ME424 HW8

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close all;clear;clc;

syms q1(t) q2(t) q = [q1(t);q2(t)]; dq = diff(q,t); syms m [1 2] syms g L postive

n = 2; Q = [0 -1 0 0;1 0 0 0;0 0 0 0;0 0 0 0]; I = cell(2,1); T = cell(3,1); U = cell(n,n); r$=cell(n,1); K = 0; P = 0; tau = cell(2,1); g=[0 g 0 0];$

for i = 1:2 r$_i$ $= [-1/2 * L; 0; 0; 1]; Ii = [1/3 * m(i) * L^2, 0, 0, -1/2 * m(i) * L^2; ...0, 0, 0, 0; 0, 0, 0, 0; -1/2 * m(i) * L^2, 0, 0, m(i)]; if i == 1 Ti = [cos(q(i)), -sin(q(i)), 0, L * cos(q(i)); ...sin(q(i)), cos(q(i)), 0, L * sin(q(i)); ...0, 0, 1, 0; 0, 0, 0, 1]; else Ti = Ti-1 * [cos(q(i)), -sin(q(i)), 0, L * cos(q(i)); ...sin(q(i)), cos(q(i)), 0, L * sin(q(i)); ...0, 0, 1, 0; 0, 0, 0, 1]; end P = P - m(i) * g_*(Ti * r_i); end P = simplify(expand(P))$ P =

$$-\frac{L\,g\,m_2\,\sin\left(q_1(t)+q_2(t)\right)}{2}-\frac{L\,g\,m_1\,\sin\left(q_1(t)\right)}{2}-L\,g\,m_2\,\sin\left(q_1(t)\right)$$

T3 = T12; U1,1 = Q*T1; U2,1 = Q*T2; U2,2 = T1*Q*T3;

for i = 1:n for p = 1:i for r = 1:i K = K+1/2*trace(Ui,p*Ii*Ui,r.'*dq(p)*dq(r)); end end end K = simplify(expand(K)) K =

$$\frac{2L^2m_1\sigma_1}{3}+\frac{7L^2m_2\sigma_1}{6}-\frac{L^3m_1\sigma_1}{2}$$
$$+\frac{2L^2m_2\sigma_2}{3}-\frac{L^3m_2\sigma_1}{2}-\frac{L^3m_2\sigma_2}{2}$$
$$+L^2m_2\cos\left(q_2(t)\right)\sigma_1-\frac{L^3m_2\cos\left(q_2(t)\right)\sigma_1}{2}$$
$$+\frac{4L^2m_2\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)}{3}-L^3m_2\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)$$
$$+L^2m_2\cos\left(q_2(t)\right)\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)-\frac{L^3m_2\cos\left(q_2(t)\right)\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)}{2}$$

where

$$\sigma_1=\left(\frac{\partial}{\partial t}q_1(t)\right)^2$$

$$\sigma_2=\left(\frac{\partial}{\partial t}q_2(t)\right)^2$$

L = simplify(K-P); dq = diff(q,t); for i = 1:n taui = diff(diff(L,dq(i)),t)-diff(L,q(i)); end L = simplify(expand(L)) L =

$$\frac{2L^2m_1\sigma_1}{3}+\frac{7L^2m_2\sigma_1}{6}-\frac{L^3m_1\sigma_1}{2}$$
$$+\frac{2L^2m_2\sigma_2}{3}-\frac{L^3m_2\sigma_1}{2}-\frac{L^3m_2\sigma_2}{2}$$
$$+\frac{L\,g\,m_2\sin\left(q_1(t)+q_2(t)\right)}{2}+\frac{L\,g\,m_1\sin\left(q_1(t)\right)}{2}+L\,g\,m_2\sin\left(q_1(t)\right)$$
$$+L^2m_2\cos\left(q_2(t)\right)\sigma_1-\frac{L^3m_2\cos\left(q_2(t)\right)\sigma_1}{2}+\frac{4L^2m_2\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)}{3}$$
$$-L^3m_2\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)+L^2m_2\cos\left(q_2(t)\right)\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)$$
$$-\frac{L^3m_2\cos\left(q_2(t)\right)\frac{\partial}{\partial t}q_2(t)\frac{\partial}{\partial t}q_1(t)}{2}$$

tau1 ans =

where

$$\sigma_1=\left(\frac{\partial}{\partial t}q_1(t)\right)^2$$

$$\sigma_2=\left(\frac{\partial}{\partial t}q_2(t)\right)^2$$

$\frac{4\,L^2\,m_1\,\sigma_1}{3} + \frac{7\,L^2\,m_2\,\sigma_1}{3} - L^3\,m_1\,\sigma_1$
$+\frac{4\,L^2\,m_2\,\sigma_2}{3} - L^3\,m_2\,\sigma_1 - L^3\,m_2\,\sigma_2 - \frac{L\,g\,m_2\,\cos(q_1(t)+q_2(t))}{2}$
$-\frac{L\,g\,m_1\,\cos(q_1(t))}{2} - L\,g\,m_2\,\cos\left(q_1\left(t\right)\right) - L^2\,m_2\,\sin\left(q_2\left(t\right)\right)\,\sigma_3$
$+\frac{L^3\,m_2\,\sin(q_2(t))\,\sigma_3}{2} + 2\,L^2\,m_2\,\cos\left(q_2\left(t\right)\right)\,\sigma_1 + L^2\,m_2\,\cos\left(q_2\left(t\right)\right)\,\sigma_2$
$-L^3\,m_2\,\cos\left(q_2\left(t\right)\right)\,\sigma_1 - \frac{L^3\,m_2\,\cos(q_2(t))\,\sigma_2}{2}$
$-2\,L^2\,m_2\,\sin\left(q_2\left(t\right)\right)\,\frac{\partial}{\partial t}\,q_2\left(t\right)\,\frac{\partial}{\partial t}\,q_1\left(t\right) + L^3\,m_2\,\sin\left(q_2\left(t\right)\right)\,\frac{\partial}{\partial t}\,q_2\left(t\right)\,\frac{\partial}{\partial t}\,q_1\left(t\right)$

tau2 ans =

where

$$\sigma_1 = \frac{\partial^2}{\partial t^2}\,q_1\left(t\right)$$

$$\sigma_2 = \frac{\partial^2}{\partial t^2}\,q_2\left(t\right)$$

$$\sigma_3 = \left(\frac{\partial}{\partial t}\,q_2\left(t\right)\right)^2$$

$\frac{4\,L^2\,m_2\,\sigma_1}{3} + \frac{4\,L^2\,m_2\,\sigma_3}{3}$
$-L^3\,m_2\,\sigma_1 - L^3\,m_2\,\sigma_3 - \frac{L\,g\,m_2\,\cos(q_1(t)+q_2(t))}{2}$
$+L^2\,m_2\,\sin\left(q_2\left(t\right)\right)\,\sigma_2 - \frac{L^3\,m_2\,\sin(q_2(t))\,\sigma_2}{2}$
$+L^2\,m_2\,\cos\left(q_2\left(t\right)\right)\,\sigma_1 - \frac{L^3\,m_2\,\cos(q_2(t))\,\sigma_1}{2}$

where

$$\sigma_1 = \frac{\partial^2}{\partial t^2}\,q_1\left(t\right)$$

$$\sigma_2 = \left(\frac{\partial}{\partial t}\,q_1\left(t\right)\right)^2$$

$$\sigma_3 = \frac{\partial^2}{\partial t^2}\,q_2\left(t\right)$$