Attribution Privacy

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1 Introduction

Peer-to-peer (P2P) networks are a common tool for sharing information. Most P2P architectures are completely open: the identities of the participants and the data being shared in the network are available to the public. When privacy is applied to P2P networks, it is usually done in a way that completely closes down the network: by making membership in the network invite-only[1].

We define a new kind of privacy for P2P networks called “attribution privacy” which exists in a traditional open network. Attribution privacy formalizes the notion that sharing public information among public peers can still leak private information. We give examples of both real world and technical applications where attribution privacy is a concern. We also classify when attribution privacy leaks can occur and measure the potential risk to peers participating in such a network.

We suggest two modifications that can be made to the P2P protocol in order to mitigate the attribution privacy leaks, and measure their efficacy as well as their performance impact. We show that these protocol modifications can effectively protect attribution privacy.

2 Related Work

This work was motivated by our attempt to implement the XRay system[2] in a collaborative, P2P environment. XRay audits online advertisements to reverse-engineer the ad targeting criteria and thus determine if a company is exploiting one’s sensitive information. It does this using a complicated system of data duplication and fake account management, but the authors of XRay note that “a collaborative approach to auditing, in which users contribute their [data] in a privacy-preserving way is a promising direction...” In such a system, users would participate in a P2P network where they freely share which ads appeared alongside which data in order to help other peers to determine the actual ad targeting (using differential correlation). We recognized that sharing such information could potentially leak the private information that was originally targeted by the ad. Formalizing this type of privacy leak lead to the definition in section 3. See section 4.2 for a technical description of collaborative XRay as an attribution privacy risk.

We found it necessary to coin the term “attribution privacy” after failing to find similar research in the field of privacy in P2P networks. Most work in this area appears to focus on protecting the identities of the users involved or protecting non-participants in the network from seeing what data are being exchanged[1]. We suspect that this is due to the strong association of P2P networks with illegal file-sharing. Although extant private P2P software would make attribution privacy a moot point (since they ensure that the other peers are trusted), trusted peers are not a requirement of attribution privacy.

Because attribution privacy applies to P2P networks, we examined possible P2P systems to which it might apply. Distributed hash tables (DHT) such as Chord[3] appeared to be a possible candidate since the idea of mapping keys to values is a kind of data attribution. However our definition of attribution (section 3) takes a more general form which does not exactly fit the key-value model of a hash table. Protocols like BitTorrent where peers freely produce and exchange information are more fitting, though we attempt to define attribution privacy as generally as possible to fit many possible P2P networks.

We also considered other network privacy methods as possible solutions to attribution privacy leaks. The anonymity provided by Tor automatically provides attribution privacy as a side-effect (if you do not know with whom you are communicating, you cannot attribute the data to anyone). However we show by definition and by example that complete anonymity is not necessary in situations where attribution privacy is needed. This coupled with the fact that Tor requires additional infrastructure to function (the relay and exit nodes) and can significantly impact performance of the network make it a heavy-handed solution to a much smaller problem. However we do note that one of our proposed solutions could be extended to a Tor-like protocol for even greater privacy gains (section 8.1).

Attribution privacy bears some similarity with differential privacy[4]: both are concerned with preserving the utility of the data while protecting the privacy of the individual. Indeed, differential privacy could even be viewed as preventing statistical data from being "attributed" to an individual. However these two types of privacy are in actuality very different. Differential privacy involves a single data set that is queried in aggregate whereas attribution privacy allows for querying individuals on the network directly. Differential privacy attempts to "hide" individuals among noise in the response whereas attribution privacy does not allow for modification of the data and only attempts to hide the data's source. Despite these differences, we do find that a technique similar to differential privacy's "randomized responses" works well for attribution privacy (section 5.2).

3 Definitions

Because we will define attribution privacy with respect to a P2P network, we first define the mechanism and purpose of the P2P network. We imagine a P2P network in which there are a fixed number of peers and there is some mechanism by which all peers are aware of all other peers in the network. These peers exchange messages via some secure channel such as HTTPS. Thus which peers are communicating with which is public information, while the contents of those messages is kept private from an observer. These assumptions are similar to a BitTorrent swarm with a central tracker.

Let $\mathcal{Q}$ be a set of public queries and let $\mathcal{D}$ be a set of public data about those queries. Imagine the elements of $\mathcal{Q}$ as the queries that a peer might make to other peers, and $\mathcal{D}$ as potential responses to those queries. Then we define a function $K : \mathcal{Q} \to 2^{\mathcal{D}}$ which maps queries to the set of possible responses to each query i.e. $\forall q \in \mathcal{Q}, K(q) \subseteq \mathcal{D}$. Note that each query can have many possible responses, and a single peer can respond with several of these responses at once. Formally, a response $D \subseteq \mathcal{D}$ to a query $q \in \mathcal{Q}$ means $D \subseteq K(q)$.

The reason why peers participate in the network is to learn more about the "knowledge function" K. For example, we can imagine $\mathcal{Q}$ as a set of file names and $\mathcal{D}$ as a set of file fragments; then K indicates which fragments correspond to which file names (and which are reused across multiple file names), so determining the value of $K(q)$ amounts to downloading file q. Although peers in the network may not yet know about K, we consider K to be public information in the sense that information about K is shared freely with any peer that requests it.

Next we define $\mathcal{S}$ to be the set of original sources of knowledge about K in the network. For every $s \in \mathcal{S}, \exists q_s \in \mathcal{Q}, D_s \subseteq \mathcal{D}$ such that $D_s \subseteq K(q_s)$. We say that a peer "has access to s" (or just "has s") if that peer knows $D_s \subseteq K(q_s)$ *without* querying another peer in the network. You can think of the network as a closed system except for these sources which inject knowledge into the network via the peers that have access to them. Thus for any response $D \subseteq K(q)$, there must be a source s that corresponds to D and q and some peer in the network must have access to s. In that case we would say that that peer is the "origin of" (or "originates") $D \subseteq K(q)$.

For attribution privacy to be a concern we need the following two conditions to hold:

1. The sources must be private. That is, a peer with access to some $s \in \mathcal{S}$ does not want it to be known publicly.

2. The relationship between $D_s \subseteq K(q_s)$ and $s \in \mathcal{S}$ must be public. That is, a peer who sees the response $D \subseteq K(q)$ will know which sources in $\mathcal{S}$ it might have originated from.

If these conditions hold, then it is possible for a public peer to share public data about a public query, and in doing so inadvertently communicate information about its private sources. In other words, it is possible for other peers to *attribute* the public data to its origin.

4 Applications

4.1 Nickelback

A simple (somewhat facetious) example is of a small town in which no one likes the Canadian rock band Nickelback. Here we imagine the townspeople as peers in the P2P network. If you (being a huge Nickelback fan) were to move to this town, you might find yourself in the uncomfortable situation where a Nickelback song comes on the radio and people are curious what the name is and who performs it. In this situation, $\mathcal{S}$ contains the property "being a Nickelback fan" (a source of knowledge about Nickelback songs). $\mathcal{Q}$ contains the unidentified song on the radio, $\mathcal{D}$ contains song names, and K maps the unidentified song to the correct name. The curious members of the town query for name of the song, and you would honestly like to provide it.

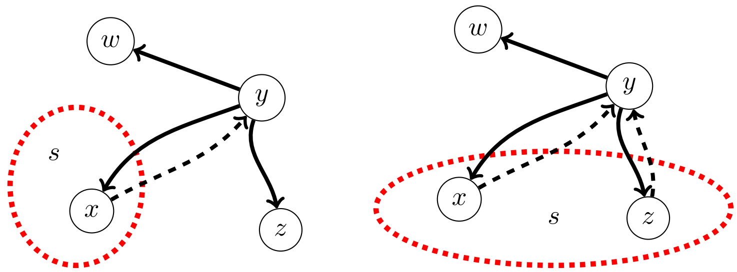
However, being that you are the only Nickelback fan in town, doing so would prove to whomever you tell that you *originate* the knowledge of the correct song name. The townspeople would be able to apply their public knowledge of the sources to discover your private information: namely that a person who knows the name of a random Nickelback song is probably a Nickelback fan. By honestly providing the requested data, you have inadvertently revealed your private source. With the methods we introduce for attribution privacy, you would be able to disclose your knowledge of Nickelback without (as much) fear of revealing your fandom.

4.2 Collaborative Advertisement Auditing

Imagine a P2P system in which peers collaborate to determine how online advertisements are targeted. This is achieved by peers sharing which ads they have seen along with the personal data those ads appeared with. By comparing which data have/have not been present when certain ads were seen, the ad targeting criteria can be determined. See section 2 for a description of XRay, the tool which inspired this application.

With such a system, $\mathcal{Q}$ is the set of online ads, $\mathcal{D}$ is the set of personal information that might be targeted, and K represents the ad targeting criteria i.e. which personal data are targeted by which ads. Interestingly, the private sources $\mathcal{S}$ in this case are identical to $\mathcal{D}$: if you originate the knowledge that a particular ad targets particular data, then informing others of that association also divulges that you possess the targeted data.

Such a P2P system could integrate our attribution privacy protocols to ensure that peers can freely share which ads they have seen without revealing the private data that those ads targeted.



(a) One response (b) Two responses

Figure 1: An attribution privacy leak. Black lines represent queries, dashed lines represent responses to a query. y gains different amounts of information about the location of source s depending on how many peers respond.

5 Privacy Risk

5.1 Baseline

To measure the attribution privacy risk in a P2P network, we start with the worst-case scenario. Suppose that for knowledge about some q there is only a single source s, only a single peer x with access to that source (i.e. a single origin), and that x has not shared any knowledge of q with the rest of the network. Another peer y who wants to learn about $K(q)$ might then query *every peer* in the network for knowledge about q. Since x has not previously shared information about q, every peer will respond with no knowledge except for x.

y now knows that of all the peers, only x has knowledge about q. Because the network is closed, y also knows that there must be a source for q among the peers who know about q. Thus x must possess a source for q and since we assumed there was only one such source, it must be s. Thus by sharing their knowledge about q, x has inadvertently shared the fact that they possess source s. This is illustrated in figure 1(a).

We can reason that this is the worst possible case as follows: if there were multiple sources related to q, then y would only know that x possesses one of those sources; it would not necessarily know which one. Also, if there were multiple peers with access to a source for q, then multiple peers would respond when y queries every peer about q (see figure 1(b)). The same would occur if x had previously shared knowledge about q with other peers in the network. In both of these cases we can quantify the information gained by y. Let S be the event that one of the peers is an origin for q (i.e. has source s) and let H be the number of peers who have the knowledge and thus respond to the query. If k peers respond to the query, then we can express $P(S|H=k)$ in terms of $P(S)$, the prior probability of a peer being an origin:

$$P(S|H=k) = P(S|\text{one of the } k \text{ is a source}) = \frac{P(S \cap \text{one of the } k \text{ is a source})}{P(\text{one of the } k \text{ is a source})}$$

$$= \frac{P(S)}{1 - P(\text{none of the } k \text{ are sources})} \tag{1}$$

$$= \frac{P(S)}{1 - (1 - P(S))^k} \tag{2}$$

Note that we can simplify to (1) because the event S is included in the event that one of the k responders is a source. Note also that $P(S|H=k) > P(S)$ for $P(S) \in (0,1)$. When $k=1$ (which is our previous assumption that there is only one origin), $P(S|H=k) = 1$ which agrees with our reasoning that y knows definitively that x has source s. However if more than one peer responds (either by multiple sources or by x sharing knowledge), y cannot be certain whether x has a source.

Going back to our original worst-case assumptions, suppose that after y queries, another peer z queries for information about q. Now we are in the previously discussed weaker case: *two* peers will respond with knowledge about q (x and y)! Thus z's information about s is only $P(S|H = 2) < 1$.

Theorem 1. *If k peers in the network have knowledge of q, then*

$$P(S|\textit{the peer responds to a query}) \leq P(S|H = k)$$

Proof. This follows from the definition in equation (2) and the previous arguments. □

The goal of a private attribution network is to make $P(S|\text{the peer responds to a query})$ smaller than $P(S|H = k)$.

5.2 Unreliable Peers

One simple approach to improving the privacy of the network is for the peers to be unreliable i.e. let $P(\text{respond to a query about } q|\text{have knowledge of } q) = r < 1$. This is similar to the "randomized response" method of differential privacy[4], but with the crucial difference that a peer without knowledge of q will never lie about it. Unlike differential privacy—which provides privacy at the expense of introducing noise into the data—we are interested in sharing only accurate data in the P2P network.

Now when y queries the entire network, they can no longer be sure that the peers who respond are all the peers who have the relevant knowledge. Thus we now have a new random variable $M \neq H$ which is the number of peers who respond to a query about q.

If m peers respond to a query, in order to calculate the posterior, y must consider how many peers in the network actually have knowledge about q, which can range from m to $n - 1$ (since y does not know about q).

$$\begin{aligned}
P(S|M = m) &= \sum_{k=m}^{n-1} P(S \cap H = k|M = m) \\
&= \sum_{k=m}^{n-1} \frac{P(S \cap H = k \cap M = m)}{P(M = m)} \frac{P(H = k \cap M = m)}{P(H = k \cap M = m)} \\
&= \sum_{k=m}^{n-1} P(S|H = k \cap M = m)P(H = k|M = m) \\
&= \sum_{k=m}^{n-1} P(S|H = k)\frac{P(M = m|H = k)P(H = k)}{P(M = m)} && (3) \\
&= \frac{1}{P(M = m)} \sum_{k=m}^{n-1} P(S|H = k)P(M = m|H = k)P(H = k) && (4)
\end{aligned}$$

where $P(S|H = k)$ is calculated as in equation (2) and we can simplify to (3) because S is independent of M if we know $H = k$. We see that now the posterior probability of a source depends on the prior distribution of knowledge in the network H. We also see the prior probability of M, but this too can be expressed in terms of H:

$$P(M = m) = \sum_{k=m}^{n-1} P(M = m \cap H = k) = \sum_{k=m}^{n-1} P(M = m|H = k)P(H = k)$$

Lastly we observe that $P(M = m|H = k)$ is simply given by the binomial distribution $b(m, k, r)$: the probability of m successes out of k trials where each trial has probability r of success. Substituting into equation (4) gives us

$$P(S|M = m) = \frac{\sum_{k=m}^{n-1} P(S|H = k)b(m, k, r)P(H = k)}{\sum_{k=m}^{n-1} b(m, k, r)P(H = k)} \qquad (5)$$

Theorem 2. *$P(S|M=m) \leq P(S|H=m)$ and this inequality is strict if $r<1$.*

Proof. Suppose that $r=1$, then $P(H=m)=1$ so equation (5) simplifies to

$$P(S|M=m) = \frac{P(S|H=m)b(m,m,r)P(H=m)}{b(m,m,r)P(H=m)} = P(S|H=m)$$

Now suppose $r<1$, by equation (2) we have that

$$P(S|H=k+1) = \frac{P(S)}{1-(1-P(S))^{k+1}} < \frac{P(S)}{1-(1-P(S))^{k}} = P(S|H=k)$$

therefore

$$\begin{aligned}
P(S|M=m) &= \frac{\sum_{k=m}^{n-1} P(S|H=k)b(m,k,r)P(H=k)}{\sum_{k=m}^{n-1} b(m,k,r)P(H=k)} \\
&< \frac{\sum_{k=m}^{n-1} P(S|H=m)b(m,k,r)P(H=k)}{\sum_{k=m}^{n-1} b(m,k,r)P(H=k)} \\
&= P(S|H=m)\frac{\sum_{k=m}^{n-1} b(m,k,r)P(H=k)}{\sum_{k=m}^{n-1} b(m,k,r)P(H=k)} \\
&= P(S|H=m)
\end{aligned}$$

□

One concern with such a change to the P2P protocol is the potential performance impact. Smaller r will increase the overall number of queries which need to be made in order for the knowledge to spread, thus decreasing the overall speed of the network. This impact is measured empirically in section 6.2.

5.3 Extra Responses

Since $P(S|H=k)$ decreases as k increases, another method of preserving privacy would be for the peers responding to queries to force k to be large. Specifically, whenever a peer responds to a query it also sends that response to each other peer with probability $\frac{t}{n-2}$. Since there are $n-2$ peers other than x and y, we expect that t extra responses will be sent.

Applying this change in protocol to our worst-case situation, we see that y still knows with $P(S|H=1)=1$ that x has source s. However, afterwards x, y, and t random peers are expected to know about q thus the next peer to make a query has an expected advantage of only $P(S|H=t+2)$. We can minimize this probability by making t even larger; $t=n-2$ means that x sends extra responses to every other peer, and then the next peer won't query at all because every peer already has the data!

But this view is oversimplified. Although this does minimize the posterior probability of x having s following a query, it just leaks the information earlier in the process. For example, consider a network of just four peers x, y, β, and γ and suppose that $t=2$. y starts as usual by querying every peer about q (x, β, and γ). Only x has knowledge of q, so it responds to y and sends extra responses with probability $t/(n-2)=1$ to peers β and γ. From β's perspective, since it received a query about q from y, it knows that x and γ were also queried. Since a queried peer always sends extra responses if it has the knowledge, β knows that every peer who sends it an extra response knows about q. It received an extra response only from x thus it can apply the same logic as y (in section 5.1) to deduce that x has s.

Clearly, $t=n-2$ is too large. But even smaller values of t leak some privacy to the peers receiving extra responses. In fact, the situation generalizes to that of section 5.2: when a query about q is received by a peer β, they expect to receive an extra response from each peer that has knowledge of q with probability $t/(n-2)$. It is as if β itself made the query and each peer responds probabilistically, which is exactly the situation with unreliable peers, only now $r=t/(n-2)$ and M is the number of extra responses received by β.

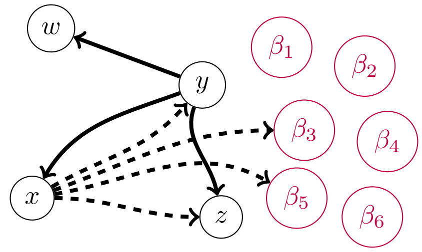


Figure 2: A flooding attack against the extra responses protocol. As in figure 1, black lines are queries, dashed lines are responses. An attacker adding malicious peers to the network increases their chance of receiving an extra response from x after y makes a query.

The point of sending out random extra responses would be defeated if $P(S|M = m) > P(S|H = m)$ because then the extra responses a peer sends out would actually leak more private information than if it were simply queried directly. However we showed with Theorem 2 that $P(S|M = m) < P(S|H = m)$ if $r < 1$ therefore we must simply choose $t < n-2$. Unfortunately, a major difference in this situation compared to unreliable peers is that the attacker has some control over r and thus M. If we have a network of n peers, an attacker can add b malicious peers to the network. These malicious peers can share information among themselves, such as who they have received extra responses from. In effect, the b peers behave like a single peer that has a higher chance of receiving extra responses. We call this a "flooding attack", as illustrated in figure 2. We would be in trouble if the attacker could use this attack to force r to be arbitrarily close to 1, because then the extra responses would leak about as much as the normal response. Thankfully this is not the case.

Theorem 3. *If extra responses are sent with probability $\frac{t}{n-2}$, then the probability that an attacker inserting malicious peers into the network receives an extra response is bounded by $1 - 1/e^t$.*

Proof. Let B be the event that at least one of the b malicious peers is sent an extra response and let n be the original number of peers in the network. Then:

$$\begin{aligned} P(B) &= 1 - P(\neg B) \\ &= 1 - (1 - P(\text{a particular malicious peer gets an extra response}))^b \\ &= 1 - \left(1 - \frac{t}{n+b-2}\right)^b \\ \lim_{b\to\infty} P(B) &= 1 - \frac{1}{e^t} \end{aligned}$$

□

Thus extra responses will always leak less information, even if an attacker adds arbitrarily many peers to the network. But even for $t = 5$ this probability bound is very close one, so we must choose a small t if we consider such an attack to be viable. Our only concern with small t might be that it does not cause the knowledge to spread "quickly enough" to provide a privacy benefit. However we will see in section 6.2 that even for $t = 1$ (which provides a limit of $1 - 1/e \approx 2/3$), knowledge propagates very quickly through the network.

Note that this leak also relies largely on the assumption that every peer queries every other peer in order to greedily determine as much private information as possible. In an actual P2P network, it is not necessary for each peer to query every other peer (see section 8.1). Thus with non-greedy peers, when a peer (or group of malicious peers) is *not* sent an extra response it is possible that it will also *not* receive the corresponding query for q—in which case it will be completely oblivious to the entire exchange and no leak will occur.

6 Knowledge Distribution

The privacy provided by both techniques from section 5 depends on $P(H)$: the prior probability distribution for the number of peers who possess knowledge about some query q. In this section we will see that $P(H)$ is formulated in much the same way as $P(S|M)$ from section 5.2 and how the design of the P2P protocol is crucial to its calculation.

Naïvely, we could assume that at the time that we query for q, no other peers have made similar queries. In that situation, the peers who possess knowledge of q would be exactly the set of origins: no queries have been made, so no knowledge has spread beyond the sources for q. Then

$$P(H = k|\text{no queries}) = P(k \text{ sources}|\text{at least one source})$$

since we need to assume sources exist in order to query for q. But this will most likely not be accurate since queries have probably occurred between other peers, thus changing the distribution of H.

However, assuming that our peers greedily query every other peer (in order to gain as much private knowledge as possible), we would know a priori how many queries have been made for q. Thus our prior should really be $P(H|Q)$ where Q is the number of previously made queries for q. Even in the non-greedy case where peers query some subset of the network, we could use the number of observed queries to probabilistically calculate the prior.

This prior can be formulated in much the same way as $P(S|M)$ from equation (5), except instead of considering possible values of H for M, we consider possible values of I for Q, where I is the number of origins that were originally in the network:

$$\begin{aligned} P(H = k|Q = u) &= \sum_{i=1}^{k} P(H = k|I = i \cap Q = u)P(I = i|Q = u) \\ &= \frac{\sum_{i=1}^{k} P(H = k|I = i \cap Q = u)P(Q = u|I = i)P(I = i)}{\sum_{i=1}^{k} P(Q = u|I = i)P(I = i)} \end{aligned} \tag{6}$$

All the terms in this formula can be calculated based on what we already know of the network. $P(I = i)$, the prior probability of there being i origins, is simply given by $P(S)$ and the binomial distribution: $P(I = i) = b(i, n - 1, P(S))$. $P(H = k|I = i \cap Q = u)$, the probability of k peers knowing about q after i origins are queried u times, can be calculated based on the rate at which knowledge spreads through the network (see section 6.1). $P(Q = u|I = i)$, the probability that u queries are made against i origins, can also be calculated using the rate at which knowledge spreads though from a different perspective (see section 6.2).

6.1 Conditional Knowledge Distribution

We showed previously how the distribution of knowledge given a certain number of queries were made depends on the knowledge distribution also assuming the number of origins: $P(H|I \cap Q)$.

The calculation of this distribution in effect measures how knowledge flows through the network given a particular querying protocol. For example, given the simple case where a peer simply responds to a query deterministically (as in section 5.1), we know that

$$P(H = i + u|I = i \cap Q = u) = 1$$

because i peers know initially and each of the u queries passes on the knowledge to one more peer.

If we have unreliable peers as in section 5.2, where a peer has probability r of responding to a query, we can easily calculate cases such as

$$P(H = i|I = i \cap Q = 1) = (1 - r)^i$$
$$P(H = i + 1|I = i \cap Q = 1) = 1 - (1 - r)^i$$

however the general calculation is far more complex as Q increases (due to multiple ways of achieving the same H). It takes the form

$$P(H = k|I = i \cap Q = u) = \sum_{\{F\}} \prod_{l=0}^{k-i-1} (1-r)^{(i+l)F_l}(1-(1-r)^{i+l})$$

where the sum is over all possible ways to have exactly $k-i$ successful queries out of u queries and the F_l represent the number of failures before the l^{th} success. This could be efficiently calculated using dynamic programming, however in practice we found that simulations of the network to empirically calculate the probability converged reasonably quickly to an accurate result (and are much simpler to implement). However we observe that this calculation only appears in equation (6) as a product with $P(Q = u|I = i)$. We will see in section 6.2 that $P(Q = u|I = i)$ will only be nonzero for a limited range of i. Thus in practice we need only calculate $P(H = k|I = i \cap Q = u)$ when this product has a chance of being nonzero. See section 7 for the result.

Similarly for the case of extra responses from section 5.3, the general probability calculation is surprisingly complex; one must consider which peers were chosen to receive extra responses and how these peers overlap with each other and the queried peers. Once again we chose to empirically calculate the distribution via simulation.

6.2 Query Distribution

The second distribution which must be calculated is $P(Q|I)$, the probability of observing a certain number of queries given a number of initial origins. We calculate this by taking into account Ω, the number of total queries about q that will be performed. Since we assume that every peer will eventually want to learn about q, this is equivalent to measuring how many queries it takes for every peer to have the knowledge.

$$P(Q = u|I = i) = \sum_{v=0}^{\infty} P(Q = u \cap \Omega = v|I = i) \tag{7}$$

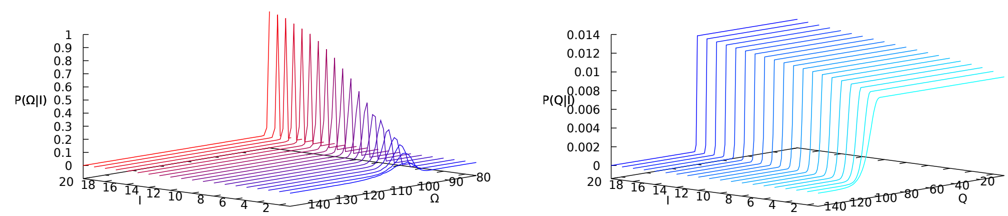
With deterministic responses as in section 5.1, if a peer were to observe the network over its lifetime, given i origins it would see $n-i-1$ queries: one for each of the peers that does not have the knowledge (not counting itself). For this protocol, equation (7) simplifies to

$$P(Q = u|I = i) = P(Q = u \cap \Omega = n-i-1|I = i) = \frac{1}{n-i-1} \tag{8}$$

for $u \in [1, n-i-1]$. The first equality follows from the fact that there must be $n-i-1$ queries. The second follows because a priori we can only assume that each of the $n-i-1$ peers is indistinguishable with respect to wanting knowledge about q, thus each query must be equally likely.

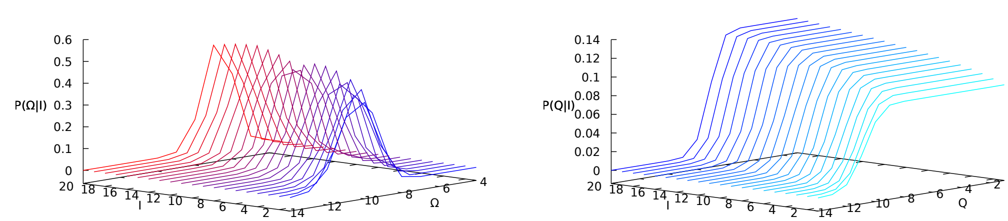
As in the previous section, the calculations are more elaborate as we change the protocol. For unreliable peers, $P(\Omega|I)$ is similar to a negative binomial distribution (the number of trials needed to get a certain number of successes in a Bernoulli process) however it is more complicated because the probability of a query succeeding increases as more queries succeed. Once again we found it more practical to simply empirically calculate $P(\Omega|I)$ via network simulation and then use the same assumption of equally likely queries from equation (8) to get $P(Q|I)$.

Figure 3 shows the result of calculating $P(\Omega|I)$ and $P(Q|I)$ in this manner for a fixed chance r of a peer responding. Note that as the number of origins increases, the probability of a query succeeding approaches 1, thus $P(\Omega = n-i|I = i) \to 1$, consequently $P(Q|I)$ approaches a uniform distribution (since this is similar to the deterministic case of equation (8)). A convenient byproduct of this calculation is that we also see the performance impact of the protocol. Because $P(\Omega|I)$ is also a sort of measure of how long it takes every peer to learn the knowledge, it is a decent measure of the worst-case performance of the network. We can see that for a small number of origins, while the smaller values of Q have the



(a) $P(\Omega|I)$ (b) $P(Q|I)$

Figure 3: Calculating $P(Q|I)$ with unreliable peers ($r = 0.2$) via network simulation ($n = 100$). Each line in (a) shows the distribution $P(\Omega|I = i)$ for different i, while lines in (b) show $P(Q|I = i)$.



(a) $P(\Omega|I)$ (b) $P(Q|I)$

Figure 4: Calculating $P(Q|I)$ with extra responses ($t = 1$) via network simulation ($n = 100$). The format is the same as figure 3, but note the difference in scale for Ω and Q.

highest probabilities, $P(Q|I)$ has a long tail, indicating that in some cases we might require significantly more queries before the knowledge has fully spread.

We can use the same technique with a different simulation to find $P(Q|I)$ for the case of extra responses. Figure 4 shows this result for a fixed t. This converges to a uniform distribution more slowly than unreliable peers does, but note that the scale for Q is much smaller: even for 1 origin, it is unlikely to take more than 14 queries for the knowledge to spread completely. This is because each successful query only slightly increases the chance of each extra response being sent, but overall the number of extra responses causes the knowledge to spread throughout the network at an (approximately) exponential rate. Since this example is for $t = 1$, we were no longer concerned with the performance impact of small t from section 5.3.

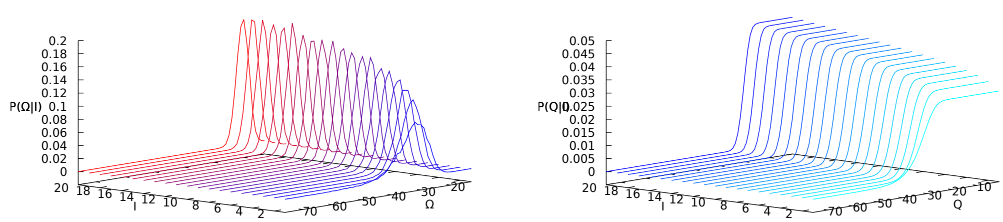
Because these calculations are done empirically, it is straightforward to extend this to the combination of the two techniques: unreliable peers where each peer sends extra responses. The result is shown in figure 5. As we might expect, the resulting distribution exhibits a mix of those from figures 3 and 4: slow convergence to a uniform distribution with a moderately long tail. We observe in section 7 that this translates to good privacy performance.

7 Evaluation

With the calculations developed in the previous sections, we can finally implement a private attribution network using the protocol changes from sections 5.2 and 5.3 and measure the resulting private knowledge leakage. We must do so empirically because of the empirical calculations in section 6.

We chose to evaluate the worst case scenario (similar to section 5.1): a single query q with a single source s possessed by a single peer x. Then for multiple iterations we simulated knowledge of q spreading through the network query-by-query, recording after each query the private information leak in terms of $P(S|x$ responds to a query).

Our first question was about the effect of differing r on the unreliable peers protocol. The results of these measurements are shown in figure 6. The "Worst case" line is the leakage in the simple deterministic case from section 5.1 where the leakage is given by $P(S|H = k)$. Note that this is equivalent to the unreliable peers protocol with $r = 1$. Because the leakage of unreliable peers is given by $P(S|M = m)$, this figure indicates that $P(S|M = m)$ converges to $P(S|H = k)$ as m and k increase and that decreasing r delays this convergence. For example, $r = 0.8$ provides a slight privacy benefit to only the first peer that is queried because it is nearly identical to the worst case by the second peer, whereas $r = 0.2$ provides a privacy benefit to the first three or four peers. Because the worst-case leakage approaches



(a) $P(\Omega|I)$ (b) $P(Q|I)$

Figure 5: Calculating $P(Q|I)$ with unreliable peers and extra responses ($r = 0.2$, $t = 1$) via network simulation ($n = 100$). The format is the same as figure 3, but note the difference in scale for Ω and Q.

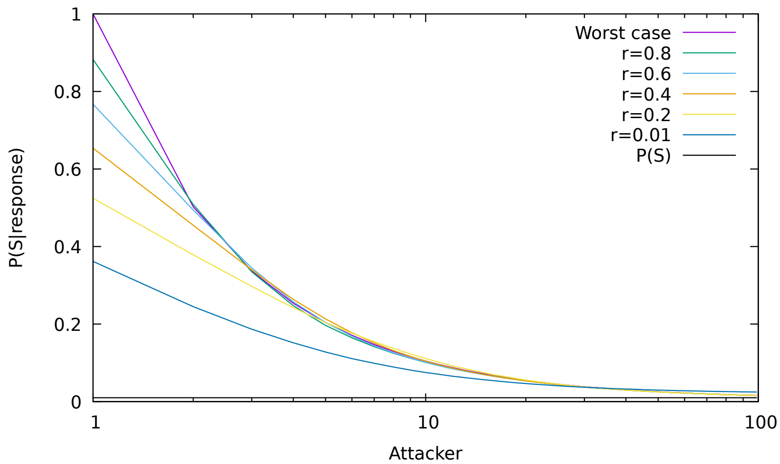


Figure 6: Comparison of the unreliable peers protocol with varying r ($n = 100$). Each line represents the private leakage as a function of when the attacker queries (ignoring failed queries). E.g. the first point represents the leakage if an attacker is the first peer to make a successful query, the second point is if they are the second peer, etc. The leakage is averaged over thousands of network simulations, so it is possible for the actual leakage to be higher or lower in a specific situation. Note the logarithmic scale on the X axis.

$P(S)$ for later queries, we can choose a point at which we deem this leakage negligible and choose r such that the convergence occurs after this point.

Next we compared the effect of the different protocols, shown in figure 7. As expected, extra responses provides no privacy benefit if the attacker is the first peer to query, since the intention is to provide higher privacy to *later* queries. Indeed the leakage decreases more quickly for extra responses than it does for unreliable peers, beating it by the third or fourth query. By the tenth or so query, the leakage is already negligible compared to $P(S)$. By that point it is likely that almost every peer has the knowledge, thus $k \approx n$ and $P(S|H = k) \approx P(S)$.

"Both" shows the effect of combining both unreliable peers and extra responses. The resulting protocol provides a good balance between the two: it has a low initial leakage for the first few peers to be queried, but also approaches $P(S)$ much quicker than the worst case. We were surprised to see that the leakage was even lower than unreliable peers with the same r. Although we are uncertain of the exact reason for this, we speculate that this is due to the $P(S|M)$ calculation involving $P(H)$ (equation (5), section 5.2). The extra responses shrink $P(H = k)$ for small k (because the knowledge spreads quicker), thus a small number of peers responding to a query are likely to be part of a larger set of peers who actually have the knowledge, which results in smaller $P(S|M)$.

Figure 7 is slightly misleading because the two protocols involving extra responses only provide a faster leakage falloff because the extra responses spread out the leakage more uniformly in the network. Thus we need to measure the distribution of this leakage in order to determine if the trade-off is worth it. The results are shown in figures 8 and 9. We can see that for early queries, this leakage is insignificant as we would expect: few peers are likely to receive an extra response at all. For later queries, the leakage becomes more uniform as the number of extra responses increases, however we see that significant leakage never extends beyond around 30% of the network ($n = 100$), and even then the leakage is already so close to $P(S)$ that it could be considered acceptable.

8 Discussion & Conclusion

We have shown that although attribution privacy can be a significant risk in certain situations, the techniques we have developed in this paper can be employed to effectively mitigate

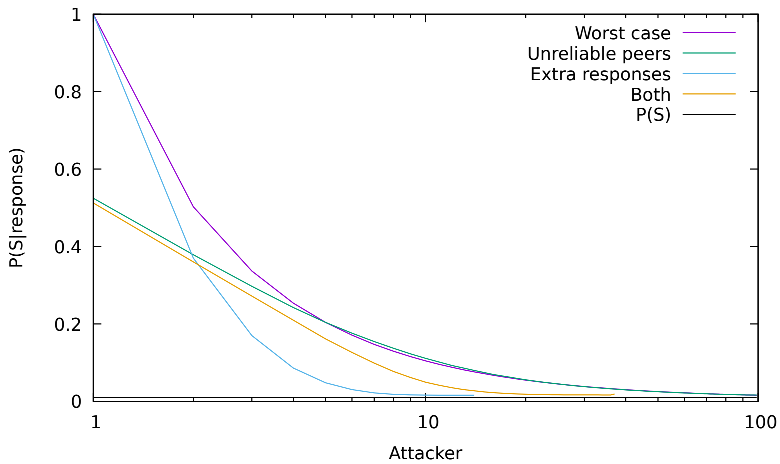


Figure 7: Comparison of different network protocols ($n = 100$, $r = 0.2$, $t = 1$). The format is the same as in figure 6.

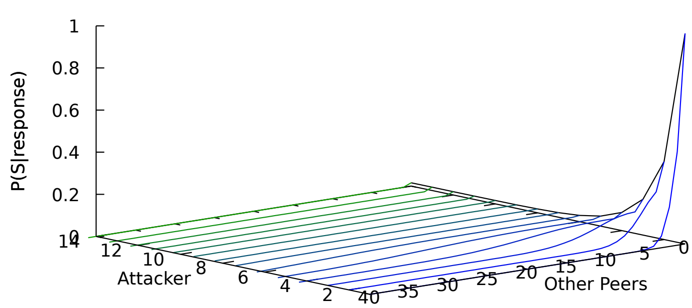


Figure 8: Distribution of privacy leakage due to extra responses. The black curve is identical to the extra responses curve in figure 7 (though mirrored). The perpendicular curves show the distribution of leakage among all other peers in the network, resulting from extra responses sent during that attacker's query.

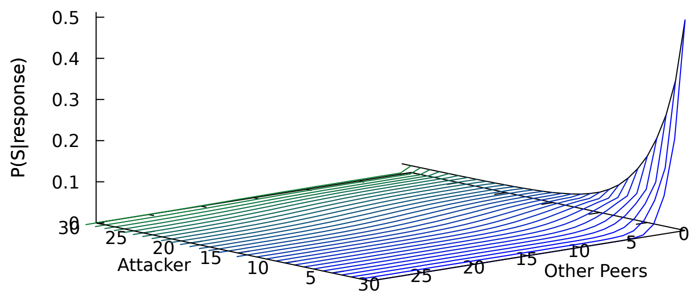


Figure 9: Distribution of privacy leakage due to extra responses with unreliable peers. The format is the same as in figure 8.

the risk. The method of unreliable peers can be applied (with a carefully chosen parameter) to ensure that a desired level of privacy is maintained even when a small number of peers respond to a query. The method of extra responses can be applied to "pay forward" privacy to other peers in the network by ensuring the knowledge spreads faster. Although extra responses make the network susceptible to an attacker that floods the network with malicious peers, the risk can be controlled by carefully choosing the parameter for the extra responses. Furthermore, we showed empirically that these two methods synergize to provide an even greater privacy benefit than either method on its own.

We conclude that the methods of unreliable peers and extra responses are a firm starting point for any P2P application in which attribution privacy is involved. They can be tuned to preserve (or increase) network performance while maintaining whatever level of privacy is demanded by the application.

8.1 Future Work

As mentioned in section 2, this research was originally part of a much larger project, and there still remain several barriers preventing a practical implementation of a private attribution network. First, we were only able to measure network performance in terms of the maximum number of queries needed for knowledge of a single query to fully spread. It is unclear what exactly the affect would be on an individual level, for a peer making many separate queries with different sources. We also assumed that every peer queries every other peer, as this simplified analysis and also matched the behavior that an attacker might use in order to learn as much private information as possible. But in a practical implementation, this behavior may not be desirable (for non-malicious peers). It would be interesting to see what the "accidental" leakage is if peers query only a subset of the network, and how this leakage is affected by the two protocols we examined. Both the performance and accidental leakage depend greatly on the specific use-case of the private attribution network, so we leave their investigation to future researchers.

We also considered a mechanism similar to extra responses, where a peer would actually not reply to the querying peer at all. Instead the peers receiving extra responses would be responsible for eventually relaying the data back to the peer that made the query. This would provide even greater privacy by masking the replying peer from the querying peer in a Tor-like way, while also spreading knowledge through the network via the extra responses. Unfortunately we were unable to quantify the privacy provided by this method during our research. This could also be extended as a method for peers to gain knowledge without needing to query the entire network, by having the queried peers send out *extra queries* on behalf of the original querying peer.

One potential attack that we did not have time to address is if an attacker makes repeated queries in order to circumvent the randomness of the unreliable peers protocol. Such an attack could be implemented similar to the flooding attack of extra responses, where multiple malicious peers all make the same query and collaborate to artificially increase r. This could be defended against by choosing r inversely proportional to n as with extra responses, or by regulating responses such that an unreliable peer will consistently lie about a query for some time after initially queried.

We described attribution privacy in an asymmetric setting where an attacker queries a potentially vulnerable peer. But this could also be extended to a situation where the queries themselves are sensitive. In our Nickelback example from section 4.1, we can imagine that someone might be equally embarrassed to ask about a Nickelback song, for fear of appearing to be a Nickelback fan themselves. Or in the advertisement example from section 4.2, someone querying about an ad might reveal that they possess the sensitive data which the ad targets. In such cases, the P2P network would need to protect both the querying peer and the queried peer.

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