

# Nernst advection and the field-generating thermal instability revisited

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It is widely held that the Nernst effect can drive instability in un-magnetised laser-plasmas by laterally compressing seed **B**-fields arising from the field-generating thermal instability [Tidman & Shanny, *Phys. Fluids*, **12**:1207 (1974)]. Indeed, for wavelike perturbations, differential compression by the Nernst mechanism is thought to be most pronounced in the limit of low wave-number  $k \rightarrow 0$ , and is considered particularly important given that it can ostensibly lead to instability when the more usual field-generating mechanism is stable. However, as part of a recent article [Bissell et al., *New J. Phys.*, **15**:025017 (2013)] we noted some irregularities to the Nernst mechanism which obscure its operation. For example, by taking characteristic density and temperature length-scales  $l_n$  and  $l_T$  respectively, we observed that consistent analytical treatment of the instability requires  $kl_{n,T} \gg 1$ , preventing the peak-growth limit  $k \rightarrow 0$ . Furthermore, the Nernst term—which compresses magnetic field perturbations—does not couple to a corresponding term acting on thermal perturbations, and as such does not describe an unstable feedback mechanism. In this article we probe the origin of such ambiguities more formally, and in so doing argue (contrary to reports existing elsewhere in the literature) that the Nernst effect does not drive instability in un-magnetised conditions, at least not in the fashion typically cited.

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## 1. Introduction

It has long been known that otherwise un-magnetised plasmas can self-generate large magnetic fields ( $\sim 100$ T) through a variety of mechanisms (Stamper et al. 1971; Pert 1977; Raven et al. 1978; Haines 1986a; Thomas et al. 2009; Li et al. 2013). These fields strongly affect electron transport by suppressing the cross-field thermal conductivity (Braginskii 1965) and are thus key to understanding a range of laser-plasma interactions, including ongoing efforts to achieve controlled inertial confinement fusion (Glenzer et al. 1999; Lindl et al. 2004; Nilson et al. 2006; Froula et al. 2007; Li et al. 2007a,b; Schurtz et al. 2007; Froula et al. 2009; Li et al. 2009, 2013). Of special importance in such contexts is the role transport effects might play in driving instabilities, especially given that such instabilities are themselves often candidate mechanisms for producing the self-generated field (Weibel 1959; Bol'shov

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et al. 1974; Tidman and Shanny 1974; Ogasawara et al. 1980; Haines 1981; Bissell et al. 2010, 2012; Gao et al. 2012; Manuel et al. 2013).

As part of a recent article (Bissell et al. 2013) we discussed how super-Gaussian transport effects arising from strong inverse bremsstrahlung (I.B.) heating can suppress growth-rates of one such candidate, the *field-generating thermal instability*, which was first reported in 1974 (Bol'shov et al. 1974; Tidman and Shanny 1974), and remains an important phenomena in laser-plasma interactions (see, for example, experimental studies of coronal plasmas by Manuel et al. (2013) reported earlier this year). It is widely held that this instability may be driven by two mechanisms which—denoting the electron temperature and density as  $T_e$  and  $n_e$  respectively, and taking  $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$  as a unit vector in the direction of the magnetic flux density  $\mathbf{B}$ —may be summarised as follows. In the first case (i) feedback acts between  $\nabla T_e \times \nabla n_e$  generated field and the consequent cross-gradient Righi-Leduc heat-flow  $\mathbf{q}_\perp \propto \mathbf{b} \times \nabla T_e$  (Braginskii 1965; Bol'shov et al. 1974; Tidman and Shanny 1974; Haines 1986a). In the second (ii) it is commonly understood that lateral advection of the field with the diffusive heat-flow  $\mathbf{q}_\perp \propto \nabla T_e$  via the Nernst effect (Nishiguchi et al. 1985; Bissell et al. 2013), i.e., with velocity  $\mathbf{v}_N \approx \mathbf{q}_\perp / (\frac{3}{2} n_e T_e)$ , can lead to exponential compression of the perturbation in  $\mathbf{B}$  (Brownell 1979; Hirao and Ogasawara 1981). In the absence of hydrodynamic effects, case (i) requires that zeroth-order temperature and density gradients be parallel, i.e.,  $l_T l_n > 0$ , where in an  $x$ -coordinate aligned geometry the length scales  $l_T$  and  $l_n$  may be defined

$$\frac{1}{l_T} = \frac{1}{T_0} \frac{\partial T_0}{\partial x} \quad \text{and} \quad \frac{1}{l_n} = \frac{1}{n_0} \frac{\partial n_0}{\partial x}, \quad (1.1)$$

respectively, with the subscripts '0' denoting zeroth-order profiles (see Sec. 3). This feature of case (i) makes the mechanism in case (ii) particularly important, since Nernst advection may both *contribute* to instability when  $l_T l_n > 0$  holds, but also (ostensibly) *drive* instability when the parallel gradient condition fails, that is, if  $l_n l_T < 0$  (Brownell 1979; Hirao and Ogasawara 1981).

Our original comments (Bissell et al. 2013) on the field-generating thermal instability focused on how super-Gaussian modifications to electron transport can suppress classically predicated instability growth-rates (by as much as  $\sim 80\%$  under both  $l_T l_n > 0$  and  $l_T l_n < 0$  conditions). For this reason, our analysis followed in the tradition of previous work in its treatment of the Nernst advection terms (Brownell 1979; Hirao and Ogasawara 1981); however, we noted a curious feature of the Nernst mechanism in that it seems to predict peak instability growth-rates as the perturbation wave-number  $k$  vanishes. When trying to develop a mathematically consistent picture of the instability, this feature is problematic because analytical treatments typically assume some local conditions  $l_{n,T} k \gg 1$  for the unstable modes, precluding  $k \rightarrow 0$  (Bol'shov et al. 1974; Tidman and Shanny 1974; Brownell 1979; Ogasawara et al. 1980; Hirao and Ogasawara 1981). Furthermore, the Nernst term (which compresses the field perturbations) does not couple to a corresponding term acting on thermal perturbations, meaning that mechanism (ii) does not account for unstable feedback in the usual way.

Given the need for clarity when establishing the stability of laser-plasma configurations, this short article revisits the impact of the Nernst effect on the field-generating thermal instability, placing special emphasis on resolving the ambiguities described above. By reviewing the basic instability theory (Secs. 2 and 3) we identify various inconsistencies of treatment which imply that Nernst advection does not

drive instability under the linear assumptions usually made (Brownell 1979; Hirao and Ogasawara 1981), and consequently (in the absence of hydrodynamic motion, and a more advanced treatment of lateral effects) that laser-plasmas should in fact be stable to the field-generating thermal instability mechanism whenever  $l_n l_T < 0$  (Sec. 4). In the light of these observations, we also comment on the role Nernst plays in driving instability more generally (Secs. 5 and 6).

## 2. Governing equations

Since we are primarily interested in Nernst advection effects we neglect hydrodynamics, and the governing equations are simply the thermal energy equation and Faraday's law,

$$\frac{3}{2}n_e \frac{\partial T_e}{\partial t} + \nabla \cdot \mathbf{q} - \mathbf{E} \cdot \mathbf{j} = \dot{U}_L \quad \text{and} \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad (2.1)$$

respectively, where  $\dot{U}_L$  describes the rate-of-change of thermal energy  $U = \frac{3}{2}n_e T_e$  due to laser heating, and  $\mathbf{j} = (\nabla \times \mathbf{B})/\mu_0$  is the current (with  $\mu_0$  as the permeability of free-space), while the electric field  $\mathbf{E}$  and total heat-flow  $\mathbf{q}$  are given by Braginskii (1965):

$$en_e \mathbf{E} = -\nabla P_e + \mathbf{j} \times \mathbf{B} + \frac{m_e}{e\tau_B} \underline{\underline{\alpha}}^c \cdot \mathbf{j} - n_e \underline{\underline{\beta}}^c \cdot \nabla T_e \quad (2.2)$$

$$\text{and} \quad \mathbf{q} = -\frac{n_e \tau_B T_e}{m_e} \underline{\underline{\kappa}}^c \cdot \nabla T_e - \underline{\underline{\psi}}' \cdot \mathbf{j} \frac{T_e}{e}. \quad (2.3)$$

Here the usual notation applies (Bissell et al. 2013), so that  $e$  is the elementary electronic charge,  $P_e = n_e T_e$  is the isotropic pressure,  $m_e$  is the electronic mass, and the Braginskii collision time  $\tau_B = c_B \tau_T$  is proportional to the thermal collision time  $\tau_T$  by the constant factor  $c_B = 3\sqrt{\pi}/4$ . By quasi-neutrality, the ion number density may be written in terms of the atomic number  $Z$  as  $n_i \approx n_e/Z$ , so that with permittivity of free space  $\epsilon_0$ , and Coulomb logarithm  $\log \Lambda_{ei} \approx 8$ ,  $\tau_T$  is itself given by  $\tau_T = (4\pi v_T^3)/(n_i [Ze^2/\epsilon_0 m_e]^2 \log \Lambda_{ei})$ , where  $v_T = (2T_e/m_e)^{1/2}$  is the mean thermal velocity and defines a thermal mean-free-path  $\lambda_T = v_T \tau_T$ . As usual in such treatments, we shall assume a geometry for which gradients and fluxes are perpendicular to  $\mathbf{B}$ , i.e. scalar  $\phi$  and vector  $\mathbf{A}$  quantities obey  $\mathbf{B} \cdot \nabla \phi = \mathbf{B} \cdot \mathbf{A} = 0$ . Under these conditions, the resistivity  $\underline{\underline{\alpha}}^c$ , conductivity  $\underline{\underline{\kappa}}^c$ , and thermoelectric tensors  $\underline{\underline{\beta}}^c$  and  $\underline{\underline{\psi}}' = \underline{\underline{\beta}}^c + [5/2]\underline{\underline{\mathbf{I}}}$  (here  $\underline{\underline{\mathbf{I}}}$  is the identity tensor) may be written in the general form

$$\underline{\underline{\eta}} \cdot \mathbf{s} = \eta_{\perp} \mathbf{s} + \eta_{\wedge} \mathbf{b} \times \mathbf{s}, \quad \text{with} \quad \eta \in \{\alpha, \beta, \kappa\} \quad \text{and} \quad \mathbf{s} \in \{\mathbf{j}, \nabla T_e\}, \quad (2.4)$$

where the components  $\eta_{\perp}$  and  $\eta_{\wedge}$  are dimensionless functions of both  $Z$  and the Hall parameter  $\chi = \omega_L \tau_B$ , with  $\omega_L = (e|\mathbf{B}|/m_e)$  as the electron Larmor frequency (Braginskii 1965; Epperlein 1984; Epperlein and Haines 1986). Note that for our un-magnetised conditions the ' $\perp$ ' components are constants, that is,  $\eta_{\perp}(0) = \eta_{\parallel}$ , while convention dictates that equation 2.4 takes the slightly different form  $\underline{\underline{\alpha}} \cdot \mathbf{j} = \alpha_{\perp} \mathbf{j} - \alpha_{\wedge} \mathbf{b} \times \mathbf{j}$  for the resistivity.

## 3. Review of the basic linear theory

Let us begin our analysis by reviewing the basic linear theory. As usual, we assume that the plasma is initially un-magnetised, so that in zeroth-order our governing equations—the thermal energy equation and induction equation (Faraday's Law and

Ohm's Law combined)—are satisfied by solutions  $\mathbf{B} = \mathbf{B}_0 = \mathbf{0}$ ,  $T_e = T_0(x, t)$  and  $n_e = n_0(x, t)$ , where gradients in the latter two quantities exist in the  $x$ -direction only, and define length-scales  $l_n$  and  $l_T$  according to equations 1.1. In addition, we note that for laser-plasmas one may take

$$\Lambda = \frac{\lambda_T}{\delta} \gg 1, \quad \text{where} \quad \delta = \frac{c}{\omega_{pe}} \quad \text{and} \quad \omega_{pe} = \left( \frac{n_e e^2}{\epsilon_0 m_e} \right)^{1/2} \quad (3.1)$$

are the collisionless-skin-depth and plasma frequency respectively, and  $c = 1/\sqrt{\mu_0 \epsilon_0}$  is the speed of light *in vacuo*.

To the zeroth-order solutions we add small wavelike perturbations with wavenumber  $k$ , frequency  $\gamma$ , and periodicity  $\propto (iky + \gamma t)$ , that is,

$$T_e = T_0 + \delta T \exp(iky + \gamma t) \quad \text{and} \quad \mathbf{B} = \delta B \exp(iky + \gamma t) \hat{\mathbf{z}}, \quad (3.2)$$

with  $\delta T \ll T_0$  and  $\delta B$  as some complex amplitudes. Hence, after substituting these perturbed forms into the governing system, subtracting zeroth-order solutions, and neglecting second-order perturbed terms or higher, the linearised energy and induction equations become

$$\Gamma + D_T K^2 - Q = +i C_E \left( \frac{e \lambda_T^2}{\tau_T} \frac{\delta B}{\delta T} \right) K \quad (3.3)$$

and

$$\Gamma + D_R K^2 - N = -i C_I \left( \frac{\tau_T}{e \lambda_T^2} \frac{\delta T}{\delta B} \right) K \quad (3.4)$$

respectively. Notice here that we have adopted the dimensionless notation

$$\Gamma = \gamma \tau_T, \quad K = \lambda_T k, \quad L_{T,n} = \frac{l_{T,n}}{\lambda_T}, \quad D_T = \frac{c_B}{3} \kappa_{\parallel}^c, \quad D_R = \frac{\alpha_{\parallel}^c}{c_B \Lambda^2}, \quad (3.5)$$

where  $D_T$  and  $D_R$  are the thermal and resistive diffusion coefficients respectively (Bissell et al. 2012). The remaining terms arise from: i) differential thermal diffusion  $Q$  lateral to the perturbation (down zeroth-order temperature gradients); ii) differential Nernst advection  $N$  lateral to the perturbation; iii) divergent Righi-Leduc heat-flow  $C_E$ ; and iv) magnetic field generation  $C_I$  by the  $\nabla T_e \times \nabla n_0$  mechanism; these are defined by

$$Q = \frac{5}{2} D_T \frac{\lambda_T^2}{n_0 \tau_T T_0} \frac{\partial}{\partial x} \left( n_0 \tau_T \frac{\partial T_0}{\partial x} \right) \sim D_T / L_{T,n}^2, \quad C_E = \frac{c_B^2}{6 L_T} \left( \frac{\partial \kappa_{\parallel}^c}{\partial \chi} \right), \quad (3.6)$$

$$N = \frac{c_B}{2} \frac{\partial \beta_{\parallel}^c}{\partial \chi} \frac{\lambda_T^2}{\tau_T T_0} \frac{\partial}{\partial x} \left( \tau_T \frac{\partial T_0}{\partial x} \right), \quad \text{and} \quad C_I = \frac{1}{L_n}. \quad (3.7)$$

At this stage in the usual analysis one assumes the local condition  $KL_{n,T} \gg 1$  (Brownell 1979; Hirao and Ogasawara 1981), so that the lateral diffusion term  $Q \sim D_T / L_{T,n}^2$  may be neglected in equation 3.3 when compared to  $D_T K^2$ , i.e.,

$$[D_T K^2 - Q] = D_T K^2 [1 - O(1/K^2 L_{T,n}^2)] \approx D_T K^2. \quad (3.8)$$

Thus, after eliminating terms in  $(\delta T/\delta B)$  from equations 3.3 and 3.4, one obtains the dispersion relation (Brownell 1979; Hirao and Ogasawara 1981; Bissell et al. 2013)

$$\Gamma_{\pm} = \frac{1}{2} \left\{ -[(D_T + D_R)K^2 - N] \pm \sqrt{[(D_T + D_R)K^2 - N]^2 + 4D_T K^2 [D_R (K_G^2 - K^2) + N]} \right\}, \quad (3.9)$$

where  $K_G$  is the source term describing coupling between the Righi-Leduc heat-flow  $C_E$  and  $\nabla T_e \times \nabla n_e$  field generation  $C_I$  as described by Tidman and Shanny (1974) and Bissell et al. (2012), *viz*

$$K_G^2 = \frac{C_E C_I}{D_T D_R} = \frac{c_B^2 \Lambda^2}{2L_T L_n \alpha_{\parallel}^c \kappa_{\parallel}^c} \left( \frac{\partial \kappa_{\parallel}^c}{\partial \chi} \right). \quad (3.10)$$

Equation 3.9 is the dispersion relation quoted by the author in our earlier context (Bissell et al. 2013) where we observed that in the absence of Nernst advection ( $N \rightarrow 0$ ) the cut-off wave-number becomes  $K_G$ , so that instability requires  $L_T L_n > 0$  (Tidman and Shanny 1974; Brownell 1979; Hirao and Ogasawara 1981). In terms of plasma stability, this result makes the contribution from the advection term  $N$  particularly important, since it ostensibly predicts instability when  $L_T L_n < 0$  provided  $N > 0$  (Brownell 1979; Hirao and Ogasawara 1981). As both Brownell (1979) and Hirao and Ogasawara (1981) describe, equation 3.9 suggests that lateral compression of the field by the advection term should be especially effective in the limit of long-wavelength perturbations  $K \rightarrow 0$  (when damping effects, such as thermal diffusion, are minimised), in which case the peak growth-rate becomes  $\Gamma = N$ . Of course, the value  $K = 0$  is not a physically meaningful one to take for the peak wave-number; however, as we noted in our original context (Bissell et al. 2013), the apparent mathematical inconsistency between the local assumption  $KL_{n,T} \gg 1$  made in the analysis, and the (ideal) conclusion that instability peaks when  $K = 0$ , invites us to treat the basic theory with some caution. What is more, Nernst compression by  $N$  does not described coupling between the energy and induction equations, and so fails to provide a mechanism for unstable feedback. For these reasons we stated that the Nernst term cannot be understood as driving instability *proper*, though it may lead to exponential compression of the local field. In what follows we consider the source of such ambiguities in more formal detail by re-examining the basic linear theory, and reflecting further on the physical basis of the Nernst effect. Ultimately, we conclude by taking the opposing view that Nernst will not drive instability in un-magnetised conditions, at least not without a more sophisticated treatment of lateral effects.

#### 4. Inconsistencies in the basic linear theory

In essence, the ambiguities described in the previous section arise from miss-application of the local approximation; however, it is often as instructive to consider the shortcomings of physical arguments as their advantages, and to this end it is worth discussing such miss-application in relative detail. Indeed, by probing related problems in the the basic linear theory (Brownell 1979; Hirao and Ogasawara 1981), one can gain insight into both the meaning of various analytical steps, and the physical processes involved. Here three issues are considered: first, physical interpretation (Sec. 4.1); second, ordering of terms (Sec. 4.2); and third, lateral effects (Sec. 4.3).

#### 4.1. Physical interpretation

Recall that the Nernst effect describes advection of the magnetic field with the diffusive heat-flow  $\mathbf{q}_\perp$ , i.e., the heat-flow representing thermal diffusion down temperature gradients (Nishiguchi et al. 1985; Bissell et al. 2013). Indeed, one may write the velocity  $\mathbf{v}_N$  of Nernst advection as

$$\mathbf{v}_N = \frac{2A_N}{3D_T} \frac{\mathbf{q}_\perp}{P_e} \approx \frac{\mathbf{q}_\perp}{3n_e T_e/2}, \quad \text{where} \quad \mathbf{q}_\perp = -\frac{3}{2}n_e D_T \frac{\lambda_T^2}{\tau_T} \nabla T_e \quad (4.1)$$

is the diffusive heat-flow,  $A_N$  is a dimensionless *Nernst advection coefficient* defined by

$$A_N = \frac{c_B}{2\chi} \beta_\wedge^c \approx D_T = \frac{c_B}{3} \kappa_\perp^c, \quad (4.2)$$

and the approximate equality follows by considering values for the transport coefficients over a range of  $\chi$  (Haines 1986b; Bissell et al. 2012). Physically, therefore, our term  $N$  in the linearised induction equation 3.4 arises from differential advection of the field perturbation by heat-flow electrons moving down the zeroth-order temperature gradient, i.e., those electrons which account for lateral diffusive transport of the temperature perturbation  $Q$  in the linearised energy equation 3.3. It seems curious that we should retain  $N$  (lateral  $\mathbf{B}$ -field advection) in the dispersion relation 3.9, whilst simultaneously arguing for the exclusion of the thermal term  $Q \sim D_T/L_{T,n}^2$  responsible for its physical origin.

#### 4.2. Ordering of terms

That such a contradiction arises may be understood by reviewing the local approximation  $KL_{n,T} \gg 1$ ; as we saw in equation 3.8, should this condition hold it is legitimate to neglect terms in  $Q \sim D_T/L_{n,T}$  when compared to those in  $D_T$  in the linearised energy equation 3.3. However, closer inspection of the Nernst term reveals that the local approximation also applies to  $N$ . Indeed, for our un-magnetised conditions the cross-field thermoelectric coefficient is directly proportional to the Hall parameter, i.e.,  $\beta_\wedge^c \propto \chi$  (Braginskii 1965), so that

$$\frac{\partial \beta_\wedge^c}{\partial \chi} = \frac{\beta_\wedge^c}{\chi} \Rightarrow N = A_N \frac{\lambda_T^2}{\tau_T T_0} \frac{\partial}{\partial x} \left( \tau_T \frac{\partial T_0}{\partial x} \right) \sim A_N / L_{T,n}^2, \quad (4.3)$$

and thus, since  $D_R \approx A_N$ , we have (cf. equation 3.8 and 3.9)

$$[D_T K^2 - N] = D_T K^2 [1 - O(1/K^2 L_{T,n}^2)] \approx D_T K^2. \quad (4.4)$$

Consistent application of the local condition  $KL_{n,T} \gg 1$  to neglect  $Q$  in equation 3.3 therefore demands that we neglect  $N$  in the dispersion relation (equation 3.9). That we cannot do so at the stage of linearising the induction equation 3.4 is a consequence of the relatively small value of the resistive diffusion coefficient  $D_R$  when compared to the thermal diffusion coefficient  $D_T$  (by the factor  $1/\Lambda^2$ , cf. equations 3.1 and 3.5).

#### 4.3. Lateral effects

Our discussion in the previous two sections demonstrates that (as a consequence of their common physical basis) the term  $Q$  arising from the divergence of the heat-flow down zeroth-order gradients has the same functional form as  $N$ . Furthermore, equations 3.8 and 4.4 establish that *both*  $Q$  and  $N$  should be properly neglected in the theory provided that the local condition  $KL_{n,T} \gg 1$  holds. It is natural to wonder, therefore, whether these terms should be retained in the analysis under conditions for

which the local approximation  $KL_{n,T} \gg 1$  does not apply, in which case we might expect  $Q$  to modify the growth-rate alongside  $N$  by acting as an additional source of instability.

To address this supposition, let us consider more carefully the role played by the local condition in the process of linearisation. Strictly speaking, the proper form for the perturbations applied in Sec. 3 should postulate some  $x$ -dependence, and this is especially true when we are interested in differential advection or diffusion, because any  $x$ -dependence will have consequences for the lateral compression (or rarefaction) of the perturbations normal to the wave-vector. To account for such lateral effects, therefore, one must set  $\delta T \equiv \delta T(x)$  and  $\delta B \equiv \delta B(x)$ , and in this case equations 3.3 and 3.4 acquire new terms. For example, after neglecting terms of order  $(\delta T)^2$  and higher, the thermal diffusion component to equation 3.3 undergoes the transformation

$$D_T K^2 \left[ 1 - \frac{Q}{D_T K^2} \right] \rightarrow D_T K^2 \left[ 1 - \frac{Q}{D_T K^2} + O \left( \frac{\lambda_T^2}{K^2 \delta T} \frac{d^2 \delta T}{dx^2} \right) + O \left( \frac{\lambda_T}{K^2 L_T \delta T} \frac{d \delta T}{dx} \right) \right]. \quad (4.5)$$

Thus, our perturbation equations 3.3 and 3.4 become coupled second-order ordinary differential equations reminiscent of those encountered in problems involving hydrodynamic stability (Chandrasekhar 1961). Indeed, although the final two differential terms describe gradients in small quantities, because the  $\nabla T_e \times \nabla n_0$  field generating mechanism responsible for driving initial growth of the instability operates over the density length-scale  $l_n$ , one expects both  $(d\delta T/dx)/\delta T \sim 1/l_n$  and  $(d^2 \delta T/dx^2)/\delta T \sim 1/l_n^2$ . The differential terms will thus be of a similar order to the lateral heat-flow term in  $Q/D_T \sim 1/L_{T,n}^2$ . In this way, we see that if the local condition does not hold, then one is obliged to retain not only  $Q$ , but also the differential terms describing gradients in  $\delta T$ , and the eigenvalue problem becomes one requiring solution of a non-linear system of coupled ordinary differential equations with eigenfunction solutions  $\delta T$  and  $\delta B$  dependent on appropriate boundary conditions (cf. Chandrasekhar 1961). In general such a problem will be non-trivial.

On the other hand, should the local approximation hold, then its utility is now clear. Provided  $KL_{n,T} \gg 1$  we can effect a ‘secondary linearisation process’ whereby we discard the differential terms in equation 4.5 (and similar), and thence solve for the growth rate  $\Gamma$  algebraically; in so doing, however, consistency requires us to surrender terms in both  $Q$  and  $N$  (as described in Sec. 4.2). To address the supposition made at the beginning of this section directly, therefore, we conclude that while it is possible for laterally divergent transport to drive instability when the local approximation does not apply, proper treatment of such effects requires a more sophisticated analysis than that considered here; it is not sufficient simply to add terms in  $Q$  and  $N$  to the dispersion relation 3.9. Indeed, substantial further investigation inclusive of numerical simulations is needed to better determine the role played by lateral effects in de-stabilising laser plasmas under such conditions.

## 5. Super-Gaussian transport and suppression of instability

Although our argument here has focused on classical (Braginskii) transport effects, it remains for us to comment briefly on how the loss of the advection mechanism  $N$  affects those results stated in our earlier context (Bissell et al. 2013) where the primary concern was super-Gaussian transport phenomena arising from strong inverse-bremsstrahlung heating. One obvious consequence is stabilisation of the plasma to the field-generating instability when  $l_T l_n < 0$ . Nevertheless, the reduction of



both  $\nabla T_e \times \nabla n_e$  field generation and the Righi-Leduc heat-flow when super-Gaussian transport applies, means that our original predictions (Bissell et al. 2013), i.e., heavy suppression of instability growth-rates due to I.B. (by as much as  $\sim 80\%$ ), remain valid for  $l_T l_n > 0$ . Of course, if hydrodynamic motion is included in the analysis, then instability can prevail without Nernst for  $l_T l_n < 0$  (Ogasawara et al. 1980); however, in this case we would continue to expect significant growth-rate suppression by I.B. heating because super-Gaussian transport phenomena are unaffected by hydrodynamic flow (Bissell et al. 2013).

## 6. Conclusion

The usual treatment of Nernst advection effects (Brownell 1979; Hirao and Ogasawara 1981) on the field-generating thermal instability (Tidman and Shanny 1974) leads to ambiguities in the dispersion relation for the growth of unstable modes. For example, and as we noted in a recent article (Bissell et al. 2013), lateral compression by the Nernst effect ostensibly yields peak-growth rates which correspond to vanishing wavenumber  $k \rightarrow 0$ , violating the local approximation  $kl_{n,T} \gg 1$  (Secs. 1 and 3). In this short article we have sought to determine the source of such ambiguities more formally, both by re-examining the basic analysis, and returning to the physical meaning of the Nernst effect (Secs. 2 and 3). In particular, we argued that consistent (and necessary) application of the local approximation requires the Nernst advection term to be omitted from the dispersion relation 3.9 (Secs. 3 and 4), meaning that for un-magnetised conditions Nernst cannot drive instability in the fashion commonly cited (Brownell 1979; Hirao and Ogasawara 1981). One consequence of such an interpretation is that—in the absence of significant hydrodynamic motion (Ogasawara et al. 1980)—un-magnetised laser-plasmas should be stable to the field-generating instability whenever zeroth-order temperature and density gradients are anti-parallel, i.e.,  $l_n l_T < 0$  (Sec. 5).

Naturally, we do not go so far as to state that Nernst advection can never drive instability, since it is possible that further investigation of lateral effects might lead to a compressive mechanism precluded by the approach taken here; though such an investigation would require solving a more complex eigenvalue problem formulated in terms of second-order differential equations, in combination with a thorough numerical investigation, and is therefore left as future research (Sec. 4.3). Indeed, theoretical and computational work reported elsewhere has shown that the Nernst effect is expected to drive a related *field-compressing magneto-thermal instability* in laser-plasmas under sufficiently magnetised conditions (Bissell et al. 2010, 2012).

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