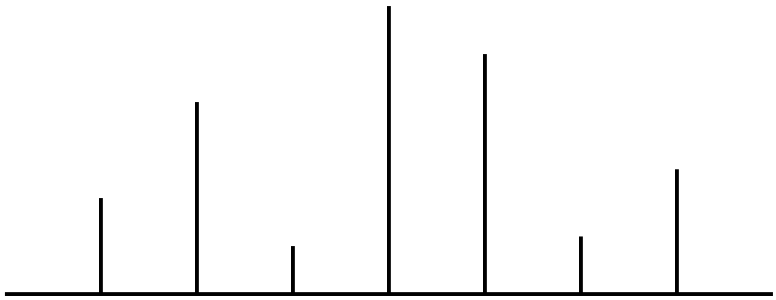
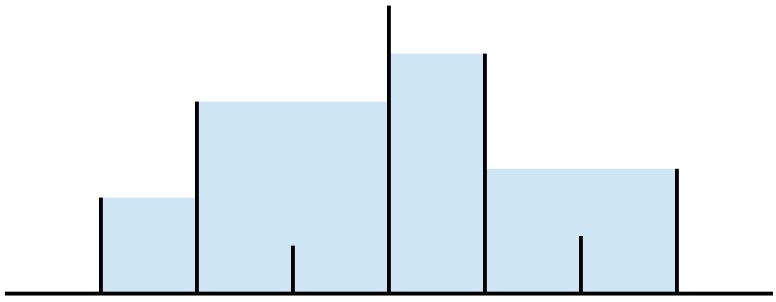
Recursion Problems: Group B

The following problems all involve recursion, memoization, or dynamic programming. Try to see if you can come up with the most efficient solutions possible!

1. Given a number n, generate all distinct ways to write n as the sum of positive integers. For example, with $n = 4$, the options are 4, 3 + 1, 2 + 2, 2 + 1 + 1, and 1 + 1 + 1 + 1.
2. In a binary tree, a *common value subtree* is a complete subtree where every node has the same value. (A complete subtree is a subtree consisting of a node and all its children). Determine the largest common value subtree in a nonempty binary tree.
3. Suppose you have a multiway tree where each node has an associated integer value. Find a set of nodes with the maximum possible sum, subject to the constraint that you cannot choose a node and any of its children at the same time.
4. Suppose that you have a rectangular box with several vertical partitions in it. The ends of the box are open. For example, here's one possible box:



Suppose you pour an enormous volume of water onto the box. Water will become trapped between the partitions in the box, as shown here:



Given the heights of the partitions, determine the heights of the water between the partitions.