ME424

September 3, 2021

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Given

$$\ddot{\theta} = \frac{MRL\sin\theta\cos\theta\cdot\dot{\theta}^2 - \tau}{MRL\cos^2\theta - L/R(2J_w + (M+2m)R^2)}$$
$$\ddot{\alpha} = \frac{MRL\sin\theta\cdot\dot{\theta}^2 - MRL\cos\theta\cdot\ddot{\theta} + \tau}{2J_w + (M+2m)R^2}$$
$$y = \begin{bmatrix}\dot{\theta}_m \\ a_m\end{bmatrix}$$

Define

$$x = \begin{bmatrix}x_1 \\ x_2 \\ x_3 \\ x_4\end{bmatrix} = \begin{bmatrix}\theta \\ \dot{\theta} \\ \alpha \\ \dot{\alpha}\end{bmatrix}$$

We have

$$\dot{x} = \begin{bmatrix}\dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4}\end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{MRL\sin x_1\cos x_1\cdot x_2^2-\tau}{MRL\cos^2 x_1 - L/R(2J_w+(M+2m)R^2)} \\ x_4 \\ \frac{MRL\sin x_1\cdot x_2^2 - MRL\cos x_1\cdot\frac{MRL\sin x_1\cos x_1\cdot x_2^2-\tau}{MRL\cos^2 x_1-L/R(2J_w+(M+2m)R^2)}+\tau}{2J_w+(M+2m)R^2}\end{bmatrix}$$
$$y = Cx = \begin{bmatrix}0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r\end{bmatrix}\begin{bmatrix}x_1 \\ x_2 \\ x_3 \\ x_4\end{bmatrix}$$

where r is radius of the wheel.

From continuous time model to descrete time model

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\begin{aligned}
x(k+1) &= x(k) + \dot{x(k)}\Delta t \\
&= \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} x_2(k) \\ \frac{MRL\sin x_1(k)\cos x_1(k)\cdot x_2^2(k)-\tau}{MRL\cos^2 x_1(k)-L/R(2J_w+(M+2m)R^2)} \\ x_4(k) \\ \frac{MRL\sin x_1(k)\cdot x_2^2(k)-MRL\cos x_1(k)\cdot\frac{MRL\sin x_1(k)\cos x_1(k)\cdot x_2^2(k)-\tau}{MRL\cos^2 x_1(k)-L/R(2J_w+(M+2m)R^2)}+\tau}{2J_w+(M+2m)R^2} \end{bmatrix} \Delta t \\
&= \begin{bmatrix} x_1(k) + x_2(k)\Delta t \\ x_2(k) + \frac{MRL\sin x_1(k)\cos x_1(k)\cdot x_2^2(k)-\tau}{MRL\cos^2 x_1(k)-L/R(2J_w+(M+2m)R^2)}\Delta t \\ x_3(k) + x_4(k)\Delta t \\ x_4(k) + \frac{MRL\sin x_1(k)\cdot x_2^2(k)-MRL\cos x_1(k)\cdot\frac{MRL\sin x_1(k)\cos x_1(k)\cdot x_2^2(k)-\tau}{MRL\cos^2 x_1(k)-L/R(2J_w+(M+2m)R^2)}+\tau}{2J_w+(M+2m)R^2}\Delta t \end{bmatrix} \\
y(k) &= Cx(k) \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}
\end{aligned}
$$

where $\Delta t = 0.001s$

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\begin{aligned}
\mathfrak{T}_{\mathrm{MINCO}} = \Big\{ & p(t) : [0, T] \mapsto \mathbb{R}^m \mid \mathbf{c} = \mathbf{c}(\mathbf{q}, \mathbf{T}), \mathbf{q} \in \mathbb{R}^{m(M-1)}, \mathbf{T} \in \mathbb{R}_{>0}^{M} \Big\} \\
\textit{where} \quad & \mathbf{q} = (q_1, \ldots, q_{M-1}), \mathbf{T} = (T_1, \ldots, T_M)^{\mathrm{T}}, \\
& p(t) = p_i\,(t - t_{i-1}), \forall t \in [t_{i-1}, t_i) \ \ \textit{and}\ t_i = \Sigma_{j=1}^{i} T_j \\
& p_i(t) = \mathbf{c}_i^{\mathrm{T}} \beta(t), t \in [0, T_i], \mathbf{c}_i \in \mathbb{R}^{2s \times m}, \\
& \beta(t) = \left(1, t, \ldots, t^N\right)^{\mathrm{T}}
\end{aligned}
$$

$q_i \in \mathbb{R}^m$ is a specified 0-order derivative at t_i, T_i is the piece time,
$\mathbf{c}_i$ is the coefficient matrix of the piece, and $\beta(t)$ is the natural basis.