Model Predictive Control

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github.com/silvasta/summary-mpc

Contents

1 Introduction

Requirements for MPC

1. A model of the system
2. A state estimator
3. Define the optimal control problem
4. Set up the optimization problem
5. Get the optimal control sequence (solve the optimization problem)

6. Verify that the closed-loop system performs as desired

1.1 Linearization

Exact solution $x(t) = e^{A^c(t-t_0)}x_0 + \int_{t_0}^{t} e^{A^c(t-\tau)}B^c u(\tau)d\tau$

Problem Most physical systems are nonlinear

Idea use First Order Taylor expansion $f(\bar{x}) + \left.\frac{\partial f}{\partial x^\top}\right|_{\bar{x}}(x-\bar{x})$

$$\begin{aligned} \dot{x_s} &= g(x_s, u_s) = 0 & \Delta\dot{x} &= \dot{x} - \dot{x_s} = A^c\Delta x + B^c\Delta u \\ y_s &= h(x_s, u_s) & \Delta y &= y - y_s = C\Delta x + D\Delta u \end{aligned}$$

$$A^c = \left.\frac{\partial g}{\partial x^T}\right|_{\substack{x_s\\u_s}} \; B^c = \left.\frac{\partial g}{\partial u^T}\right|_{\substack{x_s\\u_s}} \; C = \left.\frac{\partial h}{\partial x^T}\right|_{\substack{x_s\\u_s}} \; D = \left.\frac{\partial h}{\partial u^T}\right|_{\substack{x_s\\u_s}}$$

1.2 Discretization

For general nonlinear systems only approximate discretization methods exist, such as Euler, quality depends on sampling time

Approximation $\dot{x}^c \approx \frac{x^c(t+T_s) - x^c(t)}{T_s}$

Notation $x(k) := x^c(t_0 + kT_s)$, $u(k) := u^c(t_0 + kT_s)$

Exact Discretization of Linear Time-Invariant Models

$$x(t_{k+1}) = \underbrace{e^{A^cT_s}}_{=A}x(t_k) + \underbrace{\int_0^{T_s} e^{A^c(T_s-\tau)}B^c d\tau}_{B=(A^c)^{-1}(A-\mathbb{I})B^c}\, u(t_k)$$

$$x(k+N) = A^N x(k) + \sum_{i=0}^{N-1} A^i B u(k+N-1-i)$$

1.3 Analysis of LTI Discrete-Time Systems

Controllabe if $\text{rank}(\mathcal{C}) = n,\ \mathcal{C} = \begin{bmatrix} B & \cdots & A^{n-1}B \end{bmatrix}$
$\forall (x(0), x^*) \exists$ finite time N with inputs $\mathcal{U}$, s.t. $x(N) = x^*$

Stabilizable iff all uncontrollable modes stable

Observable if $\text{rank}(\mathcal{O}) = n,\ \begin{bmatrix} C^\top & \cdots & (CA^{n-1})^\top \end{bmatrix}^\top$
$\forall x(0) \exists$ finite time N, s.t. the measurements $y(0), \ldots, y(N-1)$ uniquely distinguish initial state $x(0)$

Detectablitiy iff all unobservable modes stable

1.4 Lyapunov

Stability is a property of an **equilibrium point** $\bar{\mathbf{x}}$ of a system

Definition 1 (Lyapunov Stability). $\bar{\mathbf{x}}$ is **Lyapunov stable** if:
$\forall\, \epsilon > 0\, \exists\, \delta(\epsilon)$ s.t. $||x(0) - \bar{x}|| < \delta(\epsilon) \to ||x(k) - \bar{x}|| < \epsilon$

Definition 2 (Globally asymptotic stability). If $\bar{\mathbf{x}}$ is Lyapunov stable and attractive, i.e., $\lim_{k\to\infty} ||x(k) - \bar{x}|| = 0,\ \forall x(0)$ then $\bar{\mathbf{x}}$ is **globally asymptotic stable**.

Definition 3 (Global Lyapunov function). For $\bar{\mathbf{x}} = 0$, function $V : \mathbb{R}^n \to \mathbb{R}$ is called **Lyapunov function** if it is continuous at the origin, finite $\forall\, x \in \mathbb{R}^n$,

$$||x|| \to \infty \Rightarrow V(x) \to \infty$$

$$V(x) > 0\, \forall\, x \in \mathbb{R}^n \setminus \{0\} \quad V(0) = 0$$
$$V(g(x)) - V(x) \leq -\alpha(x) \quad \forall x \in \mathbb{R}^n$$

where $\alpha : \mathbb{R}^n \to \mathbb{R}$ continuous positive definite

Lyapunov Theorem

Theorem 1. If a system admits a Lyapunov function $V(x)$, then $\bar{\mathbf{x}} = 0$ is **globally asymptotically stable**.

Theorem 2 (Lyapunov indirect method). For linearization of system around $\bar{\mathbf{x}} = 0$ and resulting matrix $A = \left.\frac{\partial g}{\partial x^T}\right|_{x=0}$ $|\lambda_i| < 1$

1.5 Optimal Control

Unconstrained Finite Horizon Control Problem

$$J^\star(x(0)) := \min_U x_N^\top P x_N + \sum_{i=0}^{N-1} x_i^\top Q x_i + u_i^\top R u_i$$
$$\text{subject to } x_{i+1} = Ax_i + Bu_i \quad i = 0, \dots, N-1$$
$$x_0 = x(0)$$

1.6 Batch Approach

$$\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} \mathbb{I} \\ A \\ \vdots \\ A^N \end{bmatrix} x(0) + \begin{bmatrix} 0 & \cdots & 0 \\ B & 0 & 0 \\ AB & B & 0 \\ \vdots & \ddots & 0 \\ A^{N-1}B & \cdots & B \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{bmatrix}$$

Cost
$\overline{Q} :=$ blockdiag$(Q, \dots, Q, P)$
$\overline{R} :=$ blockdiag$(R, \dots, R)$
Optimal Input

$$U^\star(x(0)) = -\big(\underbrace{(\mathcal{S}^u)^\top \overline{Q} \mathcal{S}^u + \overline{R}}_{H(\text{Hessian})^{-1}}\big) \underbrace{(\mathcal{S}^u)^\top \overline{Q} \mathcal{S}^x}_{F^\top} x(0)$$

Optimal Cost
$J^\star(x(0)) = x(0)^\top (\mathcal{S}_x^\top \overline{Q} \mathcal{S}_x - \mathcal{S}_x^\top \overline{Q} \mathcal{S}_u (\mathcal{S}_u^\top \overline{Q} \mathcal{S}_u + \overline{R})^{-1} \mathcal{S}_u^\top \overline{Q} \mathcal{S}_x) x(0)$

1.7 Recursive Approach

1.8 Infinite Horizon LQR

$$J_\infty^\star(x(k)) = \min \sum_{i=0}^{\infty} x_i^\top Q x_i + u_i^\top R u_i$$
$$\text{subj. to } x_{i+1} = Ax_i + Bu_i$$
$$x_0 = x(k)$$

2 Optimization

A mathematical optimization problem is generally formulated as:

$$\begin{aligned} &\min_{x \in \text{dom}(f)} f(x) \\ &\text{s.t. } g_i(x) \le 0, \quad i = 1, \dots, m \\ &\quad\; h_i(x) = 0, \quad i = 1, \dots, p \end{aligned} \tag{1}$$

- $x = (x_1, ..., x_n) \in \mathbb{R}^n$ decision variable
- $f : \text{dom}(f) \to \mathbb{R}$ objectivce function
- $\mathcal{X} = \{x \in \mathbb{R}^n : g(\xi) \le 0,\ h(\xi) = 0\}$ fesabile set

2.1 Convex Sets

Definition 4 (Convex Set). A set $\mathcal{C}$ is convex if and only if

$$\theta x + (1-\theta) y \in \mathcal{C} \forall\, x, y \in \mathcal{C}, \quad \forall\, \theta \in [0,1]$$

(hyperplane || half-space) $\{x \in \mathbb{R}^n \mid a^\mathsf{T} x (= \| \le) b\}$
polyhedra $\{x \in \mathbb{R}^n \mid A^{q\times n} x \preceq b^{q\times 1}, C^{r\times n} x = d^{r\times 1}\}$
polytope
Intersection $\mathcal{C}_1, \mathcal{C}_2$ cv $\Rightarrow \mathcal{C}_1 \cap \mathcal{C}_2$ convex **(cv)**
Image under affine map $\mathcal{C} \subseteq \mathbb{R}^n$ cv $\Rightarrow \{Ax + b \mid x \in \mathcal{C}\}$ cv
Inverse IoaM $\mathcal{C} \subseteq \mathbb{R}^m$ cv $\Rightarrow \{x \in \mathbb{R}^n \mid Ax + b \in \mathcal{C}\}$ cv

2.2 Convex Functions

Check Convexity f is convex if it is composition of simple convex function with convexity preserving operations or if
$f : \mathbb{R}^n \to \mathbb{R}$ twice differentiable, $\partial^2 f / \partial x^2 \succeq 0 \,\forall\, x \in \mathbb{R}^n$
$g : \mathbb{R} \to \mathbb{R}$ with $g(t) = f(x + tv)$ convex in $t \,\forall\, x, v \in \mathbb{R}^n$
$\to f$ convex (restriction to a line)

2.2.1 Operations that preserve convexity (functions)

- the point wise maximum of convex functions is convex
- the sum of convex functions is convex
- $f(Ax+b)$ is convex if f is convex

2.3 Convex Optimization Problems

Optimal value $f^\star = \inf\{f(x) \mid g_i(x) \leq 0, h_j = 0\}$
$f^\star = +\infty$ OP is infeasible, $f^\star = -\infty$ OP is unbound below

2.3.1 Linear Programming

minimize $c^\mathsf{T} x$ s.t. $Ax - b \geq 0,\ x \geq 0$
Step 1: $\mathcal{L}(x, \lambda_1, \lambda_2) = c^\mathsf{T} x - \lambda_1^\mathsf{T}(Ax-b) - \lambda_2^\mathsf{T} x,\ \lambda_i \geq 0$
Step 2: $\inf_{x\in\mathbb{R}^n} \mathcal{L} = \lambda_1^\mathsf{T} b$, if $c - A^\mathsf{T}\lambda_1 - \lambda_2 = 0$, else $-\infty$
Step 3: Dual, maximize $b^\mathsf{T}\lambda$ s.t. $c - A^\mathsf{T}\lambda \geq 0, \lambda \geq 0$ (again LP)

2.3.2 Quadratic Programming

2.4 Optimality Conditions

Lagrange Duality

We consider ...

$$f^\star = \inf_{x\in\mathbb{R}^n} f(x) \text{ s.t. } g(x) \leq 0, h(x) = 0 \qquad (2)$$

Lagrange $\mathcal{L}(x,\lambda,\nu) = f(x) + \lambda^\mathsf{T} g(x) + \nu^\mathsf{T} h(x)$
Dual Function $d(\lambda,\nu) = \inf_{x\in\mathbb{R}^n} \mathcal{L}(x,\lambda,\nu)$

Lagrange $\mathcal{L}(x,\lambda,\nu) = f(x) + \lambda^\mathsf{T} g(x) + \nu^\mathsf{T} h(x)$
Dual Function $d(\lambda,\nu) = \inf_{x\in\mathbb{R}^n} \mathcal{L}(x,\lambda,\nu)$

2.4.1 Weak and Strong Duality

Proposition 1 (Weak Duality). $d(\lambda,\nu) \leq f^\star, \forall \lambda \geq 0, \nu \in \mathbb{R}^h$
Definition 5 (Constraint qualification). $\mathcal{C}$ convex, **Slaters Condition** holds if $\exists\, \hat{x} \in \mathbb{R}^n$ s.t. $h(\hat{x}) = 0$ and $g(\hat{x}) < 0$
Proposition 2 (Strong Duality). If Slater's condition holds and (2) is convex $\Rightarrow \exists \lambda \geq 0, \nu \in \mathbb{R}^{n_h}$ s.t. $d(\lambda,\nu) = f^\star$

KKT Conditions (Karush-Kuhn-Tucker)

Theorem 3 (KKT Conditions). If Slater's condition holds and (2) is convex $\rightarrow x^\star \in \mathbb{R}^n$ is a minimizer of the primal (2) and $(\lambda^\star \geq 0, \nu^\star) \in \mathbb{R}^{n_g} \times \mathbb{R}^{n_h}$ is a maximizer of the dual $\Leftrightarrow$ is equivalent to the following statements:

KKT-1 (Stationary Lagrangian)	$\nabla_x \mathcal{L}(x^\star, \lambda^\star, \nu^\star) = 0$
KKT-2 (primal feasibility)	$g(x^\star) \leq 0, h(x^\star) = 0$
KKT-3 (dual feasibility)	$\lambda^\star, \nu^\star \in \mathbb{R}^{n_h} \geq 0$
KKT-4 (compementary slackness)	$\lambda^{\star T} g(x^\star) = 0$, $\nu^{\star T} h(x^\star) = 0$

In addition we have: $\sup_{\lambda\geq 0, \nu\in\mathbb{R}^{n_h}} q(\lambda,\nu) = \inf_{x\in\mathcal{C}} f(x)$

Remark Without Slater, KKT1-4 still implies $x^\star$ minimizes (2) and λ, ν maximizes dual, but the converse is no longer true. There can be primal-minimizer/dual-maximizer not satisfy KKT.

3 Nominal-MPC

3.1 CFTOC

CFTOC Constrained Finite Time Optimal Control problem

$$J(x(k)) = x_N^\top P x_N + \sum_{i=0}^{N-1} x_i^\top Q x_i + u_i R u_i$$

$$J^\star(x(k)) = \min_U I_f(x_N) + \sum_{i=0}^{N-1} I(x_i, u_i)$$

s.t $x_{i+1} = Ax_i + Bu_i,\ i = 0, \dots, N-1$

$x_i \in \mathcal{X},\ u_i \in \mathcal{U},\ \mathcal{X}_N \in \mathcal{X}_f,\ x_0 = x(k)$

N is the time horizon and X , U, Xf are polyhedral regions

3.2 Invariance

3.3 Feasibility and Stability

The first reason is that re-optimization provides robustness to any noise or modeling errors, while the second is that the solution at time k = 0 is sub-optimal because it is over a finite horizon. Re-optimizing can provide a control law with better performance.

MPC Mathematical Formulation

$$J^\star(x(k)) = \min \sum_{i=0}^{N-1} x_i^\top Q x_i + u_i^\top R u_i$$

subj. to $x_{i+1} = Ax_i + Bu_i$

$x_0 = x(k) \quad x_i \in \mathcal{X}, \quad u_i \in \mathcal{U}$

3.3.1 Stability of MPC - Main Result

Theorem 4. The closed-loop system under the MPC control law $u_0^\star(x)$ is asymptotically stable and the set $\mathcal{X}_f$ is positive invariant for the system $x(k+1) = Ax(k) + Bu_0^\star(x(k))$ under the following assumptions:

1. Stage cost is positive definite, i.e. it is strictly positive and only zero at the origin
2. Terminal set is **invariant** under the local control law $\kappa_f(x_i)$:

$$x_{i+1} = Ax_i + B\kappa_f(x_i) \in \mathcal{X}_f \forall x_i \in \mathcal{X}_f$$

All state and input **constraints are satisfied** in $\mathcal{X}_f$:

$$\mathcal{X}_f \in X, \kappa_f(x_i) \in U \forall x_i \in \mathcal{X}_f$$

3. Terminal cost is a continuous **Lyapunov function** in the terminal set $\mathcal{X}_f$ and satisfies:

$$I_f(x_{i+1}) - I_f(x_i) \leq -I(x_i, \kappa_f(x_i)) \quad \forall x_i \in \mathcal{X}_f$$

Finite-horizon MPC may not satisfy constraints for all time!
Finite-horizon MPC may not be stable!
- An infinite-horizon provides stability and invariance.
- Infinite-horizon *faked* by forcing final state into an invariant set for which there exists invariance-inducing controller, whose infinite-horizon cost can be expressed in closed-form.
- Extends to non-linear systems, but compute sets is difficult!

4 Practical-MPC

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5 Robust-MPC

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6 Implementation

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