PROBLEM A – PART B

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Q1: What do you observe in the histogram after applying U†?

Observation: After applying U† to Bob's qubit, the histogram shows that Bob measures the state $|0\rangle$ with approximately **100% probability** ($\geq$ 99% in simulation). The outcome $|1\rangle$ appears with near-zero probability, which is consistent with ideal noiseless simulation.

This is the expected result if teleportation succeeded: Bob's qubit has been transformed into exactly the state $|\psi\rangle$, and applying U† (the inverse of the original preparation) rotates it back to $|0\rangle$.

Q2: Why does applying U† verify teleportation?

The original unknown state was prepared as:

$$|\psi\rangle = U|0\rangle \quad \text{where } U = Rz(\varphi) \cdot Ry(\theta)$$

If teleportation worked correctly, Bob's qubit is in state $|\psi\rangle$ at the end of the correction step. Applying the inverse U† gives:

$$U^\dagger|\psi\rangle = U^\dagger(U|0\rangle) = (U^\dagger U)|0\rangle = I|0\rangle = |0\rangle$$

Since U is unitary, $U^\dagger U = I$ (identity). Therefore measuring $|0\rangle$ with 100% probability confirms that Bob holds exactly $|\psi\rangle$ — i.e., teleportation was successful. If Bob had a different state, the measurement would yield $|1\rangle$ with nonzero probability.

Q3: Why were we able to avoid sending classical bits?

In the standard protocol, Alice measures qubits 0 and 1, obtains two classical bits, and sends them to Bob over a classical channel. Bob then applies conditional Pauli corrections (X and/or Z) based on the received bits.

In this fully quantum version, we *do not* measure qubits 0 and 1. Instead, we replace the classical feed-forward with coherent controlled quantum gates: - `qc.cx(1, 2)` — replaces the classically-controlled X correction - `qc.cz(0, 2)` — replaces the classically-controlled Z correction

This works because the controlled gates act on the joint state *in superposition*: when qubit 1 is $|1\rangle$, CX applies X to qubit 2; when qubit 0 is $|1\rangle$, CZ applies Z to qubit 2. The effect is mathematically equivalent to the classical correction for all measurement outcomes simultaneously.

Key difference: The real-world protocol requires classical communication and is limited by the speed of light. This fully quantum version is a coherent, unitary circuit that works only in a *closed quantum system* (no actual communication between separated parties). It cannot be used for faster-than-light communication — the classical channel is still required in practice because Alice and Bob are physically separated. The fully quantum version is useful for circuit-based verification and quantum algorithms, not for actual physical teleportation between distant parties.