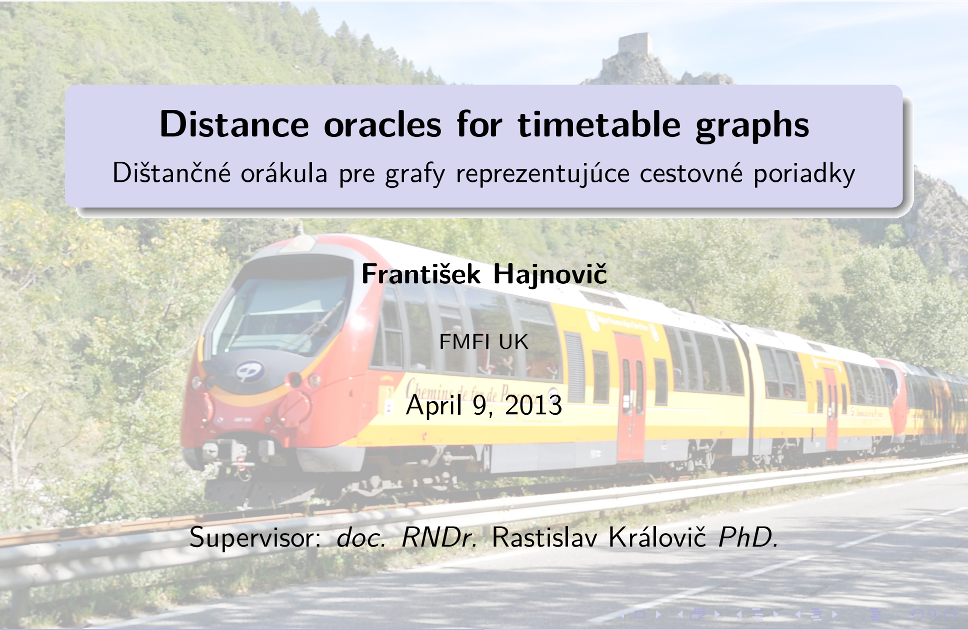

Distance oracles for timetable graphs

Dištančné orákula pre grafy reprezentujúce cestovné poriadky

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April 9, 2013

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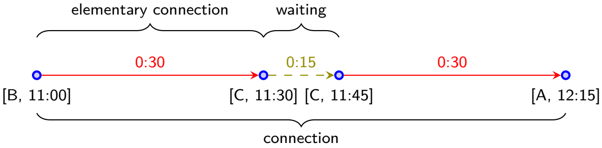
Content

Introduction

Introduction

What is it about?

- Given a timetable, we query - (a, t, b) - for
  - **Earliest arrival** (EA) - $\boldsymbol{t^*_{(a,t,b)}}$
  - **Optimal connection** (OC) - $\boldsymbol{c^*_{(a,t,b)}}$



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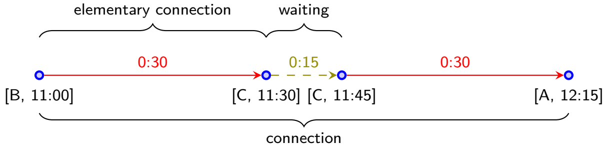
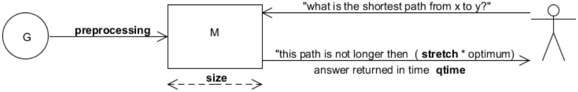


- Motivation: large-scale timetable search engines (*cp.sk*, *imhd.sk*...)
- Approach: (distance) oracle-based approach [TZ05] - pre-computation



Timetable and underlying graph

Place		Time	
From	To	Departure	Arrival
A	B	10:00	10:45
B	C	11:00	11:30
B	C	11:30	12:10
B	A	11:20	12:30
C	A	11:45	12:15

Table : **Timetable** - a set of **elementary connections** (between pairs of **cities**)

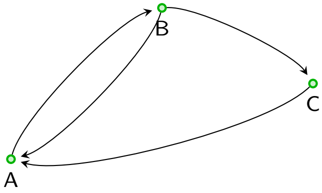


Figure : **Underlying graph**

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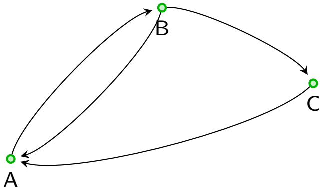


Figure : **Underlying graph**

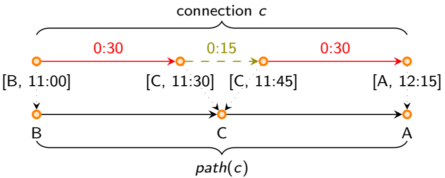
- Goals:
  - Devise methods to tackle EA/OC problem
  - Analyse properties of real-world timetables

Contribution

Contribution

Idea

- *"Usually we go through the same sequence of cities"*



Odchod	Príchod	Dĺžka cesty*	Použité linky	Zóny	Cena*
21:59	22:10	11 min	95	-	0,70 €
22:09	22:20	11 min	95	-	0,70 €
22:19	22:30	11 min	95	-	0,70 €
22:29	22:40	11 min	95	-	0,70 €
22:39	22:50	11 min	95	-	0,70 €

- p is **USP** $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP p: $expand(p) \rightarrow c^*_{(a,t,b)}$

Idea

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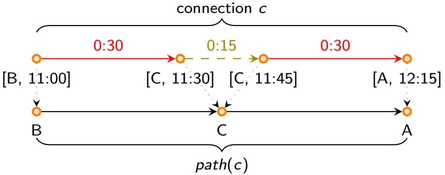

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- p is **USP** $\iff \exists t : path(c^*_{(a,t,b)}) = p$
- we have USP p: $expand(p) \rightarrow c^*_{(a,t,b)}$
- **Overtaking** [MHSWZ07] causes problems, but can be easily removed

Name	Overtaken edges (%)
air01	1%
cpru	2%
cpza	2%
montr	1%
sncf	2%
sncf-ter	2%
sncf-inter	8%
zsr	0%

USP-OR

- Pre-compute *all conn.* - space $\mathcal{O}(h\ n^2\gamma)$
    - γ - the average OC size
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  - $\boldsymbol{\tau(x,y)}$ - # of USPs between x and y

Name	avg τ	max $\tau(x,y)$	γ
air01	5.8	30	3.0
cpru	7.0	64	13.8
cpza	5.1	42	11.1
montr	4.3	30	20.3
sncf	4.3	24	10.5
sncf-inter	0.6	19	7.9
sncf-ter	6.1	33	10.8
zsr	2.5	19	13.7

Table : Daily, 200 station timetables

USP-OR	***prep***	***size***	***qtime***	***stretch***
guaranteed	$\mathcal{O}(hn^2(\log n + \delta))$	$\mathcal{O}(\tau n^2\gamma)$	avg. $\mathcal{O}(\tau\gamma)$	1
$\boldsymbol{\tau}$ **const.,** $\boldsymbol{\gamma \leq \sqrt{n}}$**,** $\boldsymbol{\delta \leq \log n}$	$\mathcal{O}(hn^2 \log n)$	$\mathcal{O}(n^{2.5})$	avg. $\mathcal{O}(\sqrt{n})$	1

Table : δ - the UG density $\frac{m}{n}$

USP-OR

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Table : δ - the UG density $\frac{m}{n}$

- Space $\mathcal{O}(n^{2.5})$ too big anyway

USP-OR - τ evolution

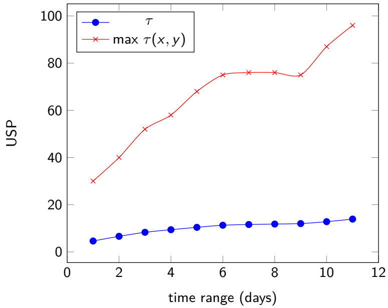


Figure : Changing of τ with increased time range in *air01* dataset. 1 day = about 800 in height

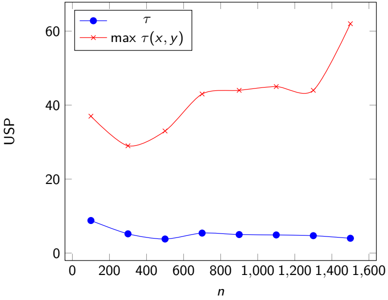
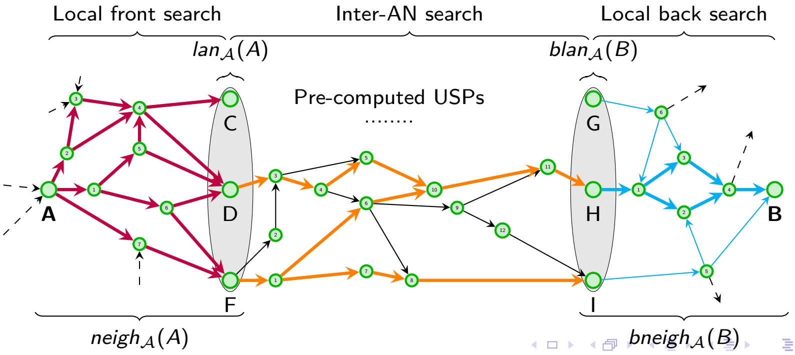


Figure : Changing of τ with increased # of stations in *sncf* dataset

USP-OR-A

- Pre-compute USPs only among *some* cities in UG: (r_1, r_2, r_3) **access node set**
  - Small size: $|\mathcal{A}| \leq r_1 \cdot \sqrt{n}$
  - Small node neighbourhoods: $avg\ (|neigh(x)|)^2 \leq r_2 \cdot n$
  - Few local access nodes: $|lan_{\mathcal{A}}(x)| \leq r_3$



USP-OR-A and access nodes

USP-OR-A	**guaranteed**	**τ, r_1, r_2, r_3 const., $\gamma \leq \sqrt{n}$, $\delta \leq \log n$**
prep	$\mathcal{O}(f(n) + (r_1 + r_2)(\delta + \log n)hn^{1.5})$	$\mathcal{O}(f(n) + hn^{1.5} \log n)$
size	$\mathcal{O}(r_2 n^{1.5} + r_1^2 \tau_{\mathcal{A}} \gamma_{\mathcal{A}} n)$	$\mathcal{O}(n^{1.5})$
qtime	avg. $\mathcal{O}(r_2 r_3 \sqrt{n}(\log(r_2 n) + \delta) + r_3^2 \tau_{\mathcal{A}} \gamma_{\mathcal{A}})$	avg. $\mathcal{O}(\sqrt{n} \log n)$
stretch	1	1

- Much depends on choosing good AN set

USP-OR-A and access nodes

USP-OR-A	guaranteed	τ, r_1, r_2, r_3 const., $\gamma \leq \sqrt{n}$, $\delta \leq \log n$
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size	$\mathcal{O}(r_2 n^{1.5} + r_1^2 \tau_{\mathcal{A}} \gamma_{\mathcal{A}} n)$	$\mathcal{O}(n^{1.5})$
qtime	avg. $\mathcal{O}(r_2 r_3 \sqrt{n}(\log(r_2 n) + \delta) + r_3^2 \tau_{\mathcal{A}} \gamma_{\mathcal{A}})$	avg. $\mathcal{O}(\sqrt{n} \log n)$
stretch	1	1

- Much depends on choosing good AN set
- Minimize $|\mathcal{A}|$ s.t. $\forall x : |neigh_{\mathcal{A}}(x)| \leq \sqrt{n} \rightarrow$ NP-complete
- Choose ANs based on degree/betweenness centrality

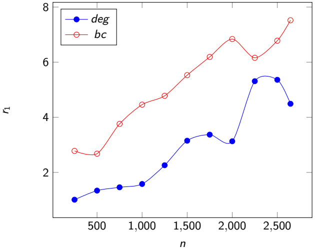


Figure : $|\mathcal{A}| = r_1\sqrt{n} \approx r_1 51$ (*sncf*).

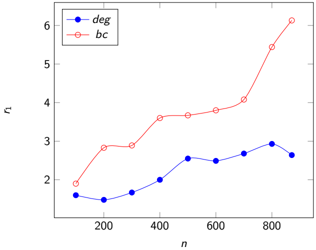
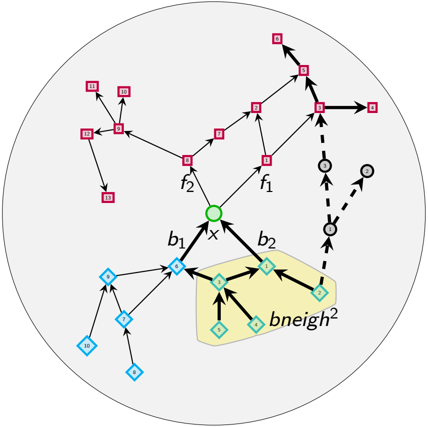
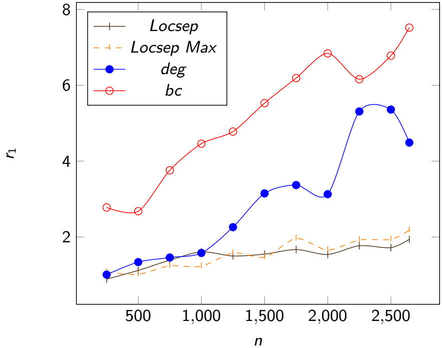


Figure : $|\mathcal{A}| = r_1\sqrt{n} \approx r_1 30$ (*cpru*).

Locsep

- Select AN that locally separates many vertices





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Results

- Time-dependent Dijkstra's algorithm with Fibonacci heap priority queue $\mathcal{O}(m + n \log n)$

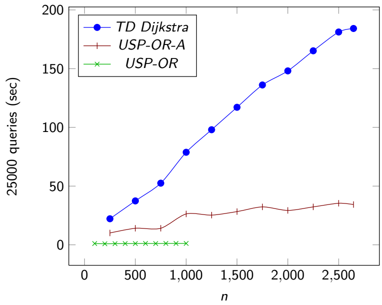


Figure : *USP-OR-A* + *Locsep* vs. *USP-OR* vs. *TD Dijkstra* on *sncf* dataset. Changing *n*.

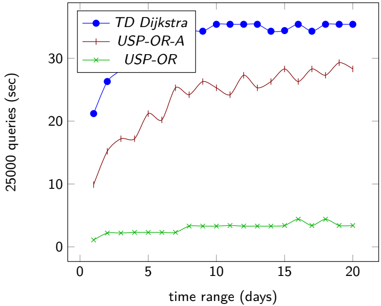


Figure : *USP-OR-A* + *Locsep* vs. *USP-OR* vs. *TD Dijkstra* on *zsr* dataset. Changing time range.

Existing methods

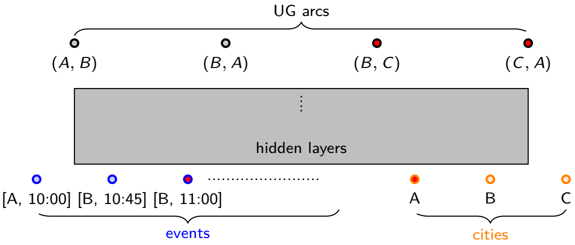
- **Time-dependent SHARC** [Del08], **Time-dependent CH** [BDSV09]
  - Speed-ups of about 26 / 1500, respectively (EA only)
  - Meant for time-dependent routing in road networks
- **Time-expanded approach** [DPW09]
  - Max speed-up of 56 (Railways with 30000 stations!)
  - Remodelling unimportant stations in TE graphs
- Theory vs. practice difference: transfers, cost of travel...

Name	*USP-OR*	*USP-OR-A*
cpru	14.5	1.7
cpza	14.3	1.7
montr	8.8	1.5
sncf	64.8	5.4
sncf-inter	27.0	3.6
sncf-ter	78.3	6.3
zsr (daily)	19.3	2.14

Table : Speed-up of *USP-OR* and *USP-OR-A* with *Locsep*

Neural network approaches

- Multi-layer perceptron, back propagation
- Input layer = events + cities.
- Output layer = arcs of UG → USP



- Tendency to remember USPs
- Long training times

Name	Conn.	Found	Was optimum (%)
air01	931	573	18.7%
cpru	481	281	48%
montr	527	346	86.7%
zsr	672	307	76.2%

Table : Timetables with 30 cities

Conclusion

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Conclusion

- Trying out **novel approaches** to find optimal connections in timetables
    - *USP-OR*: Exact and very quick answers (speed-up $\approx$ 60) but high space
    - *USP-OR-A*: Exact and quick answers (speed-up $\approx$ 6) less space-consuming
    - *NN*: Problem too challenging for NN/try different types of network

- **Application** created to carry out **analysis** of real-world timetables:
    - Degrees, connectivity, BC, high. dimension, overtaking, USPs...
    - Running & evaluating tests of oracles

Figure : It's *blazing* fast!

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Thank you for the attention