Magnetothermal instability of plasmas in a horizontal magnetic field

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The linear buoyancy instability in a magnetized plasma, generally referred to as magnetothermal instability (MTI), is investigated by considering anisotropic heat conduction. The external magnetic field is assumed to be horizontal and background heat flux is not taken into account. The general dispersion relationship of the convective instability is derived. The growth rate of the MTI in fixed boundary condition is presented and discussed. The effect of density spacial gradient on the MTI is investigated. The magnetic field is shown to suppress the MTI and even quench the instability when the magnetic field is strong enough. Under the standard Wentzel–Kramaers–Brillouin approximation, our results could be simplified to a brief form reported by one previous paper [E. Quataert, Astrophys. J. **673**, 758 (2008)].
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I. INTRODUCTION

In astrophysical plasmas in the presence of a magnetic field, the ion gyroradius is much less than the collisional mean free path. Under this circumstance, the heat is restricted to transmitting exclusively along the magnetic force lines.[1] Hence the anisotropic transport terms must be taken into account by the magnetohydrodynamic (MHD) equations to describe the dynamic behavior of plasmas.[2,3]

There is growing interest in the implication of anisotropic transport terms on the dynamics of diluted astrophysical plasmas (see, for example, Refs. 4–8). One of the most remarkable results thus far is that thermally stratified fluids are shown to be buoyantly unstable when the thermal temperature (as opposed to entropy) increases in the direction of gravity in a weakly magnetized plasma in which anisotropy electron heat conduction occurs.[2,4,9] The kind of convective instability is referred to as magnetothermal instability (MTI). The simple case in which a vertically stratified plasma with a horizontal magnetic field in the absence of a background heat flux was investigated by Balbus.[2] Later, the convective instability of a diluted astrophysical plasma with rotating flows was investigated by Balbus.[4] On the other hand, Parrish and Stone[3] used numerical methods to explore the nonlinear evolution and saturation of the MTI in two dimensions. The linear growth rates measured in the simulation agreed well with the weak-field dispersion relationship and the growth rates for stronger magnetic fields were also numerically measured. The saturation and heat transport properties of the MTI in three dimensions also be presented in Ref. 10. Recently, taking a background heat flux into account, Quataert[9] calculated the linear instability of a low-collisional and weakly magnetized plasma and gave the analytical expression of the growth rate. The presence of a background heat flux was shown to drive a buoyancy instability analogous to the MTI when the temperature decreased in the direction of gravity while this situation was magnetothermal stable according to Balbus analysis (see Ref. 2).

In this article, we describe the stability of anisotropic heat conducting in an arbitrarily magnetized plasma. The magnetic field is assumed to be horizontal. The perturbed quantities are supposed to be general z-dependent parameters. The second-order ordinary differential equation (ODE) is utilized to describe the velocity perturbation. Under the Wentzel–Kramaers–Brillouin (WKB) approximation, our results can be reduced to the earlier results in the weak-field limit.[9] In addition, the MTI in the fixed boundary condition with an exponential density distribution is discussed. The growth rate of the MTI is presented with the density spacial gradient effect. The plasma is shown to be unstable when $dT/dz<0$, that is, when the temperature increases in the direction of gravity, which is accordant with the result of Ref. 2. Moreover, the magnetic field shows stabilization effect on the MTI.

II. DISPERSION RELATION

The basic set of MHD equations with the addition of the heat flux, $\mathbf{Q}$, and a equivalent gravitational field is

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0, \tag{1}$$

$$\rho\frac{\partial\mathbf{v}}{\partial t}+\rho\mathbf{v}\cdot\nabla\mathbf{v}=-\nabla p+\frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B}+\rho\mathbf{g}, \tag{2}$$

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B}), \tag{3}$$

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$$\rho T\frac{dS}{dt} = -\nabla\cdot\mathbf{Q} + \mathbf{E}\cdot\mathbf{J} + \rho\mathbf{g}\cdot\mathbf{v}. \quad (4)$$

Here, ρ is the mass density, $\mathbf{v}$ is the fluid velocity, p is the pressure, $\mathbf{B}$ is the magnetic field, $\mathbf{g}$ is the gravitational acceleration, $\mathbf{J}=\nabla\times\mathbf{B}/\mu_0$ is the current density, $\mathbf{E}=-\mathbf{v}\times\mathbf{B}$ is the electric field, T is the thermal temperature in the energy units, and $S=3p\ \ln(p\rho^{-\gamma})/2\rho T$ is the entropy per unit mass. The adiabatic index γ is $\frac{5}{3}$. The plasma is considered to be thermally stratified in the presence of a uniform gravitational field in the vertical direction, i.e., $\mathbf{g}=-g\hat{\mathbf{z}}$. Furthermore, the general heat-loss function, resistance effect, and viscosity effect are all neglected in our model.

The contributions from electron motions (constrained to move primarily along field lines) and isotropic transport, which may arise due to photons or particles collisions driving cross field diffusion,[3] are both contained in the heat flux term. In the present paper, we concentrate on the case in which the heat is transported exclusively along the magnetic force lines. Thus,

$$\mathbf{Q} = -\chi_C\hat{\mathbf{b}}(\hat{\mathbf{b}}\cdot\nabla)T, \quad (5)$$

where χ_C is the Spitzer Coulomb conductivity,[11] $\hat{\mathbf{b}}=\mathbf{B}/B$ is a unit vector in the direction of the magnetic field.[3]

The spatial gradients of the equilibrium density, pressure, and magnetic field are all set on the opposite direction of the equivalent gravitational acceleration. Therefore, it is reasonable to suppose that the inhomogeneity occurs only in the z direction. The equilibrium profiles can be expressed in the following form:

$$\rho_0=\rho_0(z), \quad p_0=p_0(z) \quad \text{and} \quad T=T(z), \quad (6)$$

$$\mathbf{B}_0 = B_{x0}(z)\hat{e}_x + B_{y0}(z)\hat{e}_y. \quad (7)$$

Hence, there is $\mathbf{B}_0\cdot\nabla T=0$. This implies that (1) the heat flux in the initial state is zero, so, no background heat flux and (2) in the equilibrium condition, the field lines are isothermal. For brevity, we write $\chi_C=\chi$. The equilibrium condition reads

$$\frac{d}{dz}\left(p_0 + \frac{B_0^2}{2\mu_0}\right) = -\rho_0 g. \quad (8)$$

The perturbed profile $\delta\varphi$ is supposed to have the following form: $\delta\varphi=\tilde{\varphi}(z)\exp(-i\omega t+ikx)$, where ω is the wave frequency and k is the wave number. The growing modes of interest have growth times much longer than the sound crossing time of the perturbation. As a result, it is sufficient to work in the Boussinesq approximation.[12,13] Linearizing the above equations, we obtain

$$ik\delta v_x + \frac{d}{dz}\delta v_z = 0, \quad (9)$$

$$-i\omega\rho_0\delta\mathbf{v} = \delta\rho\mathbf{g} - \nabla\left(\delta p + \frac{\mathbf{B}_0\cdot\delta\mathbf{B}}{\mu_0}\right) + \frac{1}{\mu_0}(\mathbf{B}_0\cdot\nabla\delta\mathbf{B} + \delta\mathbf{B}\cdot\nabla\mathbf{B}_0), \quad (10)$$

$$-i\omega\delta\mathbf{B} = (\mathbf{B}_0\cdot i\mathbf{k})\delta\mathbf{v} - (\delta\mathbf{v}\cdot\nabla)\mathbf{B}_0, \quad (11)$$

$$\left[i\omega - \frac{2\chi T}{5p_0}(\mathbf{k}\cdot\hat{\mathbf{b}})^2\right]\frac{\delta\rho}{\rho_0} = -\frac{3}{5}\delta\mathbf{v}\cdot\nabla\ \ln p_0\rho_0^{-\gamma} + \frac{2}{5p_0}\frac{dp_0}{dz}\delta v_z + \frac{2\chi T}{5p_0B_0}[(\hat{\mathbf{b}}\cdot\nabla)(\delta\mathbf{B}\cdot\nabla\ \ln T)]. \quad (12)$$

Here the subscript 0 denotes the unperturbed variables and $\delta\hat{\mathbf{b}}=\delta(\mathbf{B}/B)=\delta\mathbf{B}/B_0-\hat{\mathbf{b}}(\delta B/B_0)$ is adopted. In view of $p=\rho T/m$, where m is the mass, we have $\delta T\simeq -T\delta\rho/\rho_0$. To obtain Eq. (12), we adopted the equilibrium condition [Eq. (8)].

Equation (11) yields the perturbed magnetic field as

$$\delta\mathbf{B} = -\frac{k}{\omega}B_{x0}\delta\mathbf{v} + \frac{\delta v_z}{i\omega}\frac{d\mathbf{B}_0}{dz}. \quad (13)$$

Accordingly, we obtain the tension provided by the magnetic field on the right-hand side of Eq. (10) as

$$\mathbf{B}_0\cdot\nabla\delta\mathbf{B} + \delta\mathbf{B}\cdot\mathbf{B}_0 = -\frac{ik^2B_{x0}^2}{\omega}\delta\mathbf{v}. \quad (14)$$

Inserting the above formula into Eq. (10), we obtain

$$\left(\rho_0\omega^2 - \frac{k^2B_{x0}^2}{\mu_0}\right)\delta\mathbf{v} = i\omega\delta\rho\mathbf{g} - i\omega\nabla\left(\delta p + \frac{\mathbf{B}_0\cdot\delta\mathbf{B}}{\mu_0}\right). \quad (15)$$

We then carry out the following manipulation $\hat{e}_y$ [$\nabla\times$ Eq. (15)]. The calculation can now be greatly simplified by eliminating the perturbed thermal pressure and magnetic pressure terms in Eq. (15) without losing the generality of results. Consequently, we have

$$\frac{d}{dz}\left[\left(\rho_0\omega^2 - \frac{k^2B_{x0}^2}{\mu_0}\right)\frac{d}{dz}\tilde{u}_z\right] = k^2\left[\left(\rho_0\omega^2 - \frac{k^2B_{x0}^2}{\mu_0}\right)\tilde{u}_z + i\omega g\tilde{\rho}\right]. \quad (16)$$

Substituting Eq. (13) in Eq. (12), we obtain the perturbed mass density,

$$\tilde{\rho} = \frac{1}{i\omega-\omega_T}\left[\frac{\omega_T}{i\omega}\frac{d}{dz}\ln T - \left(\frac{1}{5}\frac{d}{dz}\ln p_0 - \frac{d}{dz}\ln\rho_0\right)\right]\rho_0\tilde{u}_z, \quad (17)$$

with $\omega_T=2\chi T(\mathbf{k}\cdot\hat{\mathbf{b}})^2/5p_0$. Inserting the above formula into Eq. (16), we obtain the second-order ODE for the dispersion relationship,

$$\frac{d}{dz}\left[\left(\rho_0\omega^2-\frac{k^2B_{x0}^2}{\mu_0}\right)\frac{d}{dz}\tilde{u}_z\right] = k^2\left\{\left(\rho_0\omega^2-\frac{k^2B_{x0}^2}{\mu_0}\right)+\frac{i\omega\rho_0 g}{i\omega-\omega_T}\left[\frac{\omega_T}{i\omega}\frac{d}{dz}\ln T - \left(\frac{1}{5}\frac{d}{dz}\ln p_0-\frac{d}{dz}\ln\rho_0\right)\right]\right\}\tilde{u}_z. \quad (18)$$

III. ANALYSIS AND DISCUSSION

In this section, we discuss some limiting cases based on the previous result [Eq. (18)].

A. Case 1

By arranging $d\tilde{u}_z/dz=ik_z\tilde{u}_z$, implying that the WKB approximation is adopted, the ODE [Eq. (18)] is reduced to the local dispersion relationship as

$$\omega\omega^{*2}+i\omega_T\omega^{*2}-ig\omega_T\frac{k^2}{k^2+k_z^2}\frac{d}{dz}\ln T-\omega\frac{k^2\tau^2}{k^2+k_z^2} -\frac{1}{\rho_0}\frac{d}{dz}(\rho_0\omega^{*2})\frac{ik_z}{k^2+k_z^2}(\omega+i\omega_T)=0, \quad (19)$$

where we denote

$$\omega^{*2}=\omega^2-\frac{k^2B_{x0}^2}{\rho_0\mu_0}, \quad (20)$$

$$\tau^2=\frac{1}{5}g\frac{d}{dz}\ln p_0-g\frac{d}{dz}\ln\rho_0. \quad (21)$$

The WKB approximation requires $KH\gg 1$, where $K=(k^2+k_z^2)^{1/2}$ is the total wave number and H is the scale length of the local system inhomogeneity. As a result, the last term on the left-hand side of Eq. (19) can be neglected. In Ref. 9, Quataert studied this problem in the presence of a weak magnetic field and the background heat flux. The gravitational dissipation term, however, was not taken into account. That makes the expression of τ^2 here differ from its analogy, i.e., N^2 in Ref. 9. Here, we re-express τ as follows:

$$\tau^2=N^2-\frac{2}{5}g\frac{d\ln p_0}{dz}, \quad (22)$$

where $N^2=(3/5)gd\ln p_0/dz-gd\ln\rho_0/dz$. The last term on the right-hand side of Eq. (22) represents the gravitational dissipation effect. Since this term has the same order with the pressure gradient term in N^2, it cannot be neglected. However we point out that the MTI concentrates on the second and third terms while neglecting the first and fourth terms on the left-hand side of Eq. (19). In this case, the gravitational dissipation term has no significant effect on the MTI.

B. Case 2

For $\omega_T=0$, the dispersion relationship [Eq. (18)] is simplified as

$$\frac{d}{dz}\left[\left(\rho_0\omega^2-\frac{k^2B_{x0}^2}{\mu_0}\right)\frac{d}{dz}\tilde{u}_z\right] = k^2\left[\left(\rho_0\omega^2-\frac{k^2B_{x0}^2}{\mu_0}\right)+\rho_0 g\left(\frac{d}{dz}\ln\rho_0-\frac{1}{5}\frac{d}{dz}\ln p_0\right)\right]\tilde{u}_z. \quad (23)$$

This is the general dispersion relationship of the Rayleigh–Taylor (RT) instability[14,15] in the magnetized stratified plasmas. Using the incompressible limit (i.e., $c_s\to\infty$, where $c_s=\gamma p_0/\rho_0$ is the sound speed), we have

$$\frac{d\ln p_0/dz}{d\ln\rho_0/dz}\sim 0, \quad (24)$$

which means that the gradient of the pressure can be disregarded in the dispersion relationship, that is, the gravitational dissipation effect can be ignored under the incompressible approximation. Consequently, the classical RT result in the incompressible fluid is recovered.[16]

C. Case 3

In this case, we restrict ourselves to the MTI under fixed boundary condition. This boundary condition has not been analytically discussed and can help us understand the density spacial gradient effect on the MTI. According to Eq. (18), the growth rates of the instability under different boundary conditions can be obtained. To solve the second order differential equation (18), it is necessary to define the mass density distribution $\rho_0(z)$. Generally, ρ_0 has an exponential dependence on z in the stratified fluid. In the present work, we follow this hypothesis and suppose that the mass density distribution satisfies $\rho_0(z)=\rho_f\exp(z/L_D)$, where ρ_f is the mass density at $z=0$. From Eq. (8), we presume that the magnetic field $B_{x0}(z)$ has the similar z-dependent distribution as

$$B_{x0}(z)=B_f\exp(z/2L_D), \quad (25)$$

where B_f is the magnetic field component along the x direction at $z=0$. Inserting ρ_0 and B_{x0} into Eq. (18), we obtain

$$(\omega^2-k^2v_{af}^2)\frac{d^2\tilde{u}_z}{dz^2}+\frac{1}{L_D}(\omega^2-k^2v_{af}^2)\frac{d\tilde{u}_z}{dz} = k^2\left\{\omega^2-k^2v_{af}^2+\frac{i\omega g}{i\omega-\omega_T}\left[\frac{\omega_T}{i\omega}\frac{d}{dz}\ln T-\left(\frac{1}{5}\frac{d}{dz}\ln p_0-\frac{d}{dz}\ln\rho_0\right)\right]\right\}\tilde{u}_z, \quad (26)$$

where $v_{af}=(B_f^2/\mu_0\rho_f)^{1/2}$ is the Alfvén speed. Since we used the exponential density distribution, correspondingly, the fixed boundary condition $\tilde{u}_{z1}(0)=\tilde{u}_{z1}(h)=0$ should be adopted to solve the equation above. That means

$$\tilde{u}_z = \tilde{u}_0 \sin\left(\frac{n\pi z}{h}\right)\exp(\lambda z). \tag{27}$$

Substituting the above formula into Eq. (26), we obtain

$$\lambda = -\frac{1}{2L_D},$$
$$(\omega^2 - k^2 v_{af}^2)\left[\lambda^2 - k^2 + \frac{\lambda}{L_D} - \left(\frac{n\pi}{h}\right)^2\right] = \frac{i\omega g k^2}{i\omega - \omega_T}\left[\frac{\omega_T}{i\omega}\frac{d}{dz}\ln T - \left(\frac{1}{5}\frac{d}{dz}\ln p_0 - \frac{d}{dz}\ln \rho_0\right)\right]. \tag{28}$$

By inserting the value of λ into Eq. (28), we obtain the dispersion relationship as follows:

$$(\omega + i\omega_T)(\omega^2 - k^2 v_{af}^2)\left(k^2 + \frac{1}{4L_D^2} + \frac{n^2\pi^2}{h^2}\right) - i\omega_T g k^2 \frac{d}{dz}\ln T + \omega g k^2\left(\frac{d}{dz}\ln \rho_0 - \frac{1}{5}\frac{d}{dz}\ln p_0\right) = 0. \tag{29}$$

This is the general dispersion relationship of the MTI with a constant buoyancy force effect in a thermally stratified plasma with horizontal magnetic fields. We define a dynamical frequency, $\omega_d \sim \sqrt{g/H}$. We stress that this is just a scaling relationship providing a characteristic frequency to such perturbations (e.g., see Ref. 9), viz. $\omega \sim \omega_d$, but not the exact numerical expression. For $\omega_T \gg \omega_d$ (the ordering of timescales can always be achieved if the temperature is high enough such as in some astrophysical plasmas[8,9]), the dispersion relationship above is reduced to

$$\omega^2 = \frac{4L_D^2 g k^2 h^2}{h^2 + 4L_D^2 n^2\pi^2 + 4L_D^2 h^2 k^2}\frac{d}{dz}\ln T + k^2 v_{af}^2. \tag{30}$$

The above equation describes the MTI discovered by Balbus.[2] In the weak magnetic field limit, the magnetic pressure forces are not important in dynamical behavior. The single role of the magnetic field is to enforce an anisotropic transport of the heat flux. It is easy to find out that the perturbation is unstable when $dT/dz < 0$, i.e., when the temperature increases in the direction of gravity. For an arbitrary magnetic field, the growth rate $\Gamma(\omega = \omega_r + i\Gamma)$ is determined by

$$\Gamma^2 = \frac{4L_D^2 g k^2 h^2}{h^2 + 4L_D^2 n^2\pi^2 + 4L_D^2 h^2 k^2}\left(-\frac{d\ln T}{dz}\right) - k^2 v_{af}^2. \tag{31}$$

The above formula gives the growth rate of the MTI in the presence of the density spacial gradient effect. The effects of the density gradient on the instability are first discussed. Before proceeding, we define $\beta = mB_f^2/(2\mu_0\rho_f T)$. Using the equilibrium condition [Eq. (8)], we have

$$-\frac{d\ln T}{dz} = \frac{1}{L_D}(1+\beta) + \frac{mg}{T}. \tag{32}$$

Hence, the growth rate can be rewritten as

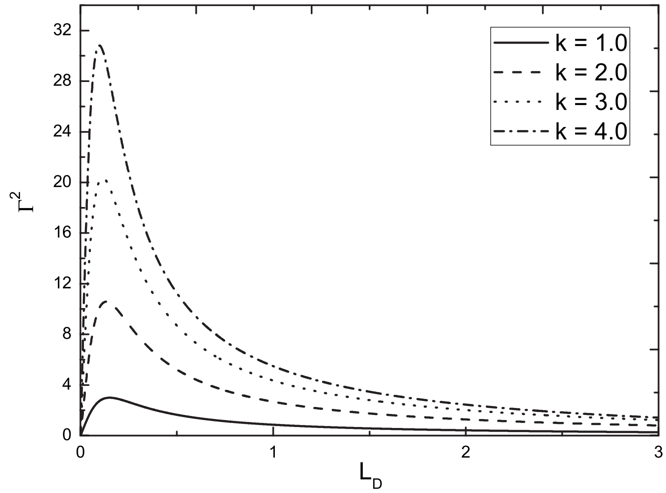

FIG. 1. The square of the growth rate against L_D when $v_{af}=0.2$. Solid curve is for $k=1.0$, dashed curve for $k=2.0$, dotted curve for $k=3.0$, and dash-dotted curve for $k=4.0$. Other parameters used in our plotting are $g=10$, $n=1$, and $h=1$ for simplicity.

$$\Gamma^2 = \frac{4L_D g k^2 h^2}{h^2 + 4L_D^2 n^2\pi^2 + 4L_D^2 h^2 k^2} - k^2 v_{af}^2, \tag{33}$$

when $\beta \ll 1$ and $mgL_D \ll T$. The formula above indicates that there is

$$\frac{\partial \Gamma^2}{\partial L_D} = 4gk^2h^2 \frac{h^2 - L_D^2(4n^2\pi^2 + 4h^2k^2)}{[h^2 + L_D^2(4n^2\pi^2 + 4h^2k^2)]^2}, \tag{34}$$

which implies that the growth rate increases monotonically as L_D increases when $L_D^2 < L_{Dc}^2$, where $L_{Dc}^2 = h^2/(4n^2\pi^2 + 4h^2k^2)$. Therefore, growth rate Γ^2 reaches its maximum value $\Gamma_{\max}^2 = 2gk^2h/(4n^2\pi^2 + 4h^2k^2)^{1/2} - k^2v_{af}^2$ at $L_D = L_{Dc}$. When $L_D > L_{Dc}$, the growth rate starts to decrease as L_D increases. These are confirmed by Fig. 1.

For short wavelengths region, where kL_D is much greater than 1, meaning that we are restricted to the local perturbations in a system which is homogeneous over several wavelengths of interest. The growth rate of the instability becomes

$$\Gamma_l = \sqrt{-g\frac{d}{dz}\ln T - k^2 v_{af}^2}. \tag{35}$$

In the weak magnetic field case, the previous results in Ref. 9 are recovered by letting $v_{af}^2 = 0$ in the formula above.

The magnetic field shows stabilizing effect on the MTI.[3] Equation (31) indicates that there is

$$\partial\Gamma^2/\partial v_{af}^2 = -k^2 < 0, \tag{36}$$

which suggests that the growth rate decreases monotonically with Alfvén speed v_{af} increases. In other words, the MTI is suppressed by the magnetic field, as confirmed by Fig. 2 which shows the relationship between the square of the growth rate [Eq. (31)] and the wave number k. With the magnetic field increases in strength, the growth rate decreases until it approaches to zero. It means that the MTI is totally quenched by the magnetic field. Physically, the present calculation confirms a clear expectation that when the Alfvén frequency of perturbations is high enough, the

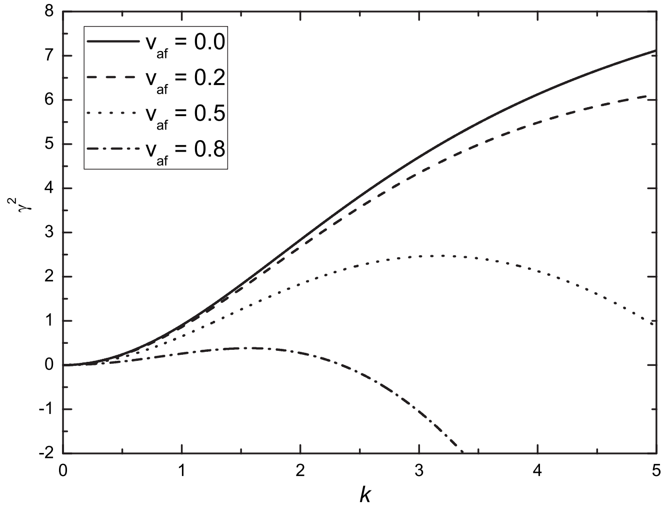

FIG. 2. The square of the growth rate against the wave number when $L_D=1$. Solid curve is for $v_{af}=0$, dashed curve for $v_{af}=0.2$, dotted curve for $v_{af}=0.5$, and dash-dotted curve for $v_{af}=0.8$. Other parameters used in our plotting are $g=10$, $n=1$, and $h=1$ for simplicity.

MTI has no time to grow and perturbations are carried away by the Alfvén waves. The critical magnetic field is determined by $\Gamma=0$ thus yielding

$$B_c = \frac{2L_D h\sqrt{\mu_0\rho_0}}{\sqrt{h^2+4L_D^2n^2\pi^2+4L_D^2h^2k^2}}\sqrt{-g\frac{d}{dz}\ln T}. \tag{37}$$

Finally, one can see that the system is magnetothermal stable if the temperature decreases in the direction of gravity, i.e., $dT/dz>0$. According to Ref. 9, the background heat flux can drive an analogous buoyancy instability for $dT/dz<0$. This is beyond the scope of this paper.

IV. CONCLUSION

In the presence of anisotropic heat conducting, the buoyancy instability in magnetized stratified plasma is investigated when the heat flux is assumed to run along the magnetic force lines. The Boussinesq approximation is adopted in our MHD model, which means that the relative changes in the pressure are assumed to be much smaller than those in the temperature or density, and in the perturbed mass conservation and momentum equations, the density variations can be neglected except when they are coupled to the gravitational term.[13] The second-order ODE was obtained to describe the perturbed velocity. Based on the ODE, the classical RT instability result can be recovered. Under fixed boundary condition, the growth rate of the MTI driven by the heat flux was obtained in the presence of the density gradient effect, which is represented by L_D, i.e., the scale length of the density gradient. Our results show that the perturbations are unstable when the thermal temperature increases in the direction of gravity, i.e., $dT/dz<0$. In hot accretion flows onto compact objects, there exists $dT/dz<0$ due to the release of gravitational potential energy and the inflow of matter and in cooling white dwarfs and neutron stars, the flow of heat outward drives $dT/dz<0$.[9] Besides, the similar instances also can be found in tokamak plasmas (see, for example, Ref. 17) and in the ablative inertial confinement fusion capsules,[18,19] in which the temperature increases along the direction of the effective gravity. Under these circumstances, the MTI is arisen. Furthermore, our results show that the growth rate increases as L_D increases at first. If L_D becomes greater than the critical value L_{Dc}, the growth rate decreases as L_D climbs. Finally, the growth rate is shown to decrease monotonically as Alfvén speed increases.

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