

# The Determinant and Arc Indices of Theta Curves and Handcuff-Graphs

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## Lower and Upper Bounds of Arc Index

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# Upper Bounds of Arc Index

## Theorem

Let  $T$  be any  $\theta$ -curve. Then,

$$\alpha(T) \leq c(T) + 3.$$

## Theorem

Let  $H$  be any handcuff graph. Then,

$$\alpha(H) \leq c(H) + 5.$$

Especially, if the constituent link of  $H$  is non-split,

$$\alpha(H) \leq c(H) + 3.$$

# Upper Bounds of Arc Index

## Theorem

Let  $T$  be any non-trivial prime  $\theta$ -curve or handcuff graph. Then,

$$\alpha(T) \leq c(T) + 3.$$