1.2

```
clc; clear;
syms x y z;
grad = jacobian(3 * x * (2 * y - z^3) + y^4 / 3, [x y z]);

f = @(x) 3 * x(1) * (2 * x(2) - x(3)^3) + x(2)^4 / 3;
x_star = [1 3 -1]';

grad_star = double(subs(grad, [x y z], x_star'))
```

```
grad_star = 1×3
    21    42    -9
```

```
delta = [1e-3 1e-3 1e-3]';

grad_hat = zeros(1, 3);

e = eye(3);

for i = 1:3
    grad_hat(i) = (f(x_star + delta(i)*e(:, i)) ...
        - f(x_star - delta(i)*e(:, i))) / (2*delta(i));
end

grad_hat
```

```
grad_hat = 1×3
   21.0000   42.0000   -9.0000
```

1.3

```
clc; clear;

x_star = [1 1 1 1 1]';

delta = [1e-5 1e-3 1e-6 1e-3 1e-3]';

grad_hat = zeros(1, 5);

e = eye(5);

for i = 1:5
    grad_hat(i) = (funcPartC(x_star + delta(i)*e(:, i)) ...
        - funcPartC(x_star - delta(i)*e(:, i))) / (2*delta(i));
end

grad_hat
```

```
grad_hat = 1×5
  117.3525  -41.3695 -636.6017   -3.8520  -11.2049
```

2

2.1

```
clc; clear; close all;

load SegwayData4KF.mat

whos
```

```
  Name        Size            Bytes  Class     Attributes

  A           4x4               128  double
  B           4x1                32  double
  C           1x4                32  double
  D           1x1                 8  double
  G           4x4               128  double
  N           1x1                 8  double
  P0          4x4               128  double
  Q           1x1                 8  double
  R           4x4               128  double
  Ts          1x1                 8  double
  t         500x1              4000  double
  u         500x1              4000  double
  x0          4x1                32  double
  y         500x1              4000  double
```

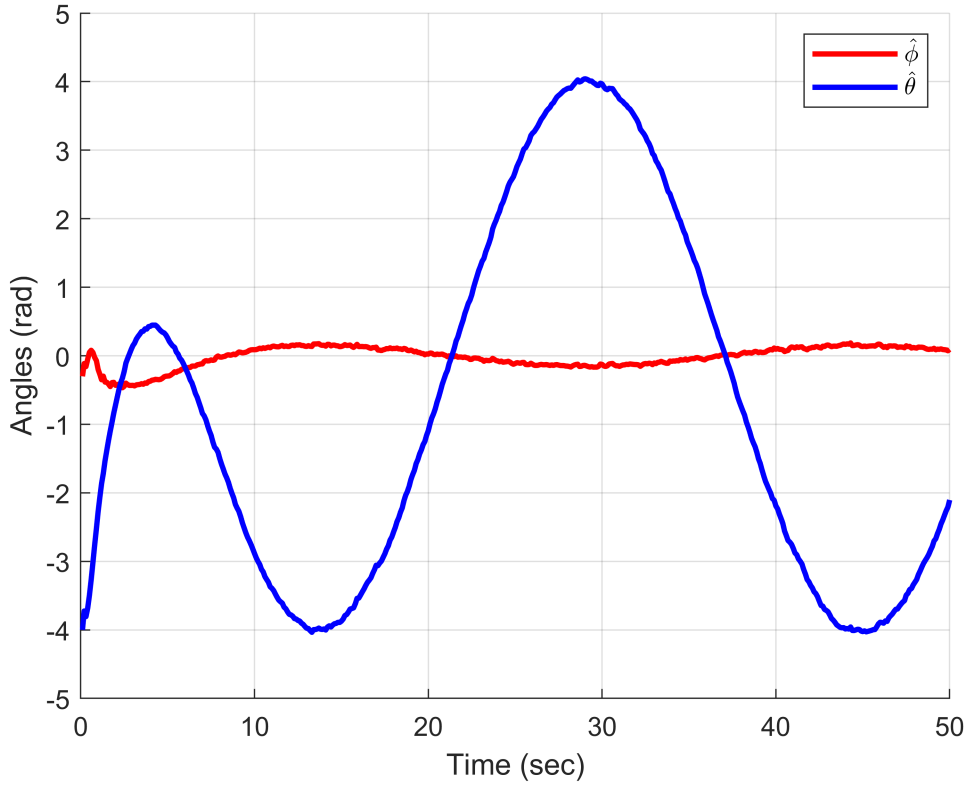
```
x_hat = zeros(length(x0), length(y));
xk = x0;
Pk = P0;
Kt = zeros(length(x0), length(y));

for i=1:length(y)
    x_hat(:, i) = xk;
    Kk = (Pk * C') * inv(C * Pk * C' + Q);
    Kt(:, i) = Kk;
    xk = A * xk + B * u(i) + A * Kk * (y(i) - C * xk);
    Pk = A * (Pk - Kk * C * Pk) * A' + G * R * G';
end
```

2.2

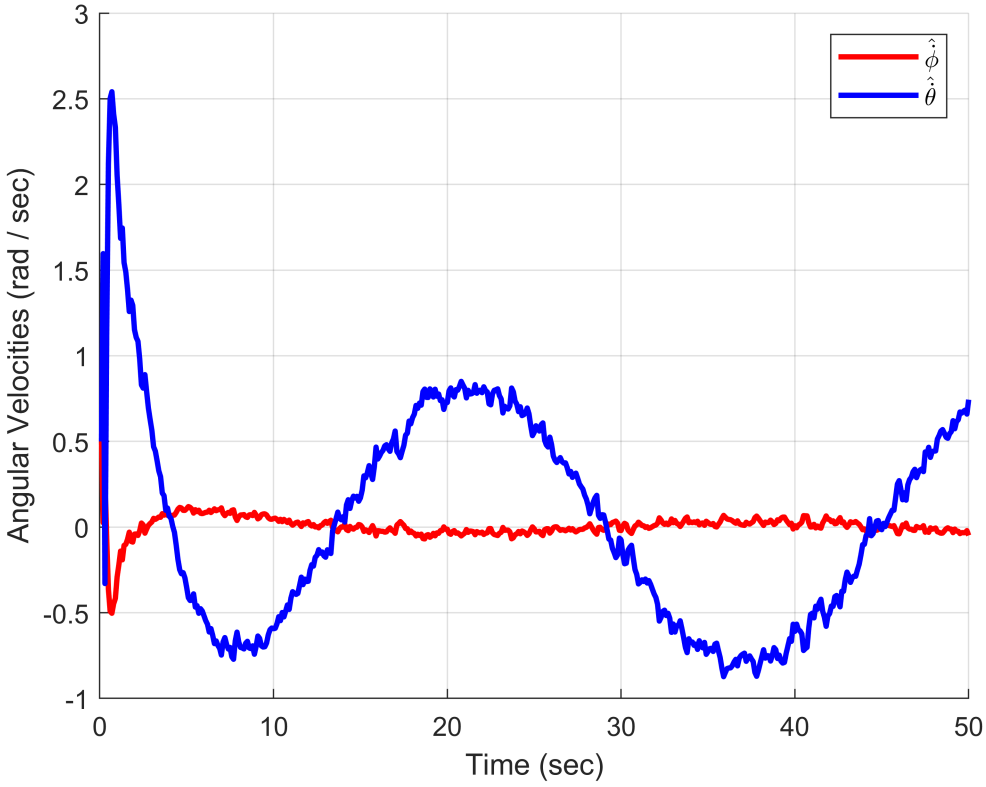
```
figure()
plot(t, x_hat(1, :), 'r', "LineWidth",2)
hold on; grid on; box off;
plot(t, x_hat(2, :), 'b', "LineWidth",2)
hold off;
legend(["$$\hat{\phi}$$", "$$\hat{\theta}$$"], 'interpreter', 'latex')

xlabel("Time (sec)")
ylabel("Angles (rad)")
```



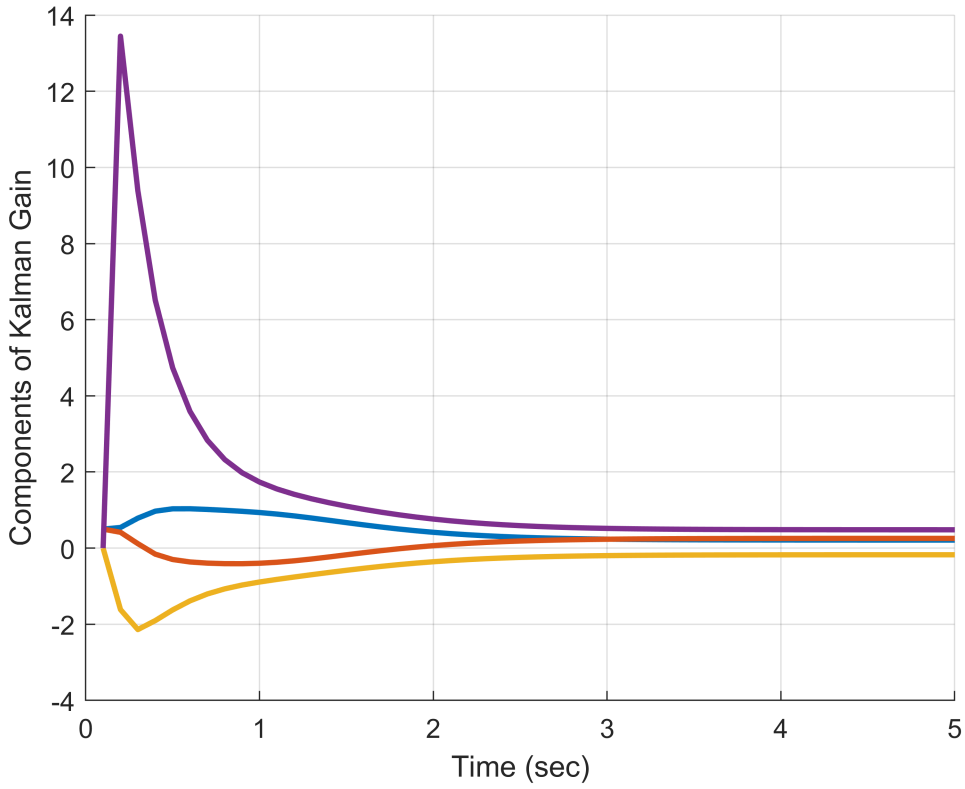

```
figure()
plot(t, x_hat(3, :), 'r', "LineWidth",2)
hold on; grid on; box off;
plot(t, x_hat(4, :), 'b', "LineWidth",2)
hold off;
legend(["$$\hat{\dot\phi}$$", "$$\hat{\dot\theta}$$"], 'interpreter', 'latex')

xlabel("Time (sec)")
ylabel("Angular Velocities (rad / sec)")
```




```
figure()
plot(t, Kt(1, :), "LineWidth",2)
hold on; grid on; box off;
plot(t, Kt(2, :), "LineWidth",2)
plot(t, Kt(3, :), "LineWidth",2)
plot(t, Kt(4, :), "LineWidth",2)
hold off;
xlim([0, 5])

xlabel("Time (sec)")
ylabel("Components of Kalman Gain")
```



2.3

```
Kk
```

```
Kk = 4×1
    0.2113
    0.2559
   -0.1744
    0.4816
```

```
[Kss,Pss] = dlqe(A,G,C,R,Q);
Kss
```

```
Kss = 4×1
    0.2113
    0.2559
   -0.1744
    0.4816
```