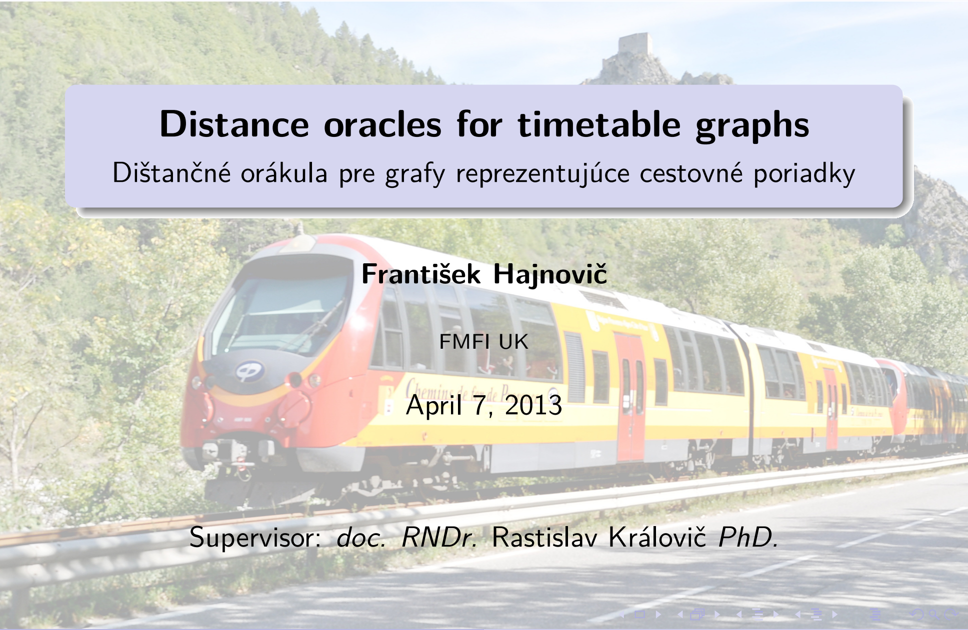


# Distance oracles for timetable graphs

Dištančné orákula pre grafy reprezentujúce cestovné poriadky

**František Hajnovič**

FMFI UK

April 7, 2013

Supervisor: *doc. RNDr.* Rastislav Královič *PhD.*

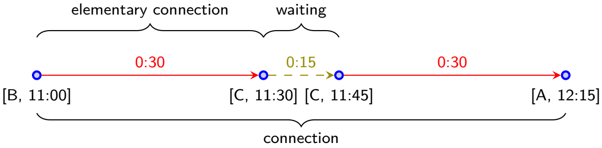
# Content

# Introduction

Introduction

# What is it about?

- Given a timetable, we query - $(a, t, b)$ - for
  - **Earliest arrival** (EA) - $\boldsymbol{t^*_{(a,t,b)}}$
  - **Optimal connection** (OC) - $\boldsymbol{c^*_{(a,t,b)}}$



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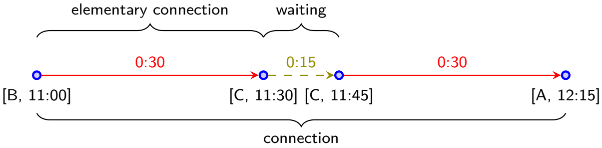
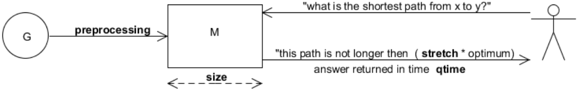



- Motivation: large-scale timetable search engines (*cp.sk*, *imhd.sk*...)
- Approach: (distance) oracle-based approach [TZ05] - pre-computation



# Timetable and underlying graph

| Place | | Time | |
|---|---|---|---|
| From | To | Departure | Arrival |
| A | B | 10:00 | 10:45 |
| B | C | 11:00 | 11:30 |
| B | C | 11:30 | 12:10 |
| B | A | 11:20 | 12:30 |
| C | A | 11:45 | 12:15 |

Table : **Timetable** - a set of **elementary connections** (between pairs of **cities**)

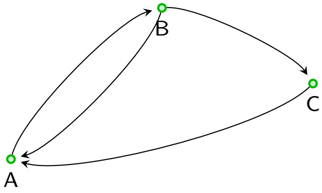



Figure : **Underlying graph**

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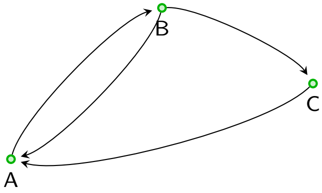



Figure : **Underlying graph**

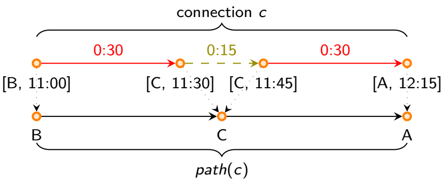
- Goals:
  - Devise methods to tackle EA/OC problem
  - Analyse properties of timetables

# Contribution

**Contribution**

# Idea

- *"Usually we go through the same sequence of cities"*




| Odchod | Príchod | Dĺžka cesty* | Použité linky | Zóny | Cena* |
|---|---|---|---|---|---|
| 21:59 | 22:10 | 11 min | 95 | - | 0,70 € |
| 22:09 | 22:20 | 11 min | 95 | - | 0,70 € |
| 22:19 | 22:30 | 11 min | 95 | - | 0,70 € |
| 22:29 | 22:40 | 11 min | 95 | - | 0,70 € |
| 22:39 | 22:50 | 11 min | 95 | - | 0,70 € |

- $p$ is **USP** $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP $\rightarrow$ reconstruct $c^*_{(a,t,b)}$

# Idea

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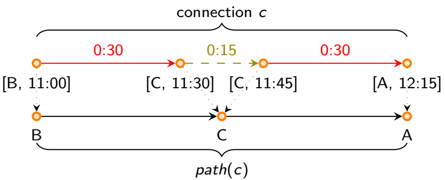


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- $p$ is **USP** $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP $\rightarrow$ reconstruct $c^*_{(a,t,b)}$
- **Overtaking** [MHSWZ07] causes problems, but can be easily removed

| Name | Overtaken edges (%) |
|---|---|
| *air01* | 1% |
| *cpru* | 2% |
| *cpza* | 2% |
| *montr* | 1% |
| *sncf* | 2% |
| *sncf-ter* | 2% |
| *sncf-inter* | 8% |
| *zsr* | 0% |

# USP-OR

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  - $\boldsymbol{\tau(x,y)}$ - # of USPs between $x$ and $y$

| Name | avg $\tau$ | max $\tau(x,y)$ | $\gamma$ |
|---|---|---|---|
| *air01* | 5.8 | 30 | 3.0 |
| *cpru* | 7.0 | 64 | 13.8 |
| *cpza* | 5.1 | 42 | 11.1 |
| *montr* | 4.3 | 30 | 20.3 |
| *sncf* | 4.3 | 24 | 10.5 |
| *sncf-inter* | 0.6 | 19 | 7.9 |
| *sncf-ter* | 6.1 | 33 | 10.8 |
| *zsr* | 2.5 | 19 | 13.7 |

Table : Daily, 200 station timetables

| ***USP-OR*** | ***prep*** | ***size*** | ***qtime*** | ***stretch*** |
|---|---|---|---|---|
| **guaranteed** | $\mathcal{O}(hn^2(\log n+\delta))$ | $\mathcal{O}(\tau n^2\gamma)$ | avg. $\mathcal{O}(\tau\gamma)$ | 1 |
| $\boldsymbol{\tau}$ **const.,** $\gamma \leq \sqrt{n}$**,** $\delta \leq \log n$ | $\mathcal{O}(hn^2\log n)$ | $\mathcal{O}(n^{2.5})$ | avg. $\mathcal{O}(\sqrt{n})$ | 1 |

Table : $\delta$ - the UG density $\frac{m}{n}$

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Table : $\delta$ - the UG density $\frac{m}{n}$

- Space $\mathcal{O}(n^{2.5})$ too big anyway

# *USP-OR* - $\tau$ evolution

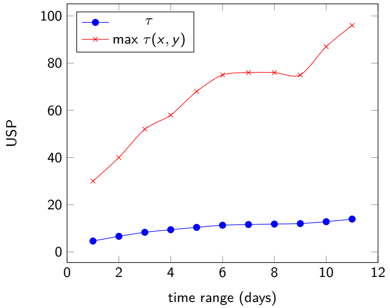



Figure : Changing of $\tau$ with increased time range in *air01* dataset. 1 day = about 800 in height

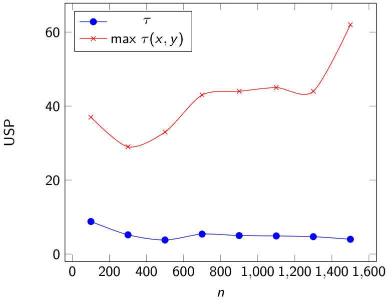
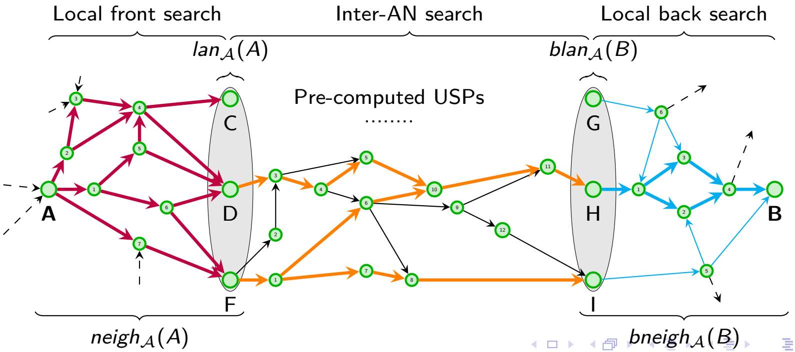



Figure : Changing of $\tau$ with increased # of stations in *sncf* dataset

# *USP-OR-A*

- Pre-compute USPs only among *some* cities in UG: $(\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3})$ **access node set**
  - Small size: $|\mathcal{A}| \leq r_1 \cdot \sqrt{n}$
  - Small node neighbourhoods: $(avg\ |neigh(x)|)^2 \leq r_2 \cdot n$
  - Few local access nodes: $|lan_{\mathcal{A}}(x)| \leq r_3$



# *USP-OR-A* and access nodes

| ***USP-OR-A*** | **guaranteed** | $\tau, r_1, r_2, r_3$ **const.**, $\gamma \leq \sqrt{n}$, $\delta \leq \log n$ |
|---|---|---|
| ***prep*** | $\mathcal{O}(f(n) + (r_1 + r_2)(\delta + \log n)hn^{1.5})$ | $\mathcal{O}(f(n) + hn^{1.5} \log n)$ |
| ***size*** | $\mathcal{O}(r_2 n^{1.5} + r_1^2 \tau_{\mathcal{A}} \gamma_{\mathcal{A}} n)$ | $\mathcal{O}(n^{1.5})$ |
| ***qtime*** | avg. $\mathcal{O}(r_2 r_3 \sqrt{n}(\log(r_2 n) + \delta) + r_3^2 \tau_{\mathcal{A}} \gamma_{\mathcal{A}})$ | avg. $\mathcal{O}(\sqrt{n} \log n)$ |
| ***stretch*** | 1 | 1 |

- Much depends on choosing good AN set

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| ***stretch*** | 1 | 1 |

- Much depends on choosing good AN set
- Minimize $|\mathcal{A}|$ s.t. $\forall x : |neigh_{\mathcal{A}}(x)| \le \sqrt{n} \rightarrow$ NP-complete
- Choose ANs based on degree/betweenness centrality

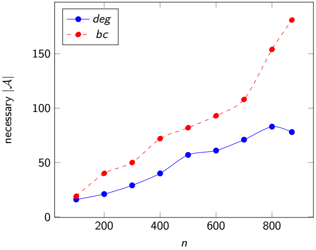



Figure : Dataset *cpru* ($\sqrt{n} \approx 30$)

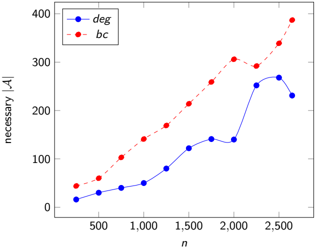
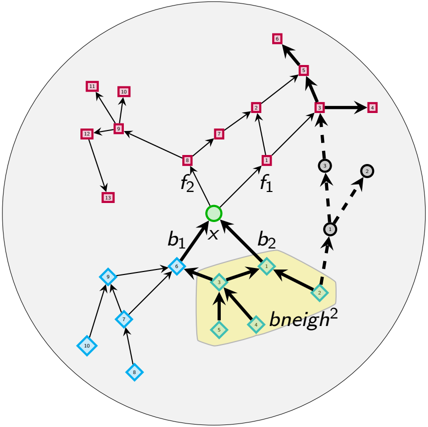



Figure : Dataset *sncf* ($\sqrt{n} \approx 51$)

# Locsep

- Select AN that locally separates many vertices



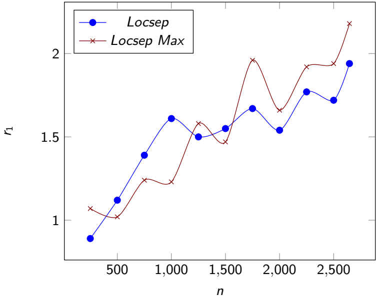




Figure : Dataset *sncf*. Found AN set with size $|\mathcal{A}| = r_1\sqrt{n} \approx r_1 51$.

# Results

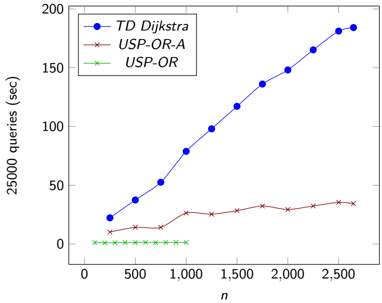



Figure : *USP-OR-A* + *Locsep* vs. *USP-OR* vs. *TD Dijkstra* on *sncf* dataset. Changing *n*.

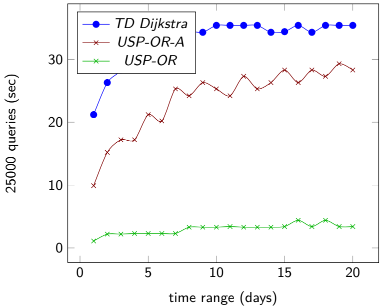



Figure : *USP-OR-A* + *Locsep* vs. *USP-OR* vs. *TD Dijkstra* on *zsr* dataset. Changing time range.

# Existing methods

- **Time-dependent SHARC** [Del08], **Time-dependent CH** [BDSV09]
  - Speed-ups of about 26 / 1500, respectively (EA only)
  - Meant for time-dependent routing in road networks
- **Time-expanded approach** [DPW09]
  - Speed-ups of about 56
  - Remodelling unimportant stations

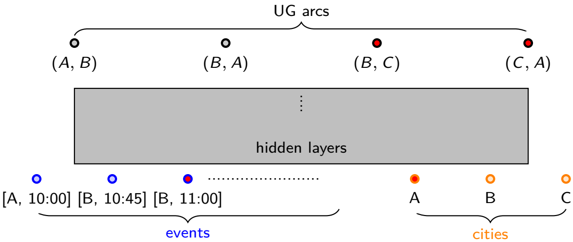
| Name | *USP-OR* | *USP-OR-A* |
|---|---|---|
| *cpru* | 14.5 | 1.7 |
| *cpza* | 14.3 | 1.7 |
| *montr* | 8.8 | 1.5 |
| *sncf* | 64.8 | 5.4 |
| *sncf-inter* | 27.0 | 3.6 |
| *sncf-ter* | 78.3 | 6.3 |
| *zsr* (daily) | 19.3 | 2.14 |

Table : Speed-up of *USP-OR* and *USP-OR-A* with *Locsep*

- Theory vs. practice difference: transfers, cost of travel...

# Neural network approaches

- Multi-layer perceptron, back propagation
- Input layer = events + cities.
- Output layer = arcs of UG → USP




- Tendency to remember USPs
- Long training times

| Name | Conn. | Found | Was optimum (%) |
|---|---|---|---|
| air01 | 931 | 573 | 18.7% |
| cpru | 481 | 281 | 48% |
| montr | 527 | 346 | 86.7% |
| zsr | 672 | 307 | 76.2% |

Table : Timetables with 30 cities

# Conclusion

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- Trying out **novel approaches** to find optimal connections in timetables
  - *USP-OR*: Exact and very quick answers (speed-up $\approx 60$) but high space
  - *USP-OR-A*: Exact and quick answers (speed-up $\approx 6$) less space-consuming
  - *NN*: Problem too challenging for NN/try different types of network

- **Application** created to carry out **analysis** of real-world timetables:
  - Degrees, connectivity, BC, high. dimension, overtaking, USPs...
  - Running & evaluating tests of oracles

Figure : It's *blazing* fast!

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# Thank you for the attention