5. Three Dimensional Transformations

Methods for geometric transformations and object modelling in 3D are extended from 2D methods by including the considerations for the z coordinate.

Basic geometric transformations are: Translation, Rotation, Scaling

5.1 Basic Transformations

<u>Translation</u>

We translate a 3D point by adding translation distances, tx, ty, and tz, to the original coordinate position (x,y,z):

$$x' = x + t_x,\ y' = y + t_y,\ z' = z + t_z$$

Alternatively, translation can also be specified by the transformation matrix in the following formula:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Exercise: translate a triangle with vertices at original coordinates (10,25,5), (5,10,5), (20,10,10) by t_x=15, t_y=5,t_z=5. For verification, roughly plot the x and y values of the original and resultant triangles, and imagine the locations of z values.

<u>Scaling With Respect to the Origin</u>

We scale a 3D object with respect to the origin by setting the scaling factors s_x, s_y and s_z, which are multiplied to the original vertex coordinate positions (x,y,z):

$$x' = x * s_x,\ y' = y * s_y,\ z' = z * s_z$$

Alternatively, this scaling can also be specified by the transformation matrix in the following formula:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Exercise: Scale a triangle with vertices at original coordinates (10,25,5), (5,10,5), (20,10,10) by s_x=1.5, s_y=2, and s_z=0.5 with respect to the origin. For verification, roughly plot the x and y values of the original and resultant triangles, and imagine the locations of z values.

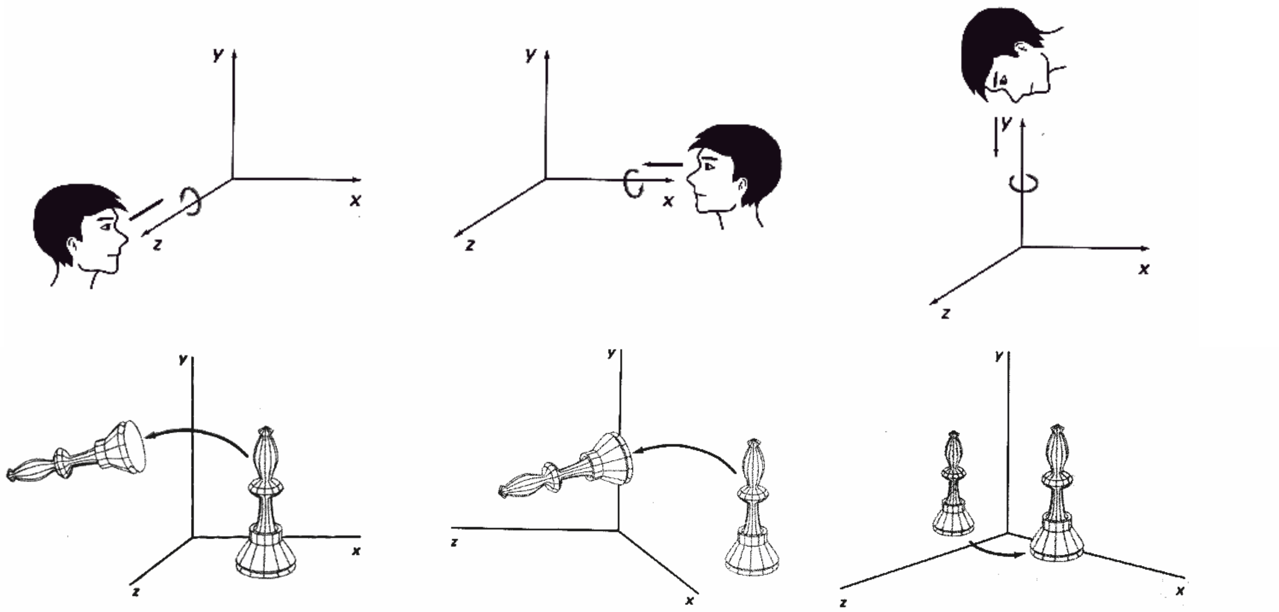
<u>Scaling with respect to a Selected Fixed Position</u>

Exercise: What are the steps to perform scaling with respect to a selected fixed position? Check your answer with the text book.

Exercise: Scale a triangle with vertices at original coordinates (10,25,5), (5,10,5), (20,10,10) by s_x=1.5, s_y=2, and s_z=0.5 with respect to the centre of the triangle. For verification, roughly plot the x and y values of the original and resultant triangles, and imagine the locations of z values.

Coordinate-Axes Rotations

A 3D rotation can be specified around any line in space. The easiest rotation axes to handle are the coordinate axes.



Z-axis rotation: x' = x cos ? - y sin ?,
y' = x sin ? + y cos ?, and
z' = z

or:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

X-axis rotation: y' = y cos ? - z sin ?,
z' = y sin ? + z cos ?, and
x' = x

or:

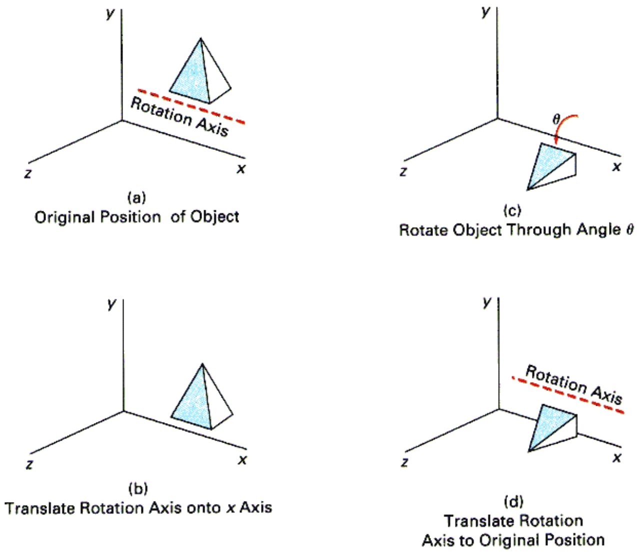
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & 0 & 0 \\ & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y-axis rotation: z' = z cos ? - x sin ?,
x' = z sin ? + x cos ?, and
y' = y

or:

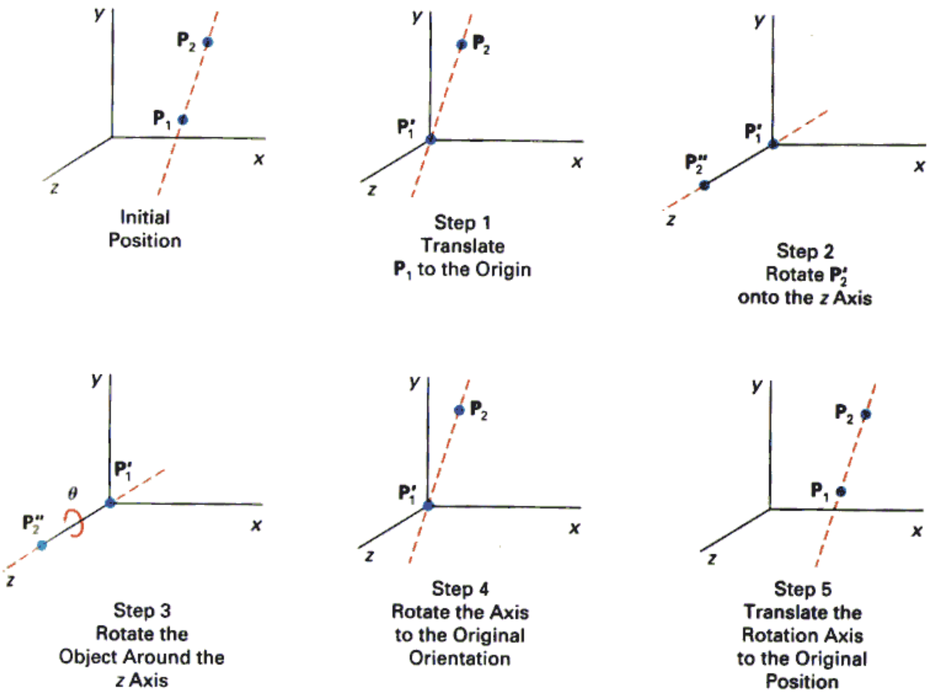
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & & \sin\theta & 0 \\ -\sin\theta & & \cos\theta & 0 \\ 0 & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Rotations About an Axis Which is Parallel to an Axis



Step 1. Translate the object so that the rotation axis coincides with the parallel coordinate axis.
Step 2. Perform the specified rotation about that axis.
Step 3. Translate the object so that the rotation axis is moved back to its original position.

General 3D Rotations



Step 1. Translate the object so that the rotation axis passes through the coordinate origin.
Step 2. Rotate the object so that the axis of rotation coincides with one of the coordinate axes.
Step 3. Perform the specified rotation about that coordinate axis.
Step 4. Rotate the object so that the rotation axis is brought back to its original orientation.
Step 5. Translate the object so that the rotation axis is brought back to its original position.