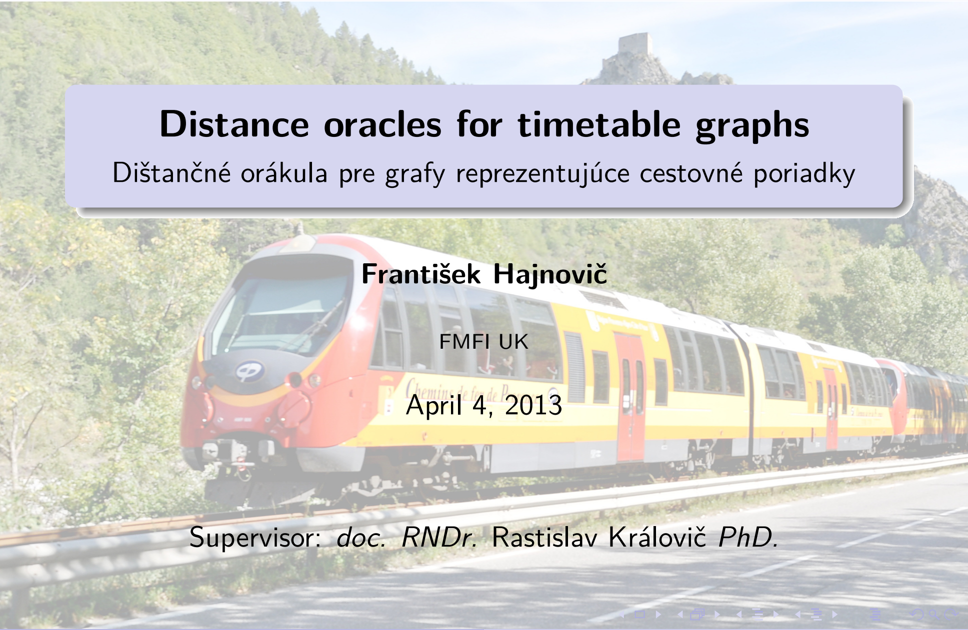

# Distance oracles for timetable graphs

## Dištančné orákula pre grafy reprezentujúce cestovné poriadky

**František Hajnovič**

FMFI UK

April 4, 2013

Supervisor: *doc. RNDr.* Rastislav Královič *PhD.*

# Content

# Introduction

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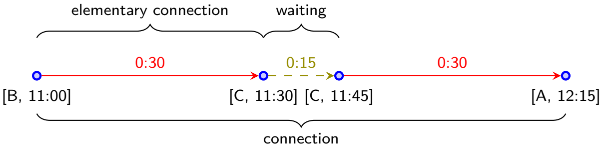
# What is it about?

- Given a timetable, we query - $(a, t, b)$ - for
  - **Earliest arrival** (EA) - $t^*_{(a,t,b)}$
  - **Optimal connection** (OC) - $c^*_{(a,t,b)}$



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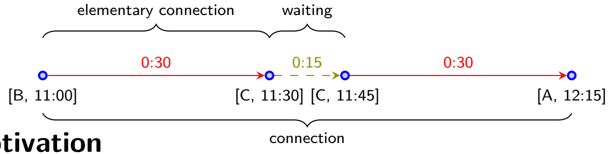
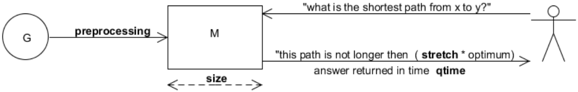



- **Motivation**
  - Large-scale timetable search engines (*cp.sk, imhd.sk*...)
- **Approach**
  - (Distance) oracle-based approach [TZ05] - pre-computation



# Timetable and underlying graph

| Place | | Time | |
|---|---|---|---|
| From | To | Departure | Arrival |
| A | B | 10:00 | 10:45 |
| B | C | 11:00 | 11:30 |
| B | C | 11:30 | 12:10 |
| B | A | 11:20 | 12:30 |
| C | A | 11:45 | 12:15 |

Table : **Timetable** - a set of **elementary connections** (between pairs of **cities**)

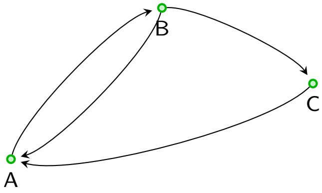



Figure : **Underlying graph**

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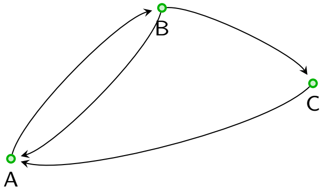


Figure : **Underlying graph**

- **Goals**
  - Devise methods to tackle EA/OC problem
  - Analyse properties of timetables

# Contribution

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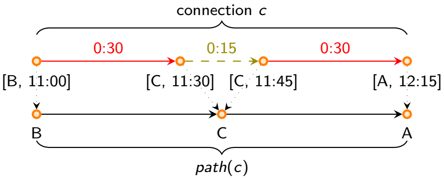
# Data

| Name | Description | El. conns. | Cities | UG arcs | Time range | Height ($h$) |
|---|---|---|---|---|---|---|
| air01 | domestic flights (US) | 601489 | 287 | 4668 | 1 month | 24374 |
| cpru | regional bus (SVK) | 37148 | 871 | 2415 | 1 day | 239 |
| cpza | regional bus (SVK) | 60769 | 1108 | 2778 | 1 day | 370 |
| montr | public transport (Montreal) | 7153 | 217 | 349 | 1 day | 363 |
| sncf | country-wide rails (FRA) | 90676 | 2646 | 7994 | 1 day | 488 |
| zsr | country-wide rails (SVK) | 932052 | 233 | 588 | 1 year | 60308 |

Table : Timetables datasets

# Idea

- *"Usually we go through the same sequence of cities"*




| Odchod | Príchod | Dĺžka cesty* | Použité linky | Zóny | Cena* |
|---|---|---|---|---|---|
| **21:59** | 22:10 | 11 min | 95 | - | 0,70 € |
| **22:09** | 22:20 | 11 min | 95 | - | 0,70 € |
| **22:19** | 22:30 | 11 min | 95 | - | 0,70 € |
| **22:29** | 22:40 | 11 min | 95 | - | 0,70 € |
| **22:39** | 22:50 | 11 min | 95 | - | 0,70 € |

- $p$ is USP $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP $\rightarrow$ reconstruct $c^*_{(a,t,b)}$

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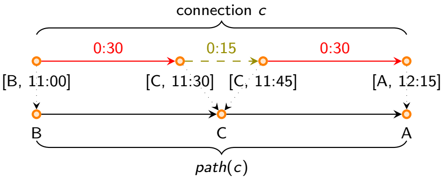


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- $p$ is USP $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP $\rightarrow$ reconstruct $c^*_{(a,t,b)}$
- **Overtaking** [MHSWZ07] causes problems, but can be easily removed

| Name | Overtaken edges (%) |
|---|---|
| air01 | 1% |
| cpru | 2% |
| cpza | 2% |
| montr | 1% |
| sncf | 2% |
| zsr | 0% |

# USP-OR

- Pre-compute *all conn.* - space $\mathcal{O}(h\ n^3)$
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  - daily height usually $200 < h < 800$
- Pre-compute *all USPs* - space $\mathcal{O}(\tau\ n^3)$
  - $\tau_{A,B}$ - # of USPs between $A$ and $B$

| Name | avg $\tau_{A,B}$ | max $\tau_{A,B}$ |
|---|---|---|
| air01 | 5.8 | 30 |
| cpru | 7.0 | 64 |
| cpza | 5.1 | 42 |
| montr | 4.3 | 30 |
| sncf | 4.3 | 24 |
| zsr | 2.5 | 19 |

Table : Daily, 200 station timetables

| Name | avg USP size |
|---|---|
| air01 | 3.0 |
| cpru | 13.8 |
| cpza | 11.1 |
| montr | 20.3 |
| sncf | 10.5 |
| zsr | 13.7 |

Table : Daily, 200 station timetables

# USP-OR

- Pre-compute *all conn.* - space $\mathcal{O}(h\ n^3)$
  - daily height usually $200 < h < 800$
- Pre-compute *all USPs* - space $\mathcal{O}(\tau\ n^3)$
  - $\tau_{A,B}$ - # of USPs between $A$ and $B$

| Prep-time | Size | Q-time | Stretch |
|---|---|---|---|
| $\mathcal{O}(hn^3)$ | $\mathcal{O}(\tau\ n^3)$ | $\mathcal{O}(\tau\ n)$ | 1 |

Table : USP-OR parameters

- $\tau$ almost constant, USP size $\approx \sqrt{n}$
- Space $\mathcal{O}(n^{2.5})$ too big anyway

| Name | avg $\tau_{A,B}$ | max $\tau_{A,B}$ |
|---|---|---|
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Table : Daily, 200 station timetables

# USP-OR - $\tau$ evolution

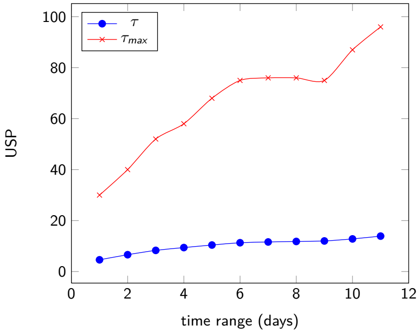



Figure : Changing of $\tau$ with increased time range in *air01* dataset. 1 day = about 800 in height

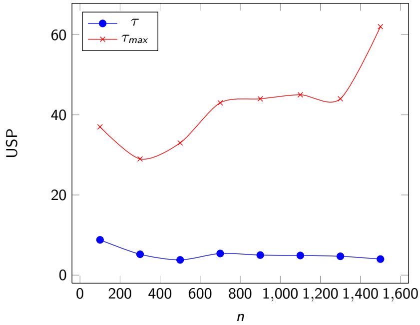
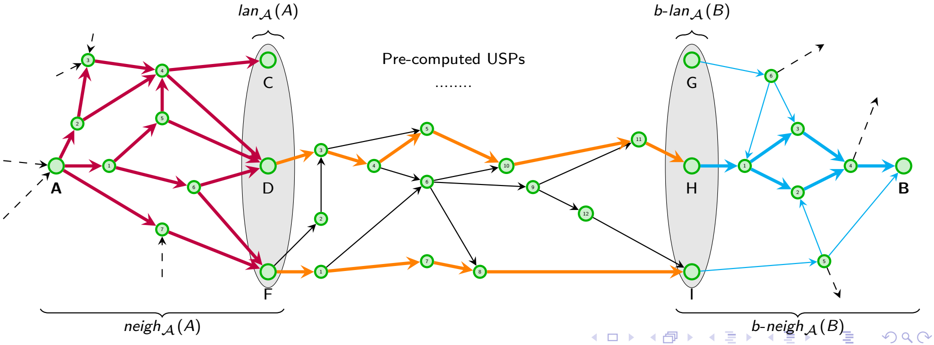



Figure : Changing of $\tau$ with increased # of stations in *sncf* dataset

# USP-OR-A

- Pre-compute USPs only among *some* cities in UG: set of **access nodes** $\mathcal{A}$
- We would like ($r1$ and $r2$ small constants w.r.t. $n$):
  - Size $|\mathcal{A}| = \mathcal{O}(\sqrt{n})$
  - Small node neighbourhoods: $\forall v \; |neigh_{\mathcal{A}}(v)| < r_1 \cdot \sqrt{n}$
  - Few local access nodes: $\forall v \; |lan_{\mathcal{A}}(v)| \leq r_2$



# Searching for optimal AN set

- NP-complete
  - Reduction to min-set-cover


Figure : text

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- NP-complete
  - Reduction to min-set-cover


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- Heuristics: node $v$ with low $|neigh_{\mathcal{A}}(v) \cap b\text{-}neigh_{\mathcal{A}}(v)|$ but high $\min\{|neigh_{\mathcal{A}}(v)|, |b\text{-}neigh_{\mathcal{A}}(v)|\}$ is good ANs

# Existing methods

- **Time-dependent SHARC** [Del08], **Time-dependent CH** [BDSV09]
  - Speed-ups of about 26 / 1500, respectively (EA only)
  - Meant for time-dependent routing in road networks
- **Time-expanded approach** [DPW09]
  - Speed-ups of about 56
  - Remodelling unimportant stations
- **Theory vs. practice** difference
  - Inclusion of transfers, cost...

# Neural network approaches

- Multi-layer perceptron, back propagation
- Input layer = events + cities. Output layer:
  1. Arcs of UG → USP
  2. Arcs of UG → routing
  3. Earliest arrival value

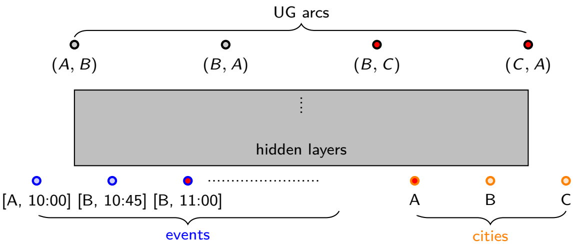


Figure : Approach 1.)

# Results

- Tendency to remember USPs
- Long training times

| Name | Conn. | Found | Was optimum (%) |
|---|---|---|---|
| air01 | 931 | 573 | 18.7% |
| cpru | 481 | 281 | 48% |
| montr | 527 | 346 | 86.7% |
| zsr | 672 | 307 | 76.2% |

Table : Tests of a trained NN on timetables with 30 cities (approach 1.)

# Timetable analyzer - TTBlazer

- Works with UG, TE, TD, TT
- Analysis ($\tau$, HD, degrees...), oracles (USP-OR, Dijkstra...), modifications (remove overtaking...), generation (subgraphs, TT $\rightarrow$ TD ...)
- Running & evaluating tests
- Easily extendible

Figure : It's *blazing* fast!

# Conclusion

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- Trying out novel approaches to solving EAP in timetables
  - *USP-OR*: Exact and quick answers but high space and time preprocessing
  - *NN*: Problem too challenging for NN/try different types of network
- Analysis of **various** real-world timetables
  - Better insight on properties of timetables
- Useful and easily extendible application

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# Thank you for the attention