Questions

1. Consider the intersection of a ray with the ellipsoid $4x^2 + 4y^2 + z^2 - 16 = 0$. Suppose that the viewpoint (i.e., the starting point of a view ray) is at $V = (1, -1, 0)^T$ and the viewing direction is $D = (0, 1, 1)^T$. Does the ray intersect the volume? If yes, compute the intersection points between them.

   Solution: We represent the ray in the parametric form:

$$\begin{aligned} R(t) &= V + Dt, \quad t \in [0, \infty) \\ &= (1, -1, 0)^T + t\,(0, 1, 1)^T \\ &= (1, t-1, t)^T \end{aligned}$$

   Substitute $R(t)$ into the ellipsoid equation, we have $5t^2 - 8t - 8 = 0$ which has two roots

$$\begin{aligned} t_0 &= \frac{4 - 2\sqrt{14}}{5} \\ t_1 &= \frac{4 + 2\sqrt{14}}{5} \end{aligned}$$

   We reject $t_0 < 0$. Hence, the ray $R(t)$ intersects the ellipsoid at one point which is given by

$$R(t_1) = (1, \frac{2\sqrt{14} - 1}{5}, \frac{4 + 2\sqrt{14}}{5}).$$

2. Consider the intersection of a ray with a triangle. The three vertices of the triangle are $A(2, 0, 2)$, $B(0, 3, -2)$, $C(-2, 3, 2)$. We shoot a ray from the origin in the direction of $(1, 1, 1)$. Does the ray intersect the triangle? If yes, compute the closest intersection point between them.

   Solution: Let P be the plane containing the triangle ABC. Then P is given by

$$N \cdot (X - X_0) = 0,$$

   where X_0 is a point on P (we take A as X_0) and N is the normal of P given by

$$\begin{aligned} N &= AB \times AC \\ &= (-2, 3, -4)^T \times (-4, 3, 0)^T \\ &= (12, 16, 6)^T \end{aligned}$$

   The parametric representation of the ray $R(t)$ is given by

$$R(t) = S + Dt, \quad t \in [0, \infty),$$

   where S is the starting point (i.e., the origin), and $D = (1, 1, 1)$.

   Substitute $R(t)$ to the plane equation, and we have

$$t = \frac{N \cdot A}{N \cdot D} = 18/17.$$

   Since $t > 0$, we have the intersection point $R(18/17) = (\frac{18}{17}, \frac{18}{17}, \frac{18}{17})$.