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R&E 참고용

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1 Introduction

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Definition 1. The grid diagram of a knot is a projection that consists only of horizontal and vertical lines, and the vertical line always passes above the horizontal line. (However, in this case, there must be exactly two points (excluding the vertices) that are vertically bent for each horizontal line (row) and vertical line (column), and the two vertices must be on the horizontal line.)

Definition The THC-Cromwell matrix is the matrix that satisfies the following conditions. ======= **Definition** The **grid diagram** of a knot is a projection that consists only of horizontal and vertical lines, and the vertical line always passes above the horizontal line. (However, in this case, there must be exactly two points (excluding the vertices) that are vertically bent for each horizontal line (row) and vertical line (column), and the two vertices must be on the horizontal line.)

Definition The **THC-Cromwell matrix** is the matrix that satisfies the following conditions. »»»> c8ae0d2edfcde996dbde17d4bd2123507474676b

1. It is a $n \times (n+1)$ matrix with entries 0 and 1.
2. It has only two '1's in every row and column, except for two rows. These two rows have three '1's.

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Theorem 2. *Every Theta knot and Hand-Cuff knot has its corresponding THC-Cromwell matrix.*

Definition Let any exception row(with three '1's) i and it's two outer '1's j, k. The Hun's Matrix of THC-matrix is $(n-1) \times (n-1)$ matrix which deleted row i and column j,k.
Theorem ======= **Theorem** Every theta-curve and handcuff knot has its corresponding THC-Cromwell matrix.

Definition Let any exception row(with three '1's) i and its two outer '1's j, k. The **H-deletion** Matrix of THC-Cromwell matrix is $(n-1)\times(n-1)$ matrix which deleted row i and column j,k. »»»> c8ae0d2edfcde996dbde17d4bd2123507474676b

2 Method of changing grid diagram of a knot to simple matrix

First, we should change grid diagram of a knot into Cromwell matrix. However, we can apply row operation of interchanging the rows. In this way, determinant of the Cromwell matrix would change only by multiplying ± 1.
By applying row operation, we should make matrix such as

$$\begin{pmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & & \\ 0 & 0 & 1 & & \vdots \\ \vdots & & & \ddots & \\ 1 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

Let matrix is $n \times n$. Next, we apply the row operation. We add the upper rows to the most bottom row. Then,

$$\begin{pmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & & \\ 0 & 0 & 1 & & \vdots \\ \vdots & & & \ddots & \\ 2 & 2 & \cdots & 2 & 2 \end{pmatrix}$$

Lastly, by subtracting $(2i-1)$th row ($i \in \mathbb{N}$) with multiplying row by 2 and substract with most bottom row, if n is even, the most bottom row only contains 0. Since it is upper triangular matrix, we can obtain determinant by trace. Hence the last entry is 0, the determinant is 0. If n is odd, the last entry of most bottom row is 2 and other entry is all 0. Since it is upper triangular matrix, we can obtain determinant by trace too. Hence the other entry is all 1, the determinant is ± 2.

3 Proof in the case of theta-curve

Erase the vertices of the row which has 3 vertices and also erase connected vertices of the left and right erased vertices in the grid diagram. Then, we can make the matrix of this grid diagram by previous section.
If end vertices is erased when we delete the row, then by what vertices is erased in the other three vertices row in the grid diagram can make different matrix.

1. **0**
   T-shaped figure is given.

2. **1 (middle vertex)**
   Line-shaped figure is given.

3. **1 (end vertex)**
   Line-shaped figure is given.

4. **2 (two end vertices)**
   T-shaped figure is given.

5. **2 (middle and end vertices)**
   Line-shaped figure is given.

1. **Line-shaped figure**
   Let the matrix is given.
   $$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$
   If you work on deleting rows and columns here, you can get the line-shaped figure of the following figure. If you convert this into a matrix according to (section n),
   $$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$
   is given. If we appropriately perform adding multiple of one row to another row and replacing the second row with the result, the determinant changes only by ± 1, and the resulting matrix will be identity matrix. If we appropriately transform other theta-curves, we can get the same result.

2. **T-shaped figure**
   Let the matrix is given.
   $$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$
   If you work on deleting rows and columns here, you can get the T-shaped figure of the following figure. If you convert this into a matrix according to (section n),
   $$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
   If we appropriately perform adding multiple of one row to another row and replacing the second row with the result, the determinant changes only by ± 1, and the resulting matrix will be identity matrix. If we appropriately transform other theta-curves, we can get the same result.

Therefore, ± 1 will be the determinant of the theta-curve's H-deletion matrix.