FYS-STK4155
Applied data analysis and machine learning

Analysis of the 2012 and 2016 US presidential elections

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Abstract

The main purpose of the project was to successfully predict the election outcome of 2012 and 2016 in the US based on certain variables, as well as identify the most important variables. The variables analyzed ranged from economics, ethnicity and education, among other variables. The data was first joined together into a design matrix and further standardized.

Various machine learning algorithms was deployed for completing the task, where accuracy score was the main metric. Support vector machine (SVM) with a kernel produced the best classification test results of 89% for the 2012 election and 96% for the 2016. Neural networks performed almost as good with 87% for 2012 and 94% for 2016, while logistic regression came out slightly worse.

The variables that contributed the most for correct classification were the age and population size of the counties. Regarding flipped counties between the elections, the applied methods hinted that the percentage of females and whites may have had a minor impact on the shift from one party to another.

Contents

1 Introduction

The idea of creating models that can predict output based on some input could be extremely valuable. The models are created by trying to find mathematical relationships in the data so that they can reproduce a desired output. Hopefully, the relationships are generally valid and make the model capable of predicting new unseen data. Models like these could be as simple as regression to the complexity of deep neural networks. The simple models are easy to understand and easy to implement, but their ability to learn complicated relationships can be to somewhat limited. That is why methods such as neural networks and support vector machines have become popular and will be representing our main methods in this project.

Models that are capable of predicting can help humans in decision making, reduce the number of experiments and observations needed and could replace humans in doing boring and repetitive tasks which humans normally are not good at. Or as in this case, reproduce results that allows us to analyze the information behind a certain outcome, such as the presidential election in the United States.

The bias-variance tradeoff is a fundamental dilemma in statistical learning theory. Bias is erroneous assumptions in a model, so that the model misses out on important relationships. This is called underfitting. Variance is error from sensitivity to fluctuations in the training data. This is called overfitting. Our

project will divide the provided data into training and test data, such that this can be observed as well.[1]

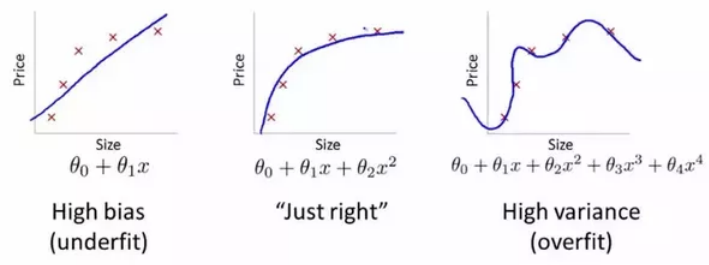


Figure 1: A simple visual example of underfitting and overfitting.

In classification, models are normally assessed based on the percentage of correct calculated labels, which will be used as metric here.

$$Accuracy = \frac{\sum_{i=1}^{n} I(t_i = y_i)}{n} \tag{1}$$

2 Theory

2.1 Data structure

The presidential election data used in this project was downloaded from GeoDa[2]. The underlying information in the dependant variables, constructing the design matrix, are all drawn from the years between 2007 and 2014. They represent sociological aspects such as education, wealth, ethnicity, population density among others. The response variable is the percentage outcome for the democrats and republicans. The response variable was in this work modified from percentage outcome into ones and zeros with a 50% cut-off. Democratic counties became ones. The problem can now be death with as supervised learning and classification.

As mentioned above, the dependent variables together construct the design matrix. The design matrix has the dimensions (3108, 51), 3108 counties and 51 variables. There are two independent variables, namely the political outcome for each election, 2012 and 2016. The independent variables are each a binary vector with the dimensions (3108, 1). After a reshuffle of the rows, or counties, 75% of the rows are used for training and cross validating a model, and 25% are left for testing on unseen data on the 2012 election. Then the model is applied on the same design matrix without splitting and reshuffling to make predictions on the 2016 election.

Since most machine learning algorithms rely on euclidean distances they do not perform well when the data is scaled differently. In this case the data is both large and low in numerical measurements, with the addition of percentage variables. Therefore, we transform the data by Scikit learn's preprocessing function 'StandardScaler'. The function subtracts the column mean and divide by the

column standard deviation for each column [3].

2.2 Cross validation

The model is first trained for all the data and model statistics are calculated. Then the model is validated and tested to assure the quality of the model. The validation technique used to validate the different models is cross validation. Cross validation is widely used when the data set is small or finding the difference between the true value and the prediction for a single or a sample of observations. When the data set is small, the resulting model is more sensitive to an arbitrary splitting of the data into training data and test data. The split could be leaving out important data from the training part. Another common validation technique is bootstrap.

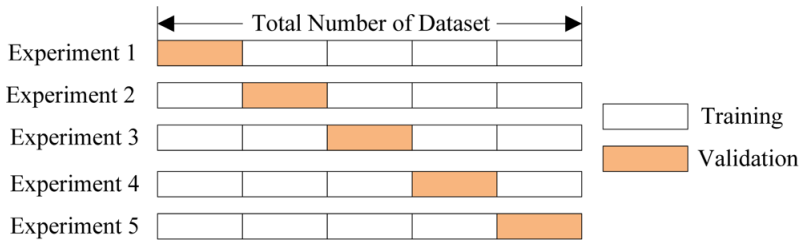


Figure 2: Cross validation explained graphically. The idea of cross validation. A small part of the data is used for training. The other larger part for testing/validation. Then another experiment and the test data changes and thus, so does the training data. In the end all data was test once but only once.

2.3 Logistic regression

A popular binary classification method is the logistic regression, especially among statisticians. This is because it's a more statistical approach to a classification problem than support vector machines or decision trees. There's no analytic expression for the coefficients, so these must be calculated numerically. How the mathematical problem is dealt with is briefly described below.

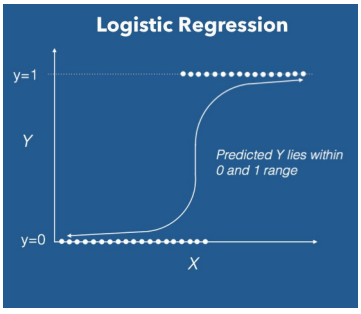


Figure 3: Logistic regression displayed graphically

The logistic function is called the Logit or the Sigmoid function (see below). It returns a value or interpreted as a probability between 0 and 1. β is the wanted coefficients and x_i is an observation. The function tries to map the product of these two to either 0 or 1, but some input values will give an output value in-between. This is where the the function sharply rises. At this range of input values, the observations do not belong to a clear category.

$$p(\beta x_i) = \frac{1}{1 + exp^{-\beta x_i}} \tag{2}$$

Moving on to deriving the cost function. The given probabilities could be either 1 or 0.

$$p(y_i = 1|x_i, \hat{\beta}) = \frac{1}{1 + exp^{-\beta \hat{x}_i}} \tag{3}$$

$$p(y_i = 0|x_i, \hat{\beta}) = 1 - \frac{1}{1 + exp^{-\hat{\beta} x_i}} \tag{4}$$

To obtain a cost function, define the total likelihood for all possible outcomes from a data set D = (y_i, x_i), with the binary labels $y_i \in$ 0,1 and where the data points are drawn independently, and then apply the Maximum Likelihood Estimation (MLE), i.e., maximizing the probability of seeing the observed data and approximating the likelihood in terms of the product of the individual probabilities of a specific outcome y_i.

$$P(D|\hat{\beta}) = \prod_{i=1}^{n} [p(y_i = 1|x_i, \hat{\beta})]^{y_i} [1 - p(y_i = 1|x_i, \hat{\beta})]^{1-y_i} \tag{5}$$

From the MLE, then the logistic regression has the following cost function.

$$C(\hat{\beta}) = \sum_{i=1}^{n} (y_i log[p(y_i = 1|x_i, \hat{\beta})] + (1 - y_i) log[1 - p(y_i = 1|x_i, \hat{\beta})]) \tag{6}$$

By defining a vector $\hat{y}$ with n elements y_i, an n$\times$p matrix $\hat{X}$ which contains the x_i values and a vector $\hat{p}$ of fitted probabilities $p(y_i|x_i, \hat{\beta})$, the first and second derivative of the cost function can be rewritten in a more compact way.

$$\frac{\partial C(\hat{\beta})}{\partial \hat{\beta})} = -\hat{X}^T (\hat{y} - \hat{p}) \tag{7}$$

$$\frac{\partial^2 C(\hat{\beta})}{\partial^2 \hat{\beta}} = \hat{X}^T \hat{W} \hat{X} \tag{8}$$

$\hat{W}$ is a diagonal matrix with the elements $p(y_i = 1|x_i, \hat{\beta})(1 - p(y_i = 1|x_i, \hat{\beta}))$

To find the minimum of the cost function, several approaches are possible. One possibility is the multivariate version of the Newton-Raphson's method. Newton-Raphson's method tries to find a root of a function f(x).

$$x^{(k+1)} = x^{(k)} - J_f(x^{(k)})^{-1} f(x^{(k)}) \tag{9}$$

where J is the Jacobian of f. A matrix of all first-order partial derivatives of the function.

The problem will have to be redefined as finding where the gradient of the function is equal to zero, $\nabla f(x) = 0$. Because we seek the roots of the derivative of the cost function.

$$x^{(k+1)} = x^{(k)} - (H_f(x^{(k)}))^{-1} \nabla f(x^{(k)}) \tag{10}$$

where H is the Hessian of f. A square matrix of second-order partial derivatives of the function.

For logistic regression, Newton-Raphson's method will then look the following way.

$$\hat{\beta}^{new} = \hat{\beta}^{old} - (\hat{X}^T \hat{W} \hat{X})^{-1} (\hat{X}^T (\hat{y} - \hat{p})) \tag{11}$$

$\hat{\beta}^{old}$ must be randomly initialized to start the iteration. $\hat{p}$ is the output from Sigmoid($\hat{\beta} X$).The iteration continues until the some criteria is reached, like the change in $\hat{\beta}$ is acceptable small.[4]

Computing the inverse of the Hessian, $\hat{X}^T \hat{W} \hat{X}^{-1}$, could be computationally expensive. Therefore, in larger datasets a gradient descent method is used instead. In practice, it means the Hessian is replaced by a scalar, also called a learning rate. Newton-Raphson's method could in addition to the second derivative also include a learning rate, but without it's known as "pure Newton". The function is convex, so Newton-Raphson's method or any gradient descent method will always converge to the minimum, a global minimum. [5]

2.4 Neural networks

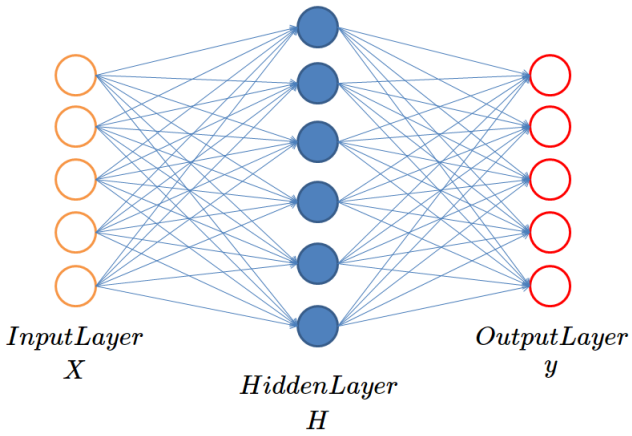


Figure 4: Neural network architecture layout

Source: `https://chunml.github.io/ChunML.github.io/project/Creating-Text-Generator-Using-Recurrent-Neural-Network/`

A neural network is constructed as shown above. In that case, the output layer contains more than one output, as it could be any desired number. One hidden layer is added, constructing a net of weights (blue lines) and biases. Additional hidden layers can be included. Each hidden layer can be assigned arbitrary many neurons (the blue circles). The idea is sprung from the logistic regression extended into a more complex configuration of interconnected elements that processes the information dynamically. It's also influenced by neuroscience, and how the human brain works. Further break down of the concept is as follows, with the encapsulated iteration process in mind:[6]

First, we choose the cross entropy function as our cost function, defined in Eq[6].

Forward phase
Weights and biases are often created randomly and normally distributed. The net input value (Z) is then calculated as:

$$Z = X \bullet W + b \tag{12}$$

Noted is that the calculations now take place as nestled matrices, where the matrix W represent all weights on both sides of a calculated layer, hence have the size (2,).

Z is then activated by chosen activation function for the interaction between the input layer and the hidden layer, and similarly by the hidden and output layer.

$$a(Z) \tag{13}$$

Backpropagation phase

An error calculation is made by use of the cost function, which is denoted $[\delta_3]$. Next step is to update the weights and biases such that the error between the target and output decreases towards a global minima. This step is the essence of a neural network and is often noted as the backpropagation algorithm, which evaluate the error and uses partial gradients to pass the necessary adjustments back into the system. For cross entropy as the choice of cost function and Sigmoid as the activation functions, the backpropagation algorithm is implemented as below:[7]

$$\delta_3 = (yp - y) \tag{14}$$

$$\delta_2 = \delta_3 \bullet W_2^T * (a(Z_1))' \tag{15}$$

$$\frac{\partial C}{\partial W_2} = a(Z)_2^T \bullet \delta_3 \tag{16}$$

$$\frac{\partial C}{\partial b_2} = sum(\delta_3) \tag{17}$$

$$\frac{\partial C}{\partial W_1} = X^T \bullet \delta_2 \tag{18}$$

$$\frac{\partial C}{\partial b_1} = sum(\delta_2) \tag{19}$$

Layers are representative by (1,2,3) = (I,H_1,O) such that the recipe can be extended if additional hidden layers are added, as (1,2,3,4) = (I,H_1,H_2,O). Note that if the cost function or the activation function are changed, one would have to derive δ_3 from scratch.

There must be highlighted that the neural network are prone to some parameters that governs entirely how well the model will preforms.[7]

Learning rate
This is a hyperparameter that controls how much the weights and biases will adjust due to the gradient of the cost function. A small value could be described as taking small, but accurate steps towards the minimum. For a large value, the path towards the minimum would be rough. To large and there is a probability of never reaching the precise minima, while to low may be stuck at a plateau. It also affects the computational time, a large learning rate would decrease the computational time and vice verse.

Regularization term
By applying a regularization term, the weights and biases (or coefficients for the linear case) is basically programmed to be small, with the assumption that a lower variance between them would represent a simpler model, hence it is a way of avoiding overfitting.

Epochs
Denotes how many times a training set is run through the neural network, both forward and backward. By increasing the number of epochs one would expect the model to obtain optimal weights and biases, only running it once would

underfit the model.

Batch size
Included in the SGD method, the training data is divided into smaller batches which run separately through the neural network. So instead of running the whole training set in one epoch, all batches runs once for one epoch. This allows us to run larger datasets. Optimal batch size is discussed, but the size will effect the convergence of the gradient. Generally a small batch size in the range of 32-512 is preferred.

Layers and neurons
Increasing the number of neurons or layers also increase the networks ability to learn detailed parts of the data, so for a complex situation more neurons and / or layers would be beneficial. This can be adjusted along the way, by doing a grid search for different numbers and compare the final score. The configuration also have impact on the proper choice of activation function.

Activation function
The choice of activation function will have impact on how the model preform as it transform the values into different domains. They may have steeper gradients and lower computational time, or be able to avoid vanishing gradients allowing deeper nets, or decrease the probability of exploding cost functions. For a network with multiple layers, one could use different functions for each layer. Without the activation functions, neural networks wouldn't preform as well as they do.

The above theory is nicely implemented in the package Tensorflow with Keras. Constructing a neural network with the package can be described with a few lines:

```
# Set up a sequential neural network
clf = tensorflow.keras.Sequential()
# Adding a hidden layer and a output layer
clf.add(tensorflow.keras.layers.Dense(...)
clf.add(tensorflow.keras.layers.Dense(...)
# Specify a gradient descent method
sgd = tensorflow.keras.optimizers.SGD(...)
# Compile model
clf.compile(optimizer=..loss=..metrics=..)
# Fit the model
clf.fit(X,y,epochs=..batch_size..)
```

2.5 Support vector machines

This is a machine learning method that is broadly used in classification problems, but it could just as well be used for regression problems. It relies on supervised data to construct a decision boundary that divides classes in space. The advantage of the algorithm lies partly in the support vectors, marked as dotted lines in figure[5]. The optimal decision boundary (red line) is obtained

by maximizing the gap between the support vectors. This feature is highly appreciable when dealing with classes that are difficult to separate. And further advantage is gained by analyzing data in higher dimensions.[8]

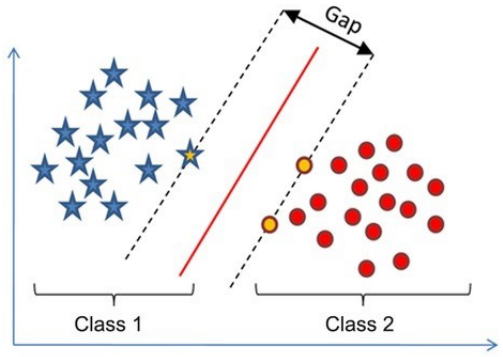


Figure 5: Visualized SVM concept

The decision boundary and support vectors are here described by:

$$y = w^T x + b = 0 \qquad , \qquad y = w^T x + b = \pm 1 \tag{20}$$

Where [w] is the normal vector to the decision boundary, [b] is the bias. The maximal margin, or gap in figure[5] is defined as:

$$max_w \frac{2}{||w||} \qquad or\ equivalently \qquad min_w ||w||^2 \tag{21}$$

Cost function:
Mentioned margin is to be optimized. But first we introduce a slack variable [ξ], which for [$\xi \geq ||w||$] a point is misclassified and for [$0 < \xi \leq ||w||$] a point is located within the margin. This is added as a term which operate as a margin softener, simply allowing for some mistakes, we have:

$$min_w ||w||^2 + C \sum_{i=1}^{m} \xi_i \quad constrained\ to \quad y_i * f(x_i) \geq 1 - \xi_i \tag{22}$$

C is the regularization parameter that regulates how much we value the mistakes and [x_i] is the support vectors. By including [$\xi_i \geq 0$] in the constraint, we can write the constraint and optimization function respectively as:

$$[\ y_i * f(x_i) \geq 1 - \xi_i\ ] \rightarrow [\ \xi_i = max(0, 1 - y_i * f(x_i))\ ] \tag{23}$$

$$min_w ||w||^2 + C \sum_{i=1}^{m} max(0, 1 - y_i * f(x_i)) \tag{24}$$

Where [$f(x_i) = w^T x_i + b$]. The summation term is denoted the hinge loss function, illustrated in figure[6].

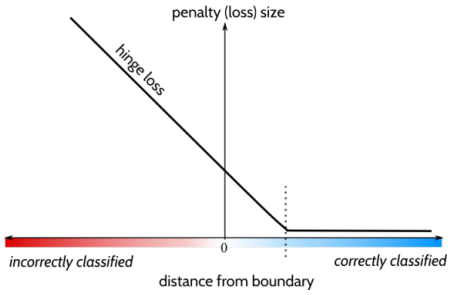


Figure 6: The hinge loss function

Since this a non-smooth function, it's needed to interpret it as a piece-wise function in order to minimize it. We can denote the term inside the summation as: [$max(0, 1 - y_i * f(x_i)) = L(x_i, y_i, w)$], by observing the two possibilities:

$$\frac{\partial(L)}{\partial(w)} = -y_i x_i \quad , \quad \frac{\partial(L)}{\partial(w)} = 0 \tag{25}$$

Further, we take use of the Lagrange method and define $\lambda = 2/(mC)$. We can now write the cost function to minimize as:

$$Cost(w) = \frac{1}{m} \sum_{i=1}^{m} (\frac{\lambda}{2} ||w||^2 + L(x_i, y_i, w)) \tag{26}$$

Kernel trick

By transforming the above primal form into dual form, the advantage of using the kernel-trick become available. The trick essentially transform a non-separable space into a separable, allowing us to obtain a decision boundary.

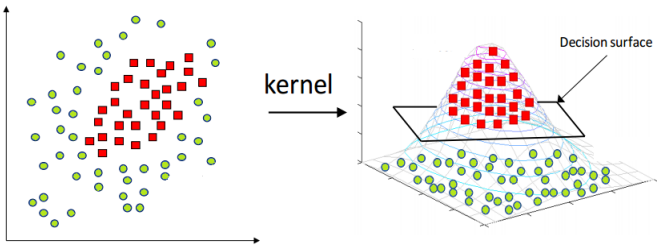


Figure 7: Kernel trick visualized

This is done by observing that the solution [w] can be expressed as a linear combination of the data:

$$w = \sum_{k=1}^{m} = \lambda_k, y_k, x_k \tag{27}$$

This is then substituted into $f(x_i)$ and the defined cost function, leading to a optimization problem over λ instead of w, with the following decision function:

$$f(x) = \sum_{i=1}^{m} \lambda_i y_i K(x_i, x) + b \tag{28}$$

And the optimization function follows as:

$$max_{(\lambda_i \geq 0)} \;\; \lambda_i - \frac{1}{2} \sum_{kj} \lambda_k \lambda_j y_k y_j K(x_k, x_j) \; for \; [0 \geq \lambda \geq C] \; and \; \sum_{i=1}^{m} \lambda_i y_i = 0 \tag{29}$$

Finally, the kernel trick can be applied by inserting chosen kernel function suitable for the specific classification problem. A few commonly functions are:

Linear

$$K(x_i, x) = x^T x_i \tag{30}$$

Polynomial

$$K(x_i, x) = (x^T x_i + \gamma)^d \tag{31}$$

Radial basis

$$K(x_i, x) = e^{-\gamma ||x - x_i||^2} \tag{32}$$

Parameters:
For being able to obtain the most efficient model of data, one need to tune some parameters. There is no golden rule, the tuning must be done by consideration for each applied situation. This is usually handled by doing a cross validation search. In addition to the parameters described below the different kernel functions also govern the final results, hence experimenting with those is also recommended.[9]

C
The parameter that governs the sensitivity of correct classifications. It's essentially a trade-off between the amount of misclassifications on training data and the maximization of the margin between the support vectors. For larger values of C only smaller margins would be allowed and for smaller values the margins would be larger and hence the decision boundary would be simpler, at the expense of accuracy. For $C = \infty$ the margin is denoted hard and do now allow any misclassifications. C is simply the regularization factor for SVM.

γ
This parameter is part of the kernel function and vary the dependence on whether the algorithm will construct a decision boundary with high consideration of points in its neighbourhood, or not. A low value would loosely fit the data, hence under-fitting it with a less complex boundary, and vice verse for a high gamma value.

The above theory is easily implemented by a two-liner in Scikit learn:

```
# Set up Support Vector Machine
clf = SVC(C=..,kernel=..,gamma=..)
# Fit the model
clf.fit(X,y)
```

3 Code implementation

The Python programming language is used for implementation of the methods. The Python packages Numpy, Scikit learn and Tensorflow with Keras are used extensively because of their rich functionality. In terms of computational time, the Tensorflow package operate a few orders of magnitude faster compared to self-implemented codes, developed directly from our understanding of the theory. Therefore, the focus has been on using the Python packages when running the calculations.

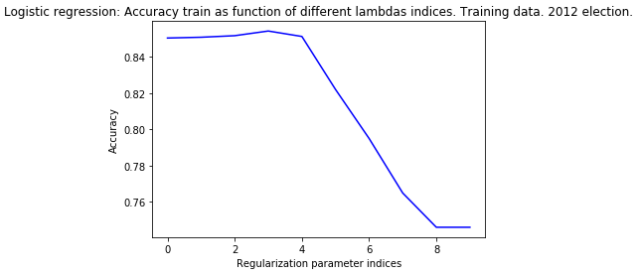
Overall code folder structure:

```
Executable files with an object oriented approach:
  * SVM and Logistic regression.py
    Calculating SVM and Logistic regression results.
  * Binary neural network.py
    Calculating neural network results.
Executable files:
  * variable_analysis.py
    Analyzing the variables.
Imported files:
  * methods.py
    Various functions needed for the project.
  * data.py
    Load the dataset properly.
  * Flipped counties.py
    Only used for drawing a map of flipped counties.
  * k_fold_CV.py
    Cross validation.
  * to_binary.py
    Changing a variable into binary variable.
  * metrics.py
    Calculating  metrics.
```

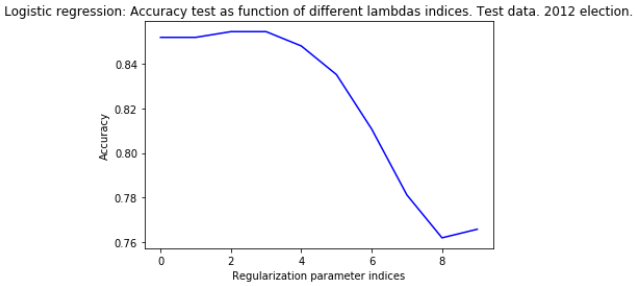
4 Results

4.1 Logistic regression

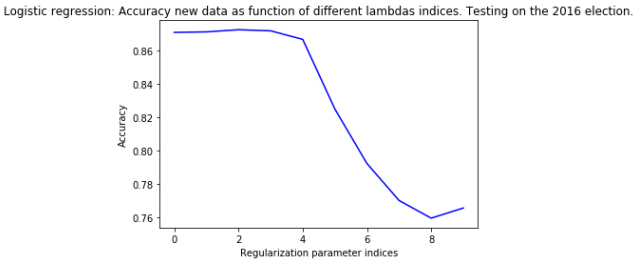
The logistic regression can up to a certain regularization value classify a satisfactory amount of counties. The calculation was made with the built in logistic regression from Scikit learn.



(a) Train data 2012



(b) Test data 2012

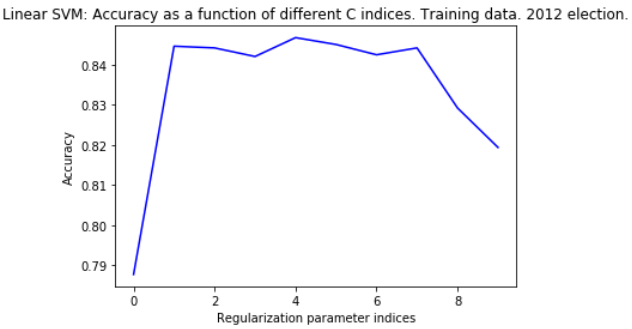


(c) Test data 2016

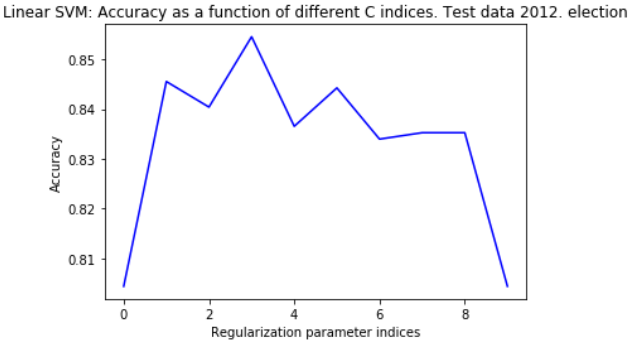
Figure 8: Accuracy graph

4.2 Linear SVM

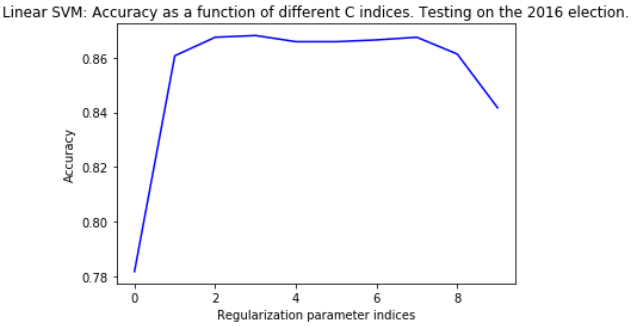
Scikit learn was also used to produce the SVM results below. The C's applied were values in the interval from 0.001 to 30. The accuracy stabilizes and peaks at C=3, index 1. The results are similar to logistic regression. This is not surprising, since they are both linear classifiers.



(a) Train data 2012



(b) Test data 2012

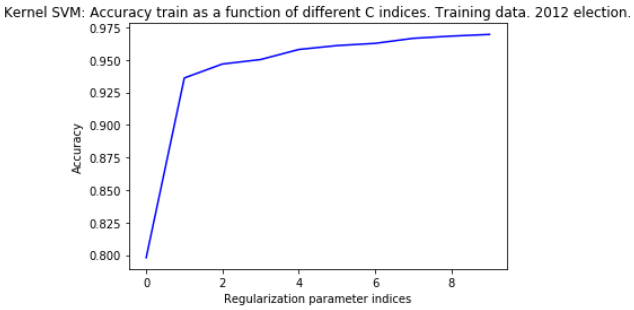


(c) Test data 2016

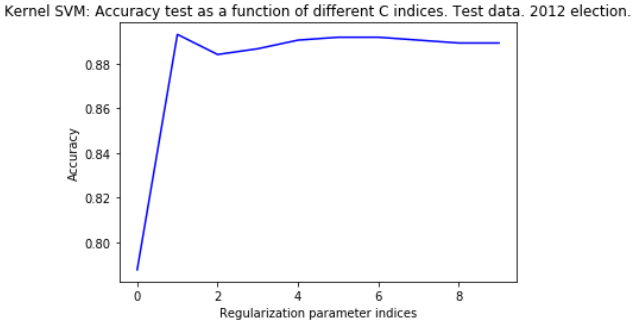
Figure 9: Accuracy graph

4.3 Kernel SVM

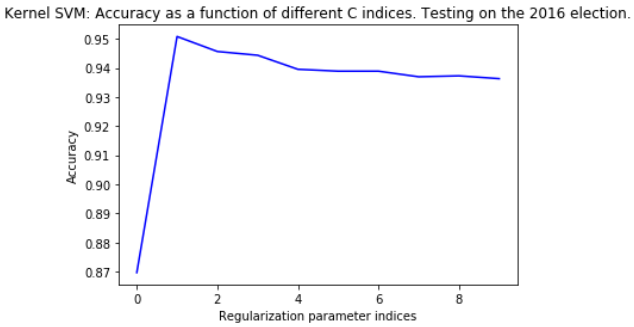
The C's values applied were the same as in linear SVM. The accuracy stabilizes and peaks at C=3, index 1. These results suggest that higher accuracy can be achieved by using the kernel trick.



(a) Train data 2012



(b) Test data 2012

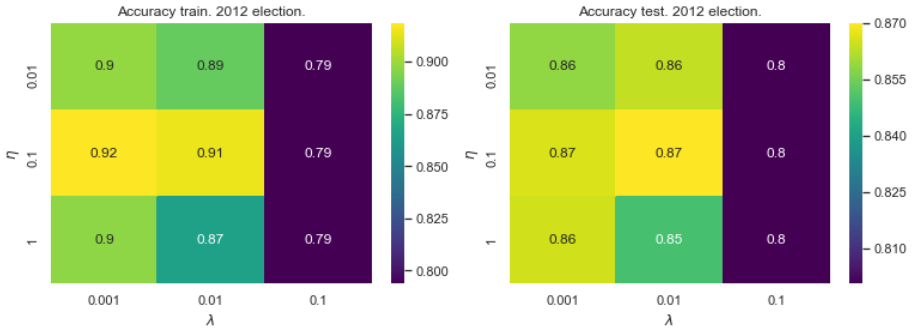


(c) Test data 2016

Figure 10: Accuracy graph

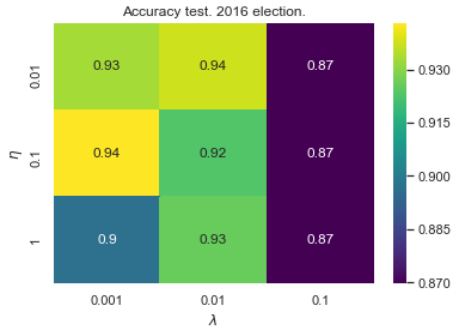
4.4 Neural network

The optimal results were given by a layer set-up of [H,H,H,H,O] Were the activation functions for the hidden layers [H] were the Relu function, while the output layer [O] had Sigmoid as the activation function. The hidden layers were assigned 1000 neurons each. The optimization technique used is the stochastic gradient descent, SGD, with a batch size of 32 and 10 epochs.



(a) Train data 2012

(b) Test data 2012



(c) Test data 2016

Figure 11: Accuracy heatmap

Validating neural network models with cross validation is computationally expensive. In order to avoid this, cross validation has been done with a low k, then less iterations. But a low k often gives worse and unreliable results, because the training data becomes significantly smaller. Cross validation results for all methods are found in the appendix [7].

4.5 Variable analysis

Below present details regarding the variables impact on the results and on the flipped states between 2012 and 2016.

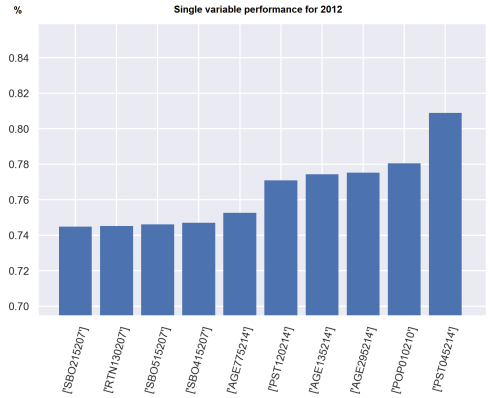


Figure 12: Most explanatory variables with accuracy score

Graphics from test data against 2012. Calculated with SVM for gamma=0.00778, c=2, and a plain guess would be 78%. This highlights the importance of the population and age variables as well as the one for firms. Further data can be found in the appendix [2].

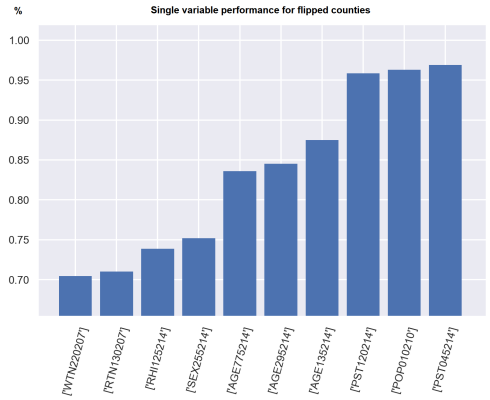


Figure 13: Most explanatory variables for flipped states with accuracy score

Graphics from test data against 2016. Calculated with SVM for gamma=0.00334, c=2, and a plain guess would be 95%.

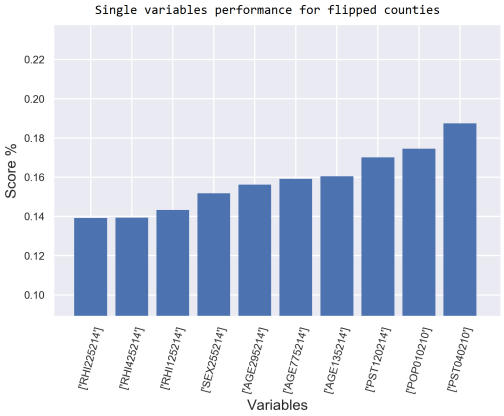
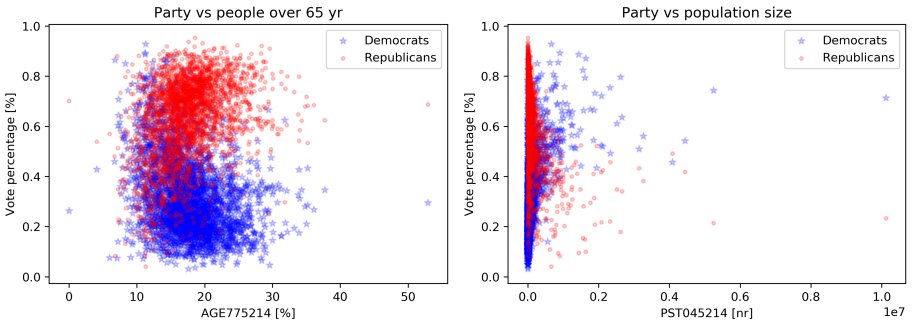


Figure 14: Most explanatory variables for flipped states with F1 score

Graphics from test data against 2016. Calculated with SVM for gamma=0.00778, c=2, and a plain guess would be 95%. For the flipped case, age and population still have the most impact, but female percentage and white percentage are showing up as well.

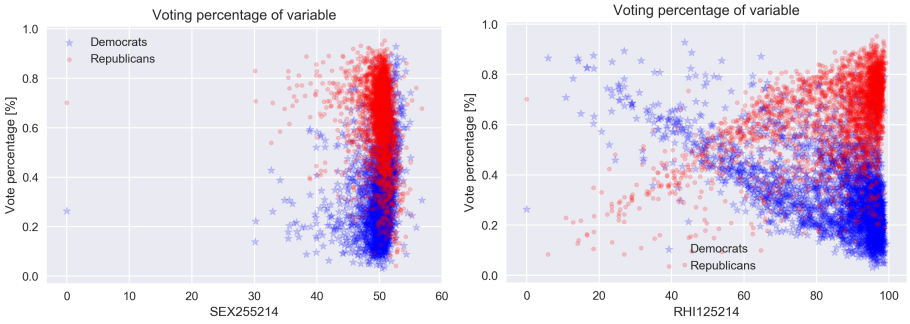


(a) Age over 65

(b) Population

Figure 15: Scatter plot of important variables

An example that highlight the difference in political support is the variables age and population. One can see that for a county with 30% percent old people, democrats are few. And with increased population, there is a decline in republicans.



(a) Females percentage

(b) White percentage

Figure 16: Scatter plot of important variables for flipped states

For the flipped counties the variables of white people percentage and female percentage stood out as slightly more important then the other variables, not considering the age and population. It is very clear how different these two groups vote.

5 Discussion

The methods of logistic regression and linear SVM produce similar results, the logistic regression seems to be more sensitive to the regularization parameter C. This could most likely be explained by the difference in loss functions, where the logistic 'cross entropy function' diverges faster than the 'hinge loss function' used in linear SVM.

A quick look at the Kernel SVM results implies that the transformation of data by the kernel trick, yields substantially better results. Especially the training results are better, which imply that the model is also overfitting to a larger extent. The trade-off in accuracy for test data is regardless favourable in this case. When it comes to the neural network performance, we did not expect the results to be greater than the SVM, since our data set is not considered large. Neural network have well-known learning limitations when it comes to small datasets. However, the method produced satisfying results in line with kernel SVM. The drawback in comparison is the computational time. What SVM was able to produce in seconds, neural network produced in several minutes, preforming the same computations without the packages would most likely scale the differences even further.

An interesting aspect of the results is the difference between the 2012 and 2016 election. In this problem, one would expect 2012 training data to fit 2012 test data better, which is not the case. An explanation could be the accuracy paradox, where the target is even more unbalanced for the 2016 election, making the overall classification prone to higher accuracy score.

There's a spatial dependence that be accounted for to further enhance the model. The spatial dependence is that the locations closer to each other are more likely

to have similar values than those that are further away. In this case, counties closer to each other are more likely to have similar class number or label. This phenomenon is known as Tobler's first law of geography. A suitable distance or neighbour weighting could implemented to take advantage of this. One approach is the Anselin Local Moran's I autocorrelation index.

Based on the variable performance, all variables included in this dataset contributes with valuable information. A variable selection test was made, and the result confirmed the statement above as it yielded worse results for selected variables, see the appendix [1]. A quick eye on the variables in figure [12], implies that the size and age of the population matters the most. This corresponds well with the reality that the most dense cities generally linked to high democratic support, as can be seen in figure [15].

As for the flipped case with 257 observations, the SVM algorithm was preferred since it is better suited for small datasets. The set for the flipped case was also heavily unbalanced, as the republicans had 95% of the flips. The combination of a small data set and unbalanced data was handled to some extent by cross validation and the balance function included in Scikit learn. However, the accuracy score varied with the percentage of democrats included in the test data. Another metric approach was tried, where we used the F1 score metric. The results were significantly different, as can be seen in figure[14]. This further highlights the accuracy paradox.

6 Conclusion

The use of machine learning for reproducing the election outcome gave valuable results that predicts the outcome with high certainty. All methods produced test results in the range of 85% to 89% for the 2012 election and in the range of 86% to 95% for the 2016 election. These results should be incorporated with the unbalanced target percentages of (79/21)% and (87/13)%, with respect to republicans and democrats. The first conclusion is that the SVM kernel method was slightly better suited than the Neural Network, and it's an obvious choice if computational time was to be considered. The addition of the kernel trick raised the results significantly when considering the unbalances in the data. We interpret the neural network as more sensitive for tuning parameters, since the results varies more than for SVM.

Population size and age of the inhabitants are the two most important features for the classification of a democrat or republican county. In the flipped case, a desire to sort out a fancy silver bullet that could explain the party shift in general. However, the results were to weak for any decisive conclusion, only vague implications that the percentage of white people and the percentage of females may have had some extra impact.

Another big takeaway, is that the accuracy score metric used throughout this project, may not be the most suitable for this dataset. The testing of the F1 score metric in the variables analysis gave a clear indication that further analysis should explore different metrics to tackle this dataset. A manual reduction of

the variables may also be of interest.

7 Appendix

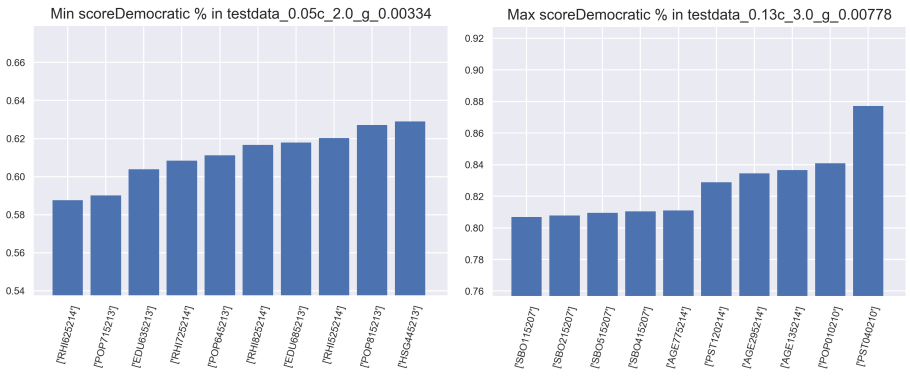
[1] Accuracy score for selection of variables:

Table 1: Accuracy score for selection [SVM:Gamma=Auto]

Original variables	Selected variables	C value
0.87	0.85	1
0.88	0.85	3
0.88	0.85	5
0.88	0.84	8

Produced by the function "ExtraTreesClassifier" by Scikit learn.

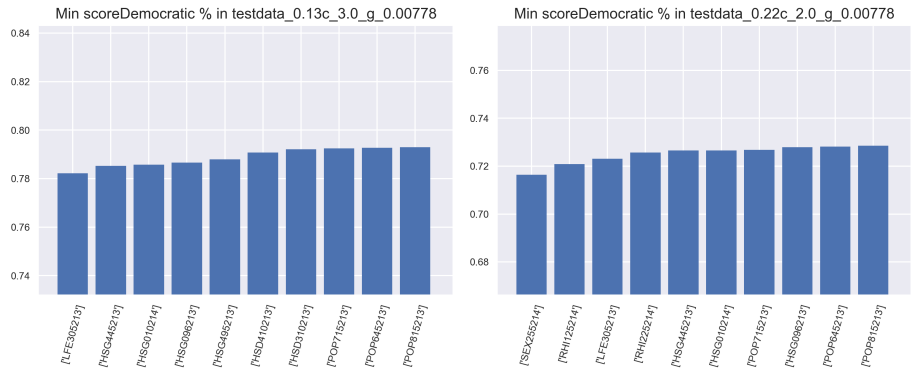
[2] Extended results for variable analysis:



(a) Minimum imapct for flipped case

(b) Maximum imapct for 2016

Figure 17: Single variables



(a) Minimum imapct for 2016 (b) Minimum imapct for 2012

Figure 18: Single variables

[3] List of variable abbreviations:
PST = Population
POP = Population
AGE = Age data
SEX = Gender data
RHI = Ethnicity data
EDU = Education data
VET = Veterans data
LFE = Travel time for work
HSG = Housing data
INC = Income data
PVY = Poverty data
BZA = Private non-farm employment
NES = Non-employer establishments
SBO = Firms owned by different ethnics
MAN = Manufactures shipping
WTN = Merchant wholesaler sales
RTN = Retail sales
AFN = Accommodation and food services sales
BSP = Building permits
LND = Land area

All variables can be found at:
`https://geodacenter.github.io/data-and-lab//county_election_2012_2016-variables/`

[4] List of previous research:
`http://www.ryan-peach.com/school-projects/2017/5/22/describing-the-2016-election-with-machine-learning`
By: Ryan Peach

[5] Map of misclassifications:

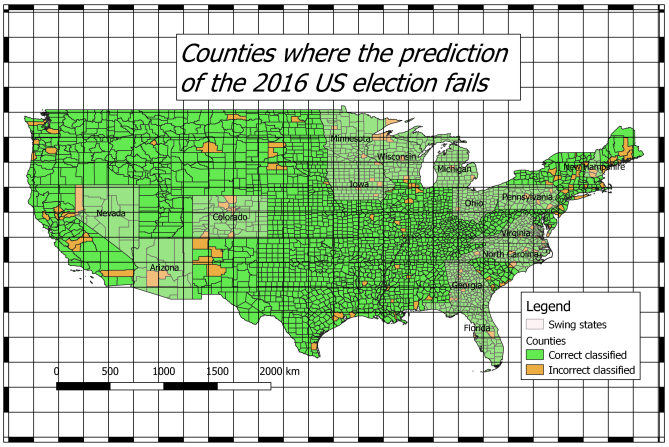


Figure 19: Counties that the SVM kernel could not correctly classify

[6] Map of flipped counties:

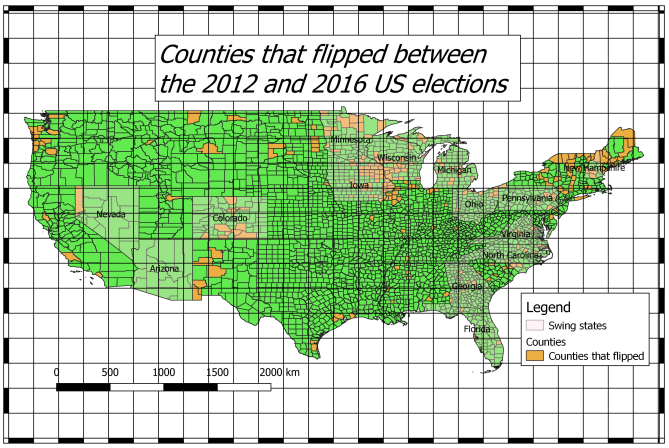


Figure 20: Map of flipped counties between 2012 and 2016

[7] Cross validation results:

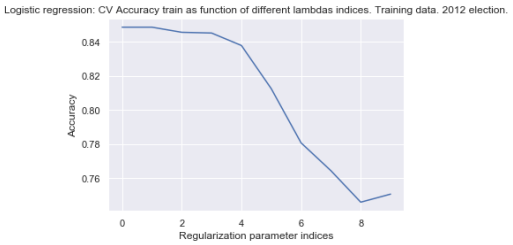


Figure 21: Cross validation results for logistic regression.

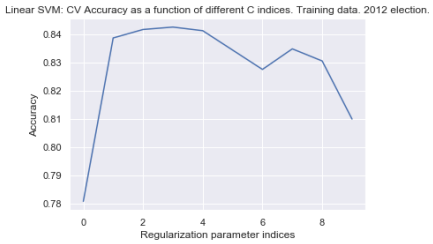


Figure 22: Cross validation results for linear SVM.

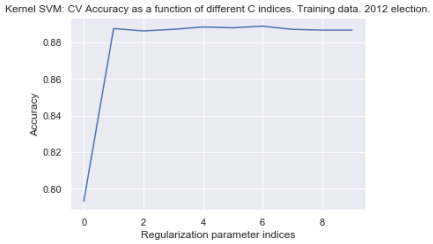


Figure 23: Cross validation results for SVM kernel.

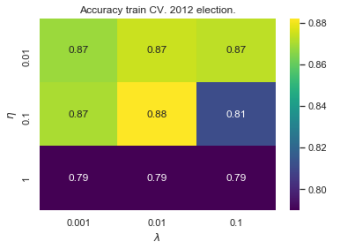


Figure 24: Cross validation results for neural network.

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[1] GeoDa. 2017. 2012 and 2016 Presidential Election. [ONLINE] Available at: https://s3.amazonaws.com/geoda/data/election.zip. [Accessed 13 November 2018].

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Link to Python scripts and latex scripts repository:
https://github.com/silverberg89/FYS-STK4155/tree/master/Project3