质心坐标系

计算机图形学

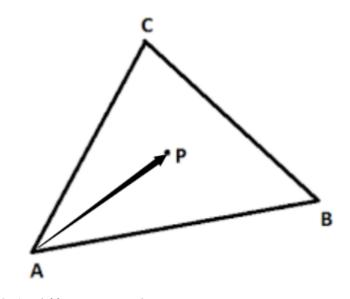
质心坐标系

以线段插值为例,P位于线段AB之间,因此P点公式为

$$P = At + B(1 - t)$$

同样的在三角形中也有对应关系

$$P = A + m(B - A) + n(C - A)$$



如何计算m和n呢?由于

$$P - A = m(B - A) + n(C - A)$$

将P-A记为向量 $\overrightarrow{v_2}$,将B-A记为 $\overrightarrow{v_0}$,将C-A记为 $\overrightarrow{v_1}$,因此公式为:

$$\overrightarrow{v_2} = m\overrightarrow{v_0} + n\overrightarrow{v_1}$$

两边分别乘以 $\overrightarrow{v_0}$ 和 $\overrightarrow{v_1}$ 得:

$$\overrightarrow{v_2} \cdot \overrightarrow{v_0} = m(\overrightarrow{v_0} \cdot \overrightarrow{v_0}) + n(\overrightarrow{v_1} \cdot \overrightarrow{v_0})$$

$$\overrightarrow{v_2} \cdot \overrightarrow{v_1} = m(\overrightarrow{v_0} \cdot \overrightarrow{v_1}) + n(\overrightarrow{v_1} \cdot \overrightarrow{v_1})$$

$$d_{20} = \overrightarrow{v_2} \cdot \overrightarrow{v_0}$$

$$d_{21} = \overrightarrow{v_2} \cdot \overrightarrow{v_1}$$

$$d_{00} = \overrightarrow{v_0} \cdot \overrightarrow{v_0}$$

$$d_{01} = \overrightarrow{v_0} \cdot \overrightarrow{v_1}$$

$$d_{11} = \overrightarrow{v_1} \cdot \overrightarrow{v_1}$$

则可简化方程式得:

$$d_{20} = md_{00} + nd_{10}$$

$$d_{21} = md_{01} + nd_{11}$$

由克莱姆定理

$$m = \frac{\begin{vmatrix} d_{20} & d_{10} \\ d_{21} & d_{11} \end{vmatrix}}{\begin{vmatrix} d_{00} & d_{10} \\ d_{01} & d_{11} \end{vmatrix}}$$

$$n = \frac{\begin{vmatrix} d_{00} & d_{20} \\ d_{01} & d_{21} \end{vmatrix}}{\begin{vmatrix} d_{00} & d_{10} \\ d_{01} & d_{11} \end{vmatrix}}$$