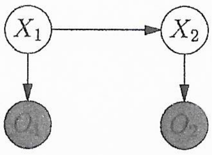
Chapter 5 - Handout 1

Consider the following Hidden Markov Model (HMM)



X_1	$\Pr(X_1)$
0	0.3
1	0.7

X_t	X_{t+1}	$\Pr(X_{t+1}\|X_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

X_t	O_t	$\Pr(O_t\|X_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that $O_1 = A$ and $O_2 = B$ is observed

a) Use the Forward algorithm to compute the probability distribution $P(X_2, O_1 = A, O_2 = B)$. Show your work

$$\begin{bmatrix} 0.1 * [0.4*0.9*0.3 + 0.8*0.5*0.7] \\ 0.5*[0.6*0.9*0.3+0.2*0.5*0.7] \end{bmatrix}$$

b) Use the Viterbi algorithm to compute the maximum probability sequence X_1, X_2. Show your work

$$\begin{bmatrix} 0.1 * \max(0.4*0.9*0.3,\ 0.8*0.5*0.7) \\ 0.5 * \max(0.6*0.9*0.3,\ 0.2*0.5*0.7) \end{bmatrix}$$

Chapter 5 - Handout 2

You have been put in charge of a Jabberwock for your friend Lewis. The Jabberwock is kept in a large tugley wood which is conveniently divided into an N × N grid. It wanders freely around the N^2 possible cells. At each time step t = 1, 2, 3, . . ., the Jabberwock is in some cell $X_t \in \{1, 2, \ldots, N\}^2$, and it moves to cell X_{t+1} randomly as follows:

- with probability $1 - \epsilon$, it chooses one of the (up to 4) valid neighboring cells uniformly at random;
- with probability ϵ, it uses its magical powers to teleport to a random cell uniformly at random among the N^2 possibilities (it might teleport to the same cell)

Suppose $\epsilon = 0.5$, N = 10 and that the Jabberwock always starts in $X_1 = (1, 1)$.

a) Compute the probability that the Jabberwock will be in $X_2 = (2, 1)$ at time step 2. What about $P(X_2 = (4, 4))$?

$P(X_2 = (2, 1)) = \underline{1/2 * 1/2 + 1/2 * 1/100 = 51/200}$

$P(X_2 = (4, 4)) = \underline{1/2 * 1/100 = 1/200}$

b) At each time step t, you don't see X_t but see E_t, which is the row that the Jabberwock is in; that is, if $X_t = (r, c)$, then $E_t = r$. You still know that $X_1 = (1, 1)$.

Suppose we see that $E_1 = 1$, $E_2 = 2$, $E_3 = 10$. Fill in the following table with the distribution over X_t after each time step, taking into consideration the evidence.

t	$P(X_t, e_{1:t-1})$	$P(X_t, e_{1:t})$	$P(X_t \mid e_{1:t})$
1	(1, 1): 1.0 others: 0.0	(1, 1): 1.0 others: 0.0	(1, 1): 1.0 others: 0.0
2	(1,2), (2,1): $\frac{51}{200}$ others: $\frac{1}{200}$	(2,1): $\frac{51}{200}$ others: $\frac{1}{200}$	(2,1): 51/60 (2, 2...10): 1/60