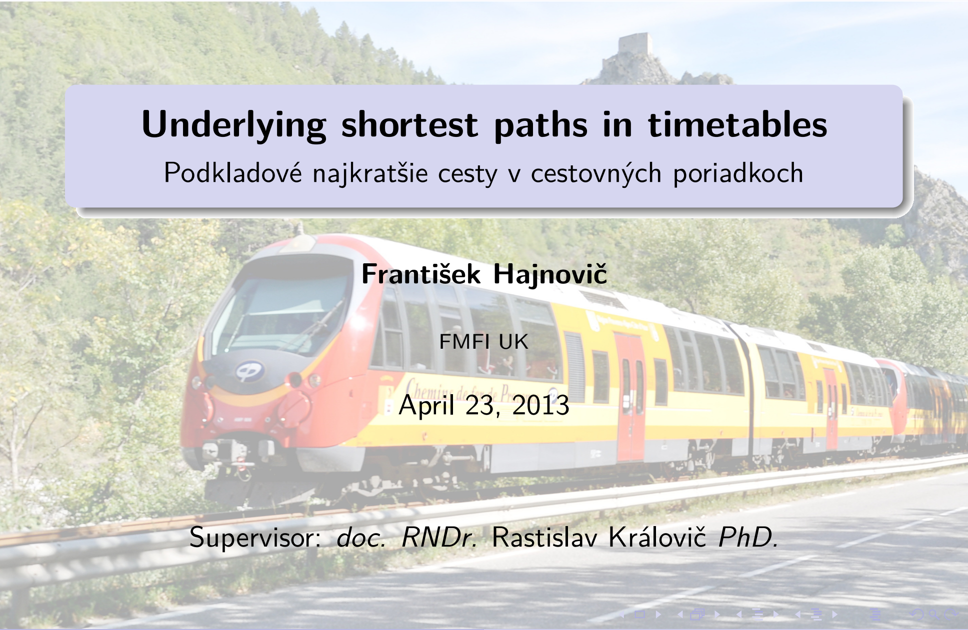


Underlying shortest paths in timetables

Podkladové najkratšie cesty v cestovných poriadkoch

František Hajnovič

FMFI UK

April 23, 2013

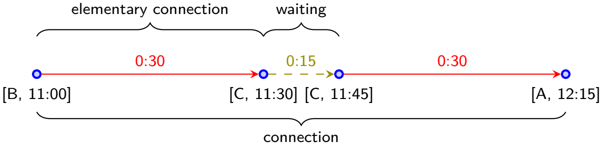
Supervisor: *doc. RNDr.* Rastislav Královič *PhD.*

Introduction

Introduction

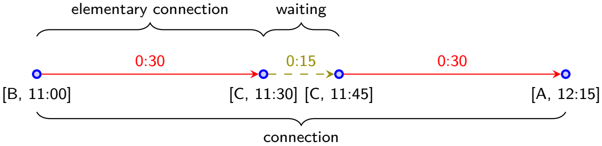
What is it about?

- Given a timetable, we query for **optimal connection** (OC): $(a, t, b) \rightarrow c^*_{(a,t,b)}$

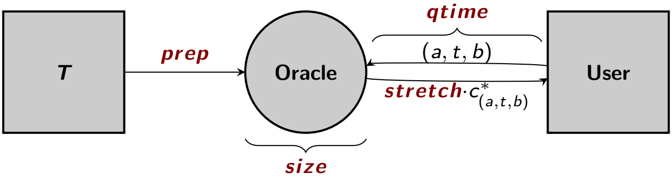


What is it about?

- Given a timetable, we query for **optimal connection** (OC): $(\boldsymbol{a}, \boldsymbol{t}, \boldsymbol{b}) \rightarrow \boldsymbol{c}^*_{(\boldsymbol{a},\boldsymbol{t},\boldsymbol{b})}$



- Motivation: large-scale timetable search engines (*cp.sk*, *imhd.sk*...)
- Approach: (distance) oracle-based approach [TZ05] - pre-computation



Timetable and underlying graph

Place		Time	
From	To	Departure	Arrival
A	B	10:00	10:45
B	C	11:00	11:30
B	C	11:30	12:10
B	A	11:20	12:30
C	A	11:45	12:15

Table : **Timetable** - a set of **elementary connections** (between pairs of **cities**).

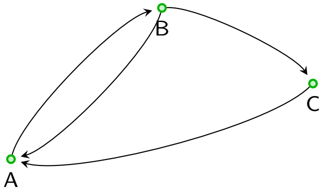


Figure : **Underlying graph**.

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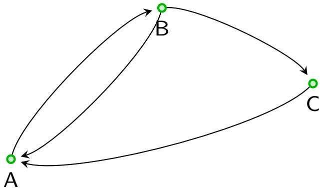


Figure : **Underlying graph**.

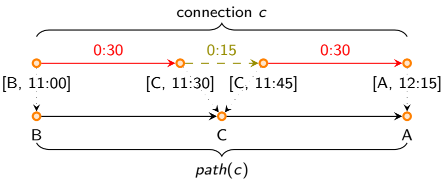
- Goals:
  - Devise methods to tackle optimal connection problem
  - Analyse properties of real-world timetables

Contribution

Contribution

Idea

- *"Usually we go through the same sequence of cities"*



Odchod	Príchod	Dĺžka cesty*	Použité linky	Zóny	Cena*
21:59	22:10	11 min	95	-	0,70 €
22:09	22:20	11 min	95	-	0,70 €
22:19	22:30	11 min	95	-	0,70 €
22:29	22:40	11 min	95	-	0,70 €
22:39	22:50	11 min	95	-	0,70 €

- p is **USP** $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP p: $expand(p) \rightarrow c^*_{(a,t,b)}$

Idea

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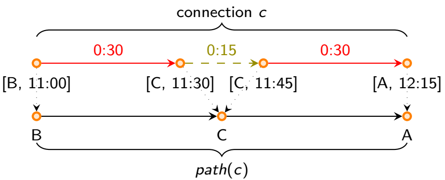

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- p is **USP** $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP p: $expand(p) \rightarrow c^*_{(a,t,b)}$
- **Overtaking** [MHSWZ07] causes problems, but can be easily removed

Name	Overtaken edges (%)
air01	1%
cpsk	2%
gb-coach	1%
gb-train	0%
montr	1%
sncf	1%
sncf-ter	1%
sncf-inter	6%
zsr	0%

USP-OR

- Pre-compute *all conn.* - space $\mathcal{O}(h\ n^2\gamma)$
  - $\boldsymbol{\gamma}$ - the average OC size
  - $\boldsymbol{h}$ - height

USP-OR

- Pre-compute *all conn.* - space $\mathcal{O}(h\ n^2\gamma)$
    - γ - the average OC size
    - $\boldsymbol{h}$ - height
- Pre-compute *all USPs* - space $\mathcal{O}(\tau\ n^2\gamma)$
    - $\boldsymbol{\tau(x,y)}$ - # of USPs between x and y
- $\boldsymbol{\delta}$ - the density of UG

Name	τ	γ	δ	h
air01	7.5	2.1	32.9	112.7
cpsk	12.1	15.9	3.9	50.7
gb-coach	5.3	5.3	5.6	48
gb-train	10.3	7.6	5.7	129.6
montr	5.1	21.0	1.9	35.0
sncf	5.6	8.6	3.4	42.4
sncf-inter	2.6	13.4	1.1	20.8
sncf-ter	5.6	12.7	3.3	34.0
zsr	3.7	13.5	2.7	21.6

Table : Daily, 200 station timetable subsets.

USP-OR	***prep***	***size***	***qtime***	***stretch***
guaranteed	$\mathcal{O}(hn^2(\log n+\delta))$	$\mathcal{O}(\tau n^2\gamma)$	avg. $\mathcal{O}(\tau\gamma)$	1
$\boldsymbol{\tau}$ **const.**, $\gamma \leq \sqrt{\boldsymbol{n}}$, $\delta \leq \log \boldsymbol{n}$	$\mathcal{O}(hn^2\log n)$	$\mathcal{O}(n^{2.5})$	avg. $\mathcal{O}(\sqrt{n})$	1

USP-OR - τ evolution

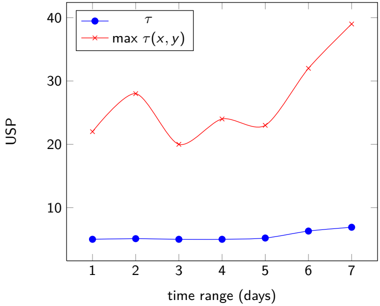

Figure : Changing of τ with increased time range in *sncf-200* dataset.

USP-OR - τ evolution

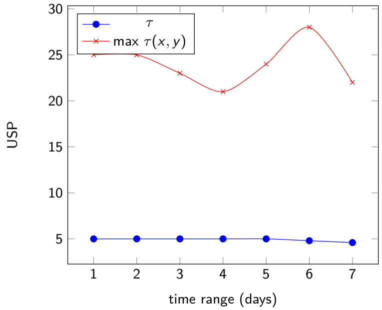

Figure : Changing of τ with increased time range in *sncf-200* dataset. **Segmentation** of the timetable to days.

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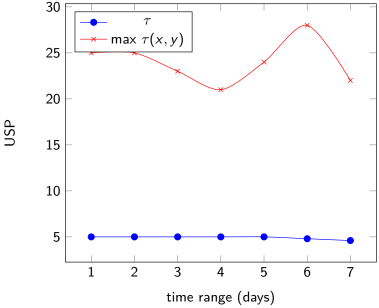

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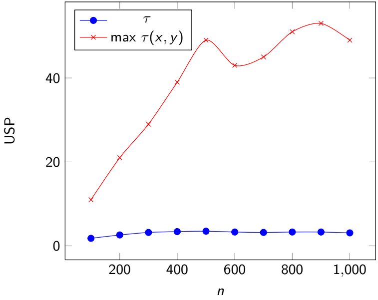

Figure : Changing of τ with increased # of stations in *gb-coach* dataset.

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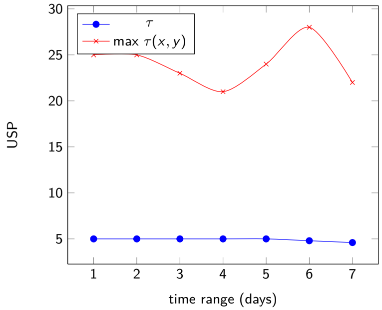

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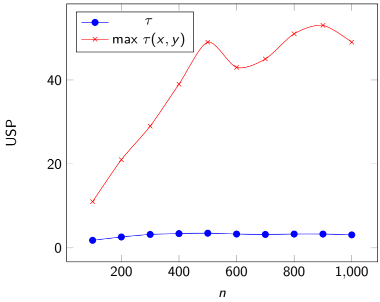
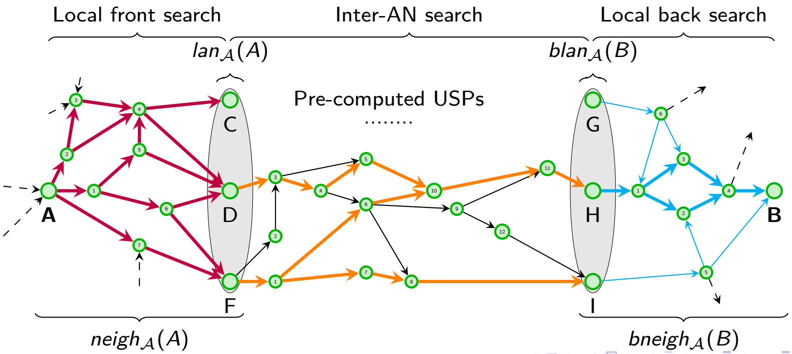


Figure : Changing of τ with increased # of stations in *gb-coach* dataset.

- Space $\mathcal{O}(n^{2.5})$ too big anyway...

USP-OR-A

- Pre-compute USPs only among *some* cities in UG: $(\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3})$ **access node set** $\mathcal{A}$
  - Small size: $|\mathcal{A}| \leq r_1 \cdot \sqrt{n}$
  - Small node neighbourhoods: $avg\ (|neigh(x)|)^2 \leq r_2 \cdot n$
  - Few local access nodes: $avg(|lan_{\mathcal{A}}(x)|)^2 \leq r_3$



USP-OR-A and access nodes

USP-OR-A	guaranteed	τ, r_1, r_2, r_3 const., $\gamma \leq \sqrt{n}$, $\delta \leq \log n$
prep	$\mathcal{O}(f(n) + (r_1 + r_2)(\delta + \log n)hn^{1.5})$	$\mathcal{O}(f(n) + hn^{1.5} \log n)$
size	$\mathcal{O}(r_2 n^{1.5} + r_1^2 \tau_{\mathcal{A}} \gamma_{\mathcal{A}} n)$	$\mathcal{O}(n^{1.5})$
qtime	avg. $\mathcal{O}(r_2 r_3 \sqrt{n}(\log(r_2 n) + \delta) + r_3 \tau_{\mathcal{A}} \gamma_{\mathcal{A}})$	avg. $\mathcal{O}(\sqrt{n} \log n)$
stretch	1	1

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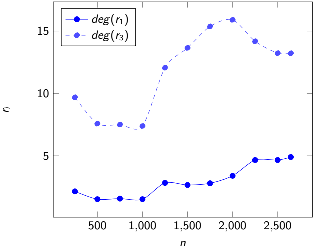

Figure : *sncf*, $r_2 \leq 1$.

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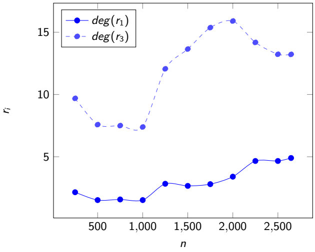

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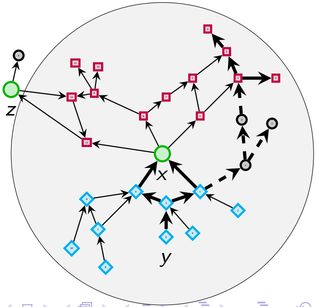

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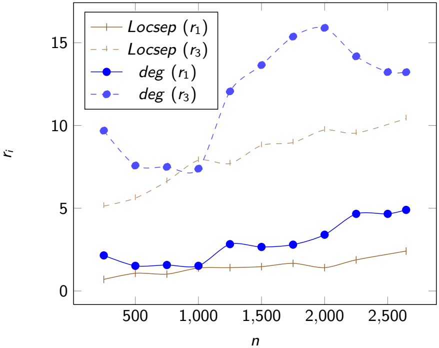

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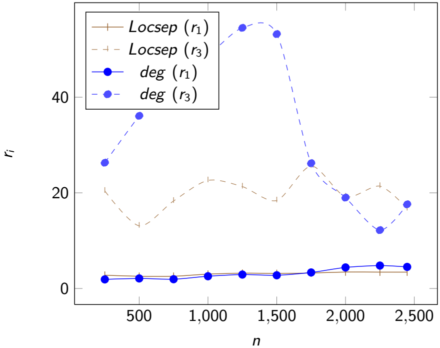

Figure : *gb-coach*, $r_2 \leq 1$.

Performance

- Time-dependent Dijkstra's algorithm with Fibonacci heap priority queue $\mathcal{O}(m + n \log n) \approx \mathcal{O}(n \log n)$ in our timetables

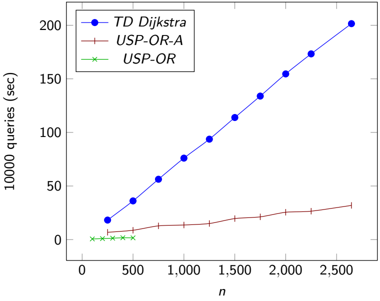

Figure : query time, *sncf*

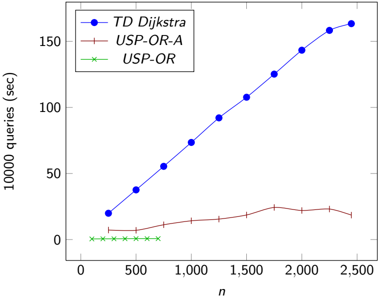

Figure : query time, *gb-coach*

Performance and existing methods

	Dataset	*sncf*		*gb-coach*		*gb-train*	
	Time range	(1 day)	(1 week)	(1 day)	(1 week)	(1 day)	(1 week)
USP-OR	***n***	700	500	1000	700	700	400
	Speed-up	29.0	23.8	**71.5**	71.3	22.4	16.6
	Size-up	271.4	228.6	169.1	116.1	138.9	80.1
USP-OR-A	***n***	2608	2646	2406	2448	2550	2555
	Speed-up	5.2	6.3	8.0	**8.8**	2.8	2.8
	Size-up	5.1	7.8	7.4	10.4	4.3	5.2

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- **Time-dependent SHARC** [Del08], **Time-dependent CH** [BDSV09]
  - Speed-ups of about 26 / 1500, respectively (earliest arrival only)
  - Meant for time-dependent routing in road networks
- **Engineering time-expanded graphs...** [DPW09]
  - Max speed-up of 56 (Railways with 30000 stations!)
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- **Engineering time-expanded graphs...** [DPW09]
  - Max speed-up of 56 (Railways with 30000 stations!)
  - Remodelling unimportant stations in time expanded graphs
- Theory vs. practice difference
  - More complicated (transfers, cost of travel...)
  - Focus on one given dataset

Conclusion

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- **Application** created to carry out **analysis** of real-world timetables:
  - Degrees, connectivity, BC, highway dimension, overtaking, USPs...
  - Running & evaluating tests of oracles

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- **Application** created to carry out **analysis** of real-world timetables:
    - Degrees, connectivity, BC, highway dimension, overtaking, USPs...
    - Running & evaluating tests of oracles
- Trying out **novel approaches** to find optimal connections in timetables
    - *USP-OR*: Exact and very quick answers (speed-up ≈) but high space
    - *USP-OR-A*: Exact and quick answers (speed-up ≈) less space-consuming
    - *NN*: Problem too challenging for NN/try different types of network

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Thank you for the attention