Notes about Code for Routing

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January 28, 2021

1 TSP

1.1 Miller–Tucker–Zemlin (MTZ) formulation

$$
\begin{aligned}
& \min \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} c_{ij} x_{ij} \\
s.t. \quad & \sum_{j=1, j\neq i}^{n} x_{ij} = 1 && i = 1, 2, \ldots, n \\
& \sum_{i=1, i\neq j}^{n} x_{ij} = 1 && j = 1, 2, \ldots, n \\
& u_j - u_i + n(1 - x_{ij}) \geq 1 && 1 \leq i \leq n, 2 \leq j \leq n, i \neq j \\
& x_{ij} \in \{0, 1\} && 1 \leq i \neq j \leq n \\
& u_1 = 1 \\
& u_i \in \{1, 2, \ldots, n\} && i = 2, \ldots, n
\end{aligned}
$$

1.2 Dantzig–Fulkerson–Johnson (DFJ) formulation

$$
\begin{aligned}
& \min \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} c_{ij} x_{ij} \\
s.t. \quad & \sum_{j=1, j\neq i}^{n} x_{ij} = 1 && i = 1, 2, \ldots, n \\
& \sum_{i=1, i\neq j}^{n} x_{ij} = 1 && j = 1, 2, \ldots, n \\
& \sum_{i\in S} \sum_{j\in S, j\neq i} x_{ij} \leq |S| - 1 && S \subset \{1, 2, \ldots, n\}, |S| \geq 2 \\
& x_{ij} \in \{0, 1\} && 1 \leq i \neq j \leq n
\end{aligned}
$$

1.3 Benders Decomposition

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in\mathbb{A}} c_{ij}x_{ij} \\
s.t. \quad & \sum_{(i,j)\in\mathbb{A}} x_{ij} = 1 && i \in \mathbb{V} \\
& \sum_{(j,i)\in\mathbb{A}} x_{ji} = 1 && i \in \mathbb{V} \\
\sum_{(i,j)\in\mathbb{A}} y_{kij} - \sum_{(j,i)\in\mathbb{A}} y_{kji} = & \begin{cases} 1 & \text{if } i = 0 \\ -1 & \text{if } i = k \\ 0 & \text{otherwise} \end{cases} && k \in \mathbb{V}\setminus\{0\}, i \in \mathbb{V} \\
& y_{kij} - x_{ij} \leq 0 && k \in \mathbb{V}\setminus\{0\}, (i,j) \in \mathbb{A} \\
& x_{ij} \in \{0,1\} && (i,j) \in \mathbb{A} \\
& y_{kij} \geq 0 && k \in \mathbb{V}\setminus\{0\}, (i,j) \in \mathbb{A}
\end{aligned}
$$