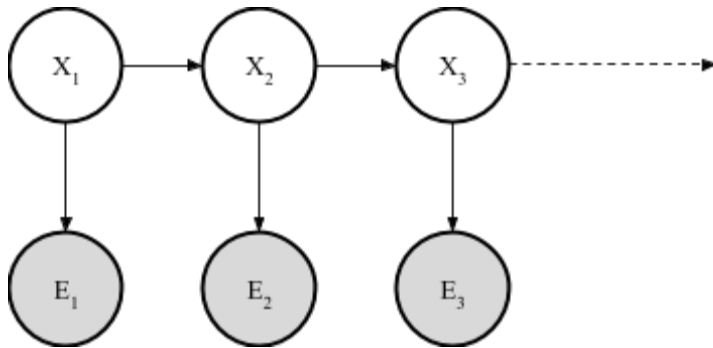


## COMP3270 Chapter 6 Teacher Notes

Slide 11



Consider the Chain Rule:  $P(x_1, x_2, x_3, \dots) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots$

It follows from the Chain Rule

$$P(x_1, e_1, x_2, e_2, x_3, e_3) =$$

$$P(x_1) P(e_1|x_1) P(x_2|x_1, e_1) P(e_2|x_1, e_1, x_2) P(x_3|x_1, e_1, x_2, e_2) P(e_3|x_1, e_1, x_2, e_2, x_3)$$

Assuming that:

$$x_2 \perp\!\!\!\perp e_1 \mid x_1$$

$$e_2 \perp\!\!\!\perp x_1, e_1 \mid x_2$$

$$x_3 \perp\!\!\!\perp x_1, e_1, e_2 \mid x_2$$

$$e_3 \perp\!\!\!\perp x_1, e_1, x_2, e_2 \mid x_3$$

It follows:

$$P(x_1, e_1, x_2, e_2, x_3, e_3) = P(x_1) P(e_1|x_1) P(x_2|x_1) P(e_2|x_2) P(x_3|x_2) P(e_3|x_3)$$

Slide 13

Does this mean that evidence variables are guaranteed to be independent?

A: No, they tend to be correlated by the hidden state

Slide 15

$X_t$	$X_{t+1}$	$P(X_{t+1} X_t)$
+x	+x	0.2
+x	-x	0.8
-x	+x	0.6
-x	-x	0.4

$X_t$	$E_t$	$P(E_t X_t)$
+x	+e	0.5
+x	-e	0.5
-x	+e	0.9
-x	-e	0.1

$X_1$	$P(X_1)$
+x	0.7
-x	0.3

Apply the Joint Distribution of an HMM formula!

$$P(X_1, E_1, X_2, E_2) = P(X_1) P(E_1|X_1) P(X_2|X_1) P(E_2|X_2)$$

$$P(X_1, E_1=+e, X_2, E_2=-e) = P(X_1) P(E_1=+e | X_1) P(X_2 | X_1) P(E_2=-e|X_2)$$

$X_1$	$X_2$	$P(X_1) P(E_1=+e   X_1) P(X_2   X_1) P(E_2=-e X_2)$
+x	+x	$0.7 * 0.5 * 0.2 * 0.5 = 0.035$
+x	-x	$0.7 * 0.5 * 0.8 * 0.1 = 0.028$
-x	+x	$0.3 * 0.9 * 0.6 * 0.5 = 0.081$
-x	-x	$0.3 * 0.9 * 0.4 * 0.1 = 0.0108$

Slide 16

$P(X_2, E_1=+e, E_2=-e) = ?$

X2	$P(X_1) P(E_1=+e   X_1) P(X_2   X_1) P(E_2=-e X_2)$
+x	$0.7 * 0.5 * 0.2 * 0.5 + 0.3 * 0.9 * 0.6 * 0.5 = 0.035 + 0.081 = 0.116$
-x	$0.7 * 0.5 * 0.8 * 0.1 + 0.3 * 0.9 * 0.4 * 0.1 = 0.028 + 0.0108 = 0.0388$

Slide 48

$$B(+r) = 0.8182$$

$$B(-r) = 0.1818$$

$$B'(X_{t+1}=+r) = 0.7 * 0.8182 + 0.3 * 0.1818 = 0.6273$$

$$B'(X_{t+1}=-r) = 0.3 * 0.8182 + 0.7 * 0.1818 = 0.3727$$

Slide 49

$$B(X_{t+1} = +r) \propto 0.9 * 0.6273 = 0.5646$$

$$B(X_{t+1} = -r) \propto 0.2 * 0.3727 = 0.0745$$

$$z = 0.5646 + 0.0745 = 0.6391$$

$$B(X_{t+1} = +r) = 0.5646 / z = 0.8834$$

$$B(X_{t+1} = -r) = 0.0745 / z = .1166$$