Computer Graphics (COMP3271)
Fractals and Transformations

Due Date: 11:59pm, Oct 8, 2019

1. (20 marks)

(a) Show that the Mandelbrot set is symmetric about the x-axis.

(b) Show that any Julia set associated with $f(z) = z^2 + c$ is symmetric about the origin.

2. (10 marks)

Derive the rotation transformation of θ degrees about the point $(a, b)^T$ in the 2D plane.

(a) Write the transformation in the form of $X' = MX + B$.

(b) Write the transformation in the form of $X' = NX$ in homogeneous coordinates.

3. (10 marks)

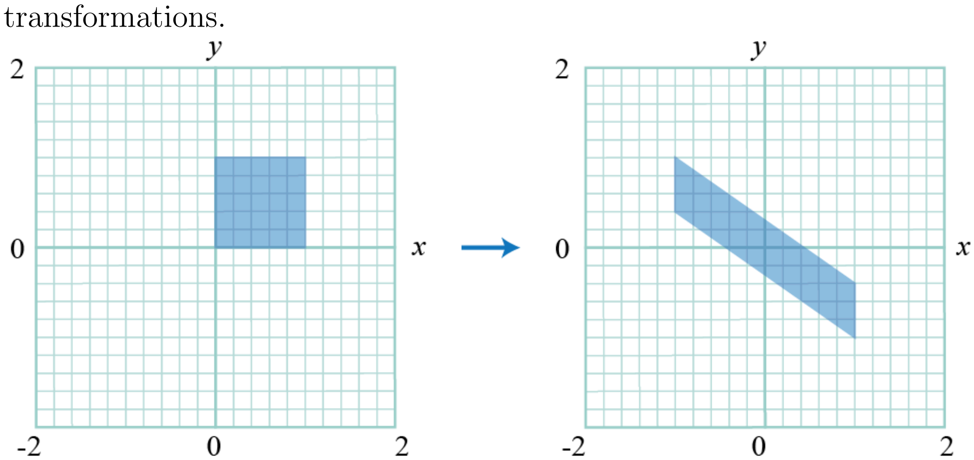
Derive the 2×2 transformation matrix for the reflection about the line $ax + by = 0$ in the $2D$ plane.

4. (10 marks)

Given two triangles ΔABC and $\Delta A'B'C'$ in 2D plane with $A = (0, 0)^T$,$B = (1, 0)^T$,$C = (0, 1)^T$, $A' = (0, 2)^T$, $B' = (-1, -1)^T$, and $C' = (2, 0)^T$. Derive the affine transformation matrix for the transformation T that maps ΔABC to $\Delta A'B'C'$ such that $T(A) = A'$, $T(B) = B'$, $T(C) = C'$.

5. (10 marks)

Give a sequence of 4 x 4 matrices that transforms the unit square in the left figure to the parallelogram in the right. Find a sequence of OpenGL function calls that implements these transformations.



6. (10 marks) Show that any sequence of rotations and translations can be replaced by a single rotation about the origin followed by a translation.

7. (10 marks) Consider the line in $\mathbb{R}^3$ given by

$$L(t) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

Let Q_1 and Q_2 be two points on this line that are at distance $\sqrt{2}$ apart from each other. We now transform the line by applying transformation T given by

$$\begin{pmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What is the distance between $T(Q_1)$ and $T(Q_2)$?

8. (20 marks)
Given an affine transformation $X' = MX + B$, where M is a 2 by 2 matrix and B is a 2D vector, find the equation of the image E of the circle $x^2 + y^2 - 1 = 0$ under this transformation. Show that E is an ellipse.