

total cards = 52

prob of pulling a red king.

2 colours = red & black

suits = 4

spade, diamond, heart, club
red

will have 2 red king

$$\frac{2}{52} =$$

$$\frac{4}{52} =$$

$\frac{2}{52}$ >> Joint prob.

=====

evidence = it's red

heart, diamond.

13 13
 ~~~~~  
 26

$$\frac{2}{26} = \left( \frac{1}{13} \right)$$

$$\frac{2}{52} \rightarrow \frac{2}{26}$$

recalibrate  
 when we get info

joint prob  
 JP :- red king  
 $\frac{2}{52}$

info | evidence =  $\frac{2}{26}$ . it's a red card

when you get evidence you recalculate the proba.

Bayes theorem

Mr. Bayesian.

Conditional proba! -

p of king

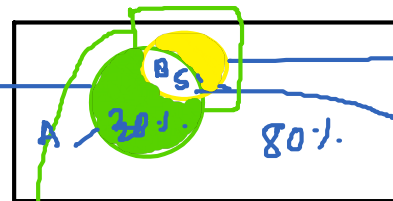
given it's red  
Info

$$\frac{2}{26} = \frac{1}{13}$$

$$\frac{0}{32} =$$

ex. flight info.

Flight delayed.

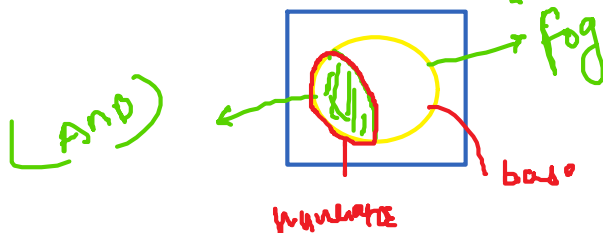


Fog but no flight delay

Flight delay & fog  
(A ∩ B) = fog.

20%  
80%

. delayed. → green.  
on the



Cond. prob.

im seeing fog at airport.  
(it's red card)

~~CP~~

CP

CP

~~JP~~

$$\begin{array}{cc} JP & CP \\ \frac{2}{52} & \frac{2}{26} \end{array}$$

Shrink

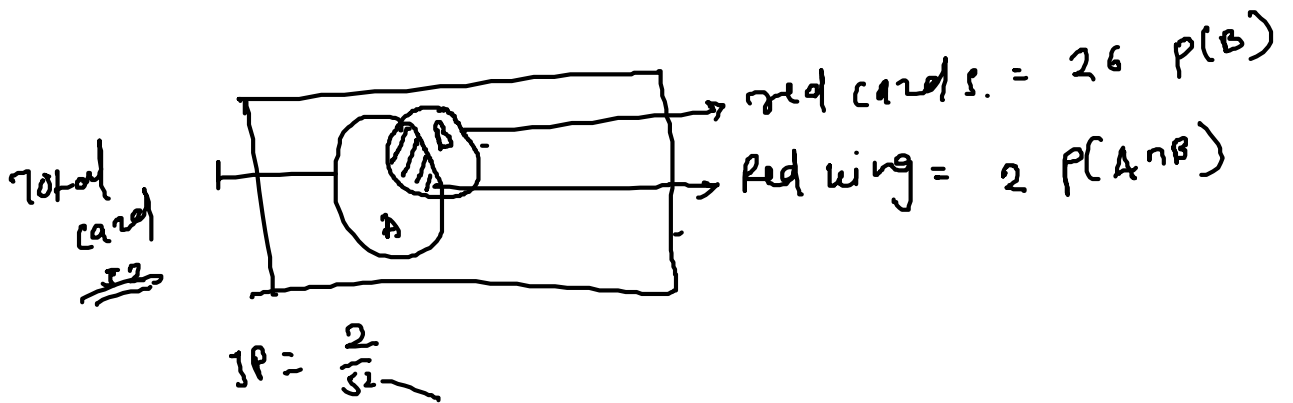
fog

ex(1) Prob of king given its red  $\frac{2}{26}$ .

ex(2) prob of flight delay given fog.

$P(\text{Flight delay given fog})$

$$P(\text{flight delay} | \text{fog}) = \frac{P(A \cap B)}{P(B)}$$



$$\begin{array}{c} P(B) \\ \downarrow \\ \frac{2}{26} \end{array}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) \times P(B) = P(A \cap B) \dots \dots \text{eqn(1)}$$

$$p(B \cap A)$$

$$p(B \cap A) = p(B|A) \times p(A)$$

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)} \quad \text{Bayesian model.}$$

ex.  $A = \text{target } \Rightarrow \text{diabetic or non-diabetic}$   
 $B = \text{ind var.}$

$p(A|B) \Rightarrow \text{diabetic for given condition}$

$p(B|A)$  - likelihood

$$\text{posterior} \rightarrow p(A|B) = \frac{p(B|A) \times p(A) \rightarrow \text{prior}}{p(B) \rightarrow \text{evidence / marginal}}$$

$A = \text{Target}$   
 $B = \text{ind. } (x_1, x_2, \dots, x_n)$



$$p(A|B) = p(\text{person being diabetic given condition})$$

$A \Rightarrow \text{diabetic}$

$B \Rightarrow \text{ind}$

20    120    150

those who are already diabetic

$p(A) \Rightarrow \text{diabetic people.}$

$P(B|A) \Rightarrow$  condition for given diabetic

| frequencies                  | freq |    |       | likelihood of freq |                |
|------------------------------|------|----|-------|--------------------|----------------|
|                              | yes  | no | total | $P(\text{yes})$    | $P(\text{no})$ |
| Flight delay                 | 4    | 16 | 20    | 4/20               | 16/20          |
| not delay                    | 1    | 79 | 80    | 1/80               | 79/80          |
| <u><math>A \cap B</math></u> | (5)  | 95 | 100   | (5/100)            | 95/100         |

$$P(\text{freq} = \text{yes} | \text{Flight delay}) = \frac{4}{20} = \underline{\underline{0.2}}$$

20% prob.

that flight is delayed given freq.

$$P(A \cap B) \Rightarrow P(\text{flight delay} | \text{freq}) = P(\text{freq} | \text{flight delay}) \times P(\text{FD})$$

$$= \frac{4}{20} \times \frac{20}{100}$$

$$= \underline{\underline{0.04}}$$

prob

(4/100)

case (1) .  $P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$

eq = 0.5 > class  
or = 0.5 < class

case (2)  $1 - P(A|B) = \frac{1 - P(B|A) \times P(A)}{P(B)}$

$$= \frac{P(B) - P(B|A) \times P(A)}{P(B)}$$

hi

###

prob. of a card is king

$$p(\text{king}) = \frac{4}{52} = \frac{1}{13}$$

$$p(\text{king} | \text{face}) = \frac{p(\text{face} | \text{king}) \times p(\text{king})}{p(\text{face})}$$

$$p(\text{face}) = \frac{12}{52}$$

$$p(\text{face} | \text{king}) = \frac{4}{4} = 1$$

$$p(\text{king} | \text{face}) = \frac{1 \times \frac{1}{13}}{\frac{12}{52}} = \frac{1}{3}$$

$$\frac{4}{12} = \frac{1}{3}$$

II

frequency table

|          | play |    | outlook |
|----------|------|----|---------|
|          | yes  | no | total   |
| overcast | 4    | 0  | 4       |
| rainy    | 3    | 2  | 5       |
| sunny    | 2    | 3  | 5       |
|          | 9    | 5  | 14      |

likelihood table

|          | play |      |      |
|----------|------|------|------|
|          | yes  | no   |      |
| overcast | 4/9  | 0/5  | 4/14 |
| rainy    | 3/9  | 2/5  | 5/14 |
| sunny    | 2/9  | 3/5  | 5/14 |
|          | 9/14 | 5/14 |      |

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)}$$

A = yes (play)

B = sunny

A|B = yes | sunny

B|A = sunny | yes

$$p(\text{yes} | \text{sunny}) = \frac{p(\text{sunny} | \text{yes}) \times p(\text{yes})}{p(\text{sunny})}$$

$P(\text{yes}) = 0.64$

$P(\text{sunny})$

$$P(\text{sunny}|\text{yes}) = \frac{2}{9} = 0.22$$

$$P(\text{sunny}) = \frac{5}{14} = 0.36$$

$$P(\text{yes}) = \frac{9}{14} = 0.64$$

hi

$$P(\text{yes}|\text{sunny}) = \frac{0.22 \times 0.64}{0.36} = 0.39$$

for NO

$$P(\text{sunny}) = 0.36$$

$$P(\text{NO}) = 0.36$$

$$P(\text{sunny}|\text{NO}) = \frac{3}{5} = 0.6$$

$$P(\text{NO}|\text{sunny}) = \frac{0.6 \times 0.36}{0.36} = 0.6$$

$$P(\text{yes}) = 0.39$$

$$P(\text{NO}) = 0.6$$

$$0.39 \approx 1$$

Humidity

|        | U | N | T  | $P(Y)$     | $P(N)$ | $P$  |
|--------|---|---|----|------------|--------|------|
| high   | 3 | 4 | 7  | 3/9        | 4/5    | 7/14 |
| normal | 6 | 1 | 7  | <u>6/9</u> | 1/5    | 7/14 |
|        | - | 5 | 14 | 9/14       | 5/14   |      |

|        |   |   |    |            |      |      |
|--------|---|---|----|------------|------|------|
| normal | 6 | 1 | 1  | <u>6/9</u> | 1/5  | 1/14 |
|        | 9 | 5 | 14 | 9/14       | 5/14 |      |

Windy

|       |   |   |    |        |        |      |
|-------|---|---|----|--------|--------|------|
|       |   |   |    | $p(u)$ | $p(w)$ | $P$  |
|       | 4 | N | T  |        |        |      |
| True  | 3 | 3 | 6  | 3/9    | 3/5    | 6/14 |
| False | 6 | 2 | 8  | 5/9    | 2/5    | 7/14 |
|       | 9 | 5 | 14 | 9/14   | 5/14   |      |

EM Outlook = rainy, Hum = normal  
windy = false. play?

$$Yes = p(\text{out} = \text{rainy} | \text{yes}) \times p(\text{Hum} = \text{normal} | \text{yes}) \times p(\text{wind} = \text{false} | \text{yes}) \times p(\text{yes})$$

$$= \frac{3}{9} \times \frac{6}{9} \times \frac{6}{5} \times \frac{3}{14}$$

$$Yes = 0.095$$

$$p(\text{yes} | x_1, x_2, x_3) = p(\text{yes} | \text{rainy, normal, false})$$

$$No = p(\text{out} = \text{rainy} | \text{no}) \times p(\text{Hum} = \text{normal} | \text{no}) \times p(\text{wind} = \text{false} | \text{no}) \times p(\text{no})$$

$$= \frac{1}{9} \times \frac{1}{5} \times \frac{2}{5} \times \frac{8}{14}$$

$$= 0.011$$

$$p(\text{yes}) = \frac{0.095}{0.095 + 0.011} = 0.896$$

$$p(\text{no}) = \frac{0.011}{0.095 + 0.011} = 0.103$$

$$+ = 1$$



$$p(\text{no}) = \frac{0.011}{0.095 + 0.011} = 0.103$$

Ans = outlook = rainy, Humidity = normal,  
wind = false

play = 8

there are 89 % chance there will  
answer today.