B0° 21B5↩

```
% 11811126 Zhuang Yulun
close all;clear;clc;
```

Phase 1

```
%% Phase 1
syms m L I a q dq [1 3]
syms r g
n = 3;
m = m.';
L = L.';
I = I.';
a = a.';
q = q.';
dq = dq.';

p = cell(n,1);
p{1} = [-(a(1)-r)*sin(q(1)),-(a(1)-r)*cos(q(1))];
p{2} = [-(L(1)-r)*sin(q(1))-a(2)*sin(q(2)),...
        -(L(1)-r)*cos(q(1))-a(2)*cos(q(2))];
p{3} = [-(L(1)-r)*sin(q(1))-L(2)*sin(q(2))-a(3)*sin(q(3)),...
        -(L(1)-r)*cos(q(1))-L(2)*cos(q(2))-a(3)*cos(q(3))];

v = cell(n,1);
K = 0;
P = 0;
for i = 1:n
    v{i} = jacobian(p{i},q)*dq;
    v{i}(1) = v{i}(1)+ r*dq1;
    K = K+1/2*m(i)*v{i}.'*v{i}+1/2*dq(i).'*I(i)*dq(i);
    P = P+m(i)*g*p{i}(2);
end

K = simplify(expand(K));
P = simplify(expand(P));

D = jacobian(K,dq).';
D = jacobian(D,dq);

G = jacobian(P,q).';

syms C real
for k=1:n
for j=1:n
        C(k,j)=0;
        for i=1:n
            C(k,j)=C(k,j)+(1/2)*(jacobian(D(k,j),q(i))...
            +jacobian(D(k,i),q(j))-jacobian(D(i,j),q(k)))*dq(i);
        end
end
end
D1 = simplify(expand(D));
C1 = simplify(expand(C));
G1 = simplify(expand(G));
disp(D1)
```

$$
\left(\begin{array}{ccc}
I_1 + {L_1}^2\, m_2 + {L_1}^2\, m_3 + {a_1}^2\, m_1 + 2\, m_1\, r^2 + 2\, m_2\, r^2 + 2\, m_3\, r^2 + 2\, m_1\, r^2 & & \\
\cos\left(q_1\right) + 2\, m_2\, r^2\, \cos\left(q_1\right) + 2\, m_3\, r^2 & & \\
\cos\left(q_1\right) - 2\, L_1\, m_2\, r - 2\, L_1\, m_3\, r - 2\, a_1\, m_1\, r - 2\, L_1\, m_2\, r & & \\
\cos\left(q_1\right) - 2\, L_1\, m_3\, r\, \cos\left(q_1\right) - 2\, a_1\, m_1\, r\, \cos\left(q_1\right) & \sigma_1 & \sigma_2 \\
\sigma_1 & m_3\, {L_2}^2 + m_2\, {a_2}^2 + I_2 & \sigma_3 \\
\sigma_2 & \sigma_3 & m_3\, {a_3}^2 + I_3
\end{array}\right)
$$

where

$$\sigma_1 = -\left(L_2\, m_3 + a_2\, m_2\right)\left(r\, \cos\left(q_2\right) - L_1\, \cos\left(q_1 - q_2\right) + r\, \cos\left(q_1 - q_2\right)\right)$$

$$\sigma_2 = -a_3\, m_3\left(r\, \cos\left(q_3\right) - L_1\, \cos\left(q_1 - q_3\right) + r\, \cos\left(q_1 - q_3\right)\right)$$

$$\sigma_3 = L_2\, a_3\, m_3\, \cos\left(q_2 - q_3\right)$$

```
disp(C1)
```

$$
\left(\begin{array}{ccc}
\mathrm{dq}_1\, r\, \sin\left(q_1\right) & & \\
\left(L_1\, m_2 + L_1\, m_3 + a_1\, m_1 - m_1\, r - m_2\, r - m_3\, r\right) & \mathrm{dq}_2\, \sigma_1 & \\
\left(r\, \sin\left(q_2\right) + L_1\, \sin\left(q_1 - q_2\right) - r\, \sin\left(q_1 - q_2\right)\right) & a_3 & \\
\mathrm{dq}_3\, m_3 & & \\
\left(r\, \sin\left(q_3\right) + L_1\, \sin\left(q_1 - q_3\right) - r\, \sin\left(q_1 - q_3\right)\right) & & \\
-\mathrm{dq}_1\, \sin\left(q_1 - q_2\right)\, \left(L_1 - r\right)\sigma_1 & 0 & L_2\, a_3\, \mathrm{dq}_3\, m_3\, \sin\left(q_2 - q_3\right) \\
-a_3\, \mathrm{dq}_1\, m_3\, \sin\left(q_1 - q_3\right)\, \left(L_1 - r\right) & -L_2\, a_3\, \mathrm{dq}_2\, m_3\, \sin\left(q_2 - q_3\right) & 0
\end{array}\right)
$$

where

$$\sigma_1 = L_2\, m_3 + a_2\, m_2$$

```
disp(G1)
```

$$
\left(\begin{array}{c}
g\, \sin\left(q_1\right)\, \left(L_1\, m_2 + L_1\, m_3 + a_1\, m_1 - m_1\, r - m_2\, r - m_3\, r\right) \\
g\, \sin\left(q_2\right)\, \left(L_2\, m_3 + a_2\, m_2\right) \\
a_3\, g\, m_3\, \sin\left(q_3\right)
\end{array}\right)
$$

Phase 2

```
%% Phase 2
syms m L I a q dq [1 2]
syms r g
n = 2;
m = m.';
L = L.';
I = I.';
a = a.';
q = q.';
dq = dq.';

p = cell(n,1);
p{1} = [-(a(1)-r)*sin(q(1)),-(a(1)-r)*cos(q(1))];
p{2} = [-(L(1)-r)*sin(q(1))-(L(1)-a(1))*sin(q(2)),...
        -(L(1)-r)*cos(q(1))-(L(1)-a(1))*cos(q(2))];

v = cell(n,1);
K = 0;
P = 0;
```

```
for i = 1:n
    v{i} = jacobian(p{i},q)*dq;
    v{i}(1) = v{i}(1)+ r*dq1;
    K = K+1/2*m(1)*v{i}.'*v{i}+1/2*dq(i).'*I(1)*dq(i);
    P = P+m(1)*g*p{i}(2);
end

K = simplify(expand(K));
P = simplify(expand(P));

D = jacobian(K,dq).';
D = jacobian(D,dq);

G = jacobian(P,q).';

syms C real
for k=1:n
for j=1:n
        C(k,j)=0;
        for i=1:n
            C(k,j)=C(k,j)+(1/2)*(jacobian(D(k,j),q(i))+...
            jacobian(D(k,i),q(j))-jacobian(D(i,j),q(k)))*dq(i);
        end
end
end

D2 = simplify(expand(D));
C2 = simplify(expand(C));
G2 = simplify(expand(G));
disp(D2)
```

$$\left(\begin{array}{cc}
I_1 + {L_1}^2\,m_1 + {a_1}^2\,m_1 + 4\,m_1 & \\
r^2 + 4\,m_1\,r^2\,\cos\left(q_1\right) - 2\,L_1\,m_1\,r - 2\,a_1\,m_1\,r - 2\,L_1\,m_1\,r & \\
\cos\left(q_1\right) - 2 & \\
a_1\,m_1\,r\,\cos\left(q_1\right) & \\
 & \sigma_1 \\
\sigma_1 & m_1\,{L_1}^2 - 2\,m_1\,L_1\,a_1 + m_1\,{a_1}^2 + I_1
\end{array}\right)$$

where

$$\sigma_1 = -m_1\,\left(L_1 - a_1\right)\,\left(r\,\cos\left(q_2\right) - L_1\,\cos\left(q_1 - q_2\right) + r\,\cos\left(q_1 - q_2\right)\right)$$

```
disp(C2)
```

$$\left(\begin{array}{cc}
\mathrm{dq}_1\,m_1\,r\,\sin\left(q_1\right)\,\left(L_1 + a_1 - 2\,r\right) & \mathrm{dq}_2 \\
m_1\,\left(L_1 - a_1\right)\,\left(r\,\sin\left(q_2\right) + L_1\,\sin\left(q_1 - q_2\right) - r\,\sin\left(q_1 - q_2\right)\right) & \\
-\mathrm{dq}_1\,m_1\,\sin\left(q_1 - q_2\right)\,\left(L_1 - a_1\right)\,\left(L_1 - r\right) & 0
\end{array}\right)$$

```
disp(G2)
```

$$\left(\begin{array}{c}
g\,m_1\,\sin\left(q_1\right)\,\left(L_1 + a_1 - 2\,r\right) \\
g\,m_1\,\sin\left(q_2\right)\,\left(L_1 - a_1\right)
\end{array}\right)$$

Knee Collision Model

$$\begin{bmatrix} \dot{\boldsymbol{q}}^{+} \\ \hat{F}_{\mathrm{k}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{D}1(\boldsymbol{q}^{+}) & \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \\ \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} & 0 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{D}1(\boldsymbol{q}^{-})\dot{\boldsymbol{q}}^{-} \\ 0 \end{bmatrix}$$

where D1 is

```
disp(D1)
```

$$\begin{pmatrix} \begin{matrix} I_1 + {L_1}^2\, m_2 + {L_1}^2\, m_3 + {a_1}^2\, m_1 + 2\, m_1\, r^2 + 2\, m_2\, r^2 + 2\, m_3\, r^2 + 2\, m_1\, r^2 \\ \cos(q_1) + 2\, m_2\, r^2\, \cos(q_1) + 2\, m_3\, r^2 \\ \cos(q_1) - 2\, L_1\, m_2\, r - 2\, L_1\, m_3\, r - 2\, a_1\, m_1\, r - 2\, L_1\, m_2\, r \\ \cos(q_1) - 2\, L_1\, m_3\, r\, \cos(q_1) - 2\, a_1\, m_1\, r\, \cos(q_1) \end{matrix} & \sigma_1 & \sigma_2 \\ \sigma_1 & m_3\, {L_2}^2 + m_2\, {a_2}^2 + I_2 & \sigma_3 \\ \sigma_2 & \sigma_3 & m_3\, {a_3}^2 + I_3 \end{pmatrix}$$

where

$$\sigma_1 = -(L_2\, m_3 + a_2\, m_2)\, (r\, \cos(q_2) - L_1\, \cos(q_1 - q_2) + r\, \cos(q_1 - q_2))$$

$$\sigma_2 = -a_3\, m_3\, (r\, \cos(q_3) - L_1\, \cos(q_1 - q_3) + r\, \cos(q_1 - q_3))$$

$$\sigma_3 = L_2\, a_3\, m_3\, \cos(q_2 - q_3)$$

```
\matlabheading{Ground Collision Model}

\begin{matlabcode}
%% Ground Collision Model
syms m L I a q dq z dz [1 2]
syms r g

n = 2;
m = m.';
L = L.';
I = I.';
a = a.';
q = [q.';z.'];
dq = [dq.';dz.'];

p = cell(n,1);
p{1} = [q(3)-(a(1)-r)*sin(q(1)),q(4)-(a(1)-r)*cos(q(1))];
p{2} = [q(3)-(L(1)-r)*sin(q(1))-(L(1)-a(1))*sin(q(2)),...
        q(4)-(L(1)-r)*cos(q(1))-(L(1)-a(1))*cos(q(2))];

v = cell(n,1);
K = 0;
for i = 1:n
    v{i} = jacobian(p{i},q)*dq;
    v{i}(1) = v{i}(1)+ r*dq1;
    K = K+1/2*m(1)*v{i}.'*v{i}+1/2*dq(i).'*I(1)*dq(i);
end

Df = hessian(K,dq);
Df = simplify(expand(Df));
```

```
Ef = [q(3)-(L(1)-r)*sin(q(1))-L(1)*sin(q(2)),...
      q(4)-(L(1)-r)*cos(q(1))-L(1)*cos(q(2))];

dEf = jacobian(Ef,q);
```

$$\begin{bmatrix} \dot{\boldsymbol{q}}_f^+ \\ \begin{bmatrix} \hat{F}_x \\ \hat{F}_y \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \boldsymbol{D}_f(\boldsymbol{q}_f^-) & -\left(\frac{\partial \boldsymbol{E}(\boldsymbol{q}_f)}{\partial \boldsymbol{q}_f}\right)'\Big|_{\boldsymbol{q}_f=\boldsymbol{q}_f^-} \\ \left(\frac{\partial \boldsymbol{E}(\boldsymbol{q}_f)}{\partial \boldsymbol{q}_f}\right)\Big|_{\boldsymbol{q}_f=\boldsymbol{q}_f^-} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}^{-1} \times \begin{bmatrix} \boldsymbol{D}_f(\boldsymbol{q}_f^-) \cdot \dot{\boldsymbol{q}}_f^- \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix}$$

where Df and $\frac{\partial E(q_f^-)}{\partial q_f^-}$ are

```
disp(Df)
```

$$\begin{pmatrix} I_1 + {L_1}^2\, m_1 + {a_1}^2\, m_1 + 4\, m_1\, r^2 + 4\, m_1\, r^2\, \cos(q_1) - 2\, L_1\, m_1\, r - 2\, a_1\, m_1\, r - 2\, L_1\, m_1\, r\, \cos(q_1) - 2\, a_1\, m_1\, r\, \cos(q_1) & \sigma_1 & \sigma_2 & \sigma_3 \\ \sigma_1 & m_1\, {L_1}^2 - 2\, m_1\, L_1\, a_1 + m_1\, {a_1}^2 + I_1 & \sigma_4 & \sigma_5 \\ \sigma_2 & \sigma_4 & 2\, m_1 & 0 \\ \sigma_3 & \sigma_5 & 0 & 2\, m_1 \end{pmatrix}$$

where

$$\sigma_1 = -m_1\,(L_1 - a_1)\,(r\, \cos(q_2) - L_1\, \cos(q_1 - q_2) + r\, \cos(q_1 - q_2))$$

$$\sigma_2 = m_1\,(2\, r - L_1\, \cos(q_1) - a_1\, \cos(q_1) + 2\, r\, \cos(q_1))$$

$$\sigma_3 = m_1\, \sin(q_1)\,(L_1 + a_1 - 2\, r)$$

$$\sigma_4 = -m_1\, \cos(q_2)\,(L_1 - a_1)$$

$$\sigma_5 = m_1\, \sin(q_2)\,(L_1 - a_1)$$

```
disp(dEf)
```

$$\begin{pmatrix} -\cos(q_1)\,(L_1 - r) & -L_1\, \cos(q_2) & 1 & 0 \\ \sin(q_1)\,(L_1 - r) & L_1\, \sin(q_2) & 0 & 1 \end{pmatrix}$$