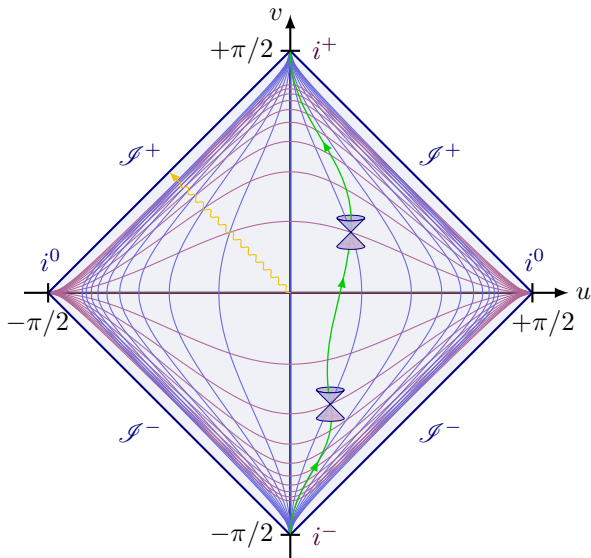
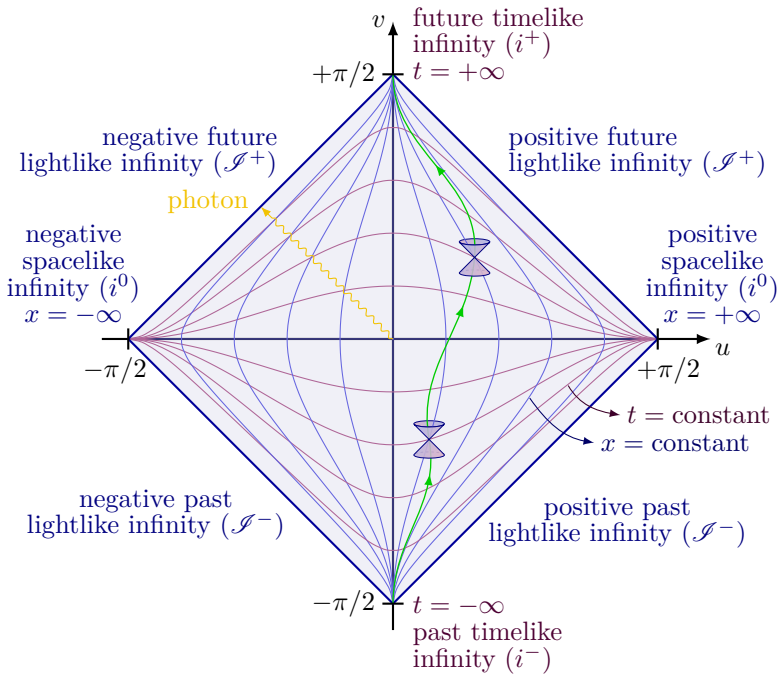


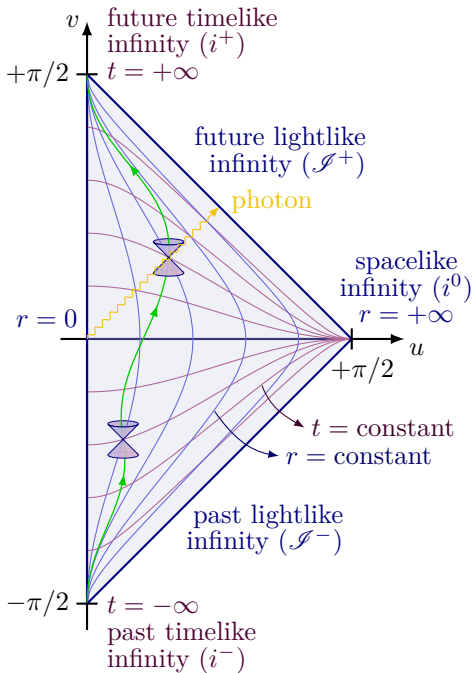
Penrose coordinates for Minkowski spacetime

$$\begin{cases} r + t = \tan(u + v) \\ r - t = \tan(u - v) \end{cases}$$

$$\Rightarrow \begin{cases} \tan v = \frac{\sqrt{(1 + \tan^2 u)^2 + 4r^2 \tan^2 u} - (1 + \tan^2 u)}{2r \tan^2 u} & \text{for constant } r \\ \tan u = \frac{\sqrt{(1 + \tan^2 v)^2 + 4t^2 \tan^2 v} - (1 + \tan^2 v)}{2t \tan^2 v} & \text{for constant } t \end{cases}$$







Kruskal-Szekeres coordinates for Schwarzschild spacetime

$$\left\{ \begin{array}{l} U = \sqrt{\frac{r}{2GM} - 1} e^{\frac{r}{4GM}} \cosh\left(\frac{t}{4GM}\right) \\ V = \sqrt{\frac{r}{2GM} - 1} e^{\frac{r}{4GM}} \sinh\left(\frac{t}{4GM}\right) \end{array} \right\} \quad \text{for } r > 2GM$$

$$\left\{ \begin{array}{l} U = \sqrt{1 - \frac{r}{2GM}} e^{\frac{r}{4GM}} \sinh\left(\frac{t}{4GM}\right) \\ V = \sqrt{1 - \frac{r}{2GM}} e^{\frac{r}{4GM}} \cosh\left(\frac{t}{4GM}\right) \end{array} \right\} \quad \text{for } r < 2GM$$

$$\Rightarrow \left\{ \begin{array}{l} U^2 - V^2 = \left(\frac{r}{2GM} - 1\right) e^{\frac{r}{2GM}} \\ V = \tanh\left(\frac{t}{4GM}\right) U \\ V = \coth\left(\frac{t}{4GM}\right) U \end{array} \right. \quad \begin{array}{l} \text{for } r > 2GM \\ \text{for } r < 2GM \end{array}$$

Penrose coordinates for Schwarzschild spacetime

$$\left\{ \begin{array}{l} U + V = \tan(u + v) \\ U - V = \tan(u - v) \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \cos(2v) = \frac{2GM + (r - 2GM) e^{\frac{r}{2GM}}}{2GM - (r - 2GM) e^{\frac{r}{2GM}}} \cos(2u) \\ \sin(2v) = \tanh\left(\frac{t}{4GM}\right) \sin(2u) \\ \sin(2v) = \cosh\left(\frac{t}{4GM}\right) \sin(2u) \end{array} \right. \quad \begin{array}{l} \\ \text{for } r > 2GM \\ \text{for } r < 2GM \end{array}$$

