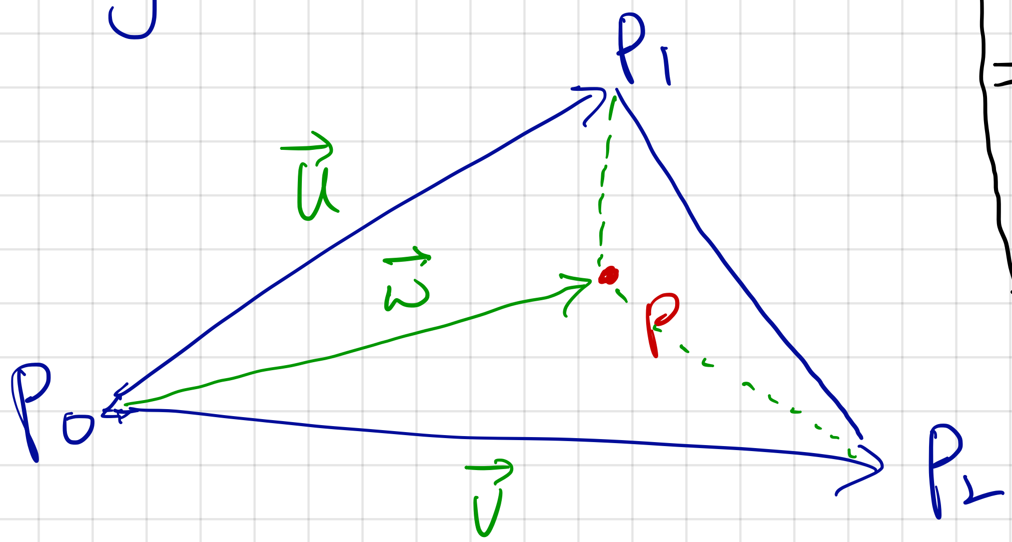
Barycentric coordinates



$\text{Area}(\Delta P_0 P P_1) = a = \frac{1}{2}\|u \times w\|$

$\text{Area}(\Delta P_0 P_2 P) = b = \frac{1}{2}\|v \times w\|$

$\text{Area}(\Delta P_0 P_1 P_2) = c = \frac{1}{2}\|u \times v\|$

To find: barycentric coordinates of P w.r.t. $\Delta P_0 P_1 P_2$

$$u = P_1 - P_0$$
$$v = P_2 - P_0$$

Equation of plane containing $\Delta P_0 P_1 P_2$:

$$P = P_0 + \alpha u + \beta v$$
$$\Rightarrow P - P_0 = \alpha u + \beta v$$
$$\Rightarrow w = \alpha u + \beta v$$

$$w = \alpha u + \beta v$$

$$\Rightarrow v \times w = v \times (\alpha u + \beta v)$$

$$= \alpha (v \times u) + \beta (v \times v)$$

$$= \alpha (v \times u)$$

Taking length of both sides, we have

$$\| v \times w \| = |\alpha| \, \| v \times u \|$$

Now, $\alpha \geq 0$ since P is inside $\Delta P_0 P_1 P_2$

and $\| v \times u \| = \| u \times v \|$

Hence, $\alpha = \dfrac{\| v \times w \|}{\| u \times v \|} = \dfrac{b}{c}$

Similarly, by considering $u \times w$, we have

$$\beta = \frac{\| u \times w \|}{\| u \times v \|} = \frac{a}{c}$$

Note that the barycentric coordinates of P inside $\triangle P_0P_1P_2$ is (t_0, t_1, t_2)

Such that $$P = t_0 P_0 + t_1 P_1 + t_2 P_2$$

where $t_0 > 0$, $t_1 > 0$, $t_2 > 0$ and $t_0 + t_1 + t_2 = 1$.

Can you derive the barycentric coordinates from α and β?

<u>Exercise</u>: Consider $\triangle P_0P_1P_2$ with

$P_0 = (1, 0, 0)$, $P_1 = (0, 2, 0)$, $P_2 = (-1, -1, 0)$

$\triangle P_0P_1P_2$ contains the point $P = (0, 0, 0)$

Find the barycentric coordinates of P.

Sol$^{\underline{n}}$: $\left(\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\right)$