box}\right)[\Gamma_u]_{rs}-\left[C^{(1)\dagger}_{\varphi q}\Gamma_u\right]_{rs}+3\left[C^{(3)\dagger}_{\varphi q}\Gamma_u\right]_{rs}\\
&+\left[\Gamma_u C^\dagger_{\varphi u}\right]_{rs}-\left[\Gamma_d C^\dagger_{\varphi ud}\right]_{rs}-2\left(\left[C^{(1)}_{qu}\right]_{rpts}+\frac{4}{3}\left[C^{(8)}_{qu}\right]_{rpts}\right)[\Gamma_u]_{pt}-\left[C^{(1)}_{\ell equ}\right]_{ptrs}[\Gamma_e]^*_{pt}\\
&+3\left[C^{(1)}_{quqd}\right]_{rspt}[\Gamma_d]^*_{pt}+\frac{1}{2}\left(\left[C^{(1)}_{quqd}\right]_{psrt}+\frac{4}{3}\left[C^{(8)}_{quqd}\right]_{psrt}\right)[\Gamma_d]^*_{pt}\Bigg],
\end{aligned}
\tag{B.87}
$$

$$
\begin{aligned}
[\beta_{\Gamma_d}]_{rs}=&\ \frac{3}{2}\left(\left[\Gamma_d\Gamma_d^\dagger\Gamma_d\right]_{rs}-\left[\Gamma_u\Gamma_u^\dagger\Gamma_d\right]_{rs}\right)+\left(\gamma_H^{(Y)}-\frac{9}{4}g^2-\frac{5}{12}g'^2-8g_s^2\right)[\Gamma_d]_{rs}\\
&+2\frac{m^2}{\Lambda^2}\Bigg[3\left[C_{d\varphi}\right]_{rs}+\frac{1}{2}\left(C_{\varphi D}-2C_{\varphi\Box}\right)[\Gamma_d]_{rs}+\left[C^{(1)\dagger}_{\varphi q}\Gamma_d\right]_{rs}+3\left[C^{(3)\dagger}_{\varphi q}\Gamma_d\right]_{rs}\\
&-\left[\Gamma_d C^\dagger_{\varphi d}\right]_{rs}-\left[\Gamma_u^* C_{\varphi ud}\right]_{rs}-2\left(\left[C^{(1)}_{qd}\right]_{rpts}+\frac{4}{3}\left[C^{(8)}_{qd}\right]_{rpts}\right)[\Gamma_d]_{pt}+\left[C^*_{\ell eqd}\right]_{ptsr}[\Gamma_e]_{pt}\\
&+3\left[C^{(1)}_{quqd}\right]_{ptrs}[\Gamma_u]^*_{pt}+\frac{1}{2}\left(\left[C^{(1)}_{quqd}\right]_{rpts}+\frac{4}{3}\left[C^{(8)}_{quqd}\right]_{rpts}\right)[\Gamma_u]^*_{tp}\Bigg],
\end{aligned}
\tag{B.88}
$$

$$
\begin{aligned}
[\beta_{\Gamma_e}]_{rs}=&\ \frac{3}{2}\left[\Gamma_e\Gamma_e^\dagger\Gamma_e\right]_{rs}+\left(\gamma_H^{(Y)}-\frac{3}{4}(3g^2+5g'^2)\right)[\Gamma_e]_{rs}\\
&+2\frac{m^2}{\Lambda^2}\Bigg[3\left[C_{e\varphi}\right]_{rs}+\frac{1}{2}\left(C_{\varphi D}-2C_{\varphi\Box}\right)[\Gamma_e]_{rs}+\left[C^{(1)\dagger}_{\varphi\ell}\Gamma_e\right]_{rs}+3\left[C^{(3)\dagger}_{\varphi\ell}\Gamma_e\right]_{rs}\\
&-\left[\Gamma_e C^\dagger_{\varphi e}\right]_{rs}-2\left[C_{\ell e}\right]_{rpts}[\Gamma_e]_{pt}+3\left[C_{\ell edq}\right]_{rspt}[\Gamma_d]_{tp}-3\left[C^{(1)}_{\ell equ}\right]_{rspt}[\Gamma_u]^*_{pt}\Bigg].
\end{aligned}
\tag{B.89}
$$

In the absence of contributions from the dim-6 operators one recovers the well-known SM RGEs [19–22]. Notice however that we do not include an $SU(5)$ normalization factor for g', and hence the usual expressions are found by replacing $g'^2 = 3/5\,g_1^2$. Finally, we also have in the SM Lagrangian

$$
\mathscr{L}_\theta=\frac{\theta' g'^2}{32\pi^2}\widetilde{B}_{\mu\nu}B^{\mu\nu}+\frac{\theta g^2}{32\pi^2}\widetilde{W}^I_{\mu\nu}W_I^{\mu\nu}+\frac{\theta_s g_s^2}{32\pi^2}\widetilde{G}^A_{\mu\nu}G_A^{\mu\nu}
\tag{B.90}
$$

with

$$\beta_{\theta'} = -\frac{128\pi^2}{g'^2}\frac{m^2}{\Lambda^2}C_{\varphi\widetilde{B}}\,,$$
$$\beta_{\theta} = -\frac{128\pi^2}{g^2}\frac{m^2}{\Lambda^2}C_{\varphi\widetilde{W}}\,,$$
$$\beta_{\theta_s} = -\frac{128\pi^2}{g_s^2}\frac{m^2}{\Lambda^2}C_{\varphi\widetilde{G}}\,. \tag{B.91}$$

C Weak Effective Theory for B physics

The Hamiltonian for the WET can be written as

$$\mathscr{H}_{\mathrm{WET}} = -\mathscr{L}^{(u,d,c,s,b,e,\mu,\tau)}_{\mathrm{QCD+QED}} - \mathscr{L}^{(6)}_{\mathrm{SM}} - \frac{4G_F}{\sqrt{2}}\sum_i \left[\, C_i\, O_i + \mathrm{h.c.}\right]\,, \tag{C.1}$$

where $\mathscr{L}^{(u,d,c,s,b,e,\mu,\tau)}_{\mathrm{QCD+QED}}$ is the usual QCD and QED Lagrangian for the light fermions, $\mathscr{L}^{(6)}_{\mathrm{SM}}$ contains pure-SM dimension-six operators and the C_i coefficients contain all beyond the Standard Model effects. The sum over the i index runs over all operators O_i defined below.[8] See Ref. [18] for definitions and conventions.

C.1 $\Delta B = \Delta S = 2$ operators

In the case of $\Delta B = \Delta S = 2$ operators, the basis is given by

$$\begin{aligned}
O_1^{sbsb} &= (\bar{s}\gamma_\mu P_L b)\,(\bar{s}\gamma^\mu P_L b)\,, & O_5^{sbsb} &= (\bar{s}_\alpha P_L b_\beta)\,(\bar{s}_\beta P_R b_\alpha)\,,\\
O_2^{sbsb} &= (\bar{s}P_L b)\,(\bar{s}P_L b)\,, & O_{1'}^{sbsb} &= (\bar{s}\gamma_\mu P_R b)\,(\bar{s}\gamma^\mu P_R b)\,,\\
O_3^{sbsb} &= (\bar{s}_\alpha P_L b_\beta)\,(\bar{s}_\beta P_L b_\alpha)\,, & O_{2'}^{sbsb} &= (\bar{s}P_R b)\,(\bar{s}P_R b)\,,\\
O_4^{sbsb} &= (\bar{s}P_L b)\,(\bar{s}P_R b)\,, & O_{3'}^{sbsb} &= (\bar{s}_\alpha P_R b_\beta)\,(\bar{s}_\beta P_R b_\alpha)\,,
\end{aligned} \tag{C.2}$$

where we denote with primed indices the operators with opposite chirality. α and β are $SU(3)_c$ indices.

C.2 $\Delta B = \Delta C = 1$ operators

The basis for the $\Delta B = \Delta C = 1$ operators is given by

$$\begin{aligned}
&O_1^{cb\ell\ell} = (\bar{c}\,P_R\,\gamma^\mu\,b)\left(\bar{\ell}\,\gamma_\mu\,\nu_\ell\right)\,, \quad O_5^{cb\ell\ell} = (\bar{c}\,P_R\,b)\left(\bar{\ell}\,\nu_\ell\right)\,, \\
&O_{1'}^{cb\ell\ell} = (\bar{c}\,P_L\,\gamma^\mu\,b)\left(\bar{\ell}\,\gamma_\mu\,\nu_\ell\right)\,, \quad O_{5'}^{cb\ell\ell} = (\bar{c}\,P_L\,b)\left(\bar{\ell}\,\nu_\ell\right)\,, \quad O_{7'}^{cb\ell\ell} = (\bar{c}\,P_L\,\sigma^{\mu\nu}\,b)\left(\bar{\ell}\,\sigma_{\mu\nu}\,\nu_\ell\right)\,,
\end{aligned} \tag{C.3}$$

with $\ell \in \{e,\mu,\tau\}$.

[8] At the moment `DsixTools` only includes a limited subset of WET operators: $\Delta B = \Delta S = 2$, $\Delta B = \Delta C = 1$ and $\Delta B = \Delta S = 1$ operators.

C.3 $\Delta B = \Delta S = 1$ operators

There are three classes of $\Delta B = \Delta S = 1$ operators: Magnetic, hadronic (4-quark) and semileptonic operators. These are chosen as:

- Magnetic penguins:

$$O_{7\gamma}^{sb} = \frac{e}{g_s^2} m_b \, (\bar{s} P_R \, \sigma_{\mu\nu} \, b) \, F^{\mu\nu}, \qquad O_{8g}^{sb} = \frac{1}{g_s} m_b \, (\bar{s} P_R \, \sigma_{\mu\nu} \, T^A \, b) \, G_A^{\mu\nu} ,$$
$$O_{7'\gamma}^{sb} = \frac{e}{g_s^2} m_b \, (\bar{s} P_L \, \sigma_{\mu\nu} \, b) \, F^{\mu\nu}, \qquad O_{8'g}^{sb} = \frac{1}{g_s} m_b \, (\bar{s} P_L \, \sigma_{\mu\nu} \, T^A \, b) \, G_A^{\mu\nu} , \tag{C.4}$$

T^A $(A = 1, \ldots, 8)$ are the $SU(3)$ generators.

- Hadronic ($q \neq s$):

$$\begin{aligned}
O_1^{sbqq} &= (\bar{s} P_R \, \gamma_\mu b) \, (\bar{q} \gamma^\mu q) , & O_2^{sbqq} &= (\bar{s} P_R \, \gamma_\mu \, T^A b) \, (\bar{q} \gamma^\mu \, T^A q) , \\
O_3^{sbqq} &= (\bar{s} P_R \, \gamma_{\mu\nu\rho} \, b) \, (\bar{q} \gamma^{\mu\nu\rho} q) , & O_4^{sbqq} &= (\bar{s} P_R \, \gamma_{\mu\nu\rho} T^A b) \, (\bar{q} \gamma^{\mu\nu\rho} \, T^A q) , \\
O_5^{sbqq} &= (\bar{s} P_R \, b)(\bar{q} \, q) , & O_6^{sbqq} &= (\bar{s} P_R \, T^A b)(\bar{q} \, T^A q) , \\
O_7^{sbqq} &= (\bar{s} P_R \, \sigma^{\mu\nu} \, b)(\bar{q} \, \sigma_{\mu\nu} \, q) , & O_8^{sbqq} &= (\bar{s} P_R \, \sigma^{\mu\nu} \, T^A b)(\bar{q} \sigma_{\mu\nu} \, T^A q) , \\
O_9^{sbqq} &= (\bar{s} P_R \, \gamma_{\mu\nu\rho\sigma} \, b) \, (\bar{q} \gamma^{\mu\nu\rho\sigma} q) , & O_{10}^{sbqq} &= (\bar{s} P_R \, \gamma_{\mu\nu\rho\sigma} \, T^A b) \, (\bar{q} \gamma^{\mu\nu\rho\sigma} \, T^A q) ,
\end{aligned} \tag{C.5}$$

where $q = \{u, d, c\}$. In the case of $q = b$, the color-octet operators $O_{2,4,6,8,10}^{sbbb}$ are Fierz-equivalent to the color-singlet ones and are not included in the basis. In addition, the analogous set with opposite chirality is needed:

$$O_{i'}^{sbqq} = O_i^{sbqq} \Big|_{P_{L,R} \to P_{R,L}} , \tag{C.6}$$

- Hadronic ($q = s$):

$$\begin{aligned}
O_1^{sbss} &= (\bar{s} \gamma_\mu \, P_L b) \, (\bar{s} \gamma^\mu \, s) , & O_{1'}^{sbss} &= (\bar{s} \gamma_\mu \, P_R b) \, (\bar{s} \gamma^\mu \, s) , \\
O_3^{sbss} &= (\bar{s} \gamma_{\mu\nu\rho} \, P_L b) \, (\bar{s} \gamma^{\mu\nu\rho} s) , & O_{3'}^{sbss} &= (\bar{s} \, \gamma_{\mu\nu\rho} \, P_R b) \, (\bar{s} \gamma^{\mu\nu\rho} \, s) , \\
O_5^{sbss} &= (\bar{s} P_L \, b)(\bar{s} s) , & O_{5'}^{sbss} &= (\bar{s} P_R \, b)(\bar{s} s) , \\
O_7^{sbss} &= (\bar{s} \sigma^{\mu\nu} \, P_L \, b)(\bar{s} \, \sigma_{\mu\nu} \, s) , & O_{7'}^{sbss} &= (\bar{s} \sigma^{\mu\nu} \, P_R \, b)(\bar{s} \, \sigma_{\mu\nu} \, s) , \\
O_9^{sbss} &= (\bar{s} \gamma_{\mu\nu\rho\sigma} \, P_L \, b) \, (\bar{s} \gamma^{\mu\nu\rho\sigma} s) , & O_{9'}^{sbss} &= (\bar{s} \gamma_{\mu\nu\rho\sigma} \, P_R \, b) \, (\bar{s} \gamma^{\mu\nu\rho\sigma} s) .
\end{aligned} \tag{C.7}$$

Again, the color-octet operators are Fierz-redundant and have been ommited.

- Semileptonic:

$$
\begin{aligned}
O_1^{sb\ell\ell} &= (\bar{s}P_R\gamma_\mu b)\,(\bar{\ell}\gamma^\mu\ell)\,, & O_{1'}^{sb\ell\ell} &= (\bar{s}P_L\gamma_\mu b)\,(\bar{\ell}\gamma^\mu\ell)\,,\\
O_3^{sb\ell\ell} &= (\bar{s}P_R\gamma_{\mu\nu\rho} b)\,(\bar{\ell}\gamma^{\mu\nu\rho}\ell)\,, & O_{3'}^{sb\ell\ell} &= (\bar{s}P_L\gamma_{\mu\nu\rho} b)\,(\bar{\ell}\gamma^{\mu\nu\rho}\ell)\,,\\
O_5^{sb\ell\ell} &= (\bar{s}P_R b)(\bar{\ell}\ell)\,, & O_{5'}^{sb\ell\ell} &= (\bar{s}P_L b)(\bar{\ell}\ell)\,,\\
O_7^{sb\ell\ell} &= (\bar{s}P_R\sigma^{\mu\nu} b)(\bar{\ell}\sigma_{\mu\nu}\ell')\,, & O_{7'}^{sb\ell\ell} &= (\bar{s}P_L\sigma^{\mu\nu} b)(\bar{\ell}\sigma_{\mu\nu}\ell)\,,\\
O_9^{sb\ell\ell} &= (\bar{s}P_R\gamma_{\mu\nu\rho\sigma} b)\,(\bar{\ell}\gamma^{\mu\nu\rho\sigma}\ell)\,, & O_{9'}^{sb\ell\ell} &= (\bar{s}P_L\gamma_{\mu\nu\rho\sigma} b)\,(\bar{\ell}\gamma^{\mu\nu\rho\sigma}\ell)\,,
\end{aligned}
\tag{C.8}
$$

with $\ell \in \{e, \mu, \tau\}$.

D DsixTools routines

In this Appendix we list the public routines implemented in `DsixTools`. The user can take advantage of them when writing a project with `DsixTools`.

D.1 General `DsixTools` routines

LoadModule[moduleName]

Loads the `DsixTools` module `moduleName`. Once this command is evaluated, the module is initialized and all its routines are available to the user.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

Arguments: In the current version of `DsixTools`, `moduleName` can either be `"SMEFTrunner"`, `"EWmatcher"` or `"WETrunner"`.

Example: `LoadModule["SMEFTrunner"]` loads the `SMEFTrunner` module.

MyPrint[string]

Prints the message `string`. It can be switched on and off by using the `DsixTools` routines `TurnOnMessages` and `TurnOffMessages`.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

Arguments: `string` must be a valid `Mathematica` string.

Example: `MyPrint["I am using DsixTools"]` prints a simple message.

TurnOnMessages

Turns on the messages written by the `DsixTools` routines.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

TurnOffMessages

Turns off the messages written by the `DsixTools` routines. The only exception are the messages written when a new module is loaded, which cannot be switched off.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

ReadInputFiles[options_file,{WCsInput_file},{SMInput_file}]

Reads all input files.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

Arguments: `options_file`, `WCsInput_file` and `SMInput_file` are the paths to the options, WCs and SM parameters input files, respectively. `WCsInput_file` can contain either SMEFT or WET WCs. In case of SMEFT WCs, the `WCsInput_file` file can be given in SLHA or WCxf formats. For `DsixTools` to identify the format as SLHA, the file must have extension `.dat`, while for the WCxf format the extension must be either `.json` or `.yaml`. Note however that reading `YAML` input files requires previous installation of a `YAML` importer for Mathematica [29]. The second and third arguments of `ReadInputFiles` are optional. The routine should be called with only one argument when it is used to read the options file `options_file` only. The routine should be called with two arguments when it is used to read WET input, since in this case the SM parameters are not required. Finally, the routine should be called with three arguments when the input WCs correspond to the SMEFT.

Example: `ReadInputFiles["Options.dat","WCsInput.dat","SMInput.dat"]` reads the content of three SMEFT input files, with the SMEFT WCs given in the native SLHA format. `ReadInputFiles["Options.dat","WCsInput.json","SMInput.dat"]` reads the content of three SMEFT input files, with the SMEFT WCs given in WCxf format in a `JSON` file. `ReadInputFiles["Options.dat","WCsInput.dat"]` reads the content of two WET input files. `ReadInputFiles["Options.dat"]` reads the content of an options card.

WriteInputFiles[options_file,WCsInput_file,{SMInput_file},data]

Creates input files with the parameter values in `data`. This routine can be used to export the input values defined by the user in a project to external text files, later to be read by `DsixTools`.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

Arguments: `options_file` and `WCsInput_file` are the paths to the options and WCs input files created by this routine. `data` is an array where the input values have a precise ordering, defined by the ordering in the `Parameters` (for SMEFT input) and `WETParameters` (for WET input) global arrays, see E. The `SMInput_file` argument is the path to the input card with the values of the SM parameters and is optional: it should be absent when this routine is used to write WET input.

Example: `WriteInputFiles["Options.dat","WCsInput.dat","SMInput.dat", data]` exports the SMEFT values in `data` to three input files. `WriteInputFiles["Options.dat", "WCsInput.dat", data]` exports the WET values in `data` to two input files. These input files can later be read by `DsixTools`.

WriteAndReadInputFiles[options_file,WCsInput_file,{SMInput_file}]

Writes data into new input files and then reads them. In essence, running this routine is equivalent to running `WriteInputFiles` and `ReadInputFiles`.

Requires: Before using the routine the data to be exported (and later read) must be declared. In case of SMEFT input, all SM parameters must be set to their input values by giving values to variables of the form `Init[SM_parameter]` and all WC values will be assumed to vanish unless they are declared in a similar way. In case of WET input, only the WC values must be declared. Finally, the options will be assumed to take default values unless set to other choices.

Arguments: `options_file` and `WCsInput_file` are the paths to the options and WCs input files created by this routine. The `SMInput_file` argument is the path to the input card with the values of the SM parameters and is optional: it should be absent when this routine is used to write and read WET input.

Example: `WriteAndReadInputFiles["Options.dat","WCsInput.dat","SMInput.dat"]` exports the previously declared values to three SMEFT input files and then reads them. `WriteAndReadInputFiles["Options.dat", "WCsInput.dat"]` exports the previously declared values to two WET input files and then reads them.

WCXFtoSLHA[WCXF_file,SLHA_file,HIGHSCALE]

Translates the SMEFT WCs file in WCxf format `WCXF_file` into an SLHA format file named `SLHA_file`.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

Arguments: `WCXF_file` and `SLHA_file` are the paths to the input and output files, respectively. The extension of the file `WCXF_file` can be `.json` or `.yaml`. `HIGHSCALE` is the high-energy scale Λ at which the WCs are generated, given in GeV. The WCs in WCxf format are dimensionful (they include the $1/\Lambda$ or $1/\Lambda^2$ factor) and hence this information is required in order to compute the dimensionless WCs internally used by `DsixTools`.

Example: `WCXFtoSLHA["WCsFile.json", "WCsFile.dat", 10000]` creates the SLHA file `WCsFile.dat` with the WCs given in WCxf format in `WCsFile.json`. The translation uses $\Lambda = 10$ TeV.

SLHAtoWCXF[SLHA_file,WCXF_file,CPV,SCALE,HIGHSCALE]

Translates the SMEFT WCs file in SLHA format `SLHA_file` into an WCxf format file named `WCXF_file`.

Requires: Nothing. This routine can be used as soon as DsixTools is loaded.

Arguments: SLHA_file and WCXF_file are the paths to the input and output files, respectively. The extension of the file WCXF_file can be .json or .yaml. SCALE is the energy scale at which the input WCs are given while HIGHSCALE is the high-energy scale Λ at which they are generated. Both energy scales must be given in GeV. CPV must be set to 0 when all the WCs are real, and to 1 when there are complex WCs.

Example: SLHAtoWCXF["WCsFile.dat", "WCsFile.json", 1, 1000, 10000] creates the WCxf file WCsFile.json with the WCs given in SLHA format in WCsFile.dat. The translation is done at 1 TeV, uses $\Lambda = 10$ TeV and considers complex WCs.

NewInput[parameter,newvalue,dispatch]

Replaces the current input (contained in the Mathematica dispatch dispatch) by a new one in which parameter takes the value newvalue. This routine is particularly useful for projects involving loops with varying parameters.

Requires: Input values must have been introduced before using this routine, for instance using the ReadInputFiles routine. Only in this way the dispatch dispatch would have been defined.

Arguments: parameter must be a valid name for a DsixTools parameter. These can be found in the global arrays Parameters (in case of SMEFT input) and WETparameters (in case of WET input), see E. newvalue must be a valid number (in general complex). dispatch is the name of the Mathematica dispatch where the input values are saved. By default, this is input. In the SMEFTrunner module there is also the internal dispatch inputSMEFTrunner.

Example: NewInput[LL[1,1,1,2],1.0,input] changes the input value (at the high-energy scale Λ) of $[C_{\ell\ell}]_{1112}$ to 1.

NewScale[scale,newvalue]

Replaces the current value of scale by newvalue. This routine is particularly useful for projects involving loops with varying energy scales.

Requires: Nothing. This routine can be used as soon as DsixTools is loaded.

Arguments: scale can be either "high" or "low". newvalue must be a valid positive real number. As with other dimensionful parameters, the scale should be given in GeV.

Example: NewScale["high",1000] changes the high energy scale Λ to 1 TeV.

H[mat]

Returns the Hermitian conjugate of the matrix `mat`. If the option `CPV` has been set to `0` then it returns the transpose.

Requires: The global variable `CPV` must have been set to either `0` (for real parameters) or `1` (for complex parameters) before using this function.

Arguments: `mat` must be a valid matrix.

Example: `H[Gu]` returns the Hermitian conjugate of the up-quarks Yukawa matrix Γ_u.

CC[x]

Returns the complex conjugate of `x`. If the option `CPV` has been set to `0` then it returns `x`.

Requires: The global variable `CPV` must have been set to either `0` (for real parameters) or `1` (for complex parameters) before using this function.

Arguments: `x` must be a valid complex number.

Example: `CC[GU[1,2]/.input]` returns $[\Gamma_u]^*_{12}$ if `CPV` has been set to `1` and $[\Gamma_u]_{12}$ if `CPV` has been set to `0`, using for $[\Gamma_u]_{12}$ its input value.

D.2 `SMEFTrunner` routines

InitializeSMEFTrunnerInput

Initializes the input for the `SMEFTrunner` module. This initialization creates the dispatches `inputSMEFTrunner` and `inputSMEFTrunnerSM`. The user does not need to use this routine, as it is automatically executed when the `SMEFTrunner` module is loaded.

Requires: The `SMEFTrunner` module must have been loaded in order to use this routine.

FindParameterSMEFT[parameter]

Returns the position of `parameter` in the `Parameters` list. If one uses a generic element name as argument, `FindParameterSMEFT` will return a list with all positions where it appears in `Parameters`.

Requires: Nothing. This routine can be used as soon as `DsixTools` is loaded.

Arguments: `parameter` can be either the name of a SM parameter or SMEFT WC, or the generic name of an element of one of the 2- or 4-fermion parameters.

Example: `FindParameterSMEFT[LQ3[2, 2, 2, 3]]` returns `{355}`, the position of $\left[C_{\ell q}^{(3)}\right]_{2223}$ in the `Parameters` list. `FindParameterSMEFT[LQ3]` returns `{327, 328, ..., 371}`, since the $C_{\ell q}^{(3)}$ WCs are saved in these positions of the `Parameters` list.

GetBeta

Computes the SMEFT β functions analytically. After running this routine they can be read by evaluating quantities of the form β`[parameter]`. For instance, the $C_{\ell q}^{(1)}$ β functions can be obtained by evaluating β`[lq1]`. Table 7 contains the names used for all SMEFT β functions.

Requires: The `SMEFTrunner` module must have been loaded in order to use this routine. Moreover, the global variable `CPV` must have been set to either `0` (for real parameters) or `1` (for complex parameters) before using this function.

LoadBetaFunctions

Constructs the SMEFT β functions (using `GetBeta` internally) or reads them from a file.

Requires: The `SMEFTrunner` module must have been loaded and the option `ReadRGEs` set in order to use this routine.

RunRGEsSMEFT

Runs the SMEFT RGEs.

Requires: The `SMEFTrunner` module must have been loaded and the option `RGEsMethod` set in order to use this routine.

RunRGEsSMEFTpython[options_file,WCsInput_file,SMInput_file]

Reads the input files and runs the SMEFT RGEs using an external `python` package written by Xuanyou Pan and David Straub [25].

Requires: The `SMEFTrunner` module must have been loaded and the `python` package [25] must be installed in order to use this routine. Some `Mac` users might be unable to run this routine in `DsixTools` because the `PATH` environment in `Mathematica` might not coincide with the usual `PATH` environment in the system terminal. In this case, the user must execute `newPATH = "cli_path"`, where `cli_path` is the path where the command-line script `smeftrunner-cli` is located, before using `RunRGEsSMEFTpython`.

Arguments: `options_file`, `WCsInput_file` and `SMInput_file` are the paths to the options, WCs and SM parameters input files, respectively. `WCsInput_file` must contain SMEFT WCs.

ExportSMEFTrunner

Exports the `SMEFTrunner` results to the output file `Output_SMEFTrunner.dat`.

Requires: The `SMEFTrunner` module must have been loaded and the option `exportSMEFTrunner` set in order to use this routine.

WriteSMEFTrunnerOutputFile[Output_file,data]

Exports the `SMEFTrunner` results in `data` to `Output_file`.

Requires: The `SMEFTrunner` module must have been loaded in order to use this routine.

Arguments: `Output_file` is the path to the `SMEFTrunner` output file. `data` is an array containing the results obtained with `SMEFTrunner`. It must contain the same of elements and ordered in exactly the same way as `outSMEFTrunner`.

Example:
`WriteSMEFTrunnerOutputFile["Output_SMEFTrunner.dat",outSMEFTrunner/.t->tLOW]` exports the data saved in the `outSMEFTrunner` array, evaluated at $\mu = \mu_{\text{EW}}$, into the text file `Output_SMEFTrunner.dat`.

D.3 `EWmatcher` routines

InitializeEWmatcherInput

Initializes the input for the `EWmatcher` module. The user does not need to use this routine, as it is automatically executed when the `EWmatcher` module is loaded.

Requires: The `EWmatcher` module must have been loaded in order to use this routine.

FindParameterWET[parameter]

Returns the position of `parameter` in the `WETParameters` list. If one uses a generic element name as argument, `FindParameterWET` will return a list with all positions where it appears in `WETParameters`.

Requires: Nothing. This routine can be used as soon as DsixTools is loaded.

Arguments: parameter must be the name of a WET WC.

Example: FindParameterWET[CBS1[μ][1]] returns {71}, the position of $C_1^{sb\mu\mu}$ within the WETParameters list. FindParameterWET[CBS1[μ]] returns {71, 72, 73, 74, 75}, since the $C_i^{sb\mu\mu}$ WCs are saved in these positions of the WETParameters list.

RotateToMassBasis

Transforms the SMEFT WCs to the fermion mass basis. It also creates the replacements array ToMassBasis, which can be used to print the numerical values of the SMEFT WCs in the fermion mass basis.

Requires: The EWmatcher module must have been loaded in order to use this routine.

RotateToWCXFBasis

Transforms the SMEFT WCs to the fermion WCXF basis. It also creates the replacements array ToWCXFBasis, which can be used to print the numerical values of the SMEFT WCs in the fermion WCXF basis.

Requires: The EWmatcher module must have been loaded in order to use this routine.

Biunitary[mat,dim]

Applies a biunitary transformation that diagonalizes the dim $\times$ dim matrix mat. It returns three outputs: the square root of the eigenvalues of mat† mat (or, equivalently, mat mat†) and the unitary matrices U_L and U_R that diagonalize mat as $U_L^\dagger\, \mathtt{mat}\, U_R = \left(\mathtt{mat}\right)_{\text{diag}}$.

Requires: The EWmatcher module must have been loaded in order to use this function.

Arguments: mat must be a valid dim $\times$ dim matrix.

Example: Biunitary[Gu/.input,3] returns the square root of the eigenvalues of $\Gamma_u^\dagger \Gamma_u$ (or, equivalently, $\Gamma_u \Gamma_u^\dagger$), as well as the matrices U_L and U_R that diagonalize Γ_u as $U_L^\dagger \Gamma_u U_R = (\Gamma_u)_{\text{diag}}$. Here Γ_u is the input value for the up-quarks Yukawa matrix.

ApplyEWmatching

Matches the SMEFT WCs onto the WET WCs. After using this routine several arrays containing the numerical values of the WET WCs at the electroweak scale are created. The result of this step can also be accessed by using the Match and MatchAnalytical functions.

Requires: The `EWmatcher` module must have been loaded in order to use this function.

Match[WC]

Returns the value of the WET Wilson coefficient `WC` after matching it to the SMEFT.

Requires: The `EWmatcher` module must have been loaded in order to use this function. Furthermore, the `ApplyEWmatching` routines must have been run before using this function.

Arguments: `WC` must be the name of a valid SMEFT WC.

Example: `Match[CBS1[d][1]]` prints the numerical value of the C_1^{sbdd} WC.

MatchAnalytical[WC]

Returns the analytical expression of the WET Wilson coefficient `WC` after matching it to the SMEFT. Only the energy scales and the SM parameters are replaced by numerical values in this function.

Requires: The `EWmatcher` module must have been loaded in order to use this function. Furthermore, the `ApplyEWmatching` routines must have been run before using this function.

Arguments: `WC` must be the name of a valid SMEFT WC.

Example: `Match[CBS1[d][1]]` prints the analytical expression of the C_1^{sbdd} WC.

WriteWCsOutputFile[Output_file,basis,format]

Exports the SMEFT WCs in the fermion basis `basis` to the file `Output_file`, which has format `format`.

Requires: The `EWmatcher` module must have been loaded in order to use this function. Furthermore, the routine to compute the WCs in the fermion basis `basis` (`RotateToMassBasis` or `RotateToWCXFBasis`) must have been run before using this function.

Arguments: `Output_file` is the path to the text file where the SMEFT WCs will be exported. The argument `basis` must be either `"MassBasis"` or `"WCXF"` and selects the specific fermion basis for the WCs. The argument `format` can be either `"SLHA"`, `"JSON"` or `"YAML"` and selects the output file type. In the first case this will be given in SLHA format, whereas in the last two cases it will be given in WCxf format.

Example: `WriteWCsOutputFile["Output_WCsMassBasis.dat","WCXF","SLHA"]` will export the SMEFT WCs in the fermion WCXF basis to the SLHA file `"Output_WCsMassBasis.dat"`.

ExportEWmatcher

Exports the `EWmatcher` results to the output file `Output_EWmatcher.dat`.

Requires: The `EWmatcher` module must have been loaded and the option `exportEWmatcher` set in order to use this routine.

WriteEWmatcherOutputFile[Output_file,data]

Exports the `EWmatcher` results in `data` to `Output_file`.

Requires: The `EWmatcher` module must have been loaded in order to use this routine.

Arguments: `Output_file` is the path to the `EWmatcher` output file. `data` is an array containing the results obtained with `EWmatcher`. The precise ordering of the WET WCs in `data` is given by that in `WETParameters`, see E.

Example: `WriteEWmatcherOutputFile["Output_EWmatcher.dat",dataOutput]` exports the data saved in the `dataOutput` array to the `Output_EWmatcher.dat` text file.

D.4 `WETrunner` routines

InitializeWETrunnerInput

Initializes the input for the `WETrunner` module. The user does not need to use this routine, as it is automatically executed when the `WETrunner` module is loaded.

Requires: The `WETrunner` module must have been loaded in order to use this routine.

RunRGEsWET

Runs the WET RGEs.

Requires: The `WETrunner` module must have been loaded in order to use this routine.

ExportWETrunner

Exports the `WETrunner` results to the output file `Output_WETrunner.dat`.

Requires: The `WETrunner` module must have been loaded and the option `exportWETrunner` set in order to use this routine.

WriteWETrunnerOutputFile[Output_file,data,scale]

Exports the `WETrunner` results in `data` to `Output_file` after evaluating them at $\mu = $ `scale`.

Requires: The `WETrunner` module must have been loaded in order to use this routine.

Arguments: `Output_file` is the path to the `WETrunner` output file. `data` is an array containing the results obtained with `WETrunner`. The precise ordering of the WET WCs in `data` is given by `BS2Low-BC1Low-BS1unprimedLow-BS1primedLow`, following exactly the ordering in `WETParameters` (see E), and they must be evaluated at $t = \log_{10}$ `scale`.

Example:
`WriteWETrunnerOutputFile["Output_WETrunner.dat",dataOutput,mb]` will export the data saved in the `dataOutput` array, evaluated at $\mu = m_b$, into the `Output_WETrunner.dat` text file.

E DsixTools parameters

In this Appendix we provide additional details about the SMEFT and WET parameters used in `DsixTools`. These can be useful to properly read or write some variables in a `Mathematica` session using `DsixTools`.

E.1 SMEFT parameters

Table 7 provides a complete list of the SMEFT parameters used in `DsixTools`. In addition to the SMEFT WCs, this includes the SM parameters (gauge couplings, Yukawa matrices and scalar and θ parameters). This table is particularly useful to identify the names given to the elements of 2- and 4-fermion WCs, as well as the β functions used in the `SMEFTrunner` module.

Table 7: SMEFT parameters. *Position* denotes the position of the parameter (or parameters for 2- and 4-fermion objects) in the `Parameters` global array. The column β *function* gives the name of the β function (obtained with β`[parameter]` after using the `GetBeta` routine). *Type* indicates the type of parameter (with nF standing for n-fermion) and *Category* denotes the index symmetry category of the coefficient, being relevant for 2- and 4-fermion WCs.

Position	Parameter(s)	DsixTools name	Elements	β function	Type	Category
1	g	g	-	β[g]	0F	0
2	g'	gp	-	β[gp]	0F	0
3	g_s	gs	-	β[gs]	0F	0
4	λ	λ	-	β[λ]	0F	0
5	m^2 [GeV^2]	m2	-	β[m2]	0F	0
6-14	Γ_u	Gu	GU[i,j]	β[GuX]	2F	1
15-23	Γ_d	Gd	GD[i,j]	β[GdX]	2F	1
24-32	Γ_e	Ge	GE[i,j]	β[GeX]	2F	1
33	θ	θ	-	β[θ]	0F	0
34	θ'	θp	-	β[θp]	0F	0
35	θ_s	θs	-	β[θs]	0F	0
36	C_G	G	-	β[G]	0F	0
37	$C_{\widetilde{G}}$	Gtilde	-	β[Gtilde]	0F	0
38	C_W	W	-	β[W]	0F	0
39	$C_{\widetilde{W}}$	Wtilde	-	β[Wtilde]	0F	0
40	C_φ	φ	-	β[φ]	0F	0
41	$C_{\varphi\Box}$	$\varphi\Box$	-	β[$\varphi\Box$]	0F	0
42	$C_{\varphi D}$	φDD	-	β[φD]	0F	0
43	$C_{\varphi G}$	φG	-	β[φG]	0F	0
44	$C_{\varphi B}$	φB	-	β[φB]	0F	0
45	$C_{\varphi W}$	φW	-	β[φW]	0F	0
46	$C_{\varphi WB}$	φWB	-	β[φWB]	0F	0
47	$C_{\varphi\widetilde{G}}$	φGtilde	-	β[φGtilde]	0F	0
48	$C_{\varphi\widetilde{B}}$	φBtilde	-	β[φBtilde]	0F	0
49	$C_{\varphi\widetilde{W}}$	φWtilde	-	β[φWtilde]	0F	0
50	$C_{\varphi\widetilde{W}B}$	φWtildeB	-	β[φWtildeB]	0F	0
51-59	$C_{u\varphi}$	WC[uφ]	Uφ[i,j]	β[uφ]	2F	1
60-68	$C_{d\varphi}$	WC[dφ]	Dφ[i,j]	β[dφ]	2F	1
69-77	$C_{e\varphi}$	WC[eφ]	Eφ[i,j]	β[eφ]	2F	1
78-86	C_{eW}	WC[eW]	EW[i,j]	β[eW]	2F	1
87-95	C_{eB}	WC[eB]	EB[i,j]	β[eB]	2F	1
96-104	C_{uG}	WC[uG]	UG[i,j]	β[uG]	2F	1
105-113	C_{uW}	WC[uW]	UW[i,j]	β[uW]	2F	1
114-122	C_{uB}	WC[uB]	UB[i,j]	β[uB]	2F	1
123-131	C_{dG}	WC[dG]	DG[i,j]	β[dG]	2F	1
132-140	C_{dW}	WC[dW]	DW[i,j]	β[dW]	2F	1
141-149	C_{dB}	WC[dB]	DB[i,j]	β[dB]	2F	1
150-155	$C_{\varphi\ell}^{(1)}$	WC[φl1]	φL1[i,j]	β[φl1]	2F	2
156-161	$C_{\varphi\ell}^{(3)}$	WC[φl3]	φL3[i,j]	β[φl3]	2F	2
162-167	$C_{\varphi e}$	WC[φe]	φE[i,j]	β[φe]	2F	2
168-173	$C_{\varphi q}^{(1)}$	WC[φq1]	φQ1[i,j]	β[φq1]	2F	2
174-179	$C_{\varphi q}^{(3)}$	WC[φq3]	φQ3[i,j]	β[φq3]	2F	2

180-185	$C_{\varphi u}$	`WC[`φ`u]`	φ`U[i,j]`	β`[`φ`u]`	2F	2
186-191	$C_{\varphi d}$	`WC[`φ`d]`	φ`D[i,j]`	β`[`φ`d]`	2F	2
192-200	$C_{\varphi ud}$	`WC[`φ`ud]`	φ`UD[i,j]`	β`[`φ`ud]`	2F	1
201-227	$C_{\ell\ell}$	`WC[ll]`	`LL[i,j,k,l]`	β`[ll]`	4F	4
228-254	$C_{qq}^{(1)}$	`WC[qq1]`	`QQ1[i,j,k,l]`	β`[qq1]`	4F	4
255-281	$C_{qq}^{(3)}$	`WC[qq3]`	`QQ3[i,j,k,l]`	β`[qq3]`	4F	4
282-326	$C_{\ell q}^{(1)}$	`WC[lq1]`	`LQ1[i,j,k,l]`	β`[lq1]`	4F	5
327-371	$C_{\ell q}^{(3)}$	`WC[lq3]`	`LQ3[i,j,k,l]`	β`[lq3]`	4F	5
372-392	C_{ee}	`WC[ee]`	`EE[i,j,k,l]`	β`[ee]`	4F	6
393-419	C_{uu}	`WC[uu]`	`UU[i,j,k,l]`	β`[uu]`	4F	4
420-446	C_{dd}	`WC[dd]`	`DD[i,j,k,l]`	β`[dd]`	4F	4
447-491	C_{eu}	`WC[eu]`	`EU[i,j,k,l]`	β`[eu]`	4F	5
492-536	C_{ed}	`WC[ed]`	`ED[i,j,k,l]`	β`[ed]`	4F	5
537-581	$C_{ud}^{(1)}$	`WC[ud1]`	`UD1[i,j,k,l]`	β`[ud1]`	4F	5
582-626	$C_{ud}^{(8)}$	`WC[ud8]`	`UD8[i,j,k,l]`	β`[ud8]`	4F	5
627-671	$C_{\ell e}$	`WC[le]`	`LE[i,j,k,l]`	β`[le]`	4F	5
672-716	$C_{\ell u}$	`WC[lu]`	`LU[i,j,k,l]`	β`[lu]`	4F	5
717-761	$C_{\ell d}$	`WC[ld]`	`LD[i,j,k,l]`	β`[ld]`	4F	5
762-806	C_{qe}	`WC[qe]`	`QE[i,j,k,l]`	β`[qe]`	4F	5
807-851	$C_{qu}^{(1)}$	`WC[qu1]`	`QU1[i,j,k,l]`	β`[qu1]`	4F	5
852-896	$C_{qu}^{(8)}$	`WC[qu8]`	`QU8[i,j,k,l]`	β`[qu8]`	4F	5
897-941	$C_{qd}^{(1)}$	`WC[qd1]`	`QD1[i,j,k,l]`	β`[qd1]`	4F	5
942-986	$C_{qd}^{(8)}$	`WC[qd8]`	`QD8[i,j,k,l]`	β`[qd8]`	4F	5
987-1067	$C_{\ell edq}$	`WC[ledq]`	`LEDQ[i,j,k,l]`	β`[ledq]`	4F	3
1068-1148	$C_{quqd}^{(1)}$	`WC[quqd1]`	`QUQD1[i,j,k,l]`	β`[quqd1]`	4F	3
1149-1229	$C_{quqd}^{(8)}$	`WC[quqd8]`	`QUQD8[i,j,k,l]`	β`[quqd8]`	4F	3
1230-1310	$C_{\ell equ}^{(1)}$	`WC[lequ1]`	`LEQU1[i,j,k,l]`	β`[lequ1]`	4F	3
1311-1391	$C_{\ell equ}^{(3)}$	`WC[lequ3]`	`LEQU3[i,j,k,l]`	β`[lequ3]`	4F	3
1392-1472	$C_{duq\ell}$	`WC[duql]`	`DUQL[i,j,k,l]`	β`[duql]`	4F	3
1473-1526	C_{qque}	`WC[qque]`	`QQUE[i,j,k,l]`	β`[qque]`	4F	7
1527-1583	$C_{qqq\ell}$	`WC[qqql]`	`QQQL[i,j,k,l]`	β`[qqql]`	4F	8
1584-1664	C_{duue}	`WC[duue]`	`DUUE[i,j,k,l]`	β`[duue]`	4F	3
1665-1670	$C_{\ell\ell\varphi\varphi}$	`WC[ll`$\varphi\varphi$`]`	`LL`$\varphi\varphi$`[i,j]`	β`[ll`$\varphi\varphi$`]`	2F	9

It is well known that some of the 2- and 4-fermion operators in the SMEFT posess specific symmetries under the exchange of flavor indices. This translates into an index symmetry for the corresponding WCs. For instance, the Wilson coefficient $C_{\varphi e}$ is a Hermitian matrix, hence following the symmetry relation $\left[C_{\varphi e}\right]_{ij} = \left[C_{\varphi e}\right]^*_{ji}$. More complicated index symmetries exist for some of the 4-fermion WCs. In all these cases, the number of independent WCs gets reduced. For example, the C_{ee} 4-fermion WC does not contain 81 ($= 3^4$) independent complex WCs, but just 21 real and 15 imaginary independent components. Therefore, it is convenient to restrict the number of parameters considered in SMEFT calculations to just the independent ones. In `DsixTools` we have followed this approach, dropping redundant WCs in all calculations. This is what motivates the introduction of an index symmetry category column in Table 7. The meaning of the different categories is given by:

Category	Meaning
0	0F scalar object
1	2F general 3×3 matrix
2	2F Hermitian matrix
3	4F general $3 \times 3 \times 3 \times 3$ object
4	4F two identical $\bar{\psi}\psi$ currents
5	4F two independent $\bar{\psi}\psi$ currents
6	4F two identical $\bar{\psi}\psi$ currents - special case C_{ee}
7	4F Baryon-number-violating - special case C_{qque}
8	4F Baryon-number-violating - special case C_{qqql}
9	2F symmetric matrix

We see that, apart from the WCs in categories 0, 1 and 3, all other WCs have index symmetries. Furthermore, there are three dimension-six WCs with special symmetries, not shared by any other WC: C_{ee}, C_{qque} and C_{qqql}. Similarly, the dimension-five WC $C_{\ell\ell\varphi\varphi}$ is the only symmetric matrix. In Table 8 we list the independent WCs contained in each category. This, combined with Table 7, completely allows the user to determine the position of a given parameter in the `Parameters` array. In any case, we remind the reader that the function `FindParametersSMEFT` can also be used for this purpose.

Table 8: Independent SMEFT WCs in each category. Elements in red denote real WCs.

	1	2	3	4	5	6	7	8	9
1	{1, 1}	{1, 1}	{1, 1, 1, 1}	{1, 1, 1, 1}	{1, 1, 1, 1}	{1, 1, 1, 1}	{1, 1, 1, 1}	{1, 1, 1, 1}	{1, 1}
2	{1, 2}	{1, 2}	{1, 1, 1, 2}	{1, 1, 1, 2}	{1, 1, 1, 2}	{1, 1, 1, 2}	{1, 1, 1, 2}	{1, 1, 1, 2}	{1, 2}
3	{1, 3}	{1, 3}	{1, 1, 1, 3}	{1, 1, 1, 3}	{1, 1, 1, 3}	{1, 1, 1, 3}	{1, 1, 1, 3}	{1, 1, 1, 3}	{1, 3}
4	{2, 1}	{2, 2}	{1, 1, 2, 1}	{1, 1, 2, 2}	{1, 1, 2, 2}	{1, 1, 2, 2}	{1, 1, 2, 1}	{1, 1, 2, 1}	{2, 2}
5	{2, 2}	{2, 3}	{1, 1, 2, 2}	{1, 1, 2, 3}	{1, 1, 2, 3}	{1, 1, 2, 3}	{1, 1, 2, 2}	{1, 1, 2, 2}	{2, 3}
6	{2, 3}	{3, 3}	{1, 1, 2, 3}	{1, 1, 3, 3}	{1, 1, 3, 3}	{1, 1, 3, 3}	{1, 1, 2, 3}	{1, 1, 2, 3}	{3, 3}
7	{3, 1}		{1, 1, 3, 1}	{1, 2, 1, 2}	{1, 2, 1, 1}	{1, 2, 1, 2}	{1, 1, 3, 1}	{1, 1, 3, 1}	
8	{3, 2}		{1, 1, 3, 2}	{1, 2, 1, 3}	{1, 2, 1, 2}	{1, 2, 1, 3}	{1, 1, 3, 2}	{1, 1, 3, 2}	
9	{3, 3}		{1, 1, 3, 3}	{1, 2, 2, 1}	{1, 2, 1, 3}	{1, 2, 2, 2}	{1, 1, 3, 3}	{1, 1, 3, 3}	
10			{1, 2, 1, 1}	{1, 2, 2, 2}	{1, 2, 2, 1}	{1, 2, 2, 3}	{1, 2, 1, 1}	{1, 2, 1, 1}	
11			{1, 2, 1, 2}	{1, 2, 2, 3}	{1, 2, 2, 2}	{1, 2, 3, 2}	{1, 2, 1, 2}	{1, 2, 1, 2}	
12			{1, 2, 1, 3}	{1, 2, 3, 1}	{1, 2, 2, 3}	{1, 2, 3, 3}	{1, 2, 1, 3}	{1, 2, 1, 3}	
13			{1, 2, 2, 1}	{1, 2, 3, 2}	{1, 2, 3, 1}	{1, 3, 1, 3}	{1, 2, 2, 1}	{1, 2, 2, 1}	
14			{1, 2, 2, 2}	{1, 2, 3, 3}	{1, 2, 3, 2}	{1, 3, 2, 3}	{1, 2, 2, 2}	{1, 2, 2, 2}	

15	{1, 2, 2, 3}	{1, 3, 1, 3}	{1, 2, 3, 3}	{1, 3, 3, 3}	{1, 2, 2, 3}	{1, 2, 2, 3}
16	{1, 2, 3, 1}	{1, 3, 2, 2}	{1, 3, 1, 1}	{2, 2, 2, 2}	{1, 2, 3, 1}	{1, 2, 3, 1}
17	{1, 2, 3, 2}	{1, 3, 2, 3}	{1, 3, 1, 2}	{2, 2, 2, 3}	{1, 2, 3, 2}	{1, 2, 3, 2}
18	{1, 2, 3, 3}	{1, 3, 3, 1}	{1, 3, 1, 3}	{2, 2, 3, 3}	{1, 2, 3, 3}	{1, 2, 3, 3}
19	{1, 3, 1, 1}	{1, 3, 3, 2}	{1, 3, 2, 1}	{2, 3, 2, 3}	{1, 3, 1, 1}	{1, 3, 1, 1}
20	{1, 3, 1, 2}	{1, 3, 3, 3}	{1, 3, 2, 2}	{2, 3, 3, 3}	{1, 3, 1, 2}	{1, 3, 1, 2}
21	{1, 3, 1, 3}	{2, 2, 2, 2}	{1, 3, 2, 3}	{3, 3, 3, 3}	{1, 3, 1, 3}	{1, 3, 1, 3}
22	{1, 3, 2, 1}	{2, 2, 2, 3}	{1, 3, 3, 1}		{1, 3, 2, 1}	{1, 3, 2, 1}
23	{1, 3, 2, 2}	{2, 2, 3, 3}	{1, 3, 3, 2}		{1, 3, 2, 2}	{1, 3, 2, 2}
24	{1, 3, 2, 3}	{2, 3, 2, 3}	{1, 3, 3, 3}		{1, 3, 2, 3}	{1, 3, 2, 3}
25	{1, 3, 3, 1}	{2, 3, 3, 2}	{2, 2, 1, 1}		{1, 3, 3, 1}	{1, 3, 3, 1}
26	{1, 3, 3, 2}	{2, 3, 3, 3}	{2, 2, 1, 2}		{1, 3, 3, 2}	{1, 3, 3, 2}
27	{1, 3, 3, 3}	{3, 3, 3, 3}	{2, 2, 1, 3}		{1, 3, 3, 3}	{1, 3, 3, 3}
28	{2, 1, 1, 1}		{2, 2, 2, 2}		{2, 2, 1, 1}	{2, 1, 2, 1}
29	{2, 1, 1, 2}		{2, 2, 2, 3}		{2, 2, 1, 2}	{2, 1, 2, 2}
30	{2, 1, 1, 3}		{2, 2, 3, 3}		{2, 2, 1, 3}	{2, 1, 2, 3}
31	{2, 1, 2, 1}		{2, 3, 1, 1}		{2, 2, 2, 1}	{2, 1, 3, 1}
32	{2, 1, 2, 2}		{2, 3, 1, 2}		{2, 2, 2, 2}	{2, 1, 3, 2}
33	{2, 1, 2, 3}		{2, 3, 1, 3}		{2, 2, 2, 3}	{2, 1, 3, 3}
34	{2, 1, 3, 1}		{2, 3, 2, 1}		{2, 2, 3, 1}	{2, 2, 2, 1}
35	{2, 1, 3, 2}		{2, 3, 2, 2}		{2, 2, 3, 2}	{2, 2, 2, 2}
36	{2, 1, 3, 3}		{2, 3, 2, 3}		{2, 2, 3, 3}	{2, 2, 2, 3}
37	{2, 2, 1, 1}		{2, 3, 3, 1}		{2, 3, 1, 1}	{2, 2, 3, 1}
38	{2, 2, 1, 2}		{2, 3, 3, 2}		{2, 3, 1, 2}	{2, 2, 3, 2}
39	{2, 2, 1, 3}		{2, 3, 3, 3}		{2, 3, 1, 3}	{2, 2, 3, 3}
40	{2, 2, 2, 1}		{3, 3, 1, 1}		{2, 3, 2, 1}	{2, 3, 1, 1}
41	{2, 2, 2, 2}		{3, 3, 1, 2}		{2, 3, 2, 2}	{2, 3, 1, 2}
42	{2, 2, 2, 3}		{3, 3, 1, 3}		{2, 3, 2, 3}	{2, 3, 1, 3}
43	{2, 2, 3, 1}		{3, 3, 2, 2}		{2, 3, 3, 1}	{2, 3, 2, 1}
44	{2, 2, 3, 2}		{3, 3, 2, 3}		{2, 3, 3, 2}	{2, 3, 2, 2}
45	{2, 2, 3, 3}		{3, 3, 3, 3}		{2, 3, 3, 3}	{2, 3, 2, 3}
46	{2, 3, 1, 1}				{3, 3, 1, 1}	{2, 3, 3, 1}
47	{2, 3, 1, 2}				{3, 3, 1, 2}	{2, 3, 3, 2}
48	{2, 3, 1, 3}				{3, 3, 1, 3}	{2, 3, 3, 3}
49	{2, 3, 2, 1}				{3, 3, 2, 1}	{3, 1, 3, 1}
50	{2, 3, 2, 2}				{3, 3, 2, 2}	{3, 1, 3, 2}
51	{2, 3, 2, 3}				{3, 3, 2, 3}	{3, 1, 3, 3}
52	{2, 3, 3, 1}				{3, 3, 3, 1}	{3, 2, 3, 1}
53	{2, 3, 3, 2}				{3, 3, 3, 2}	{3, 2, 3, 2}
54	{2, 3, 3, 3}				{3, 3, 3, 3}	{3, 2, 3, 3}
55	{3, 1, 1, 1}					{3, 3, 3, 1}
56	{3, 1, 1, 2}					{3, 3, 3, 2}

57	{3, 1, 1, 3}	{3, 3, 3, 3}
58	{3, 1, 2, 1}	
59	{3, 1, 2, 2}	
60	{3, 1, 2, 3}	
61	{3, 1, 3, 1}	
62	{3, 1, 3, 2}	
63	{3, 1, 3, 3}	
64	{3, 2, 1, 1}	
65	{3, 2, 1, 2}	
66	{3, 2, 1, 3}	
67	{3, 2, 2, 1}	
68	{3, 2, 2, 2}	
69	{3, 2, 2, 3}	
70	{3, 2, 3, 1}	
71	{3, 2, 3, 2}	
72	{3, 2, 3, 3}	
73	{3, 3, 1, 1}	
74	{3, 3, 1, 2}	
75	{3, 3, 1, 3}	
76	{3, 3, 2, 1}	
77	{3, 3, 2, 2}	
78	{3, 3, 2, 3}	
79	{3, 3, 3, 1}	
80	{3, 3, 3, 2}	
81	{3, 3, 3, 3}	

E.2 WET parameters

Table 9 provides a complete list of the WET parameters considered in `DsixTools`.

Table 9: WET parameters. *Position* denotes the position of the parameter in the `WETParameters` global array.

Position	Parameter	`DsixTools` name	Position	Parameter	`DsixTools` name
1	C_1^{sbsb}	`CBS2[1]`	70	C_9^{sbee}	`CBS1[e][9]`
2	C_2^{sbsb}	`CBS2[2]`	71	$C_1^{sb\mu\mu}$	`CBS1[μ][1]`
3	C_3^{sbsb}	`CBS2[3]`	72	$C_3^{sb\mu\mu}$	`CBS1[μ][3]`
4	C_4^{sbsb}	`CBS2[4]`	73	$C_5^{sb\mu\mu}$	`CBS1[μ][5]`
5	C_5^{sbsb}	`CBS2[5]`	74	$C_7^{sb\mu\mu}$	`CBS1[μ][7]`
6	$C_{1'}^{sbsb}$	`CBS2p[1]`	75	$C_9^{sb\mu\mu}$	`CBS1[μ][9]`

7	$C^{sbsb}_{2'}$	CBS2p[2]	76	$C^{sb\tau\tau}_{1}$	CBS1[τ][1]
8	$C^{sbsb}_{3'}$	CBS2p[3]	77	$C^{sb\tau\tau}_{3}$	CBS1[τ][3]
9	C^{cbee}_{1}	CBC1[e][1]	78	$C^{sb\tau\tau}_{5}$	CBS1[τ][5]
10	C^{cbee}_{5}	CBC1[e][5]	79	$C^{sb\tau\tau}_{7}$	CBS1[τ][7]
11	$C^{cbee}_{1'}$	CBC1p[e][1]	80	$C^{sb\tau\tau}_{9}$	CBS1[τ][9]
12	$C^{cbee}_{5'}$	CBC1p[e][5]	81	$C^{sbuu}_{1'}$	CBS1p[u][1]
13	$C^{cbee}_{7'}$	CBC1p[e][7]	82	$C^{sbuu}_{2'}$	CBS1p[u][2]
14	$C^{cb\mu\mu}_{1}$	CBC1[μ][1]	83	$C^{sbuu}_{3'}$	CBS1p[u][3]
15	$C^{cb\mu\mu}_{5}$	CBC1[μ][5]	84	$C^{sbuu}_{4'}$	CBS1p[u][4]
16	$C^{cb\mu\mu}_{1'}$	CBC1p[μ][1]	85	$C^{sbuu}_{5'}$	CBS1p[u][5]
17	$C^{cb\mu\mu}_{5'}$	CBC1p[μ][5]	86	$C^{sbuu}_{6'}$	CBS1p[u][6]
18	$C^{cb\mu\mu}_{7'}$	CBC1p[μ][7]	87	$C^{sbuu}_{7'}$	CBS1p[u][7]
19	$C^{cb\tau\tau}_{1}$	CBC1[τ][1]	88	$C^{sbuu}_{8'}$	CBS1p[u][8]
20	$C^{cb\tau\tau}_{5}$	CBC1[τ][5]	89	$C^{sbuu}_{9'}$	CBS1p[u][9]
21	$C^{cb\tau\tau}_{1'}$	CBC1p[τ][1]	90	$C^{sbuu}_{10'}$	CBS1p[u][10]
22	$C^{cb\tau\tau}_{5'}$	CBC1p[τ][5]	91	$C^{sbdd}_{1'}$	CBS1p[d][1]
23	$C^{cb\tau\tau}_{7'}$	CBC1p[τ][7]	92	$C^{sbdd}_{2'}$	CBS1p[d][2]
24	C^{sbuu}_{1}	CBS1[u][1]	93	$C^{sbdd}_{3'}$	CBS1p[d][3]
25	C^{sbuu}_{2}	CBS1[u][2]	94	$C^{sbdd}_{4'}$	CBS1p[d][4]
26	C^{sbuu}_{3}	CBS1[u][3]	95	$C^{sbdd}_{5'}$	CBS1p[d][5]
27	C^{sbuu}_{4}	CBS1[u][4]	96	$C^{sbdd}_{6'}$	CBS1p[d][6]
28	C^{sbuu}_{5}	CBS1[u][5]	97	$C^{sbdd}_{7'}$	CBS1p[d][7]
29	C^{sbuu}_{6}	CBS1[u][6]	98	$C^{sbdd}_{8'}$	CBS1p[d][8]
30	C^{sbuu}_{7}	CBS1[u][7]	99	$C^{sbdd}_{9'}$	CBS1p[d][9]
31	C^{sbuu}_{8}	CBS1[u][8]	100	$C^{sbdd}_{10'}$	CBS1p[d][10]
32	C^{sbuu}_{9}	CBS1[u][9]	101	$C^{sbcc}_{1'}$	CBS1p[c][1]
33	C^{sbuu}_{10}	CBS1[u][10]	102	$C^{sbcc}_{2'}$	CBS1p[c][2]
34	C^{sbdd}_{1}	CBS1[d][1]	103	$C^{sbcc}_{3'}$	CBS1p[c][3]
35	C^{sbdd}_{2}	CBS1[d][2]	104	$C^{sbcc}_{4'}$	CBS1p[c][4]
36	C^{sbdd}_{3}	CBS1[d][3]	105	$C^{sbcc}_{5'}$	CBS1p[c][5]
37	C^{sbdd}_{4}	CBS1[d][4]	106	$C^{sbcc}_{6'}$	CBS1p[c][6]
38	C^{sbdd}_{5}	CBS1[d][5]	107	$C^{sbcc}_{7'}$	CBS1p[c][7]
39	C^{sbdd}_{6}	CBS1[d][6]	108	$C^{sbcc}_{8'}$	CBS1p[c][8]
40	C^{sbdd}_{7}	CBS1[d][7]	109	$C^{sbcc}_{9'}$	CBS1p[c][9]
41	C^{sbdd}_{8}	CBS1[d][8]	110	$C^{sbcc}_{10'}$	CBS1p[c][10]
42	C^{sbdd}_{9}	CBS1[d][9]	111	$C^{sbss}_{1'}$	CBS1p[s][1]
43	C^{sbdd}_{10}	CBS1[d][10]	112	$C^{sbss}_{3'}$	CBS1p[s][3]
44	C^{sbcc}_{1}	CBS1[c][1]	113	$C^{sbss}_{5'}$	CBS1p[s][5]
45	C^{sbcc}_{2}	CBS1[c][2]	114	$C^{sbss}_{7'}$	CBS1p[s][7]

46	C_3^{sbcc}	`CBS1[c][3]`	115	$C_{9'}^{sbss}$	`CBS1p[s][9]`
47	C_4^{sbcc}	`CBS1[c][4]`	116	$C_{1'}^{sbbb}$	`CBS1p[b][1]`
48	C_5^{sbcc}	`CBS1[c][5]`	117	$C_{3'}^{sbbb}$	`CBS1p[b][3]`
49	C_6^{sbcc}	`CBS1[c][6]`	118	$C_{5'}^{sbbb}$	`CBS1p[b][5]`
50	C_7^{sbcc}	`CBS1[c][7]`	119	$C_{7'}^{sbbb}$	`CBS1p[b][7]`
51	C_8^{sbcc}	`CBS1[c][8]`	120	$C_{9'}^{sbbb}$	`CBS1p[b][9]`
52	C_9^{sbcc}	`CBS1[c][9]`	121	$C_{7'\gamma}$	`CBS1p[M][7]`
53	C_{10}^{sbcc}	`CBS1[c][10]`	122	$C_{8'g}$	`CBS1p[M][8]`
54	C_1^{sbss}	`CBS1[s][1]`	123	$C_{1'}^{sbee}$	`CBS1p[e][1]`
55	C_3^{sbss}	`CBS1[s][3]`	124	$C_{3'}^{sbee}$	`CBS1p[e][3]`
56	C_5^{sbss}	`CBS1[s][5]`	125	$C_{5'}^{sbee}$	`CBS1p[e][5]`
57	C_7^{sbss}	`CBS1[s][7]`	126	$C_{7'}^{sbee}$	`CBS1p[e][7]`
58	C_9^{sbss}	`CBS1[s][9]`	127	$C_{9'}^{sbee}$	`CBS1p[e][9]`
59	C_1^{sbbb}	`CBS1[b][1]`	128	$C_{1'}^{sb\mu\mu}$	CBS1p[μ][1]
60	C_3^{sbbb}	`CBS1[b][3]`	129	$C_{3'}^{sb\mu\mu}$	CBS1p[μ][3]
61	C_5^{sbbb}	`CBS1[b][5]`	130	$C_{5'}^{sb\mu\mu}$	CBS1p[μ][5]
62	C_7^{sbbb}	`CBS1[b][7]`	131	$C_{7'}^{sb\mu\mu}$	CBS1p[μ][7]
63	C_9^{sbbb}	`CBS1[b][9]`	132	$C_{9'}^{sb\mu\mu}$	CBS1p[μ][9]
64	$C_{7\gamma}$	`CBS1[M][7]`	133	$C_{1'}^{sb\tau\tau}$	CBS1p[τ][1]
65	C_{8g}	`CBS1[M][8]`	134	$C_{3'}^{sb\tau\tau}$	CBS1p[τ][3]
66	C_1^{sbee}	`CBS1[e][1]`	135	$C_{5'}^{sb\tau\tau}$	CBS1p[τ][5]
67	C_3^{sbee}	`CBS1[e][3]`	136	$C_{7'}^{sb\tau\tau}$	CBS1p[τ][7]
68	C_5^{sbee}	`CBS1[e][5]`	137	$C_{9'}^{sb\tau\tau}$	CBS1p[τ][9]
69	C_7^{sbee}	`CBS1[e][7]`			

F SMEFT operators in the mass basis

After electroweak symmetry breaking, the Warsaw basis operators can be rotated to the fermion mass basis. This is achieved by performing unitary transformations of the fermion fields in order to diagonalize the fermion mass matrices,

$$\begin{aligned} u_L &\to V_{u_L} u_L\,, \qquad d_L \to V_{d_L} d_L\,, \qquad u_R \to V_{u_R} u_R\,, \qquad d_R \to V_{d_R} d_R\,, \\ e_L &\to V_{e_L} e_L\,, \qquad \nu_L \to V_{\nu_L} \nu_L\,, \qquad e_R \to V_{e_R} e_R\,. \end{aligned} \tag{F.1}$$

In this way, the Dirac mass term

$$m_\psi^{\rm diag} \equiv V_{\psi_L}^\dagger m_\psi V_{\psi_R} \tag{F.2}$$

is a diagonal and positive matrix corresponding to the physical fermion masses. The dimension-5 Weinberg operator, which leads to Majorana neutrino masses, also takes a diagonal form after these transformations. We note that these definitions imply that the CKM matrix is given by $V = V_{u_L}^{\dagger} V_{d_L}$ whereas the PMNS matrix is $U = V_{e_L}^{\dagger} V_{\nu_L}$. The resulting operators after applying these unitary transformations to the SMEFT operators are given below. These results agree with those in [17] and extend them by including the baryon-number-violating operators and the transformations of the leptonic fields.

$\boxed{\psi^2 \varphi^3}$

Operator	Definition in the mass basis
$Q_{u\varphi}$	$\left[\widetilde{C}_{u\varphi}\right]_{ij} (\varphi^{\dagger}\varphi) \left[\overline{u}_L^i u_R^j (\varphi^0)^* - V_{ik}^* \overline{d}_L^k u_R^j \varphi^-\right]$ $\left[\widetilde{C}_{u\varphi}\right]_{ij} = [C_{u\varphi}]_{mn} [V_{u_L}^{\dagger}]_{im} [V_{u_R}]_{nj}$
$Q_{d\varphi}$	$\left[\widetilde{C}_{d\varphi}\right]_{ij} (\varphi^{\dagger}\varphi) \left[V_{ki} \overline{u}_L^k d_R^j \varphi^+ + \overline{d}_L^i d_R^j \varphi^0\right]$ $\left[\widetilde{C}_{d\varphi}\right]_{ij} = [C_{d\varphi}]_{mn} \left[V_{d_L}^{\dagger}\right]_{im} [V_{d_R}]_{nj}$
$Q_{e\varphi}$	$\left[\widetilde{C}_{e\varphi}\right]_{ij} (\varphi^{\dagger}\varphi) \left[U_{ik}^* \overline{\nu}_L^k e_R^j \varphi^+ + \overline{e}_L^i e_R^j \varphi^0\right]$ $\left[\widetilde{C}_{e\varphi}\right]_{ij} = [C_{e\varphi}]_{mn} [V_{e_L}^{\dagger}]_{im} [V_{e_R}]_{nj}$

$\boxed{\psi^2 X\varphi}$

Operator	Definition in the mass basis
Q_{eW}	$\left[\widetilde{C}_{eW}\right]_{ij}\left[U^*_{ik}\overline{\nu}^k_L\sigma^{\mu\nu}e^j_R\varphi^+ - \overline{e}^i_L\sigma^{\mu\nu}e^j_R\varphi^0\right]W^3_{\mu\nu}+\dots$ $\left[\widetilde{C}_{eW}\right]_{ij}=[C_{eW}]_{mn}\left[V^\dagger_{e_L}\right]_{im}[V_{e_R}]_{nj}$
Q_{eB}	$\left[\widetilde{C}_{eB}\right]_{ij}\left[U^*_{ik}\overline{\nu}^k_L\sigma^{\mu\nu}e^j_R\varphi^+ + \overline{e}^i_L\sigma^{\mu\nu}e^j_R\varphi^0\right]B_{\mu\nu}$ $\left[\widetilde{C}_{eB}\right]_{ij}=[C_{eB}]_{mn}\left[V^\dagger_{e_L}\right]_{im}[V_{e_R}]_{nj}$
Q_{uG}	$\left[\widetilde{C}_{uG}\right]_{ij}\left[\overline{u}^i_L\sigma^{\mu\nu}T^A u^j_R\left(\varphi^0\right)^* - V^*_{ik}\overline{d}^k_L\sigma^{\mu\nu}T^A u^j_R\varphi^-\right]G^A_{\mu\nu}$ $\left[\widetilde{C}_{uG}\right]_{ij}=[C_{uG}]_{mn}\left[V^\dagger_{u_L}\right]_{im}[V_{u_R}]_{nj}$
Q_{uW}	$\left[\widetilde{C}_{uW}\right]_{ij}\left[\overline{u}^i_L\sigma^{\mu\nu}u^j_R\left(\varphi^0\right)^* + V^*_{ik}\overline{d}^k_L\sigma^{\mu\nu}u^j_R\varphi^-\right]W^3_{\mu\nu}+\dots$ $\left[\widetilde{C}_{uW}\right]_{ij}=[C_{uW}]_{mn}\left[V^\dagger_{u_L}\right]_{im}[V_{u_R}]_{nj}$
Q_{uB}	$\left[\widetilde{C}_{uB}\right]_{ij}\left[\overline{u}^i_L\sigma^{\mu\nu}u^j_R\left(\varphi^0\right)^* - V^*_{ik}\overline{d}^k_L\sigma^{\mu\nu}u^j_R\varphi^-\right]B_{\mu\nu}$ $\left[\widetilde{C}_{uB}\right]_{ij}=[C_{uB}]_{mn}\left[V^\dagger_{u_L}\right]_{im}[V_{u_R}]_{nj}$
Q_{dG}	$\left[\widetilde{C}_{dG}\right]_{ij}\left[V_{ki}\overline{u}^k_L\sigma^{\mu\nu}T^A d^j_R\varphi^+ + \overline{d}^i_L\sigma^{\mu\nu}T^A d^j_R\varphi^0\right]G^A_{\mu\nu}$ $\left[\widetilde{C}_{dG}\right]_{ij}=[C_{dG}]_{mn}\left[V^\dagger_{d_L}\right]_{im}[V_{d_R}]_{nj}$
Q_{dW}	$\left[\widetilde{C}_{dW}\right]_{ij}\left[V_{ki}\overline{u}^k_L\sigma^{\mu\nu}d^j_R\varphi^+ - \overline{d}^i_L\sigma^{\mu\nu}d^j_R\varphi^0\right]W^3_{\mu\nu}+\dots$ $\left[\widetilde{C}_{dW}\right]_{ij}=[C_{dW}]_{mn}\left[V^\dagger_{d_L}\right]_{im}[V_{d_R}]_{nj}$
Q_{dB}	$\left[\widetilde{C}_{dB}\right]_{ij}\left[V_{ki}\overline{u}^k_L\sigma^{\mu\nu}d^j_R\varphi^+ + \overline{d}^i_L\sigma^{\mu\nu}d^j_R\varphi^0\right]B_{\mu\nu}$ $\left[\widetilde{C}_{dB}\right]_{ij}=[C_{dB}]_{mn}\left[V^\dagger_{d_L}\right]_{im}[V_{d_R}]_{nj}$

$\boxed{\psi^2\varphi^2 D}$

Operator	Definition in the mass basis
$Q_{\varphi\ell}^{(1)}$	$\left[\widetilde{C}_{\varphi\ell}^{(1)}\right]_{ij}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)\left[U_{im}^* U_{jn}\overline{\nu}_L^m\gamma^\mu\nu_L^n+\overline{e}_L^i\gamma^\mu e_L^j\right]$ $\left[\widetilde{C}_{\varphi\ell}^{(1)}\right]_{ij}=\left[C_{\varphi\ell}^{(1)}\right]_{mn}\left[V_{e_L}^\dagger\right]_{im}\left[V_{e_L}\right]_{nj}$
$Q_{\varphi\ell}^{(3)}$	$\left[\widetilde{C}_{\varphi\ell}^{(3)}\right]_{ij}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu^1\varphi\right)\left[U_{im}^*\overline{\nu}_L^m\gamma^\mu e_L^j+U_{jn}\overline{e}_L^i\gamma^\mu\nu_L^n\right]+\dots$ $\left[\widetilde{C}_{\varphi\ell}^{(3)}\right]_{ij}=\left[C_{\varphi\ell}^{(3)}\right]_{mn}\left[V_{e_L}^\dagger\right]_{im}\left[V_{e_L}\right]_{nj}$
$Q_{\varphi e}$	$\left[\widetilde{C}_{\varphi e}\right]_{ij}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)\overline{e}_R^i\gamma^\mu e_R^j$ $\left[\widetilde{C}_{\varphi e}\right]_{ij}=\left[C_{\varphi e}\right]_{mn}\left[V_{e_R}^\dagger\right]_{im}\left[V_{e_R}\right]_{nj}$
$Q_{\varphi q}^{(1)}$	$\left[\widetilde{C}_{\varphi q}^{(1)}\right]_{ij}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)\left[V_{mi}V_{nj}^*\overline{u}_L^m\gamma^\mu u_L^n+\overline{d}_L^i\gamma^\mu d_L^j\right]$ $\left[\widetilde{C}_{\varphi q}^{(1)}\right]_{ij}=\left[C_{\varphi q}^{(1)}\right]_{mn}\left[V_{d_L}^\dagger\right]_{im}\left[V_{d_L}\right]_{nj}$
$Q_{\varphi q}^{(3)}$	$\left[\widetilde{C}_{\varphi q}^{(3)}\right]_{ij}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu^1\varphi\right)\left[V_{mi}\overline{u}_L^m\gamma^\mu d_L^j+V_{nj}^*\overline{d}_L^i\gamma^\mu u_L^n\right]+\dots$ $\left[\widetilde{C}_{\varphi q}^{(3)}\right]_{ij}=\left[C_{\varphi q}^{(3)}\right]_{mn}\left[V_{d_L}^\dagger\right]_{im}\left[V_{d_L}\right]_{nj}$
$Q_{\varphi u}$	$\left[\widetilde{C}_{\varphi u}\right]_{ij}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)\overline{u}_R^i\gamma^\mu u_R^j$ $\left[\widetilde{C}_{\varphi u}\right]_{ij}=\left[C_{\varphi u}\right]_{mn}\left[V_{u_R}^\dagger\right]_{im}\left[V_{u_R}\right]_{nj}$
$Q_{\varphi d}$	$\left[\widetilde{C}_{\varphi d}\right]_{ij}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)\overline{d}_R^i\gamma^\mu d_R^j$ $\left[\widetilde{C}_{\varphi d}\right]_{ij}=\left[C_{\varphi d}\right]_{mn}\left[V_{d_R}^\dagger\right]_{im}\left[V_{d_R}\right]_{nj}$
$Q_{\varphi ud}$	$\left[\widetilde{C}_{\varphi ud}\right]_{ij}\left(\tilde{\varphi}^\dagger iD_\mu\varphi\right)\overline{u}_R^i\gamma^\mu d_R^j$ $\left[\widetilde{C}_{\varphi ud}\right]_{ij}=\left[C_{\varphi ud}\right]_{mn}\left[V_{u_R}^\dagger\right]_{im}\left[V_{d_R}\right]_{nj}$

$\boxed{(\overline{L}L)\,(\overline{L}L)}$

Operator	Definition in the mass basis
$Q_{\ell\ell}$	$\left[\widetilde{C}_{\ell\ell}\right]_{ijkl}\left(U^*_{im}U_{jn}\overline{\nu}^m_L\gamma^\mu\nu^n_L+\overline{e}^i_L\gamma^\mu e^j_L\right)\left(U^*_{km}U_{ln}\overline{\nu}^m_L\gamma^\mu\nu^n_L+\overline{e}^k_L\gamma^\mu e^l_L\right)$ $\left[\widetilde{C}_{\ell\ell}\right]_{ijkl}=[C_{\ell\ell}]_{pqrs}\,[V^\dagger_{e_L}]_{ip}\,[V_{e_L}]_{qj}\,[V^\dagger_{e_L}]_{kr}\,[V_{e_L}]_{sl}$
$Q^{(1)}_{qq}$	$\left[\widetilde{C}^{(1)}_{qq}\right]_{ijkl}\left(V_{mi}V^*_{nj}\overline{u}^m_L\gamma^\mu u^n_L+\overline{d}^i_L\gamma^\mu d^j_L\right)\left(V_{mk}V^*_{nl}\overline{u}^m_L\gamma^\mu u^n_L+\overline{d}^k_L\gamma^\mu d^l_L\right)$ $\left[\widetilde{C}^{(1)}_{qq}\right]_{ijkl}=\left[C^{(1)}_{qq}\right]_{pqrs}\left[V^\dagger_{d_L}\right]_{ip}[V_{d_L}]_{qj}\left[V^\dagger_{d_L}\right]_{kr}[V_{d_L}]_{sl}$
$Q^{(3)}_{qq}$	$\left[\widetilde{C}^{(3)}_{qq}\right]_{ijkl}\left(V_{mi}V^*_{nj}\overline{u}^m_L\gamma^\mu u^n_L-\overline{d}^i_L\gamma^\mu d^j_L\right)\left(V_{mk}V^*_{nl}\overline{u}^m_L\gamma^\mu u^n_L-\overline{d}^k_L\gamma^\mu d^l_L\right)+\dots$ $\left[\widetilde{C}^{(3)}_{qq}\right]_{ijkl}=\left[C^{(3)}_{qq}\right]_{pqrs}\left[V^\dagger_{d_L}\right]_{ip}[V_{d_L}]_{qj}\left[V^\dagger_{d_L}\right]_{kr}[V_{d_L}]_{sl}$
$Q^{(1)}_{\ell q}$	$\left[\widetilde{C}^{(1)}_{\ell q}\right]_{ijkl}\left(U^*_{ip}U_{jq}\overline{\nu}^p_L\gamma^\mu\nu^q_L+\overline{e}^i_L\gamma^\mu e^j_L\right)\left(V_{pk}V^*_{ql}\overline{u}^p_L\gamma^\mu u^q_L+\overline{d}^k_L\gamma^\mu d^l_L\right)$ $\left[\widetilde{C}^{(1)}_{\ell q}\right]_{ijkl}=\left[C^{(1)}_{\ell q}\right]_{pqrs}[V^\dagger_{e_L}]_{ip}[V_{e_L}]_{qj}\left[V^\dagger_{d_L}\right]_{kr}[V_{d_L}]_{sl}$
$Q^{(3)}_{\ell q}$	$\left[\widetilde{C}^{(3)}_{\ell q}\right]_{ijkl}\left(U^*_{ip}U_{jq}\overline{\nu}^p_L\gamma^\mu\nu^q_L-\overline{e}^i_L\gamma^\mu e^j_L\right)\left(V_{pk}V^*_{ql}\overline{u}^p_L\gamma^\mu u^q_L-\overline{d}^k_L\gamma^\mu d^l_L\right)+\dots$ $\left[\widetilde{C}^{(3)}_{\ell q}\right]_{ijkl}=\left[C^{(3)}_{\ell q}\right]_{pqrs}[V^\dagger_{e_L}]_{ip}[V_{e_L}]_{qj}\left[V^\dagger_{d_L}\right]_{kr}[V_{d_L}]_{sl}$

$\boxed{(\overline{R}R)\,(\overline{R}R)}$

Operator	Definition in the mass basis
Q_{ee}	$\left[\widetilde{C}_{ee}\right]_{ijkl}\left(\overline{e}_R^i\gamma^\mu e_R^j\right)\left(\overline{e}_R^k\gamma^\mu e_R^l\right)$ $\left[\widetilde{C}_{ee}\right]_{ijkl}=[C_{ee}]_{pqrs}\left[V_{e_R}^\dagger\right]_{ip}[V_{e_R}]_{qj}\left[V_{e_R}^\dagger\right]_{kr}[V_{e_R}]_{sl}$
Q_{uu}	$\left[\widetilde{C}_{uu}\right]_{ijkl}\left(\overline{u}_R^i\gamma^\mu u_R^j\right)\left(\overline{u}_R^k\gamma^\mu u_R^l\right)$ $\left[\widetilde{C}_{uu}\right]_{ijkl}=[C_{uu}]_{pqrs}\left[V_{u_R}^\dagger\right]_{ip}[V_{u_R}]_{qj}\left[V_{u_R}^\dagger\right]_{kr}[V_{u_R}]_{sl}$
Q_{dd}	$\left[\widetilde{C}_{dd}\right]_{ijkl}\left(\overline{d}_R^i\gamma^\mu d_R^j\right)\left(\overline{d}_R^k\gamma^\mu d_R^l\right)$ $\left[\widetilde{C}_{dd}\right]_{ijkl}=[C_{dd}]_{pqrs}\left[V_{d_R}^\dagger\right]_{ip}[V_{d_R}]_{qj}\left[V_{d_R}^\dagger\right]_{kr}[V_{d_R}]_{sl}$
Q_{eu}	$\left[\widetilde{C}_{eu}\right]_{ijkl}\left(\overline{e}_R^i\gamma^\mu e_R^j\right)\left(\overline{u}_R^k\gamma^\mu u_R^l\right)$ $\left[\widetilde{C}_{eu}\right]_{ijkl}=[C_{eu}]_{pqrs}\left[V_{e_R}^\dagger\right]_{ip}[V_{e_R}]_{qj}\left[V_{u_R}^\dagger\right]_{kr}[V_{u_R}]_{sl}$
Q_{ed}	$\left[\widetilde{C}_{ed}\right]_{ijkl}\left(\overline{e}_R^i\gamma^\mu e_R^j\right)\left(\overline{d}_R^k\gamma^\mu d_R^l\right)$ $\left[\widetilde{C}_{ed}\right]_{ijkl}=[C_{ed}]_{pqrs}\left[V_{e_R}^\dagger\right]_{ip}[V_{e_R}]_{qj}\left[V_{d_R}^\dagger\right]_{kr}[V_{d_R}]_{sl}$
$Q_{ud}^{(1)}$	$\left[\widetilde{C}_{ud}^{(1)}\right]_{ijkl}\left(\overline{u}_R^i\gamma^\mu u_R^j\right)\left(\overline{d}_R^k\gamma^\mu d_R^l\right)$ $\left[\widetilde{C}_{ud}^{(1)}\right]_{ijkl}=\left[C_{ud}^{(1)}\right]_{pqrs}\left[V_{u_R}^\dagger\right]_{ip}[V_{u_R}]_{qj}\left[V_{d_R}^\dagger\right]_{kr}[V_{d_R}]_{sl}$
$Q_{ud}^{(8)}$	$\left[\widetilde{C}_{ud}^{(8)}\right]_{ijkl}\left(\overline{u}_R^iT^A\gamma^\mu u_R^j\right)\left(\overline{d}_R^kT^A\gamma^\mu d_R^l\right)$ $\left[\widetilde{C}_{ud}^{(8)}\right]_{ijkl}=\left[C_{ud}^{(8)}\right]_{pqrs}\left[V_{u_R}^\dagger\right]_{ip}[V_{u_R}]_{qj}\left[V_{d_R}^\dagger\right]_{kr}[V_{d_R}]_{sl}$

$\boxed{(\overline{L}L)\,(\overline{R}R)}$

Operator	Definition in the mass basis
$Q_{\ell e}$	$\left[\widetilde{C}_{\ell e}\right]_{ijkl}\left(U^*_{im}U_{jn}\overline{\nu}^m_L\gamma^\mu\nu^n_L+\overline{e}^i_L\gamma^\mu e^j_L\right)\left(\overline{e}^k_R\gamma^\mu e^l_R\right)$ $\left[\widetilde{C}_{\ell e}\right]_{ijkl}=[C_{\ell e}]_{pqrs}\left[V^\dagger_{e_L}\right]_{ip}[V_{e_L}]_{qj}\left[V^\dagger_{e_R}\right]_{kr}[V_{e_R}]_{sl}$
$Q_{\ell u}$	$\left[\widetilde{C}_{\ell u}\right]_{ijkl}\left(U^*_{im}U_{jn}\overline{\nu}^m_L\gamma^\mu\nu^n_L+\overline{e}^i_L\gamma^\mu e^j_L\right)\left(\overline{u}^k_R\gamma^\mu u^l_R\right)$ $\left[\widetilde{C}_{\ell u}\right]_{ijkl}=[C_{\ell u}]_{pqrs}\left[V^\dagger_{e_L}\right]_{ip}[V_{e_L}]_{qj}\left[V^\dagger_{u_R}\right]_{kr}[V_{u_R}]_{sl}$
$Q_{\ell d}$	$\left[\widetilde{C}_{\ell d}\right]_{ijkl}\left(U^*_{im}U_{jn}\overline{\nu}^m_L\gamma^\mu\nu^n_L+\overline{e}^i_L\gamma^\mu e^j_L\right)\left(\overline{d}^k_R\gamma^\mu d^l_R\right)$ $\left[\widetilde{C}_{\ell d}\right]_{ijkl}=[C_{\ell d}]_{pqrs}\left[V^\dagger_{e_L}\right]_{ip}[V_{e_L}]_{qj}\left[V^\dagger_{d_R}\right]_{kr}[V_{d_R}]_{sl}$
Q_{qe}	$\left[\widetilde{C}_{qe}\right]_{ijkl}\left(V_{mi}V^*_{nj}\overline{u}^m_L\gamma^\mu u^n_L+\overline{d}^i_L\gamma^\mu d^j_L\right)\left(\overline{e}^k_R\gamma^\mu e^l_R\right)$ $\left[\widetilde{C}_{qe}\right]_{ijkl}=\left[C_{qe}\right]_{pqrs}\left[V^\dagger_{d_L}\right]_{ip}[V_{d_L}]_{qj}\left[V^\dagger_{e_R}\right]_{kr}[V_{e_R}]_{sl}$
$Q^{(1)}_{qu}$	$\left[\widetilde{C}^{(1)}_{qu}\right]_{ijkl}\left(V_{mi}V^*_{nj}\overline{u}^m_L\gamma^\mu u^n_L+\overline{d}^i_L\gamma^\mu d^j_L\right)\left(\overline{u}^k_R\gamma^\mu u^l_R\right)$ $\left[\widetilde{C}^{(1)}_{qu}\right]_{ijkl}=\left[C^{(1)}_{qu}\right]_{pqrs}\left[V^\dagger_{d_L}\right]_{ip}[V_{d_L}]_{qj}\left[V^\dagger_{u_R}\right]_{kr}[V_{u_R}]_{sl}$
$Q^{(8)}_{qu}$	$\left[\widetilde{C}^{(8)}_{qu}\right]_{ijkl}\left(V_{mi}V^*_{nj}\overline{u}^m_L\gamma^\mu T^A u^n_L+\overline{d}^i_L\gamma^\mu T^A d^j_L\right)\left(\overline{u}^k_R\gamma^\mu T^A u^l_R\right)$ $\left[\widetilde{C}^{(8)}_{qu}\right]_{ijkl}=\left[C^{(8)}_{qu}\right]_{pqrs}\left[V^\dagger_{d_L}\right]_{ip}[V_{d_L}]_{qj}\left[V^\dagger_{u_R}\right]_{kr}[V_{u_R}]_{sl}$
$Q^{(1)}_{qd}$	$\left[\widetilde{C}^{(1)}_{qd}\right]_{ijkl}\left(V_{mi}V^*_{nj}\overline{u}^m_L\gamma^\mu u^n_L+\overline{d}^i_L\gamma^\mu d^j_L\right)\left(\overline{d}^k_R\gamma^\mu d^l_R\right)$ $\left[\widetilde{C}^{(1)}_{qd}\right]_{ijkl}=\left[C^{(1)}_{qd}\right]_{pqrs}\left[V^\dagger_{d_L}\right]_{ip}[V_{d_L}]_{qj}\left[V^\dagger_{d_R}\right]_{kr}[V_{d_R}]_{sl}$
$Q^{(8)}_{qd}$	$\left[\widetilde{C}^{(8)}_{qd}\right]_{ijkl}\left(V_{mi}V^*_{nj}\overline{u}^m_L\gamma^\mu T^A u^n_L+\overline{d}^i_L\gamma^\mu T^A d^j_L\right)\left(\overline{d}^k_R\gamma^\mu T^A d^l_R\right)$ $\left[\widetilde{C}^{(8)}_{qd}\right]_{ijkl}=\left[C^{(8)}_{qd}\right]_{pqrs}\left[V^\dagger_{d_L}\right]_{ip}[V_{d_L}]_{qj}\left[V^\dagger_{d_R}\right]_{kr}[V_{d_R}]_{sl}$

$\boxed{\left(\overline{L}R\right)\left(\overline{R}L\right)}$

Operator	Definition in the mass basis
$Q_{\ell edq}$	$\left[\widetilde{C}_{\ell edq}\right]_{ijkl}\left[V^*_{nl}U^*_{im}\left(\overline{\nu}^m_L e^j_R\right)\left(\overline{d}^k_R u^n_L\right)+\left(\overline{e}^i_L e^j_R\right)\left(\overline{d}^k_R d^l_L\right)\right]$
	$\left[\widetilde{C}_{\ell edq}\right]_{ijkl}=\left[C_{\ell edq}\right]_{pqrs}\left[V^\dagger_{e_L}\right]_{ip}\left[V_{e_R}\right]_{qj}\left[V^\dagger_{d_R}\right]_{kr}\left[V_{d_L}\right]_{sl}$

$\boxed{\left(\overline{L}R\right)\left(\overline{L}R\right)}$

Operator	Definition in the mass basis
$Q^{(1)}_{quqd}$	$\left[\widetilde{C}^{(1)}_{quqd}\right]_{ijkl}\left[\left(\overline{u}^i_L u^j_R\right)\left(\overline{d}^k_L d^l_R\right)-V^*_{im}V_{nk}\left(\overline{d}^m_L u^j_R\right)\left(\overline{u}^n_L d^l_R\right)\right]$
	$\left[\widetilde{C}^{(1)}_{quqd}\right]_{ijkl}=\left[C^{(1)}_{quqd}\right]_{pqrs}\left[V^\dagger_{u_L}\right]_{ip}\left[V_{u_R}\right]_{qj}\left[V^\dagger_{d_L}\right]_{kr}\left[V_{d_R}\right]_{sl}$
$Q^{(8)}_{quqd}$	$\left[\widetilde{C}^{(8)}_{quqd}\right]_{ijkl}\left[\left(\overline{u}^i_L T^A u^j_R\right)\left(\overline{d}^k_L T^A d^l_R\right)-V^*_{im}V_{nk}\left(\overline{d}^m_L T^A u^j_R\right)\left(\overline{u}^n_L T^A d^l_R\right)\right]$
	$\left[\widetilde{C}^{(1)}_{quqd}\right]_{ijkl}=\left[C^{(1)}_{quqd}\right]_{pqrs}\left[V^\dagger_{u_L}\right]_{ip}\left[V_{u_R}\right]_{qj}\left[V^\dagger_{d_L}\right]_{kr}\left[V_{d_R}\right]_{sl}$
$Q^{(1)}_{\ell equ}$	$\left[\widetilde{C}^{(1)}_{\ell equ}\right]_{ijkl}\left[V^*_{km}U^*_{in}\left(\overline{\nu}^n_L e^j_R\right)\left(\overline{d}^m_L u^l_R\right)-\left(\overline{e}^i_L e^j_R\right)\left(\overline{u}^k_L u^l_R\right)\right]$
	$\left[\widetilde{C}^{(1)}_{\ell equ}\right]_{ijkl}=\left[C^{(1)}_{\ell equ}\right]_{pqrs}\left[V^\dagger_{e_L}\right]_{ip}\left[V_{e_R}\right]_{qj}\left[V^\dagger_{u_L}\right]_{kr}\left[V_{u_R}\right]_{sl}$
$Q^{(3)}_{\ell equ}$	$\left[\widetilde{C}^{(3)}_{\ell equ}\right]_{ijkl}\left[V^*_{km}U^*_{in}\left(\overline{\nu}^n_L \sigma^{\mu\nu} e^j_R\right)\left(\overline{d}^m_L \sigma_{\mu\nu} u^l_R\right)-\left(\overline{e}^i_L \sigma^{\mu\nu} e^j_R\right)\left(\overline{u}^k_L \sigma_{\mu\nu} u^l_R\right)\right]$
	$\left[\widetilde{C}^{(3)}_{\ell equ}\right]_{ijkl}=\left[C^{(3)}_{\ell equ}\right]_{pqrs}\left[V^\dagger_{e_L}\right]_{ip}\left[V_{e_R}\right]_{qj}\left[V^\dagger_{u_L}\right]_{kr}\left[V_{u_R}\right]_{sl}$

$\boxed{Baryon-number-violating}$

Operator	Definition in the mass basis
$Q_{duq\ell}$	$\left[\widetilde{C}_{duq\ell}\right]_{ijkl}\left[\left(d_R^i\right)^\intercal C u_R^j\right]\left\{V_{mk}^* U_{tl}^*\left[\left(u_L^m\right)^\intercal C e_L^t\right]-\left[\left(d_L^k\right)^\intercal C \nu_L^l\right]\right\}$ $\left[\widetilde{C}_{duq\ell}\right]_{ijkl}=\left[C_{duq\ell}\right]_{pqrs}\left[V_{d_R}\right]_{pi}\left[V_{u_R}\right]_{qj}\left[V_{d_L}\right]_{rk}\left[V_{\nu_L}\right]_{sl}$
Q_{qque}	$\left[\widetilde{C}_{qque}\right]_{ijkl}\left\{V_{mi}^* V_{jn}\left[\left(u_L^m\right)^\intercal C d_L^n\right]-\left[\left(d_L^i\right)^\intercal C u_L^j\right]\right\}\left[\left(u_R^k\right)^\intercal C e_R^l\right]$ $\left[\widetilde{C}_{qque}\right]_{ijkl}=\left[C_{qque}\right]_{pqrs}\left[V_{d_L}\right]_{pi}\left[V_{u_L}\right]_{qj}\left[V_{u_R}\right]_{rk}\left[V_{e_R}\right]_{sl}$
$Q_{qqq\ell}$	$\left[\widetilde{C}_{qqq\ell}\right]_{ijkl}\left\{U_{la}\left[\left(d_L^i\right)^\intercal C d_L^j\right]\left[\left(u_L^k\right)^\intercal C \nu_L^a\right]+V_{mi}^* V_{nj}^* V_{kt}\left[\left(u_L^m\right)^\intercal C u_L^n\right]\left[\left(d_L^t\right)^\intercal C e_L^l\right]\right.$ $\left.-V_{mi}^*\left[\left(u_L^m\right)^\intercal C d_L^j\right]\left[\left(u_L^k\right)^\intercal C e_L^l\right]-V_{nj}^* V_{kt} U_{la}\left[\left(d_L^i\right)^\intercal C u_L^n\right]\left[\left(d_L^t\right)^\intercal C \nu_L^a\right]\right\}$ $\left[\widetilde{C}_{qqq\ell}\right]_{ijkl}=\left[C_{qqq\ell}\right]_{pqrs}\left[V_{d_L}\right]_{pi}\left[V_{d_L}\right]_{qj}\left[V_{u_L}\right]_{rk}\left[V_{e_L}\right]_{sl}$
Q_{duue}	$\left[\widetilde{C}_{duue}\right]_{ijkl}\left[\left(d_R^i\right)^\intercal C u_R^j\right]\left[\left(u_R^k\right)^\intercal C e_R^l\right]$ $\left[\widetilde{C}_{duue}\right]_{ijkl}=\left[C_{duue}\right]_{pqrs}\left[V_{d_R}\right]_{pi}\left[V_{u_R}\right]_{qj}\left[V_{u_R}\right]_{rk}\left[V_{e_R}\right]_{sl}$

$\boxed{Dimension-5}$

Operator	Definition in the mass basis
$Q_{\ell\ell\varphi\varphi}$	$\left[\widetilde{C}_{\ell\ell\varphi\varphi}\right]_{ij}\left[U_{ik}\left(\nu_L^k\right)^\intercal\varphi^0-\left(e_L^i\right)^\intercal\varphi^+\right]C\left(U_{jl}\nu_L^l\varphi^0-e_L^j\varphi^+\right)$ $\left[\widetilde{C}_{\ell\ell\varphi\varphi}\right]_{ij}=\left[C_{\ell\ell\varphi\varphi}\right]_{mn}\left[V_{e_L}\right]_{mi}\left[V_{e_L}\right]_{nj}$

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