Solucion Ejercicio 1

$W_o = \frac{2\pi}{T} = 2\pi F_o \qquad T = t_f - t_i = \frac{1}{2F_o} + \frac{1}{2F_o} = \frac{1}{F_o}$

$F_o = \frac{1}{T} \qquad T = \frac{1}{F_o}$

$t = \frac{-1}{2F_o}$

$x(t) = |A\cos(2\pi F_o t)|^2$

$x(t) |A\cos(W_o t)|^2 = A^2\cos^2(W_o t)$

$\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$

$$x(t) = A^2\left(\frac{1+\cos(2W_o t)}{2}\right) = \frac{A^2}{2} + \frac{A^2\cos(2W_o t)}{2}$$

Sabemos que es simetria par por lo cual

$b_n = 0$

$$\frac{A^2}{2} + \frac{A^2\cos(2W_o t)}{2} = a_o + \sum_{n=-N}^{N} a_n\cos(nW_o t)$$

$a_0 = \frac{A^2}{2}$

$a_2 = \frac{A^2}{2}$

$$a_n = \begin{cases} \frac{A^2}{2} & n = 0 \\ \frac{A^2}{2} & n = 2 \\ 0 & \forall n \setminus \{0, 2\} \end{cases}$$

$C_n = \frac{a_n - jb_n}{2} \qquad C_o = a_o$

$$C_n = \begin{cases} \frac{A^2}{2} & n = 0 \\ \frac{A^2}{4} & n = \{2, -2\} \\ 0 & \forall n \setminus \{0, 6, -6\} \end{cases}$$

$$P_x = \frac{1}{T}\int_{t_1}^{t_f} |x(t)|^2 dt$$

$$T = \frac{1}{F_0} \qquad F_0 = \frac{W_0}{2\pi}$$

$$x(t) = \frac{A^2}{2} + \frac{A^2 \operatorname{Cos}(2W_0 t)}{2} \qquad \frac{1}{\frac{1}{F_0}}$$

$$P_x = \frac{1}{T}\int_{-1/2F_0}^{1/2F_0} \left|\frac{A^2}{2} + \frac{A^2\operatorname{Cos}(2W_0t)}{2}\right|^2 dt$$

$$P_x = F_0\int_{-1/2F_0}^{1/2F_0} \frac{A^4}{4} + \frac{2A^4\operatorname{Cos}(2W_0t)}{4} + \frac{A^4\operatorname{Cos}^2(2W_0t)}{4}\, dt$$

$$P_x = F_0\int_{-1/2F_0}^{1/2F_0} \frac{A^4}{4}\left[1 + 2\operatorname{Cos}(2W_0t) + \operatorname{Cos}^2(2W_0t)\right] dt$$

$$P_x = \frac{A^4F_0}{4}\int_{-1/2F_0}^{1/2F_0} 1 + 2\operatorname{Cos}(2W_0t) + \frac{1+\operatorname{Cos}(4W_0t)}{2}\, dt$$

$$P_x = \frac{A^4F_0}{4}\int_{-1/2F_0}^{1/2F_0} 1 + 2\operatorname{Cos}(2W_0t) + \frac{1}{2} + \frac{\operatorname{Cos}(4W_0t)}{2}\, dt$$

$$P_x = \cancel{\frac{A^4F_0}{4}\int_{-1/2F_0}^{1/2F_0}}$$

$$P_x = \frac{A^4F_0}{4}\left(t + \frac{2\operatorname{Sin}(2W_0t)}{2W_0} + \frac{t}{2} + \frac{\operatorname{Sin}(4W_0t)}{8W_0}\, dt\right]_{-1/2F_0}^{1/2F_0}$$

$$P_x = \frac{A^4F_0}{4}\left(\frac{1}{2F_0} + \frac{2\sin(2W_0/2F_0)}{2W_0} + \frac{1}{4F_0} + \frac{\sin(4W_0/2F_0)}{8W_0}\right.$$

$$\left. - \left(-\frac{1}{2F_0} + \frac{2\sin(-2W_0/2F_0)}{2W_0} - \frac{1}{4F_0} + \frac{\sin(-4W_0/2F_0)}{8W_0}\right)\right)$$

$$P_x = \frac{A^4F_0}{4}\left(\frac{1}{2F_0} + \frac{2\sin(W_0/F_0)}{2W_0} + \frac{1}{4F_0} + \frac{\sin(2W_0/F_0)}{8W_0}\right.$$

$$\left. + \frac{1}{2F_0} - \frac{2\sin(-W_0/F_0)}{2W_0} + \frac{1}{4F_0} - \frac{\sin(-2W_0/F_0)}{8W_0}\right)$$

$$P_x = \frac{A^4F_0}{4}\left(\frac{2}{2F_0} + \frac{2\sin(W_0/F_0) - 2\sin(-W_0/F_0)}{2W_0} + \frac{2}{4F_0}\right.$$

$$\left. + \frac{\sin(2W_0/F_0) - \sin(-2W_0/F_0)}{8W_0}\right)$$

$$W_0 = 2\pi F_0 \quad F_0 = \frac{W_0}{2\pi}$$

$$P_x = \frac{A^4F_0}{4}\left(\frac{2\cdot 1}{2F_0} + \frac{\sin(2\pi) - \sin(-2\pi)}{2W_0} + \frac{1}{2F_0}\right.$$

$$\left. + \frac{\sin(4\pi) - \sin(-4\pi)}{8W_0}\right)$$

$$P_x = \frac{A^4F_0}{4}\left(\frac{3}{2F_0} + \cancel{\overset{0}{\frac{\sin(2\pi) - \sin(-2\pi)}{2W_0}}} + \cancel{\overset{0}{\frac{\sin(4\pi) - \sin(-4\pi)}{8W_0}}}\right)$$

$$P_x = \frac{A^4F_0}{4}\left(\frac{3}{2F_0}\right) = \frac{3A^4\cancel{F_0}}{8\cancel{F_0}} = \frac{3A^4}{8}$$

$$P_x = \frac{3A^4}{8}$$

$$E_x(\%) = \left(1 - \frac{|C_{-2}|^2 + |C_0|^2 + |C_2|^2}{P_x}\right) 100\%$$

$$E_x(\%) \left(1 - \left(\frac{A^4}{64} + \frac{A^4}{4} + \frac{A^4}{64}\right)\left(\frac{1}{P_x}\right)\right) 100\%$$

$$E_x(\%) = \left(1 - \left(\frac{9A^4}{32}\right)\left(\frac{1}{\frac{3A^4}{8}}\right)\right) 100\%$$

$$E_x(x) = \left(1 - \left(\frac{9A^4}{32}\right)\left(\frac{8}{3A^4}\right)\right) 100\%$$

$$E(x) = \left(1 - \left(\frac{72}{96}\right)\right) 100\% = \left(1 - \frac{19}{24}\right) 100\%$$

$$\cancel{E(x) = (1 - 0.}$$

$$E(x) = 20{,}83\%$$

$$x(t) = C_{-2}e^{-j2t} + C_0 e^{0} + C_2 e^{j2t}$$

$$= \frac{-A^2}{4}(\cos(2t) - j\sin(2t)) + \frac{A^2}{2} - \frac{A^2}{2}(\cos(2t) + j\sin(2t))$$

$$= \frac{A^2}{2} + \frac{A^2\cos(2w_0t)}{2}$$