Questions

1. Given Euler angles $\theta_x, \theta_y, \theta_z$, the corresponding rotation matrix is given by

$$R = R_x R_y R_z = \begin{pmatrix} C_y C_z & -C_y S_z & S_y \\ S_x S_y C_z + C_x S_z & -S_x S_y S_z + C_x C_z & -S_x C_y \\ -C_x S_y C_z + S_x S_z & C_x S_y S_z + S_x C_z & C_x C_y \end{pmatrix}$$

   where

$$\begin{aligned} C_x &= \cos\theta_x \quad S_x = \sin\theta_x \\ C_y &= \cos\theta_y \quad S_y = \sin\theta_y \\ C_z &= \cos\theta_z \quad S_z = \sin\theta_z \end{aligned}$$

   Consider the matrix

$$R = \begin{pmatrix} 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$$

   Find angles $\theta_x, \theta_y, \theta_z$ such that $R = R_x R_y R_z$. Are there more choices of $\theta_x, \theta_y, \theta_z$?

   Solution: One possible solution is $(45°, 90°, 0°)$. This is not the only possibility.

2. Given an axis $\mathbf{r}$ (unit vector) and an angle θ, the matrix representing the rotation of θ around $\mathbf{r}$ is given by

$$R_{\mathbf{r}\theta} = \begin{pmatrix} tx^2 + c & txy - sz & txz + sy \\ txy + sz & ty^2 + c & tyz - sx \\ txz - sy & tyz + sx & tz^2 + c \end{pmatrix}$$

   where

$$\begin{aligned} \mathbf{r} &= (x, y, z) \\ c &= \cos\theta \\ s &= \sin\theta \\ t &= 1 - \cos\theta \end{aligned}$$

   Consider the rotation matrix

$$R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

   Determine the rotation angle and rotation axis.

   Solution: $180°, (1, 0, 1)^T$ (your answer may differ by a positive scaling factor)

3. Determine the quaternion that corresponds to rotation around the axis $(-1, -2, -2)^T$ with $\theta = 270°$.

   Solution: $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{6}, -\frac{\sqrt{2}}{3}, -\frac{\sqrt{2}}{3})$

4. Find the quaternion q that corresponds to rotation around the axis $(1,1,0)$ with $90°$. Consider the point $(1,1,1)$. Let $p_1 = (0,1,1,1)$ be the corresponding quaternion. Calculate qp_1q^{-1}. Let $p_2 = (0,1,1,0)$. Argue that qp_2q^{-1} equals p_2. Show this by performing the exact calculations.

Solution: $(\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2}, 0)$