COMP3270 1920 Chapter 4 Teacher Notes

Slide 36:

d^n

Slide 38:

Probability that it’s hot and sunny
0.4

Probability that it’s hot
0.5

Probability that it’s hot or sunny
0.7

Slide 39:

P(+x, +y) =
0.2

P(+x) =
0.5

P(-y OR +x) =
0.6

Slide 41:

P(X)
0.5
0.5

P(Y)
0.6
0.4

Slide 42:

Supplement with this equation: $P(a,b) = P(a \cap b) = P(a|b)P(b)$

Slide 44:

Conditional probability formula:

$P(+x \mid +y) = P(+x, +y) / P(+y) = 0.2 / (0.2+0.4) = 1 / 3$
$P(X = true \mid Y = true) = ...$

$P(-x \mid +y) = P(-x, +y) / P(+y) = 0.4 / (0.2+0.4) = 2 / 3$
(this also follows from the above, i.e., $1-P(+x \mid +y)$)

$P(-y \mid +x) = = P(-y, +x) / P(+x) = 0.3 / (0.2+0.3) = 0.6$

Slide 48:

X	Y	P(X, -y)
+x	-y	0.3
-x	-y	0.1

X	Y	P(X| -y)
+x	-y	0.75
-x	-y	0.25

Slide 59

D	W	P(D,W)
wet	sun	.8 * .1 = .08
dry	sun	.8 * .9 = .72

wet	rain	.2 * .7 = .14
dry	rain	.2 * .3 = .06

Slide 60:

Conditional Probability: $$P(x|y) = \frac{P(x,y)}{P(y)}$$

Why is P(x1) P(x2 | x1) P(x3 | x1, x2) = P(x1, x2, x3) ?

Note that P(x2 | x1) = P(x2, x1) / P(x1)

It follows that P(x1) P(x2 | x1) P(x3 | x1, x2) = P(x1) (P(x2, x1) / P(x1)) P(x3 | x1, x2)

Note that P(x3 | x1, x2) = P(x3, x1, x2) / P(x1, x2)

It follows that P(x1) P(x2 | x1) P(x3 | x1, x2) = P(x1) * P(x2, x1) / P(x1) * P(x3, x1, x2) / P(x1, x2) = P(x3, x2, x1) = P(x1, x2, x3)

What does this notation mean: $$P(x_1, x_2, \ldots x_n) = \prod_i P(x_i | x_1 \ldots x_{i-1})$$?

Consider n = 4

P(x1) [i=1]
P(x2 | x1)[i=2]
P(x3 | x1, x2)[i=3]
P(x4 | x1, x2, x3)[i=4]

Putting it all together: P(x1, x2, …, x4) = P(x1) P(x2 | x1)P(x3 | x1, x2)P(x4 | x1, x2, x3)

Similar to the above case we can prove that this is always true.

Slide 63:

Given: $P(+m) = 0.0001$, $P(+s \mid +m) = 0.8$, $P(+s) = 0.01$

$P(+m \mid +s) = P(+s \mid +m) P(+m) / P(+s) = 0.8 * 0.0001 / 0.01 = 0.008$

Note: posterior probability of meningitis still very small, why should you still get stiff necks checked out?

Slide 64:

D	W	P(D|W)
wet	sun	0.1
dry	sun	0.9
wet	rain	0.7
dry	rain	0.3

W	P(W)
sun	0.8
rain	0.2

<u>Method 1:</u>

P(sun | dry) = P(dry | sun) / P(dry) * P(sun)

P(dry) = P(dry|sun)P(sun) + P(dry|rain)P(rain) = 0.9 * 0.8 + 0.3 * 0.2 = 0.78

P(sun | dry) = P(dry | sun) * P(sun) / P(dry) = 0.9 * 0.8 / 0.78 = 0.72 / 0.78
P(rain | dry) = P(dry | rain) * P(rain) / P(dry) = 0.3 * 0.2 / 0.78 = 0.06 / 0.78

<u>Method 2:</u>

Note: P(sun | dry) + P(rain | dry) should be 1.0 We need to normalize!
Let use z instead of P(dry) to normalize.

P(sun | dry) = P(dry | sun) / z * P(sun) = 0.9 * 0.8 / z = 0.72 / z
P(rain | dry) = P(dry | rain) / z * P(rain) = 0.3 * 0.2 / z = 0.06 / z
z = 0.78

Slide 72:
$T \perp\!\!\!\perp C$
$W \perp\!\!\!\perp C$

Tasmania

Slide 77:

We want to show that
$P(x,y|z) = P(x|z) P(y|z)$
Is the same as
$P(x|z,y) = P(x|z)$

Let's assume $P(x,y|z) = P(x|z) P(y|z)$ is given.

Consider $P(x|z,y) = P(x,z,y) \backslash P(z,y)$ from the definition of conditional probability

Using $P(x,z,y) = P(x,y|z) P(z)$ we get

$P(x|z,y) = P(x,z,y) \backslash P(z,y) = P(x,y|z) P(z) \backslash P(z,y)$

Using our assumption $P(x,y|z) = P(x|z) P(y|z)$ we get

$P(x|z,y) = P(x|z) P(y|z)P(z) \backslash P(z,y)$

Note that $P(y|z)P(z) = P(z,y)$ (product rule)

It follows that $P(x|z,y) = P(x|z)$

Slide 79:

$T \perp\!\!\!\perp U \mid R$

If we know whether it is raining or not, then the umbrella doesn't tell us anything more about traffic

$A \perp\!\!\!\perp F \mid S$

Alarm is independent of fire given smoke. Once I know there is smoke, fire doesn't tell me anything more about whether the alarm will go off or not.

Slide 82:

1.

$P(+g, +a, +b, +s) = P(+g)P(+a|+g)P(+b|+g, +a)P(+s|+g,+a,+b)$

Assuming: $G \perp\!\!\!\perp S \mid A$, $A \perp\!\!\!\perp B$, $G \perp\!\!\!\perp B$

$P(+g, +a, +b, +s) = P(+g)P(+a|+g)P(+b)P(+s|+a,+b)$

$P(+g, +a, +b, +s) = P(+g)P(+a| + g)P(+b)P(+s| + b, +a) = (0.1)(1.0)(0.4)(1.0) = 0.04$

2.

$P(+a) = P(+a| +g)P(+g) + P(+a| -g)P(-g) = (1.0)(0.1) + (0.1)(0.9) = 0.19$

3.

$P(+a| + b) = P(+a) = 0.19$

4.

$P(+a| + s, +b) = P(+a,+b,+s) / (P(+a,+b,+s)+P(-a,+b,+s)) = (0.19)(0.4)(1.0) / ((0.19)(0.4)(1.0)+(0.81)(0.4)(0.8)) = 0.2267$

$P(+g| + a) = P(+g)P(+a|+g) / (P(+g)P(+a|+g)+P(-g)P(+a|-g)) = (0.1)(1.0) / ((0.1)(1.0)+(0.9)(0.1)) = 0.5263$

$P(+g| + b) = P(+g) = 0.1$

Slide 83:

P(−e, −s, −m, −b) = P(−e)P(−m)P(−s| − e, −m)P(−b| − m) = (0.6)(0.9)(0.9)(0.9) = 0.4374

P(+b) = P(+b| + m)P(+m) + P(+b| − m)P(−m) = (1.0)(0.1) + (0.1)(0.9) = 0.19

P(+m| + b) = P (+b|+m)P (+m) / P (+b) = 0.5263

P(+m| + s, +b, +e) = P(+m, +s, +b, +e) / (sum_m P(m, +s, +b, +e)) = (0.4)(0.1)(1.0)(1.0) / ((0.4)(0.1)(1.0)(1.0) + (0.4)(0.9)(0.8)(0.1)) = 0.5814

P(+e| + m) = P(+e) = 0.4

Slide 87:

According to the chain rule slide 59

P(X1, X2, X3, X4) = P(X1)P(X2|X1)P(X3|X2, X1)P(X4|X3, X2, X1)

<=> because X3 ⫫ X1 | X2

P(X1, X2, X3, X4) = P(X1)P(X2|X1)P(X3|X2)P(X4|X3, X2, X1)

<=> because X4 ⫫ {X1,X2} | X3

P(X1, X2, X3, X4) = P(X1)P(X2|X1)P(X3|X2)P(X4|X3)

Slide 90:

Assume: $X_3 \perp\!\!\!\perp X_1 \mid X_2$ and $X_4 \perp\!\!\!\perp X_1, X_2 \mid X_3$

$$\begin{aligned} P(X_1 \mid X_2, X_3, X_4) &= \frac{P(X_1, X_2, X_3, X_4)}{P(X_2, X_3, X_4)} \\ &= \frac{P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_2)P(X_4 \mid X_3)}{\sum_{x_1} P(x_1)P(X_2 \mid x_1)P(X_3 \mid X_2)P(X_4 \mid X_3)} \\ &= \frac{P(X_1, X_2)}{P(X_2)} \\ &= P(X_1 \mid X_2) \end{aligned}$$

First = follows from conditional probability definition

Second = follows from conditional independence assumptions made and marginal dist. definition

Third = follows from product rule and marginal dist. definition

Fourth = follows from the definition of conditional probability

Slide 93:

Why is $P(X2=sun) = P(X2|sun)\ P(X1=sun)P(X1=sun) + P(X2=sun|X1=rain)P(X1=rain)$?

According to the product rule: $P(X1, X2) = P(X2|X1)P(X1)$
Then get the marginal: $P(X2) = P(X2|X1=sun)P(X1=sun) + P(X2|X1=rain)P(X1=rain)$

Slide 94:

$P(X3 = sun) = P(X3 = sun \mid X2 = sun)P(X2 = sun) + P(X3 = sun \mid X2 = rain)P(X2 = rain)$
$P(X3 = sun) = .9 * 0.9 + 0.3 * 0.1 = 0.84$
$P(X3 = rain) = P(X3 = rain \mid X2 = sun)P(X2 = sun) + P(X3 = rain \mid X2 = rain)P(X2 = rain)$
$P(X3 = rain) = .1 * 0.9 + 0.7 * 0.1 = 0.16$